

Eletromagnetismo Avançado

1º ciclo
Aula de 3 setembro

Ondas eletromagnéticas

$$\nabla^2 \vec{\mathbf{E}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

$$\nabla^2 \vec{\mathbf{B}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2}$$

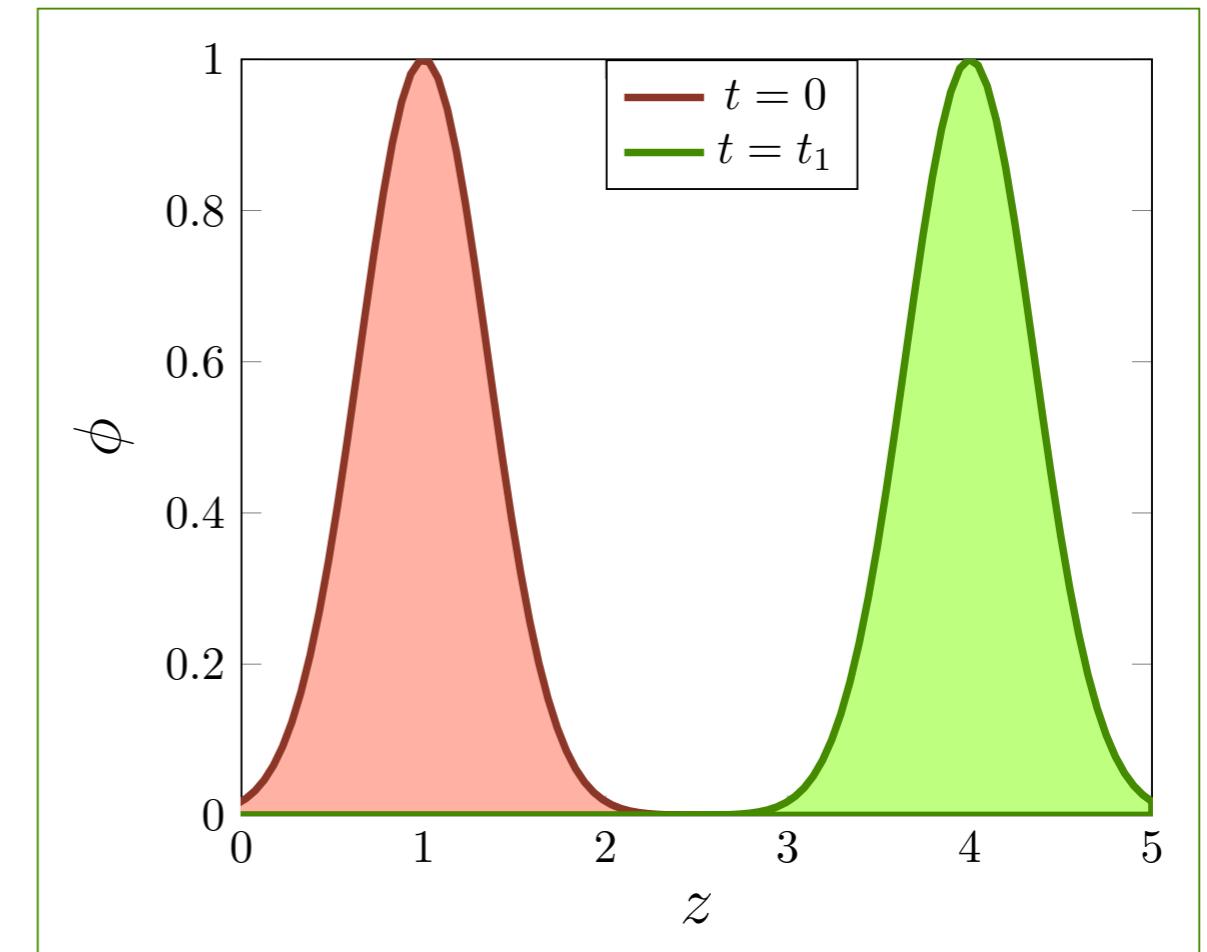
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$$\nabla^2 \phi = \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2}$$

$$\phi_{\vec{\mathbf{k}}}(\vec{\mathbf{r}}, t) = f(\vec{\mathbf{k}} \cdot \vec{r} - \omega t) \quad (\omega = kv)$$



