

Eletromagnetismo Avançado

1º ciclo
Aula de 22 setembro

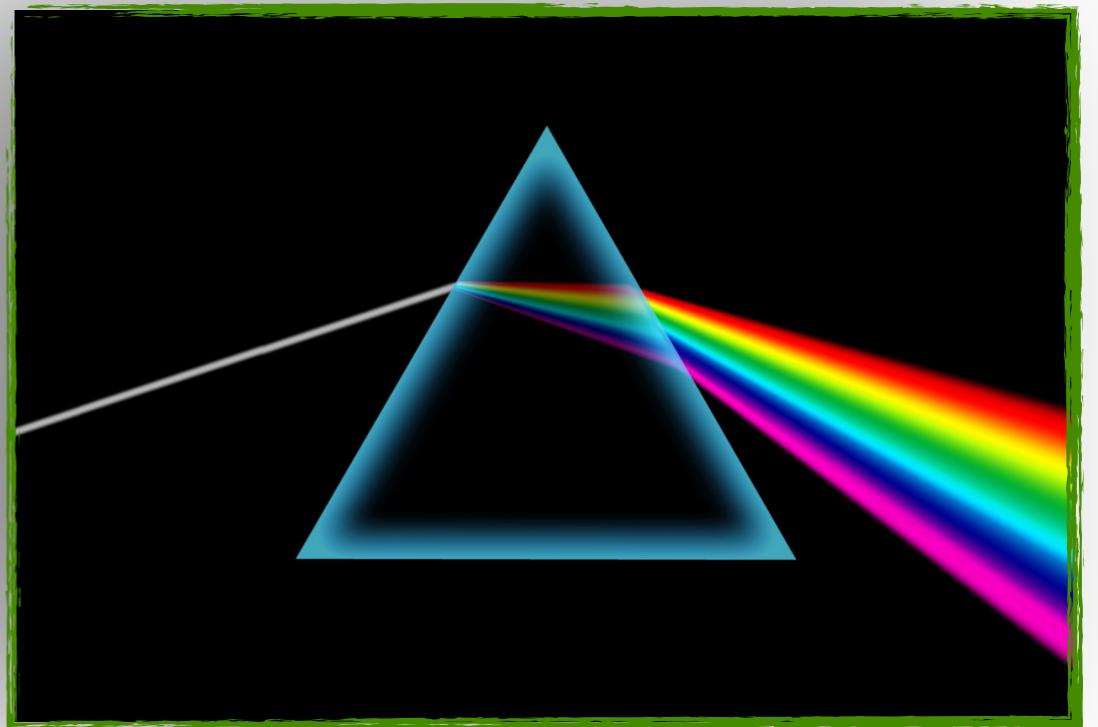
Ondas em meios lineares

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \vec{\nabla} \times \vec{B} = \mu\epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2 \vec{B} = \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$



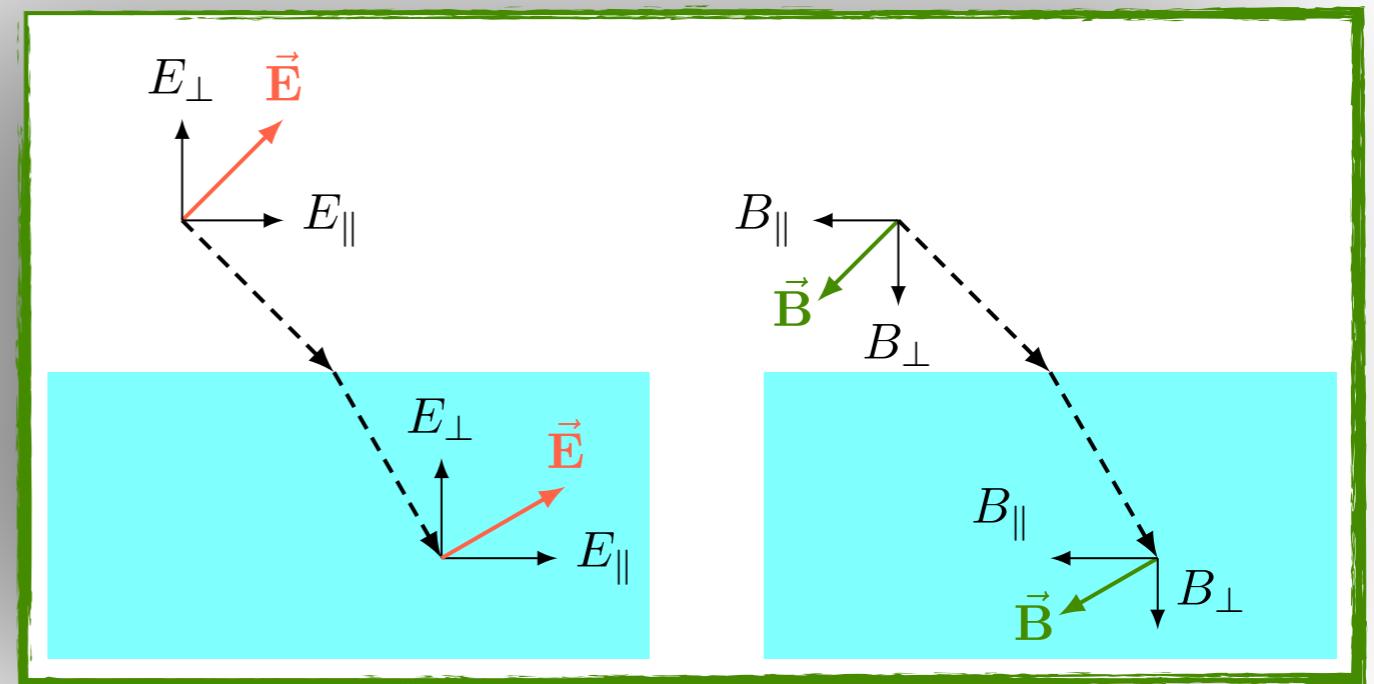
Ondas em meios lineares

$$\vec{\nabla} \cdot \vec{D} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

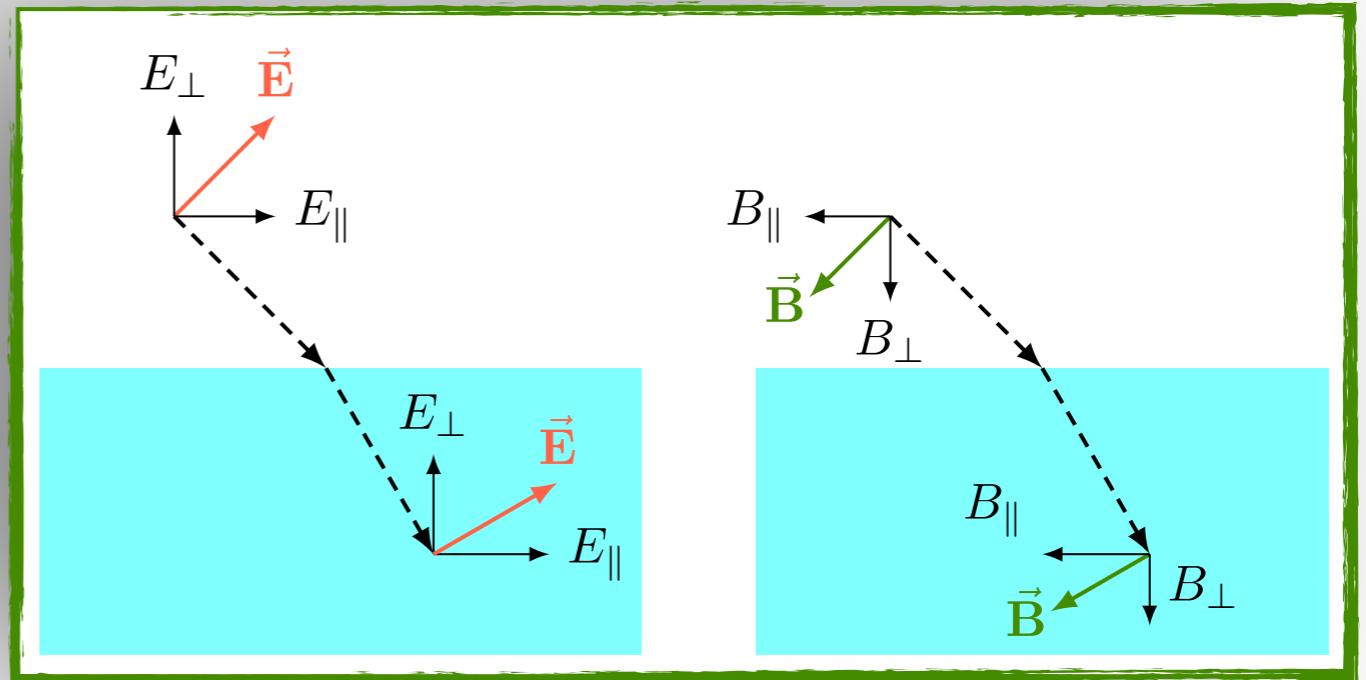
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$



Ondas em meios lineares

$$\vec{\nabla} \cdot \vec{D} = 0 \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \cdot \vec{B} = 0 \quad \vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$



$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

$$E_{1\parallel} = E_{2\parallel}$$

$$B_{1\perp} = B_{2\perp}$$

$$\frac{1}{\mu_1} B_{1\parallel} = \frac{1}{\mu_2} B_{2\parallel}$$

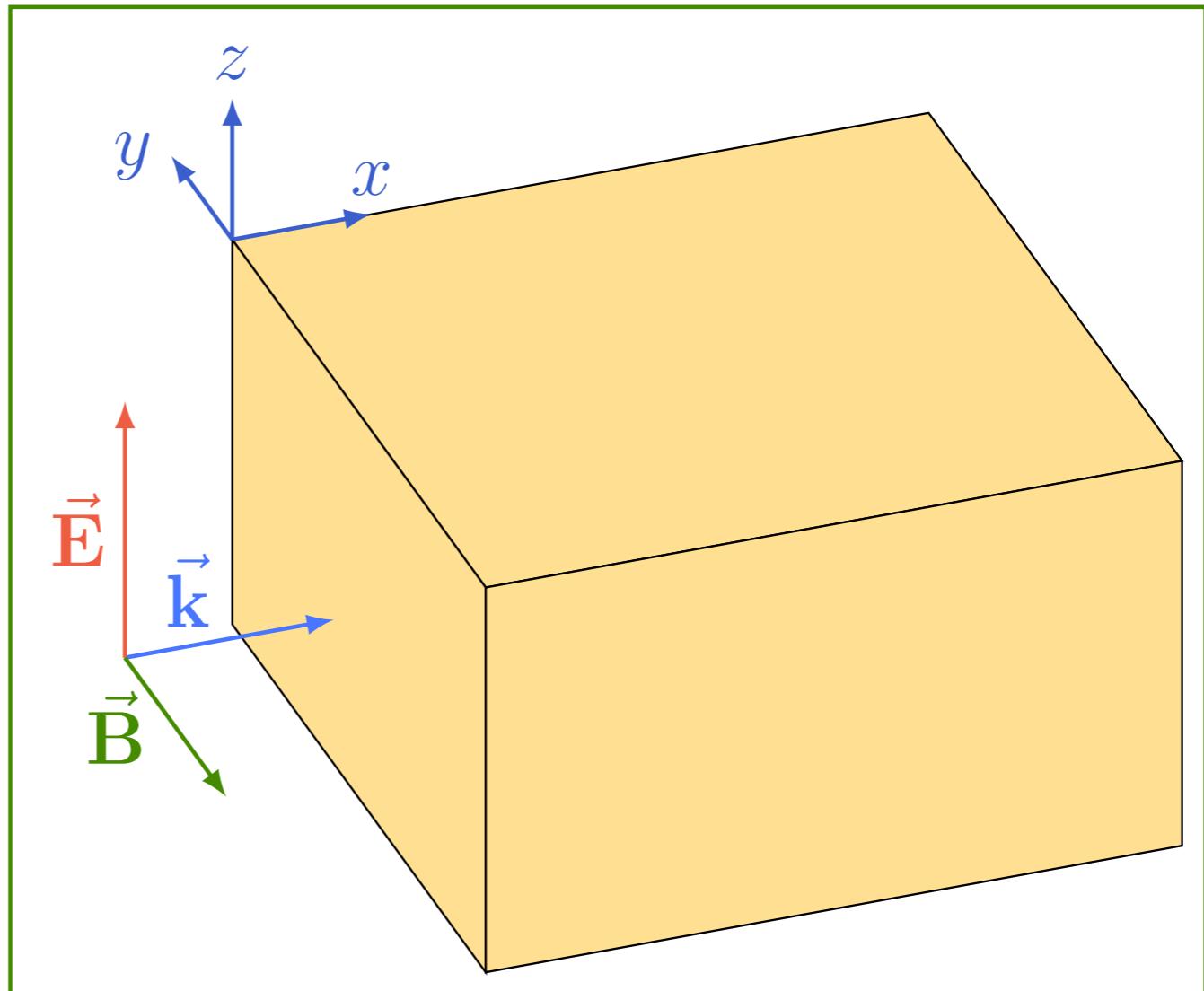
Pratique o que aprendeu

$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

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Pratique o que aprendeu

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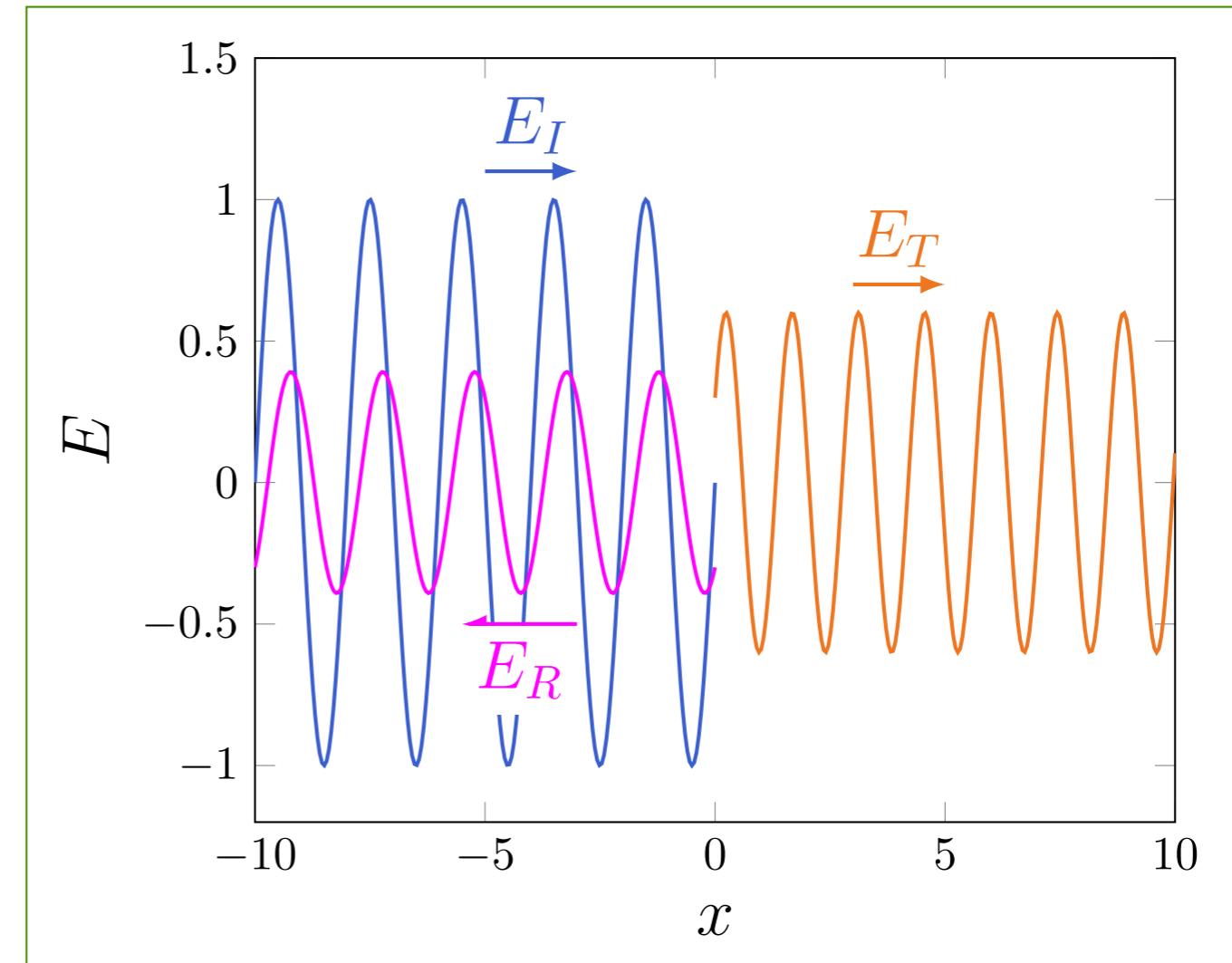
$$E_{1\parallel} = E_{2\parallel}$$

$$\frac{1}{\mu_1} B_{1\parallel} = \frac{1}{\mu_2} B_{2\parallel}$$

$$\tilde{E}_I = \tilde{E}_0 e^{i(kx - \omega t)}$$

$$\tilde{E}_T = \tilde{E}_{0T} e^{i(qx - \omega t)}$$

$$\tilde{E}_R = \tilde{E}_{0R} e^{-i(kx + \omega t)}$$



Pratique o que aprendeu

$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

$$B_{1\perp} = B_{2\perp}$$

$$E_{1\parallel} = E_{2\parallel}$$

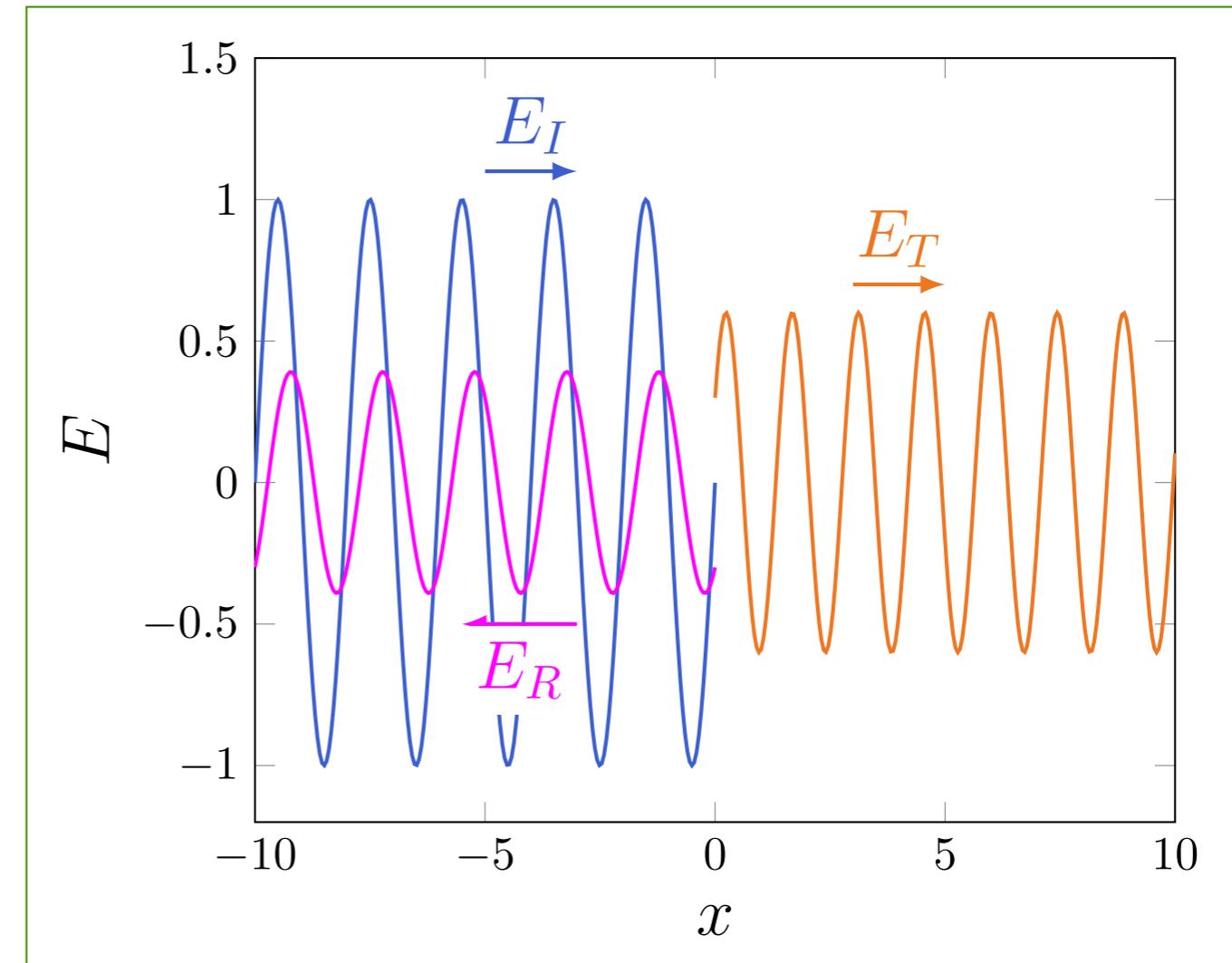
$$\frac{1}{\mu_1} B_{1\parallel} = \frac{1}{\mu_2} B_{2\parallel}$$

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$$\tilde{E}_R = \tilde{E}_{0R} e^{-i(kx + \omega t)}$$

$$\tilde{E}_0 + \tilde{E}_{0R} = \tilde{E}_{0T}$$



Pratique o que aprendeu

$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

$$B_{1\perp} = B_{2\perp}$$

$$E_{1\parallel} = E_{2\parallel}$$

$$\frac{1}{\mu_1} B_{1\parallel} = \frac{1}{\mu_2} B_{2\parallel}$$

$$\tilde{E}_I = \tilde{E}_0 e^{i(kx - \omega t)}$$

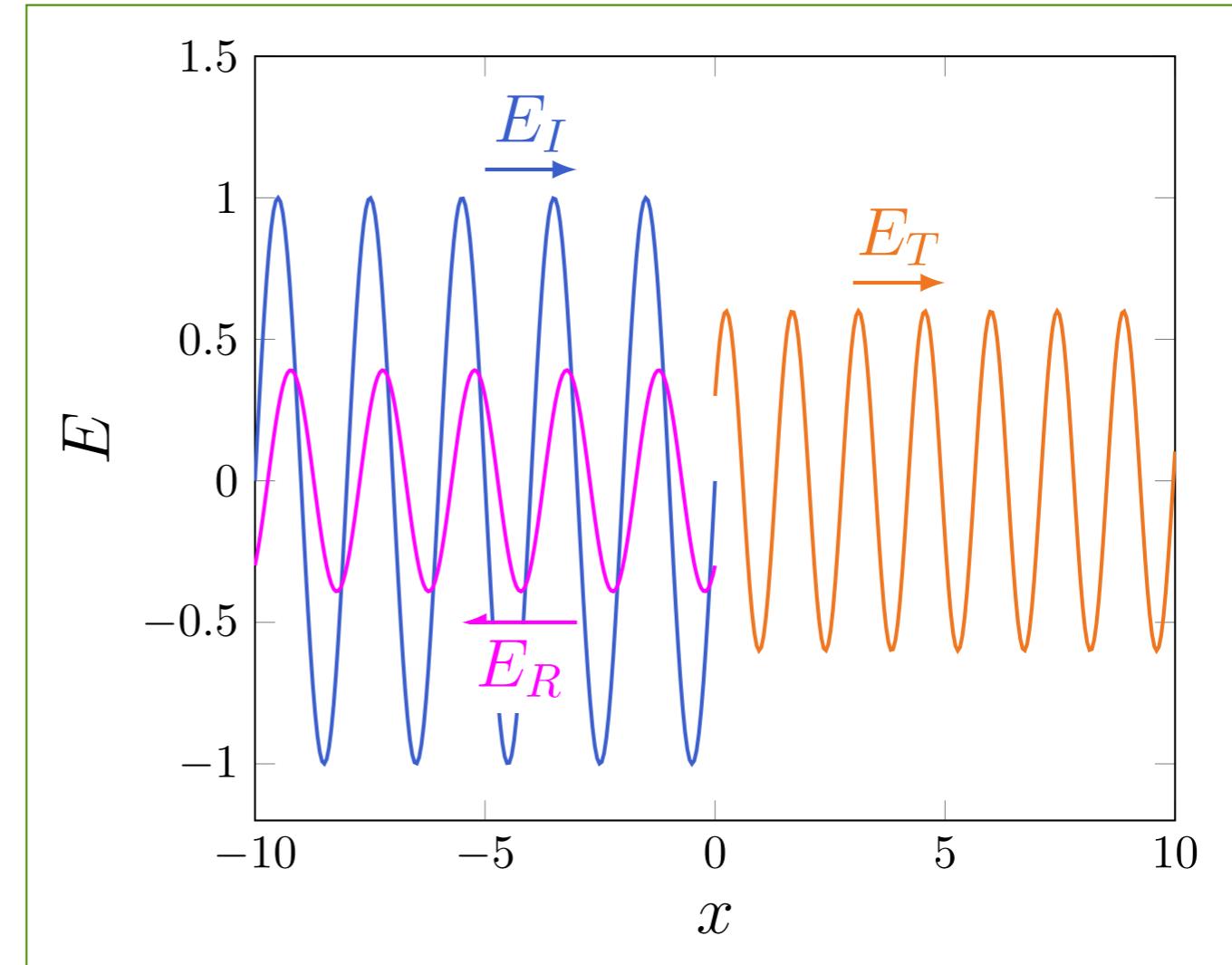
$$\tilde{B}_I = \frac{\tilde{E}_0}{c} e^{i(kx - \omega t)}$$

$$\tilde{E}_T = \tilde{E}_{0T} e^{i(qx - \omega t)}$$

$$\tilde{B}_T = \frac{n \tilde{E}_{0T}}{c} e^{i(qx - \omega t)}$$

$$\tilde{E}_R = \tilde{E}_{0R} e^{-i(kx + \omega t)}$$

$$\tilde{B}_R = -\frac{\tilde{E}_{0R}}{c} e^{i(-kx - \omega t)}$$



$$\tilde{E}_0 + \tilde{E}_{0R} = \tilde{E}_{0T}$$

Pratique o que aprendeu

$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

$$B_{1\perp} = B_{2\perp}$$

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$$\tilde{E}_I = \tilde{E}_0 e^{i(kx - \omega t)}$$

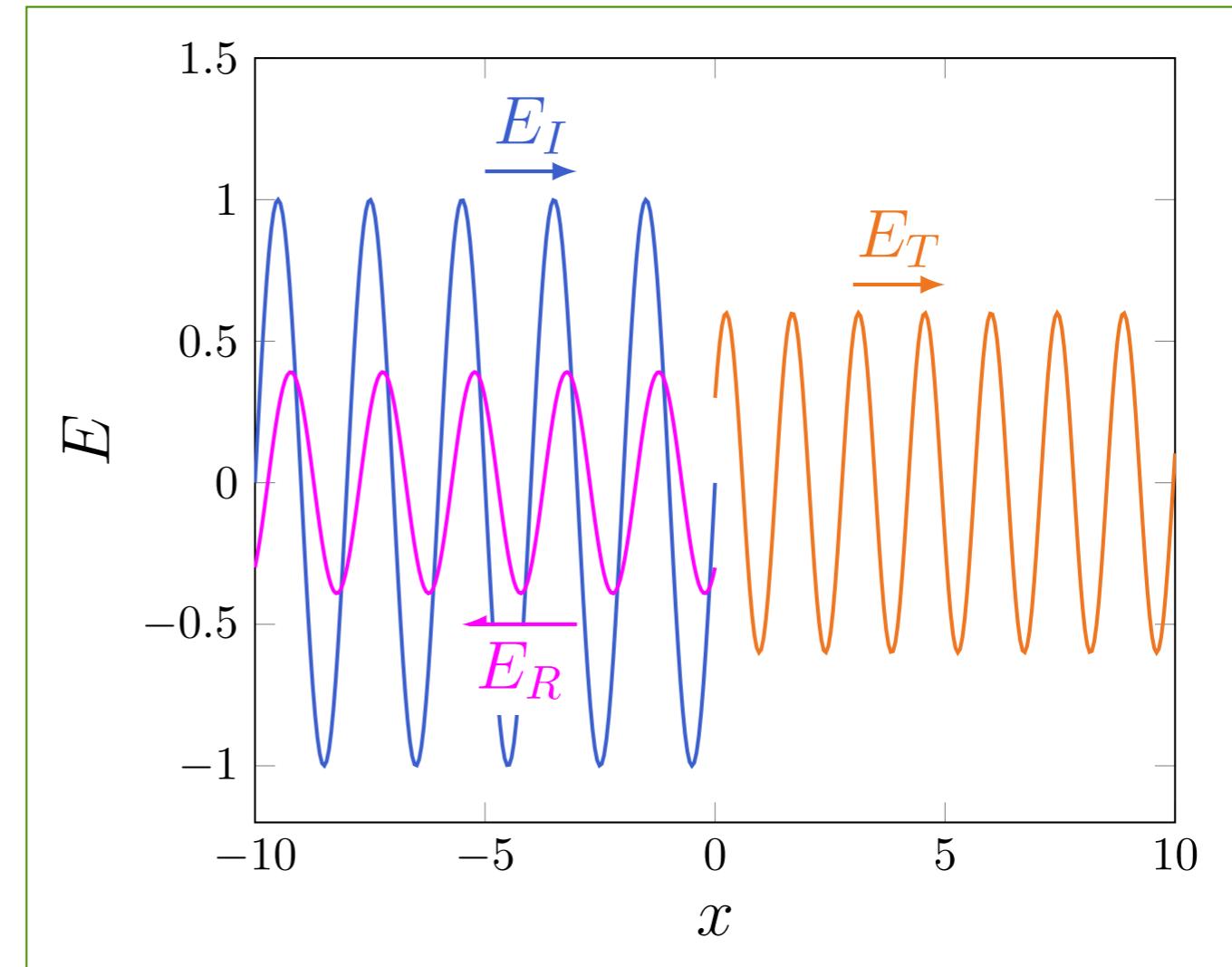
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$$\tilde{E}_T = \tilde{E}_{0T} e^{i(qx - \omega t)}$$