Ondas eletromagnéticas

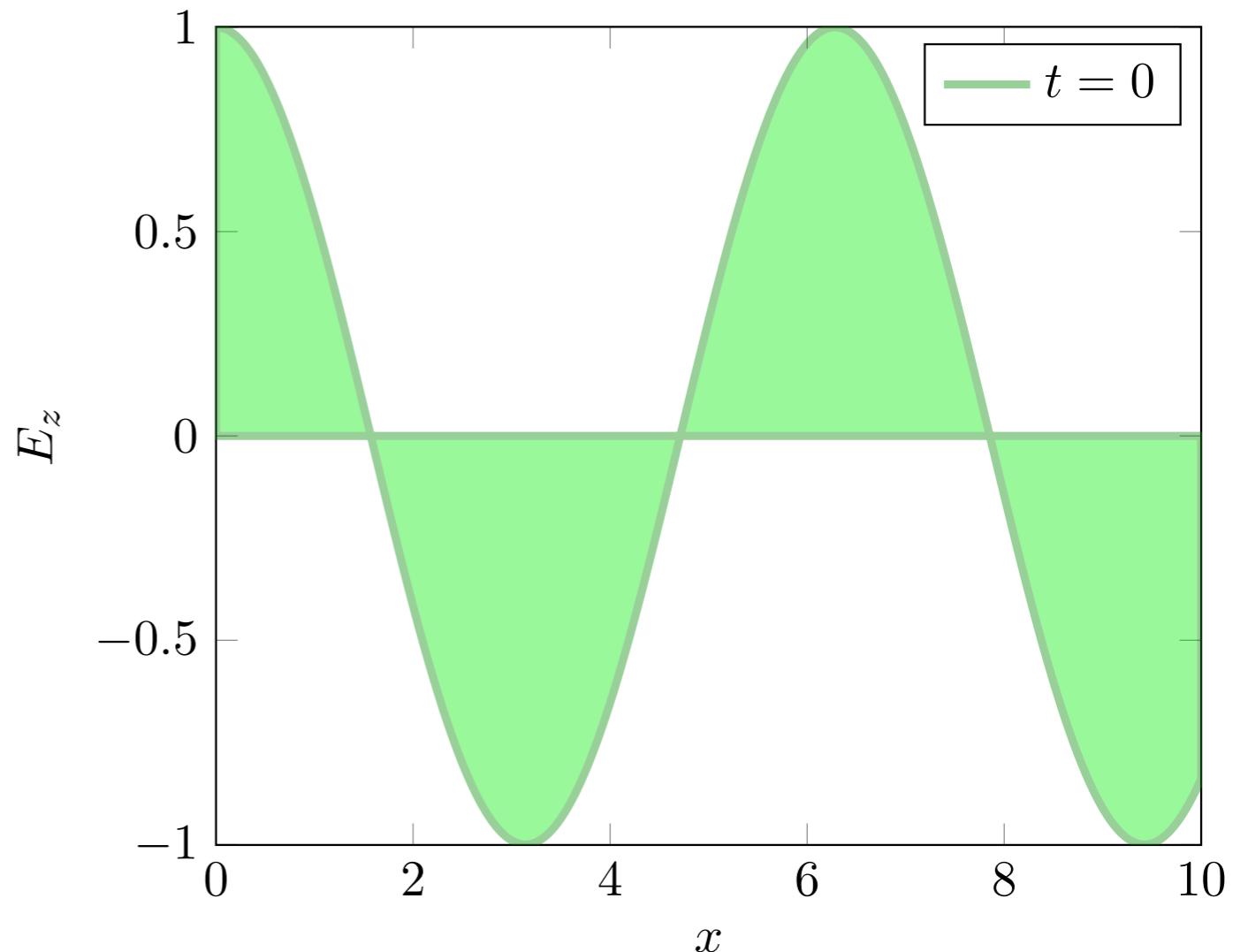
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Ondas planas