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$$\tilde{E}_R = \tilde{E}_{0R} e^{-i(kx + \omega t)}$$

$$\tilde{B}_R = -\frac{\tilde{E}_{0R}}{c} e^{i(-kx - \omega t)}$$



$$\tilde{E}_0 + \tilde{E}_{0R} = \tilde{E}_{0T}$$

$$\tilde{E}_0 - \tilde{E}_{0R} = \frac{n\mu_0}{\mu} \tilde{E}_{0T}$$

Pratique o que aprendeu

$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

$$B_{1\perp} = B_{2\perp}$$

$$E_{1\parallel} = E_{2\parallel}$$

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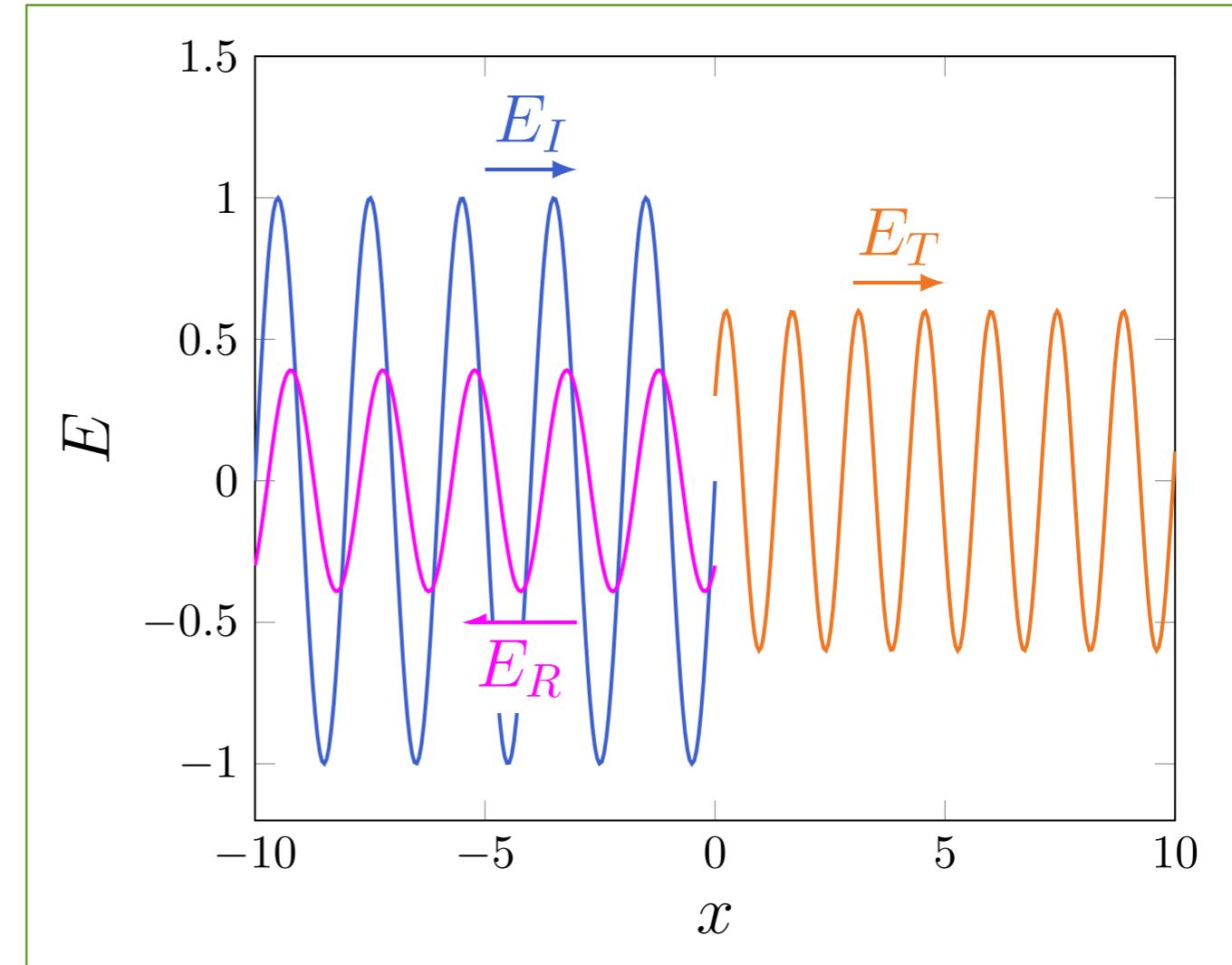
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$$\tilde{E}_R = \tilde{E}_{0R} e^{-i(kx + \omega t)}$$

$$\tilde{B}_R = -\frac{\tilde{E}_{0R}}{c} e^{i(-kx - \omega t)}$$



$$\tilde{E}_0 + \tilde{E}_{0R} = \tilde{E}_{0T}$$

$$\tilde{E}_0 - \tilde{E}_{0R} = n \tilde{E}_{0T}$$

$$n \approx \sqrt{\frac{\epsilon}{\epsilon_0}}$$

Pratique o que aprendeu

$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

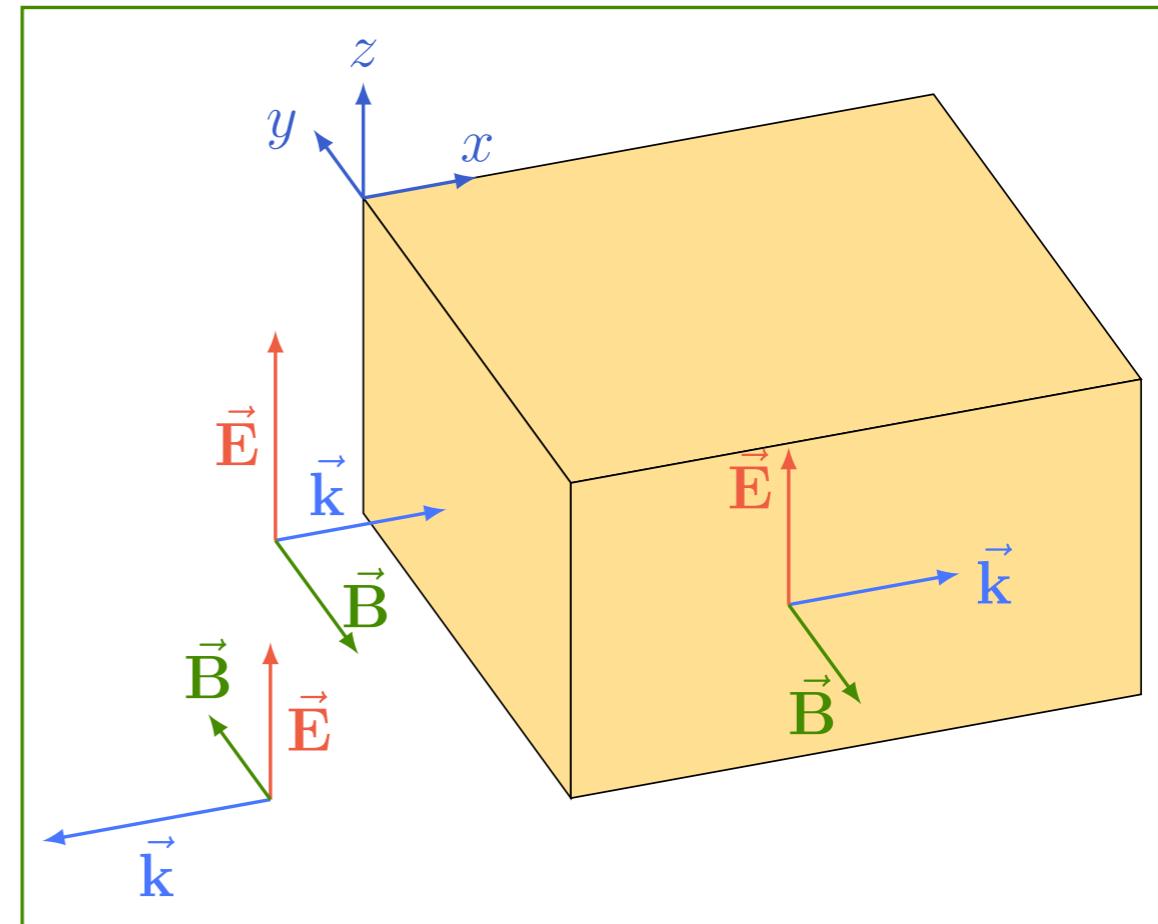
$$B_{1\perp} = B_{2\perp}$$

$$E_{1\parallel} = E_{2\parallel}$$

$$\frac{1}{\mu_1} B_{1\parallel} = \frac{1}{\mu_2} B_{2\parallel}$$

$$\tilde{E}_0 + \tilde{E}_{0R} = \tilde{E}_{0T}$$

$$\tilde{E}_0 - \tilde{E}_{0R} = n \tilde{E}_{0T}$$



Pratique o que aprendeu

$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

$$B_{1\perp} = B_{2\perp}$$

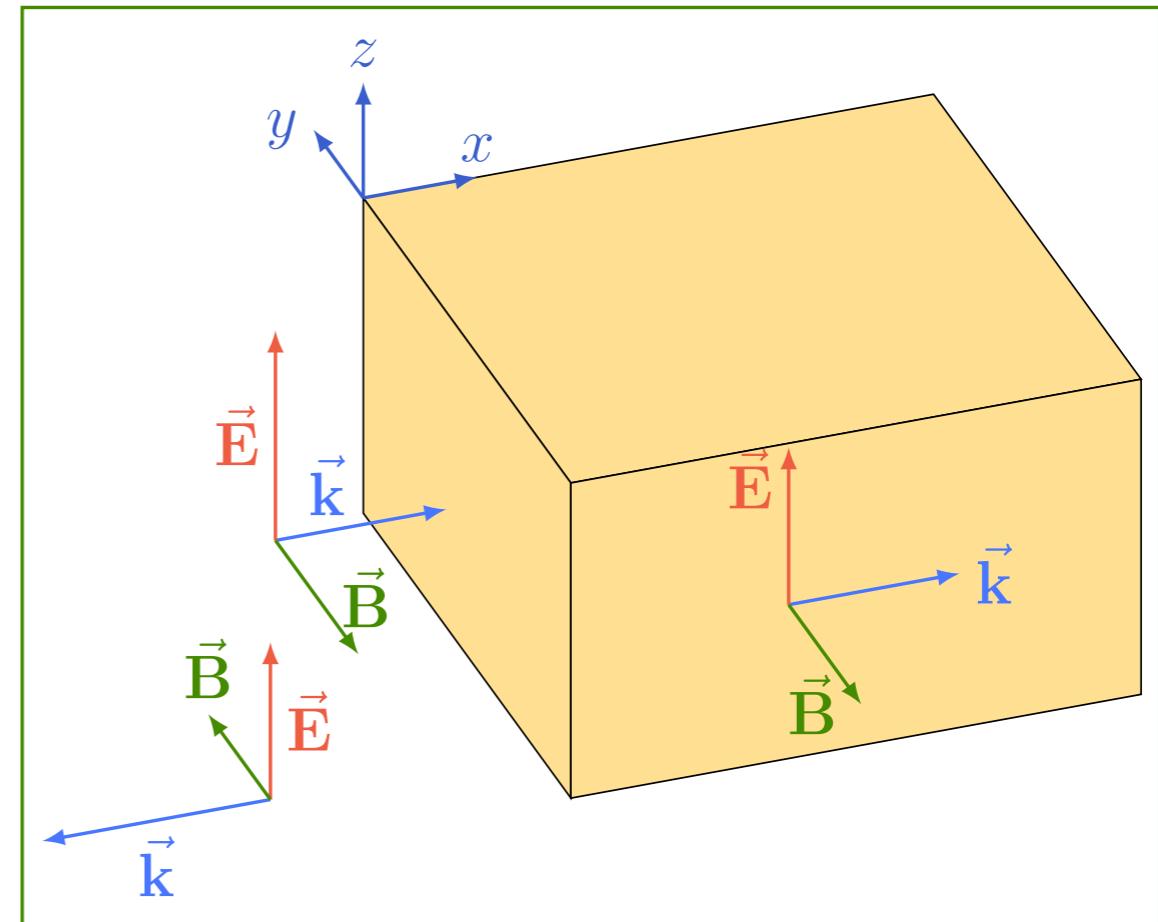
$$E_{1\parallel} = E_{2\parallel}$$

$$\frac{1}{\mu_1} B_{1\parallel} = \frac{1}{\mu_2} B_{2\parallel}$$

$$\tilde{E}_0 + \tilde{E}_{0R} = \tilde{E}_{0T}$$

$$\tilde{E}_0 - \tilde{E}_{0R} = n \tilde{E}_{0T}$$

$$\tilde{E}_{0T} = \frac{2}{1+n} \tilde{E}_0$$



Pratique o que aprendeu

$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

$$B_{1\perp} = B_{2\perp}$$

$$E_{1\parallel} = E_{2\parallel}$$

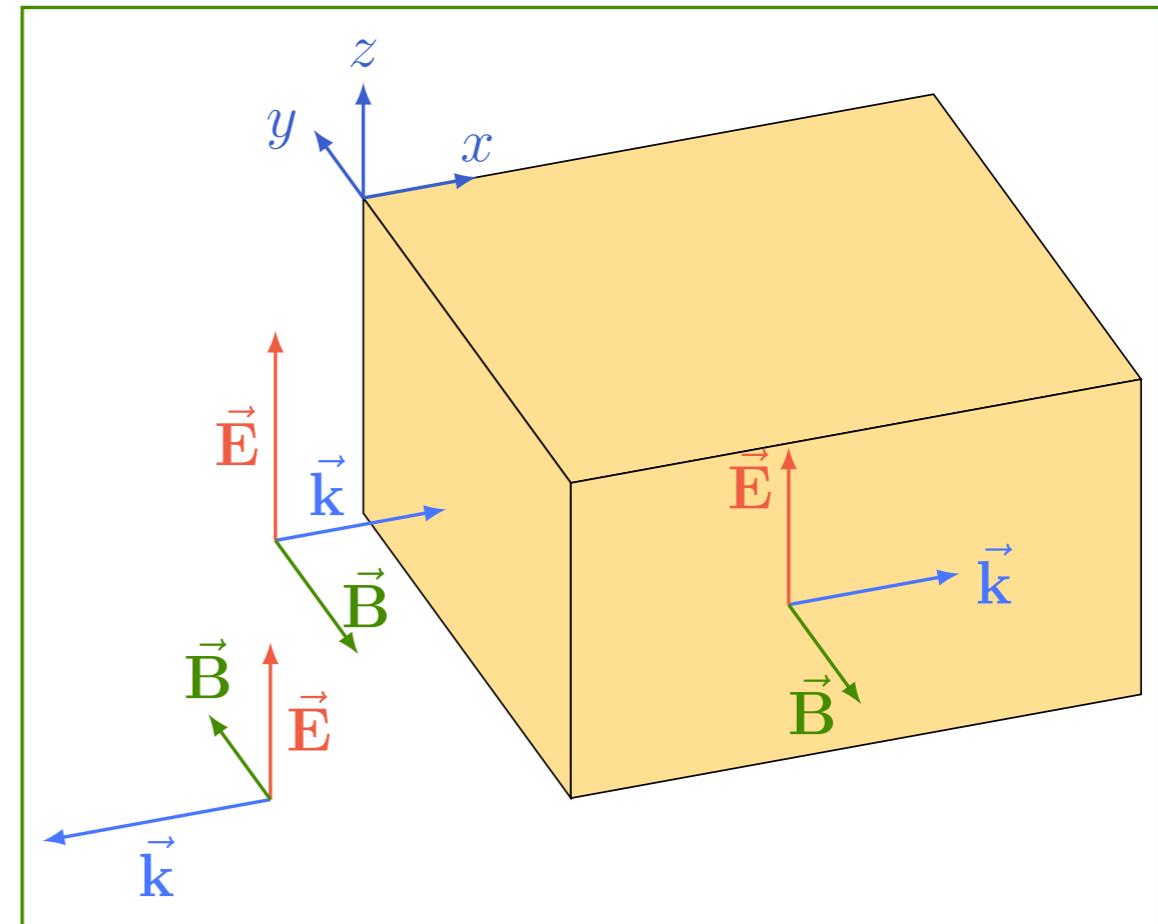
$$\frac{1}{\mu_1} B_{1\parallel} = \frac{1}{\mu_2} B_{2\parallel}$$

$$\tilde{E}_0 + \tilde{E}_{0R} = \tilde{E}_{0T}$$

$$\tilde{E}_0 - \tilde{E}_{0R} = n \tilde{E}_{0T}$$

$$\tilde{E}_{0T} = \frac{2}{1+n} \tilde{E}_0$$

$$\tilde{E}_{0R} = \frac{1-n}{1+n} \tilde{E}_0$$



Pratique o que aprendeu

$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

$$B_{1\perp} = B_{2\perp}$$

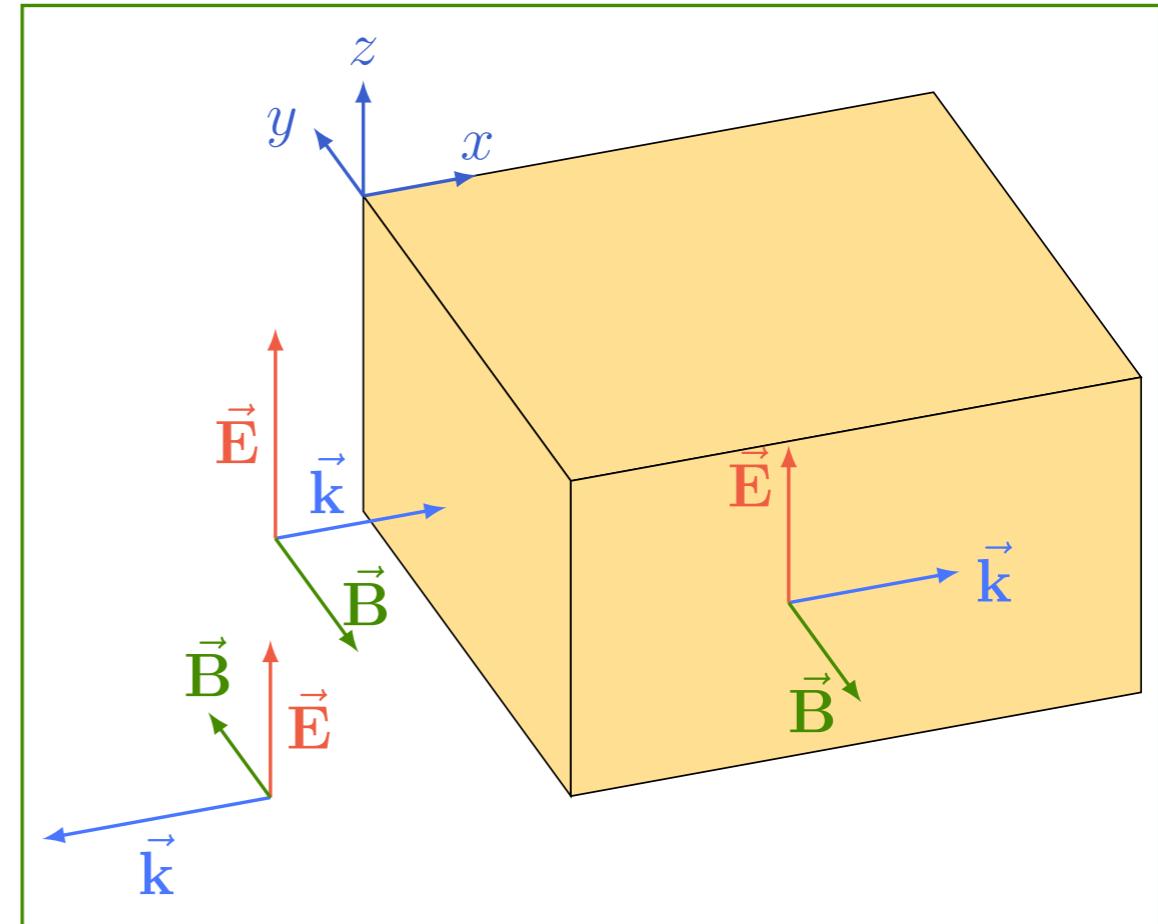
$$E_{1\parallel} = E_{2\parallel}$$

$$\frac{1}{\mu_1} B_{1\parallel} = \frac{1}{\mu_2} B_{2\parallel}$$

$$\tilde{E}_{0T} = \frac{2}{1+n} \tilde{E}_0$$

$$\tilde{E}_{0R} = \frac{1-n}{1+n} \tilde{E}_0$$

$$I = \frac{1}{2} v \epsilon E_0^2$$



Pratique o que aprendeu

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$$B_{1\perp} = B_{2\perp}$$

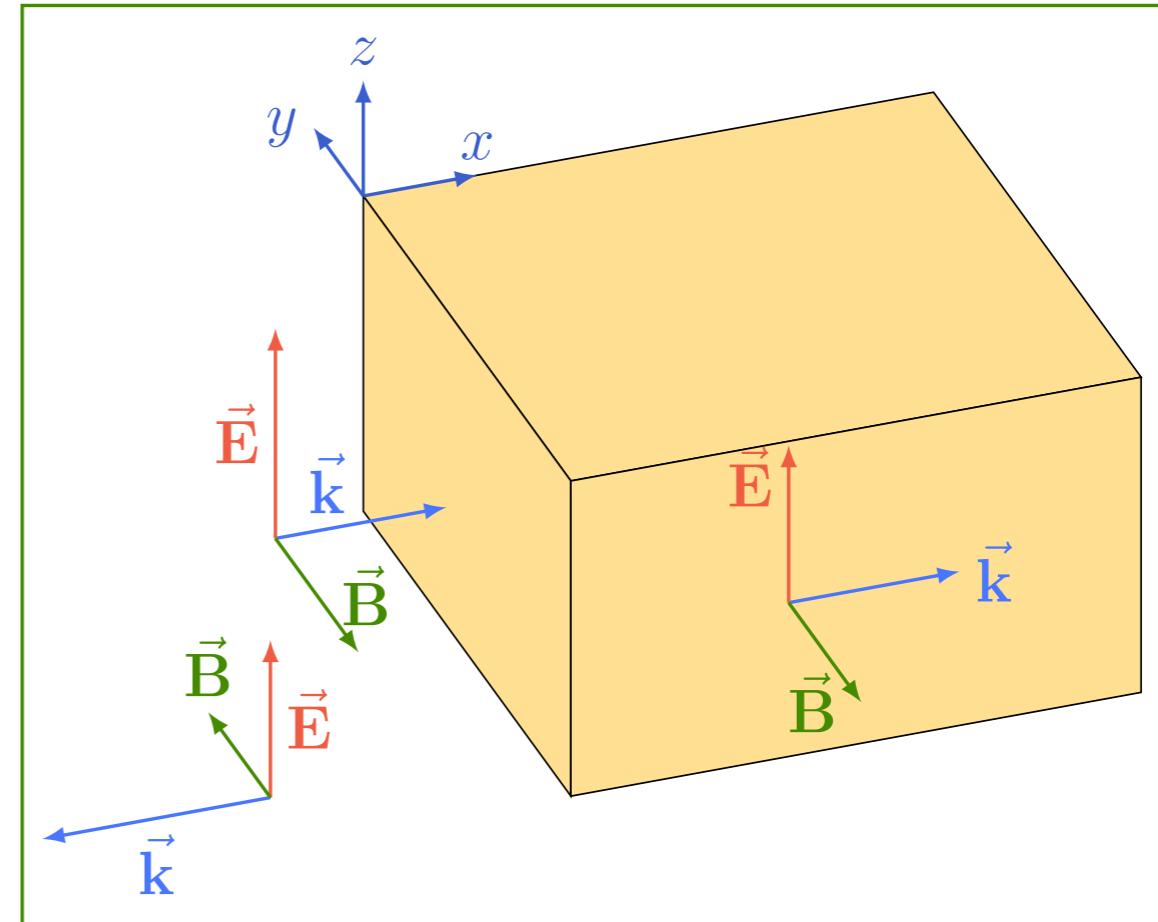
$$E_{1\parallel} = E_{2\parallel}$$

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$$\tilde{E}_{0T} = \frac{2}{1+n} \tilde{E}_0$$

$$\tilde{E}_{0R} = \frac{1-n}{1+n} \tilde{E}_0$$

$$I = \frac{1}{2} v \epsilon E_0^2 = n \frac{1}{2} c \epsilon_0 E_0^2$$



Pratique o que aprendeu

$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

$$B_{1\perp} = B_{2\perp}$$

$$E_{1\parallel} = E_{2\parallel}$$

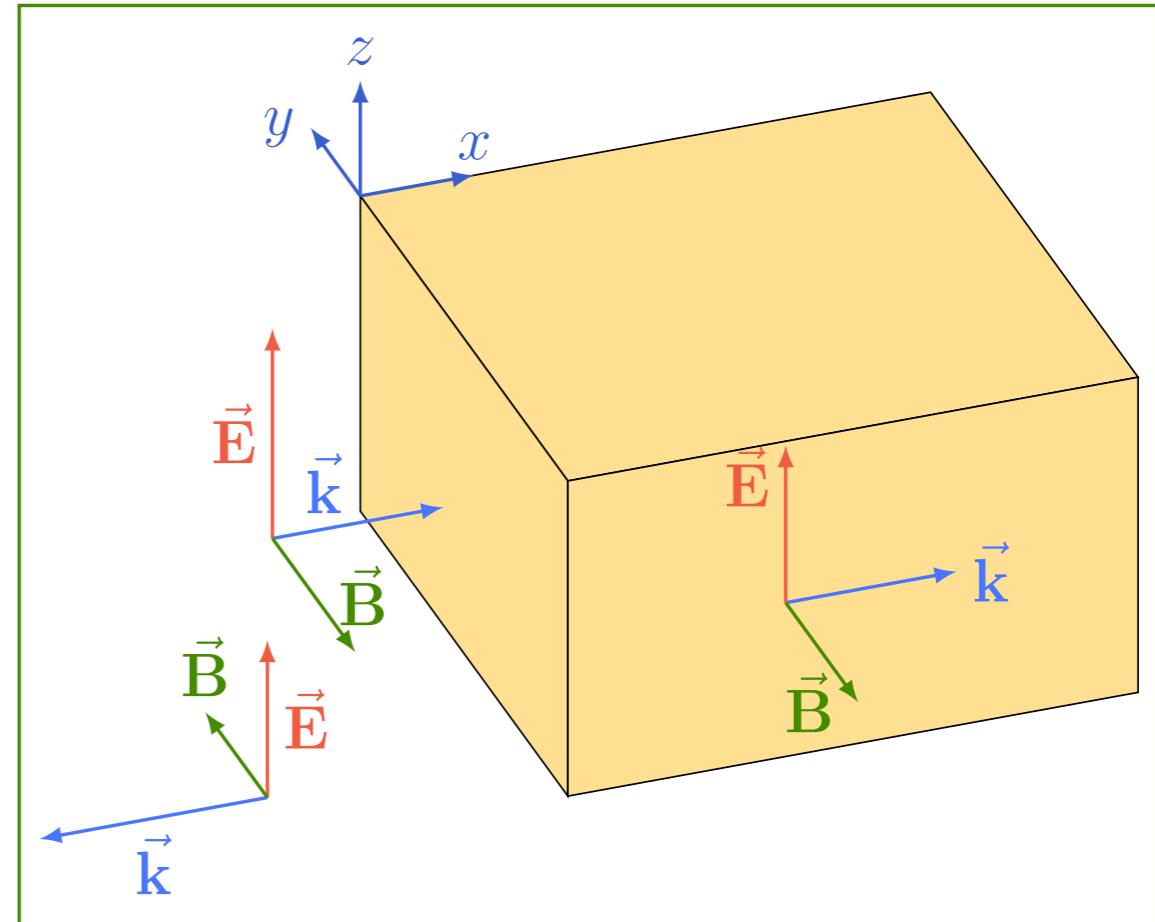
$$\frac{1}{\mu_1} B_{1\parallel} = \frac{1}{\mu_2} B_{2\parallel}$$

$$\tilde{E}_{0T} = \frac{2}{1+n} \tilde{E}_0$$

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$$I = \frac{1}{2} v \epsilon E_0^2 = n \frac{1}{2} c \epsilon_0 E_0^2$$

$$I_R = \left(\frac{1-n}{1+n} \right)^2 I_0$$



Pratique o que aprendeu

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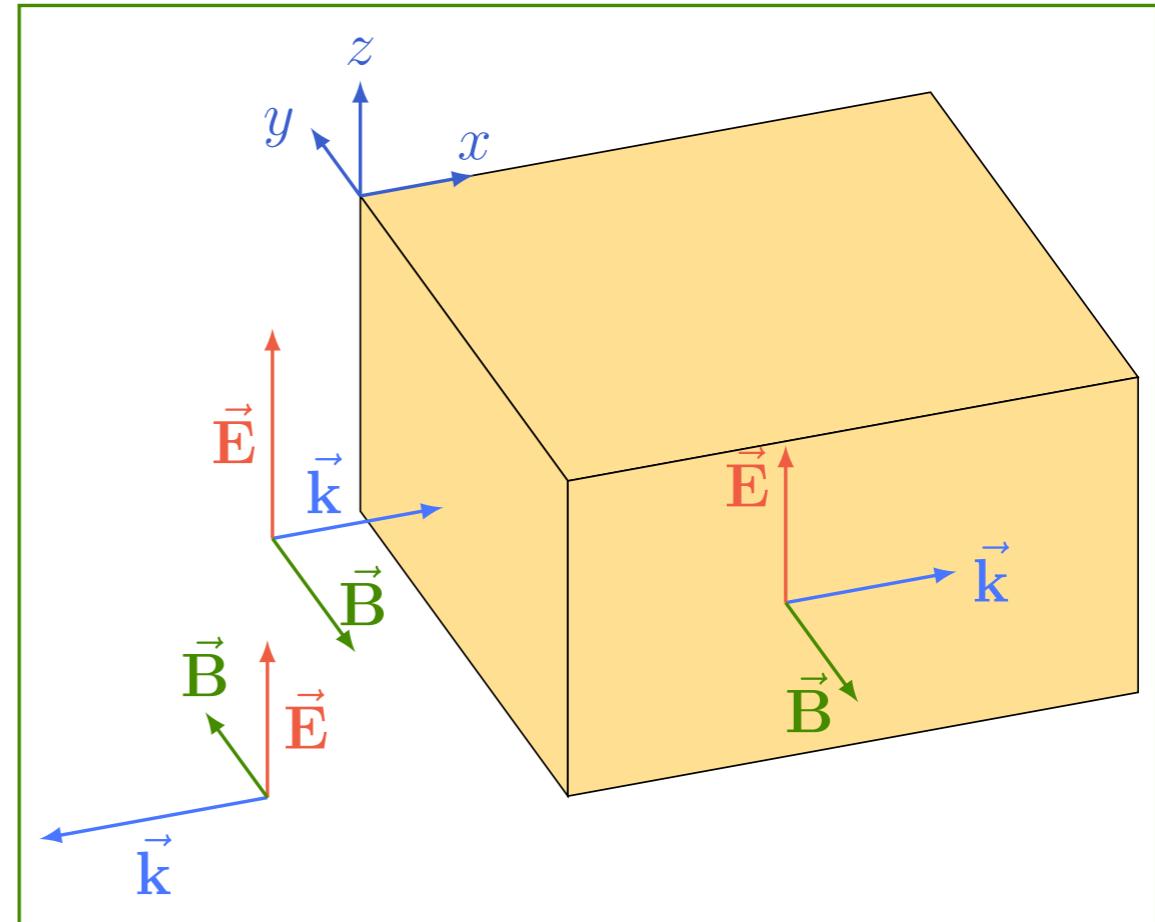
$$\tilde{E}_{0T} = \frac{2}{1+n} \tilde{E}_0$$

$$\tilde{E}_{0R} = \frac{1-n}{1+n} \tilde{E}_0$$

$$I = \frac{1}{2} v \epsilon E_0^2 = n \frac{1}{2} c \epsilon_0 E_0^2$$

$$I_R = \left(\frac{1-n}{1+n} \right)^2 I_0$$

$$I_T = \frac{4n}{(1+n)^2} I_0$$



Pratique o que aprendeu

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$$B_{1\perp} = B_{2\perp}$$

$$E_{1\parallel} = E_{2\parallel}$$

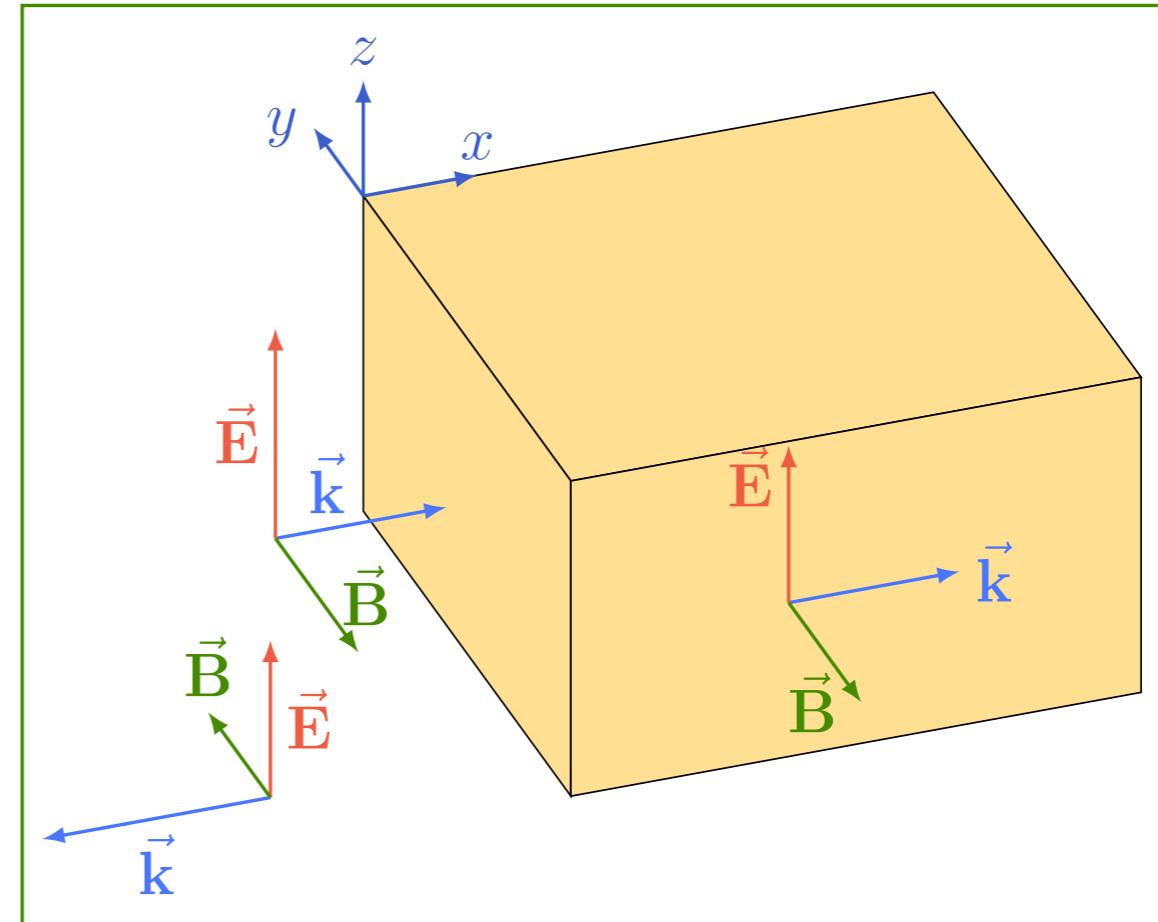
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$$\tilde{E}_{0T} = \frac{2}{1+n} \tilde{E}_0$$

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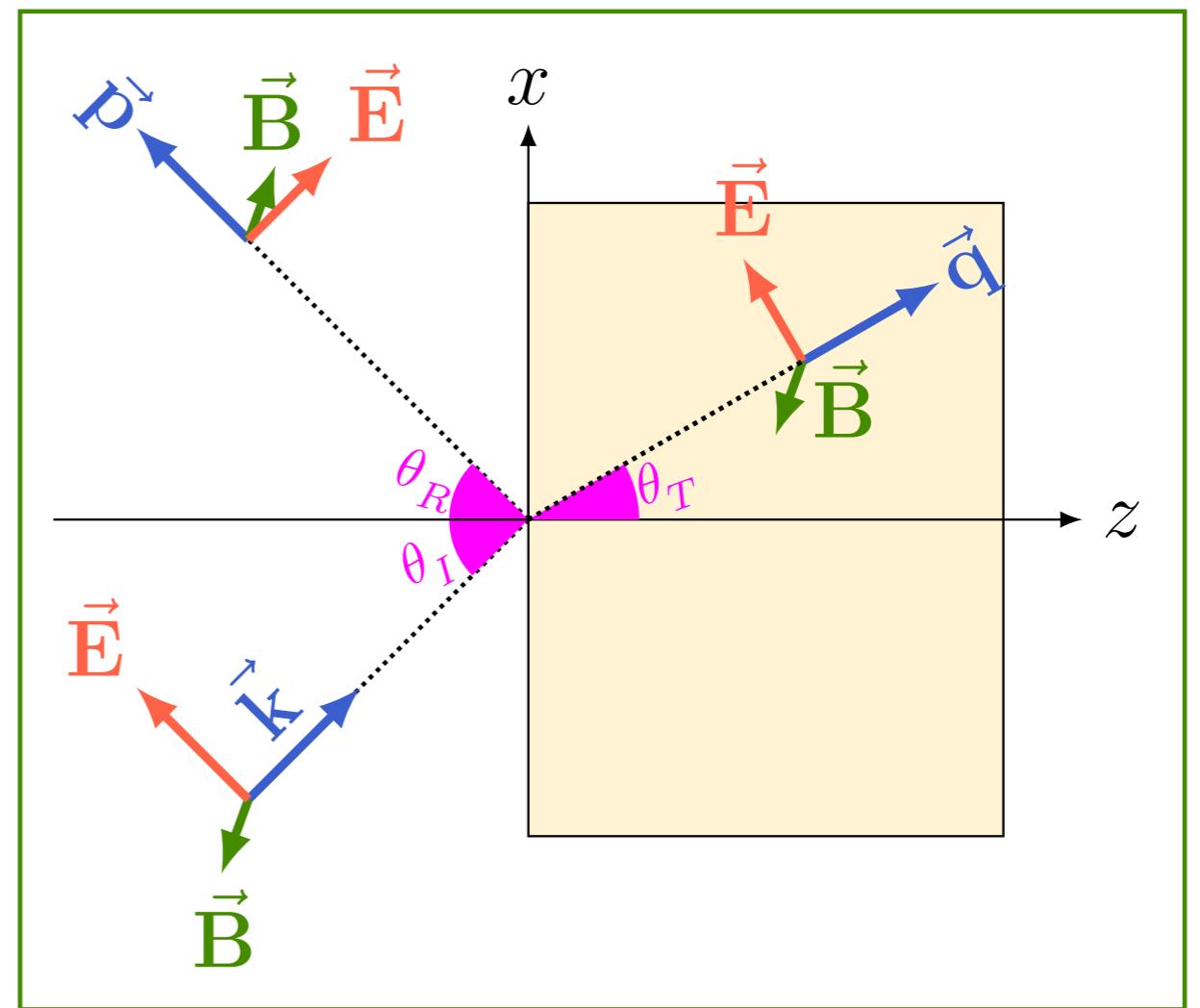
$$I = \frac{1}{2} v \epsilon E_0^2 = n \frac{1}{2} c \epsilon_0 E_0^2$$

$$\left. \begin{aligned} I_R &= \left(\frac{1-n}{1+n} \right)^2 I_0 \\ I_T &= \frac{4n}{(1+n)^2} I_0 \end{aligned} \right\} I_T + I_R = I_0$$



Incidência oblíqua

$$\tilde{\mathbf{E}}_I = \tilde{\mathbf{E}}_{0I} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

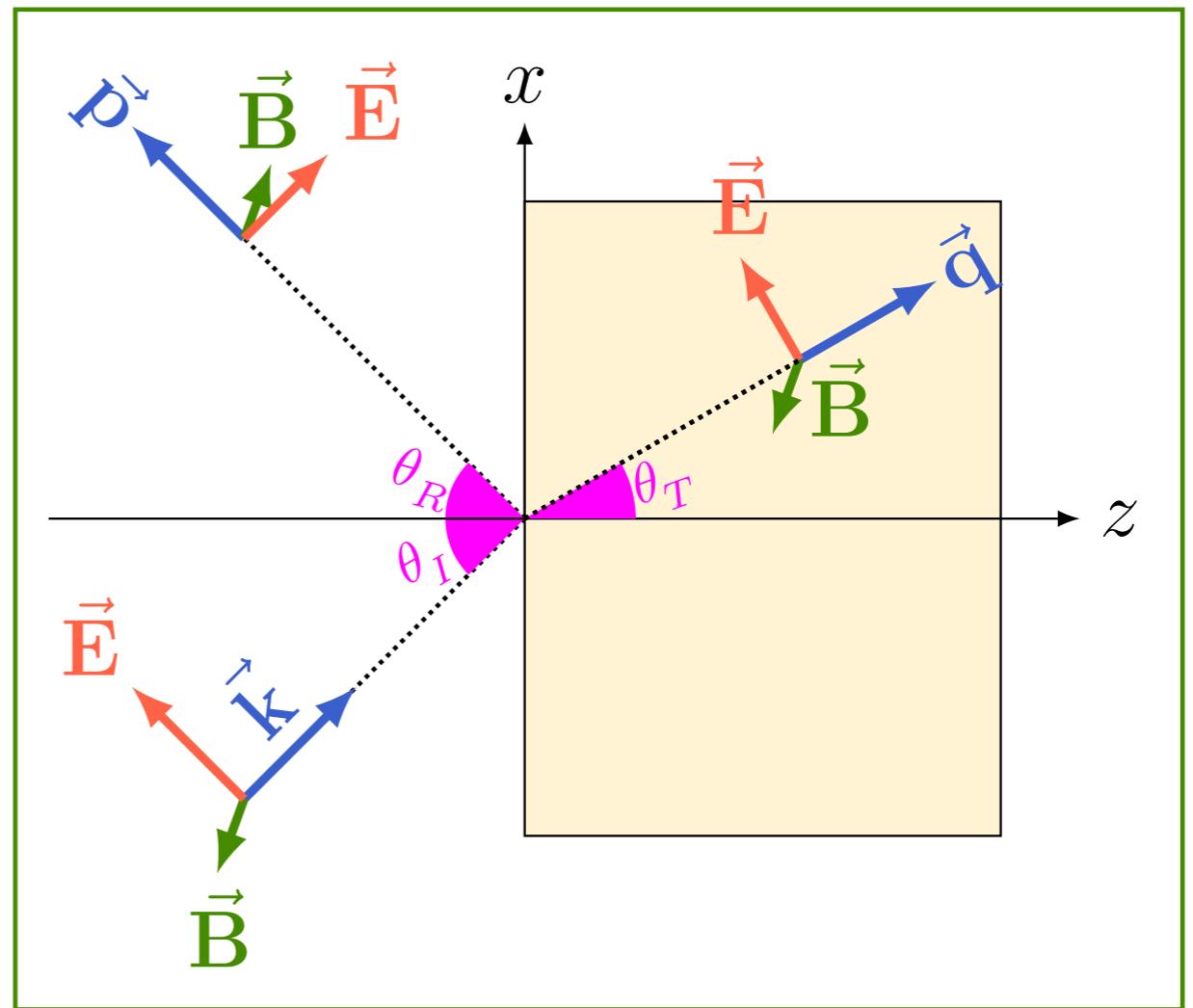


Incidência oblíqua

$$\tilde{\mathbf{E}}_I = \tilde{\mathbf{E}}_{0I} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\tilde{\mathbf{E}}_T = \tilde{\mathbf{E}}_{0T} e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

$$\tilde{\mathbf{E}}_R = \tilde{\mathbf{E}}_{0R} e^{i(\vec{p} \cdot \vec{r} - \omega t)}$$



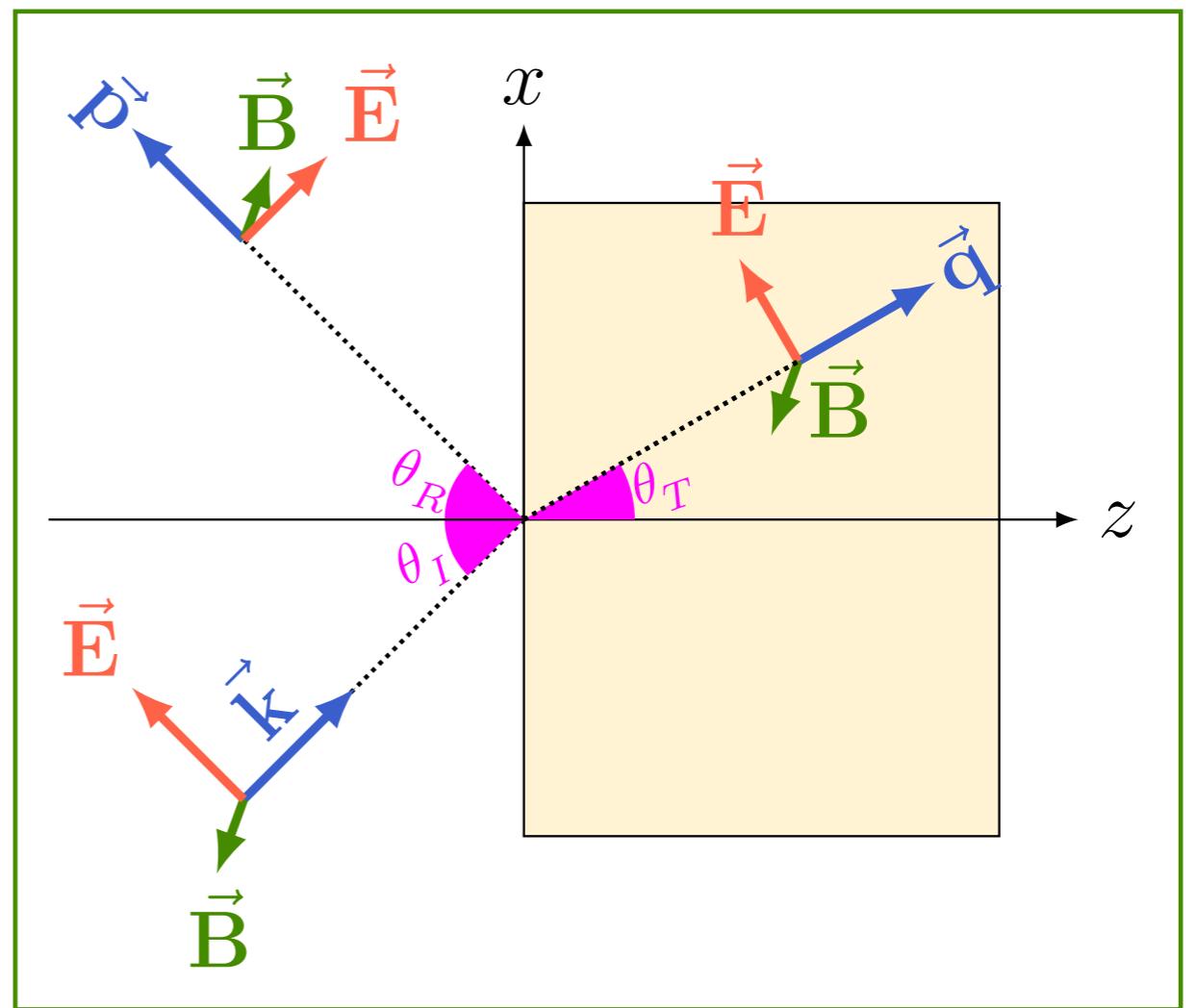
Incidência oblíqua

$$\tilde{\mathbf{E}}_I = \tilde{\mathbf{E}}_{0I} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\tilde{\mathbf{E}}_T = \tilde{\mathbf{E}}_{0T} e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

$$\tilde{\mathbf{E}}_R = \tilde{\mathbf{E}}_{0R} e^{i(\vec{p} \cdot \vec{r} - \omega t)}$$

$$kc = pc = q \frac{c}{n}$$



Incidência oblíqua

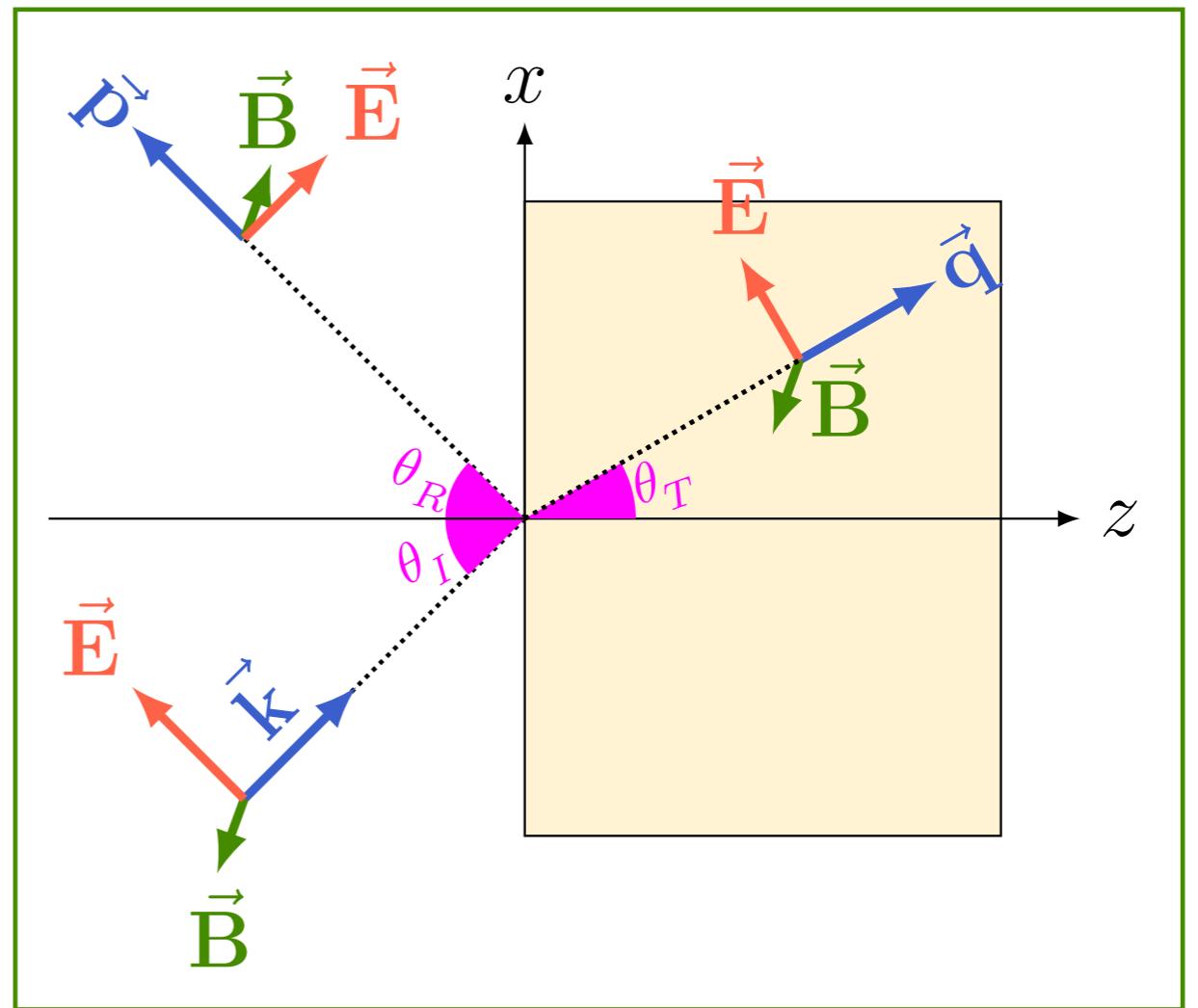
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$$\tilde{\mathbf{E}}_R = \tilde{\mathbf{E}}_{0R} e^{i(\vec{p} \cdot \vec{r} - \omega t)}$$

$$k = p = \frac{q}{n}$$

$$k_x x = p_x x = q_x x$$



Incidência oblíqua

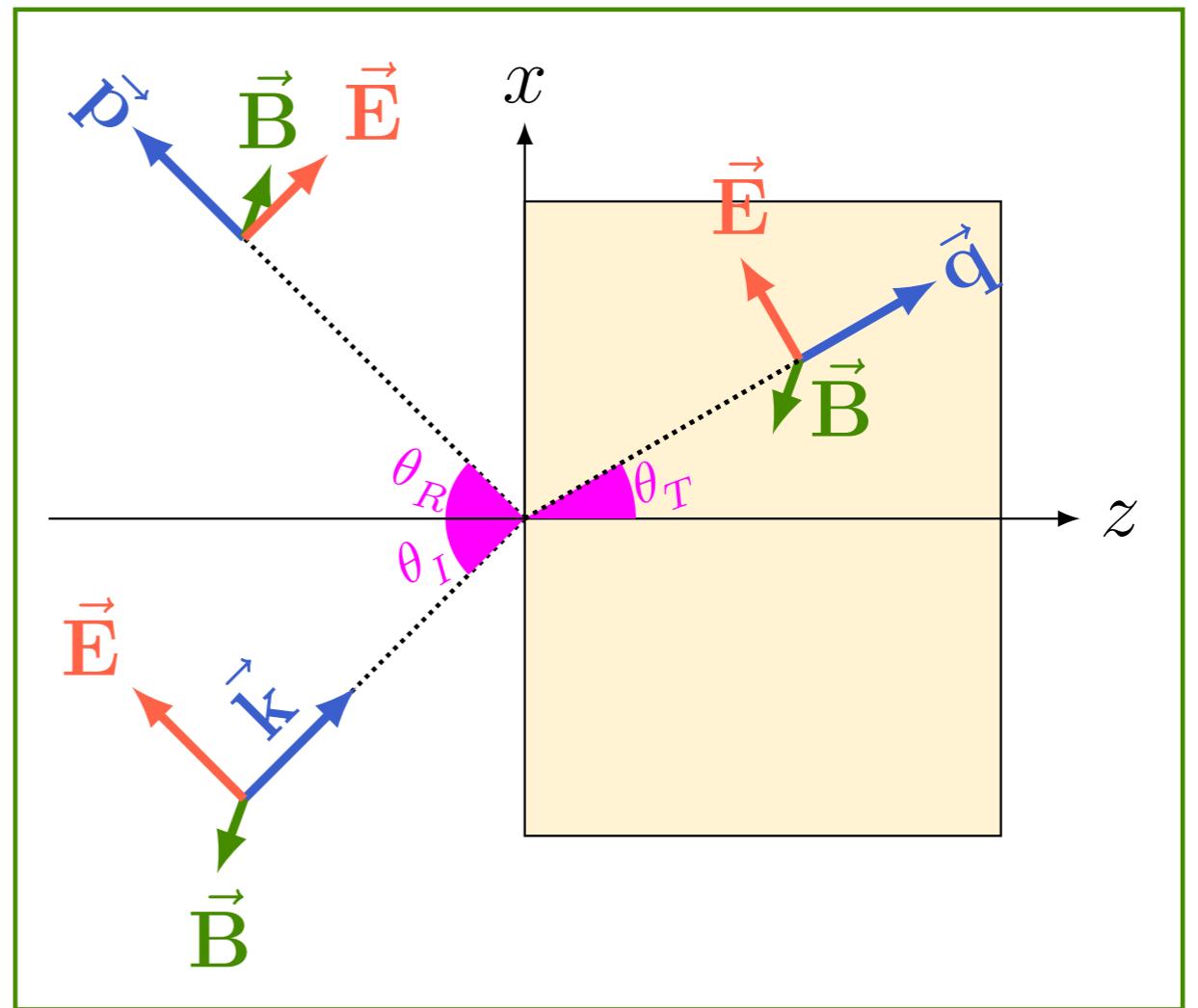
$$\tilde{\mathbf{E}}_I = \tilde{\mathbf{E}}_{0I} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

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$$\tilde{\mathbf{E}}_R = \tilde{\mathbf{E}}_{0R} e^{i(\vec{p} \cdot \vec{r} - \omega t)}$$

$$k = p = \frac{q}{n}$$

$$k_x = p_x = q_x$$



Incidência oblíqua

$$\tilde{\mathbf{E}}_I = \tilde{\mathbf{E}}_{0I} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\tilde{\mathbf{E}}_T = \tilde{\mathbf{E}}_{0T} e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

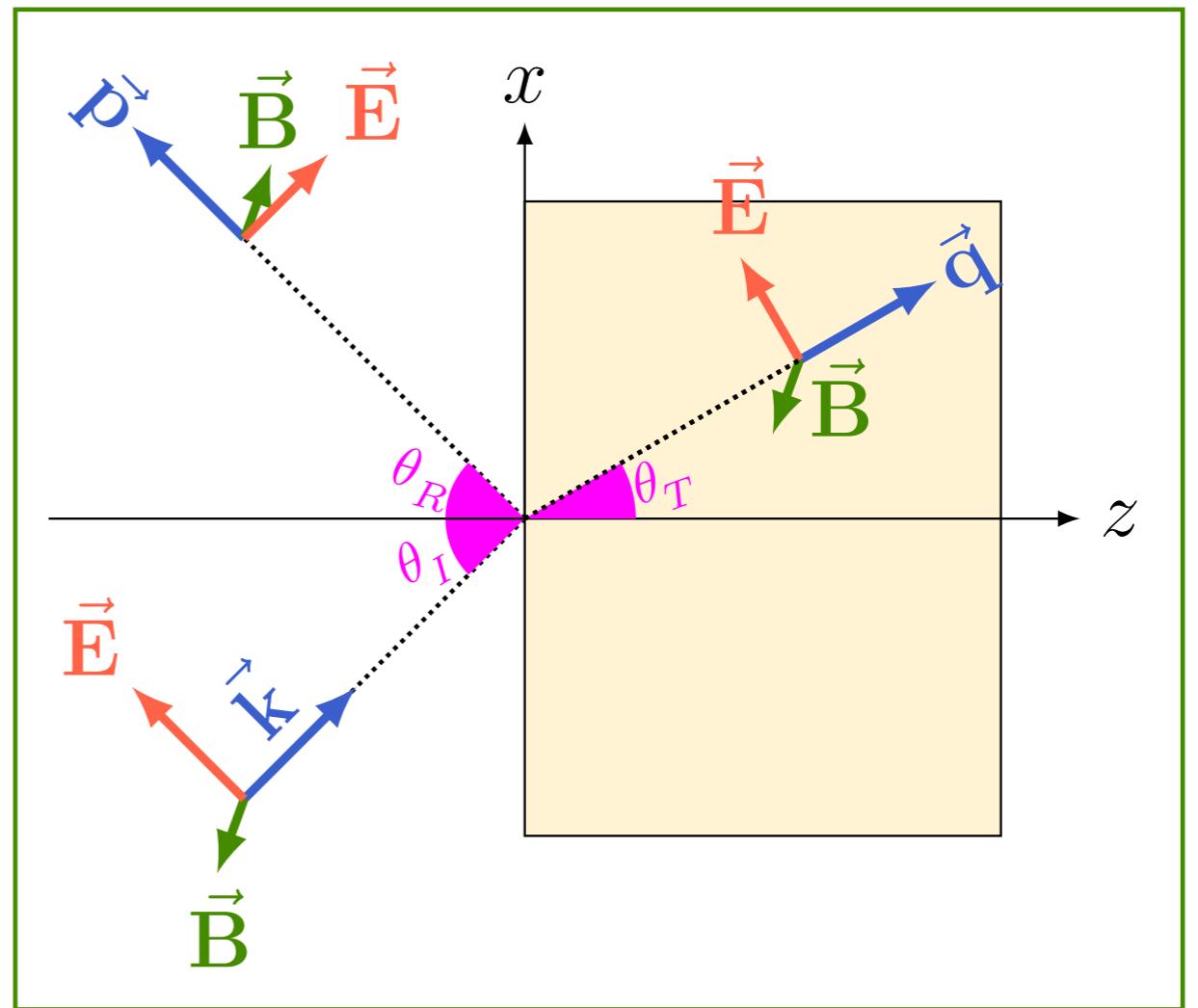
$$\tilde{\mathbf{E}}_R = \tilde{\mathbf{E}}_{0R} e^{i(\vec{p} \cdot \vec{r} - \omega t)}$$

$$k = p = \frac{q}{n}$$

$$k_x = p_x = q_x$$

$$\theta_I = \theta_R$$

$$\sin \theta_I = n \sin \theta_T$$

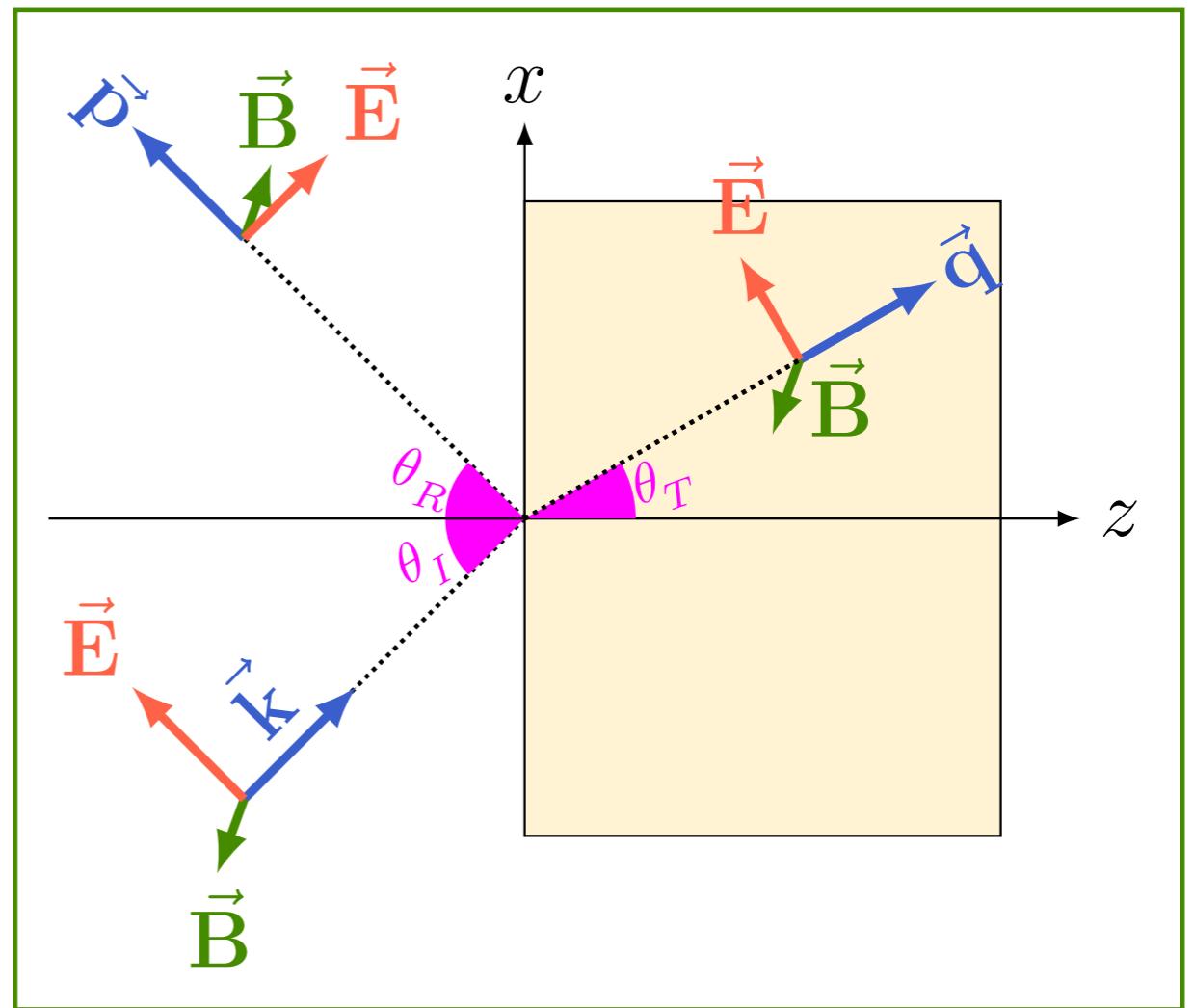


Incidência oblíqua

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$$\tilde{\mathbf{E}}_R = \tilde{\mathbf{E}}_{0R} e^{i(\vec{p} \cdot \vec{r} - \omega t)}$$



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$$\frac{1}{\mu_1} B_{1\parallel} = \frac{1}{\mu_2} B_{2\parallel}$$

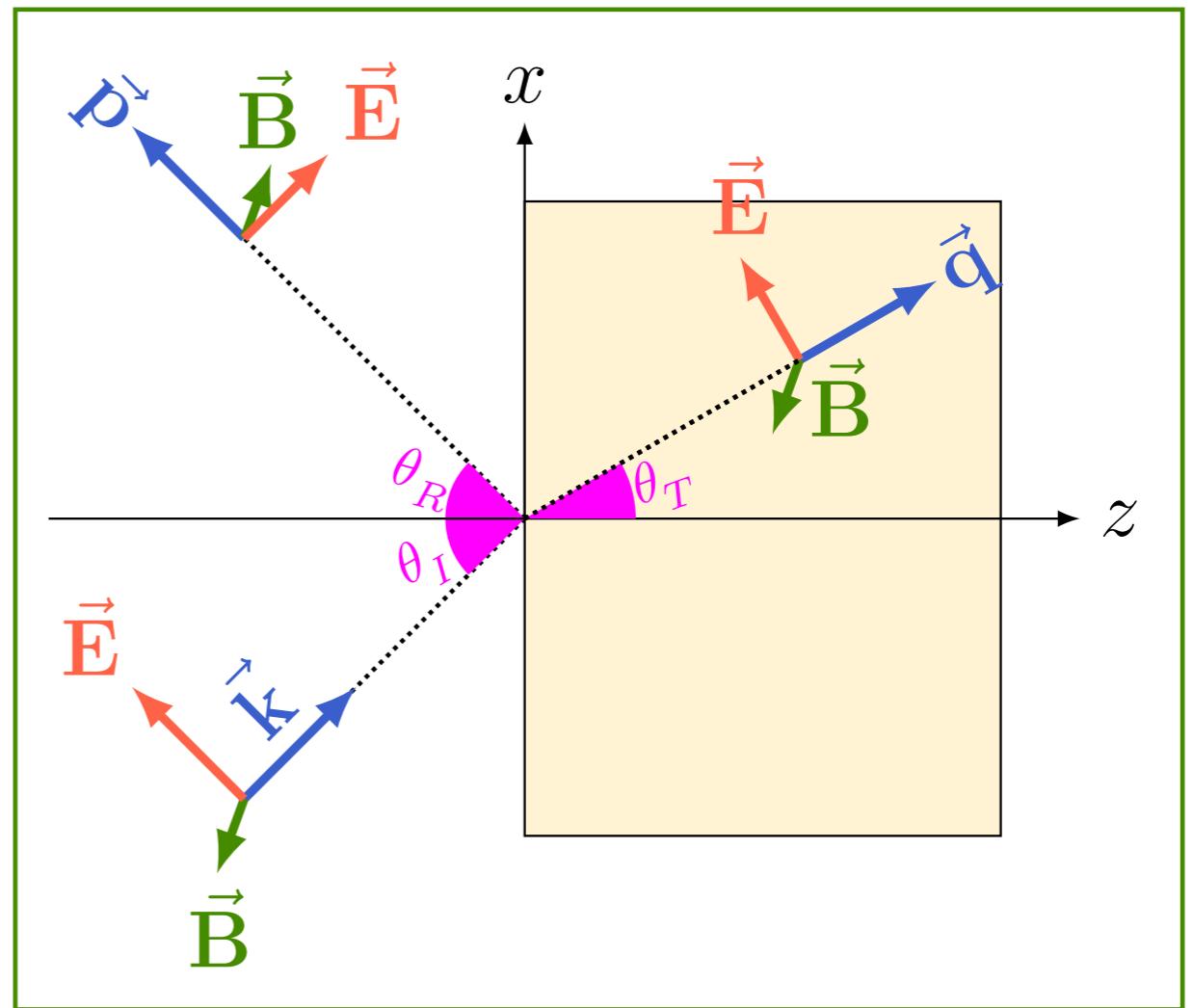
Incidência oblíqua

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$$\tilde{\mathbf{E}}_R = \tilde{\mathbf{E}}_{0R} e^{i(\vec{p} \cdot \vec{r} - \omega t)}$$

$$\epsilon_0 (\tilde{E}_{0z} + \tilde{E}_{0Rz}) = \epsilon \tilde{E}_{0Tz}$$



$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

$$E_{1\parallel} = E_{2\parallel}$$

$$B_{1\perp} = B_{2\perp}$$

$$\frac{1}{\mu_1} B_{1\parallel} = \frac{1}{\mu_2} B_{2\parallel}$$

Incidência oblíqua

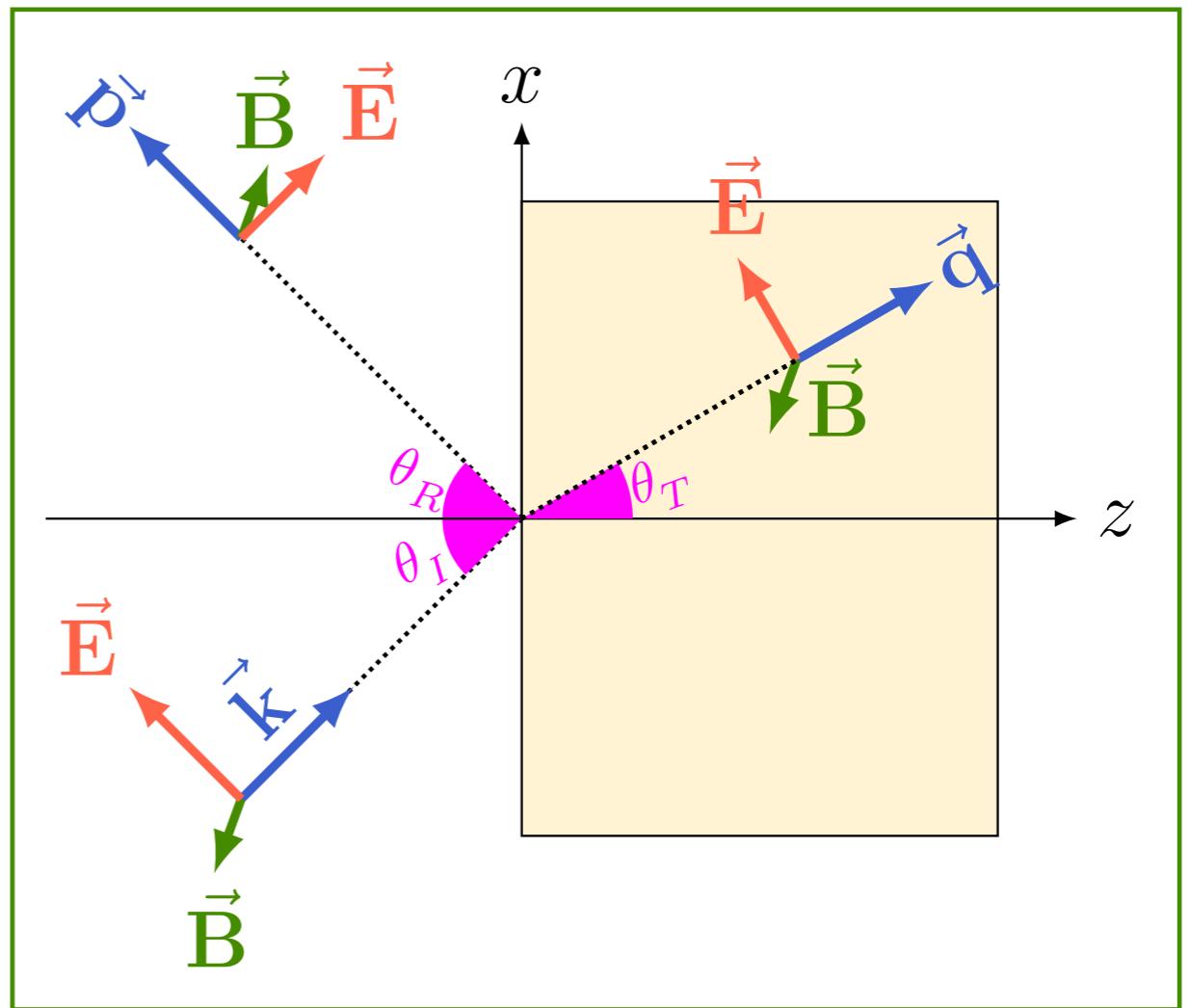
$$\tilde{\mathbf{E}}_I = \tilde{\mathbf{E}}_{0I} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\tilde{\mathbf{E}}_T = \tilde{\mathbf{E}}_{0T} e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

$$\tilde{\mathbf{E}}_R = \tilde{\mathbf{E}}_{0R} e^{i(\vec{p} \cdot \vec{r} - \omega t)}$$

$$\epsilon_0 (\tilde{E}_{0z} + \tilde{E}_{0Rx}) = \epsilon \tilde{E}_{0Tz}$$

$$\tilde{E}_{0x} + \tilde{E}_{0Rx} = \tilde{E}_{0Tx}$$



$$\epsilon_1 E_{1\perp} = \epsilon_2 E_{2\perp}$$

$$E_{1\parallel} = E_{2\parallel}$$

$$B_{1\perp} = B_{2\perp}$$

$$\frac{1}{\mu_1} B_{1\parallel} = \frac{1}{\mu_2} B_{2\parallel}$$

Incidência oblíqua

$$\tilde{\mathbf{E}}_I = \tilde{\mathbf{E}}_{0I} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

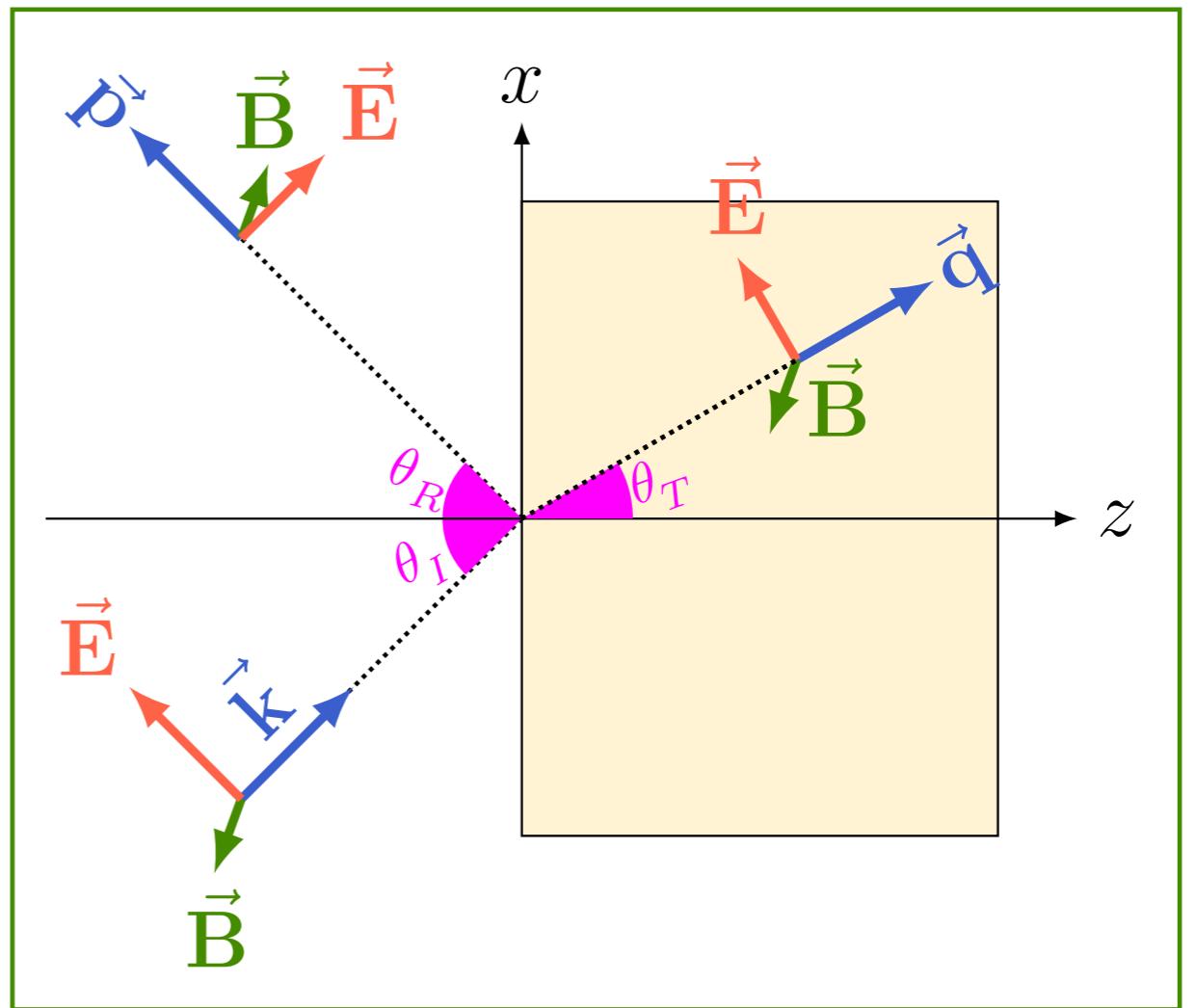
$$\tilde{\mathbf{E}}_T = \tilde{\mathbf{E}}_{0T} e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

$$\tilde{\mathbf{E}}_R = \tilde{\mathbf{E}}_{0R} e^{i(\vec{p} \cdot \vec{r} - \omega t)}$$

$$\epsilon_0 (\tilde{E}_{0z} + \tilde{E}_{0Rx}) = \epsilon \tilde{E}_{0Tz}$$

$$\tilde{E}_{0x} + \tilde{E}_{0Rx} = \tilde{E}_{0Tx}$$

$$-E_0 \sin \theta + E_{0R} \sin \theta = n^2 E_{0T} \sin \theta_T$$



Incidência oblíqua

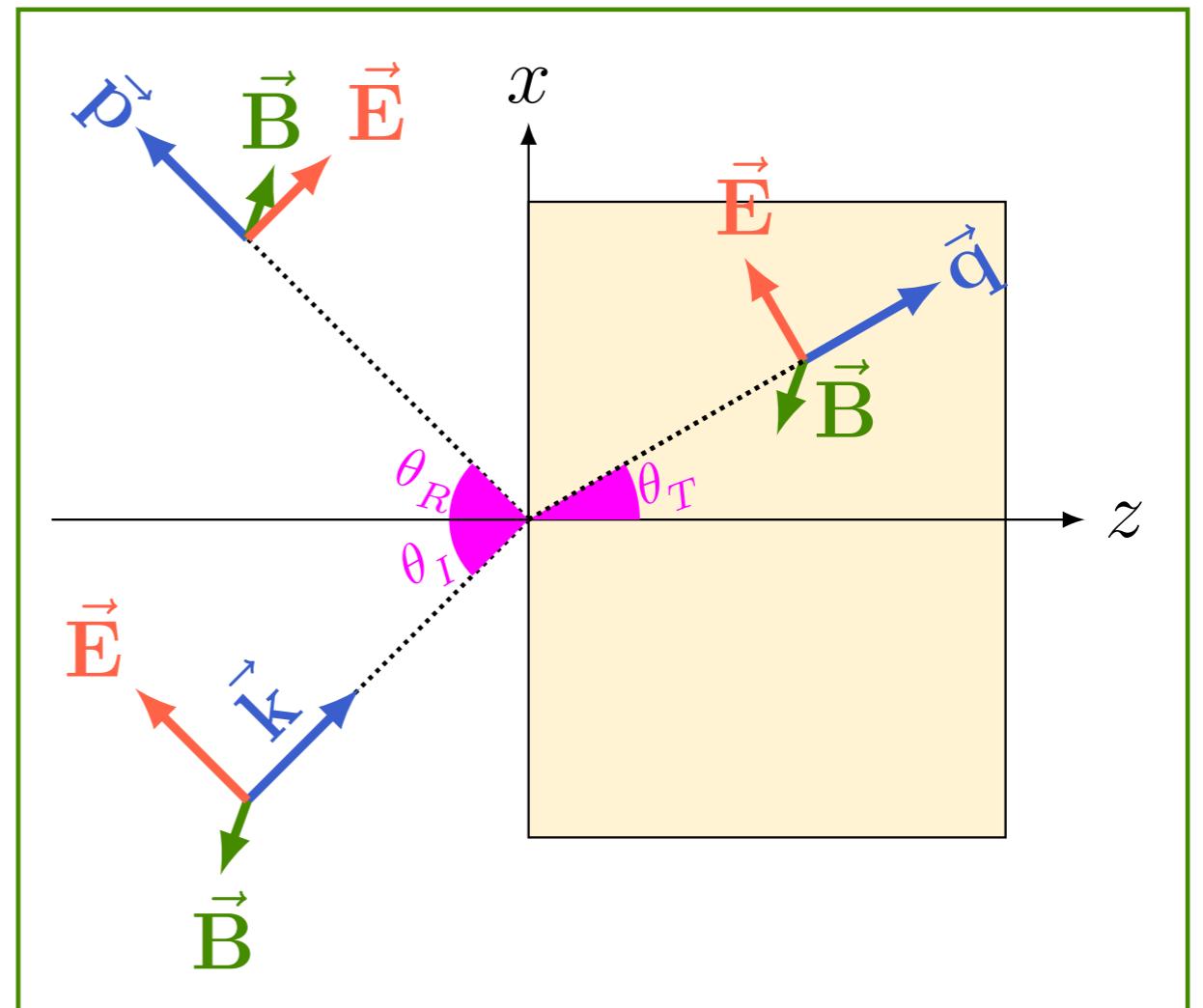
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$$\tilde{\mathbf{E}}_R = \tilde{\mathbf{E}}_{0R} e^{i(\vec{p} \cdot \vec{r} - \omega t)}$$

$$\epsilon_0 (\tilde{E}_{0z} + \tilde{E}_{0Rz}) = \epsilon \tilde{E}_{0Tz}$$

$$\tilde{E}_{0x} + \tilde{E}_{0Rx} = \tilde{E}_{0Tx}$$



$$E_{0R} - E_0 = nE_{0T}$$

Incidência oblíqua

$$\tilde{\mathbf{E}}_I = \tilde{\mathbf{E}}_{0I} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\tilde{\mathbf{E}}_T = \tilde{\mathbf{E}}_{0T} e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

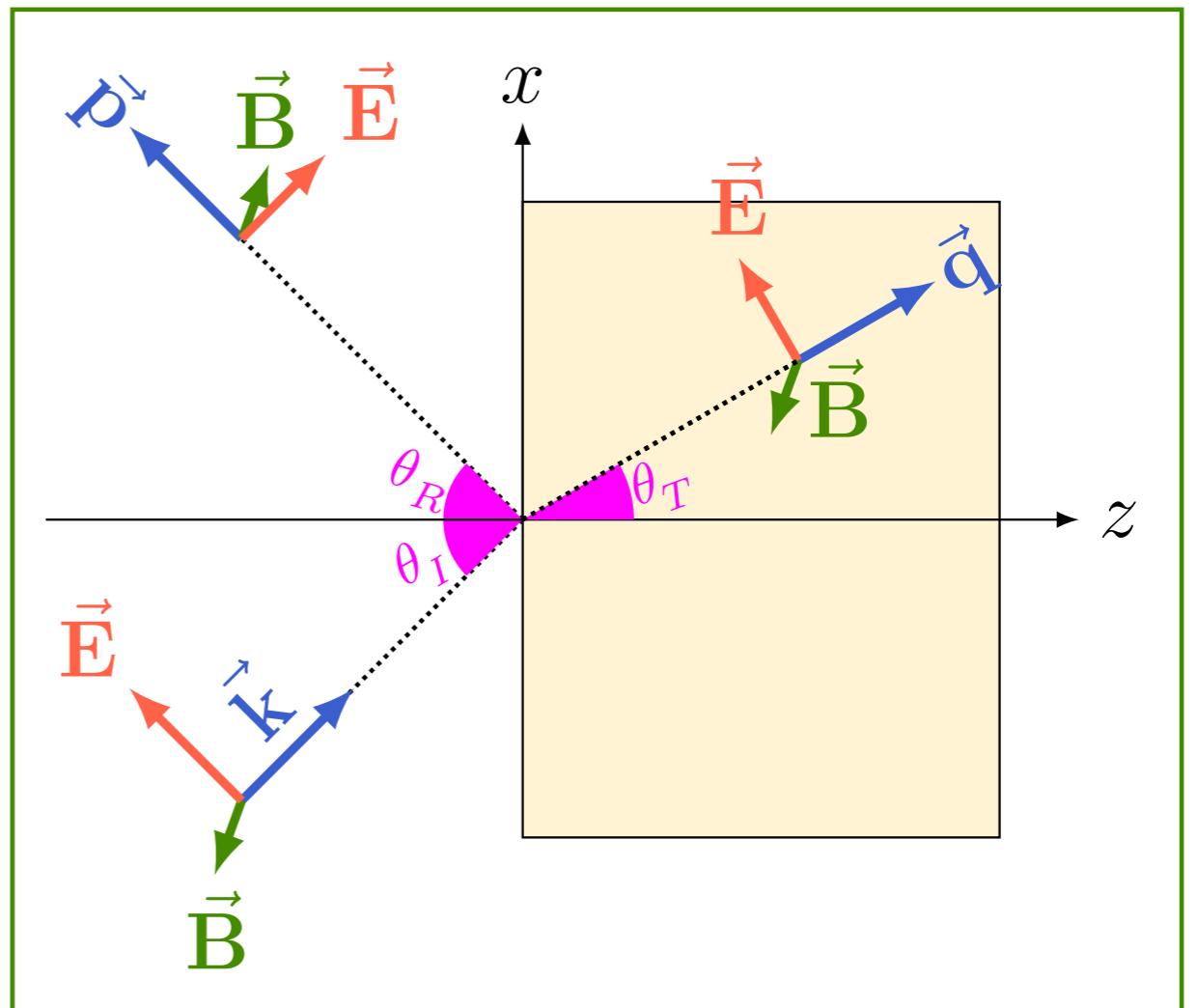
$$\tilde{\mathbf{E}}_R = \tilde{\mathbf{E}}_{0R} e^{i(\vec{p} \cdot \vec{r} - \omega t)}$$

$$\epsilon_0 (\tilde{E}_{0z} + \tilde{E}_{0Rz}) = \epsilon \tilde{E}_{0Tz}$$

$$\tilde{E}_{0x} + \tilde{E}_{0Rx} = \tilde{E}_{0Tx}$$

$$E_0 \cos \theta + E_{0R} \cos \theta = E_{0T} \cos \theta_T$$

$$E_{0R} - E_0 = n E_{0T}$$



Incidência oblíqua

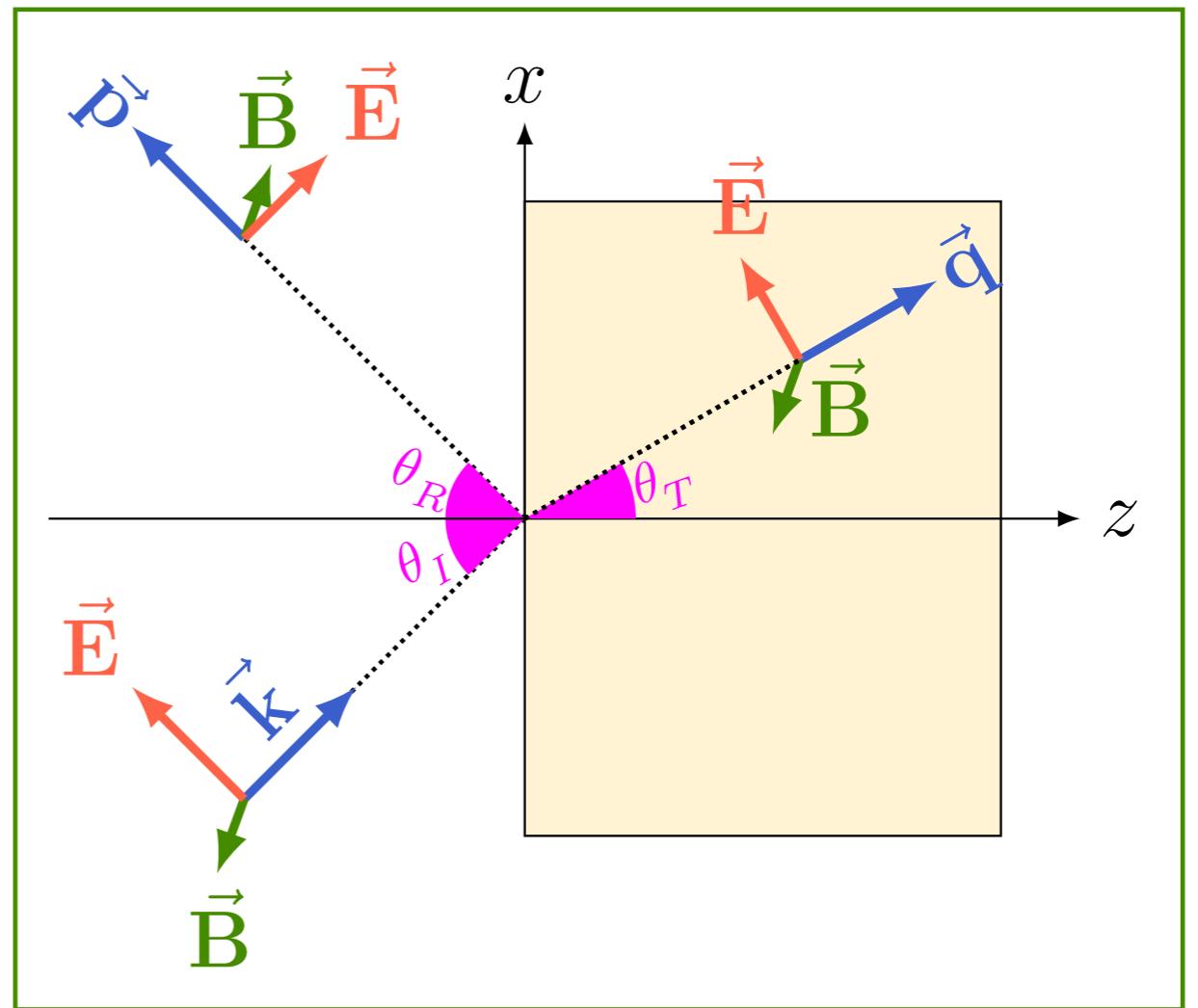
$$\tilde{\mathbf{E}}_I = \tilde{\mathbf{E}}_{0I} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\tilde{\mathbf{E}}_T = \tilde{\mathbf{E}}_{0T} e^{i(\vec{q} \cdot \vec{r} - \omega t)}$$

$$\tilde{\mathbf{E}}_R = \tilde{\mathbf{E}}_{0R} e^{i(\vec{p} \cdot \vec{r} - \omega t)}$$

$$\epsilon_0 (\tilde{E}_{0z} + \tilde{E}_{0Rx}) = \epsilon \tilde{E}_{0Tz}$$

$$\tilde{E}_{0x} + \tilde{E}_{0Rx} = \tilde{E}_{0Tx}$$

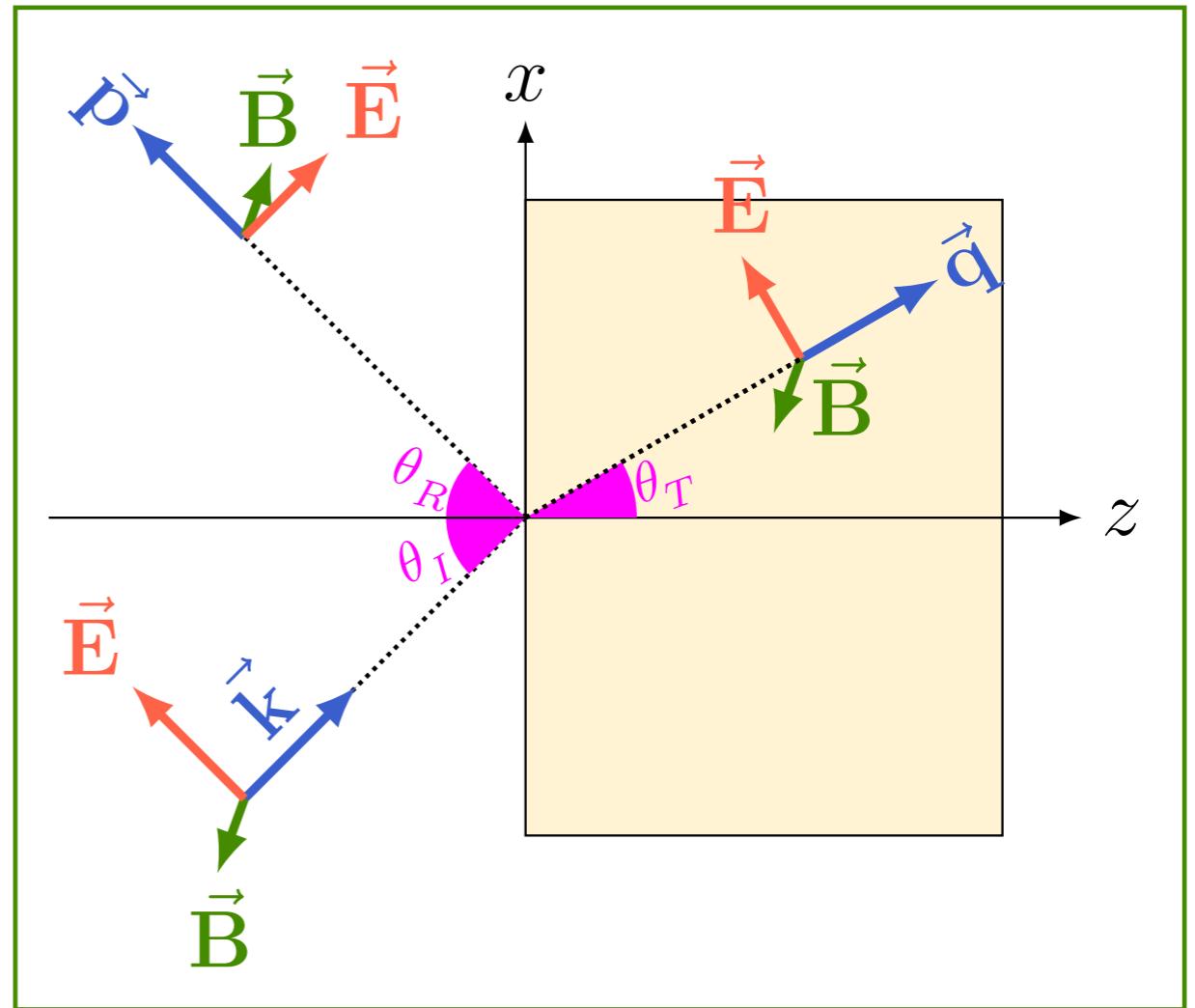


$$E_{0R} - E_0 = n E_{0T}$$

$$E_0 + E_{0R} = \alpha E_{0T}$$

$$\alpha \equiv \frac{\cos \theta_T}{\cos \theta}$$

Incidência oblíqua



$$E_{0R} - E_0 = nE_{0T}$$

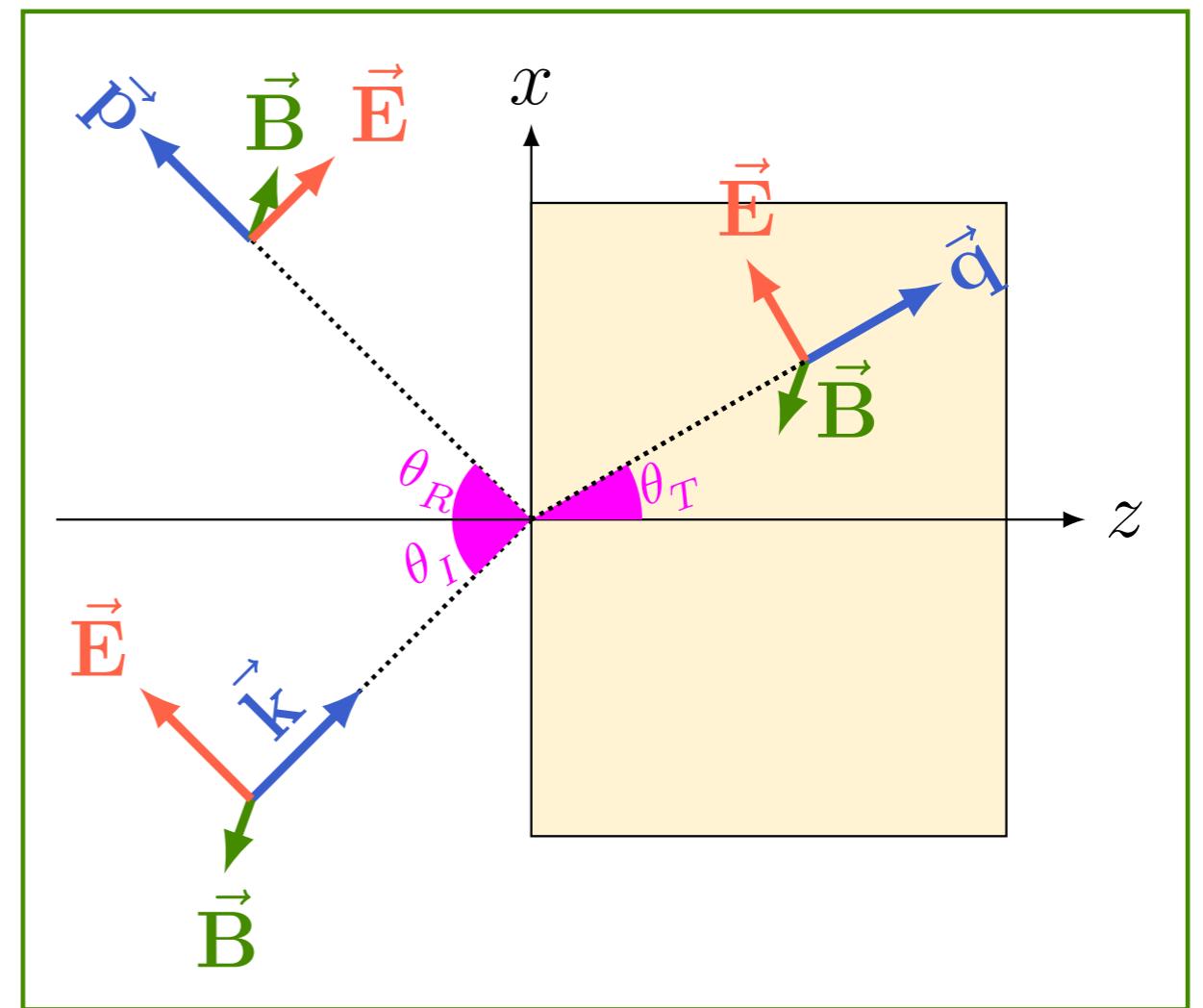
$$E_0 + E_{0R} = \alpha E_{0T}$$

$$\alpha \equiv \frac{\cos \theta_T}{\cos \theta}$$

Incidência oblíqua

$$\tilde{E}_{0R} = \frac{\alpha - n}{\alpha + n} \tilde{E}_0$$

$$\tilde{E}_{0T} = \frac{2}{\alpha + n} \tilde{E}_0$$



$$E_{0R} - E_0 = nE_{0T}$$

$$E_0 + E_{0R} = \alpha E_{0T}$$

$$\alpha \equiv \frac{\cos \theta_T}{\cos \theta}$$