$$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$$

$$\vec{B} = \vec{B}_0 e^{i(kz - \omega t)}$$



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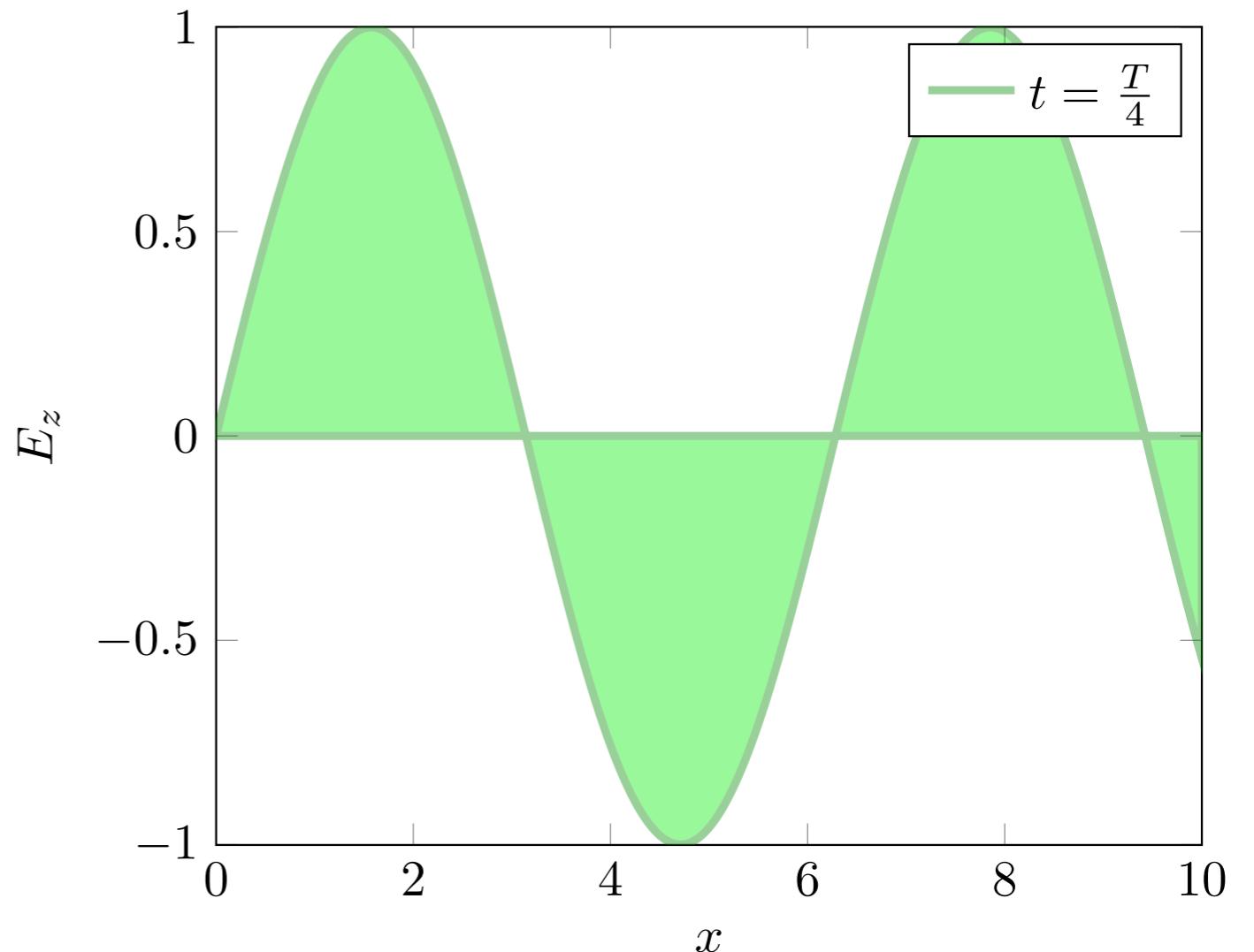
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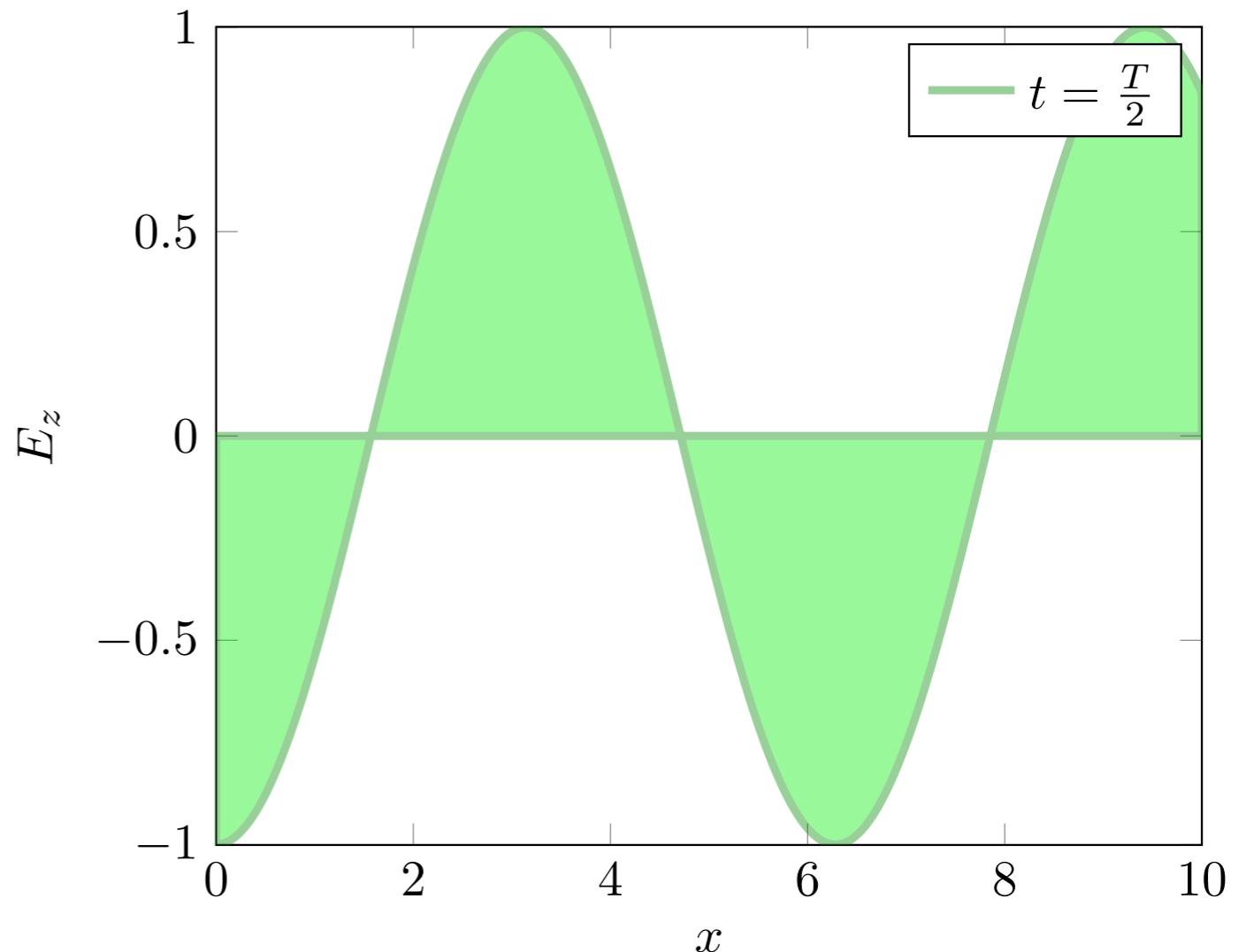
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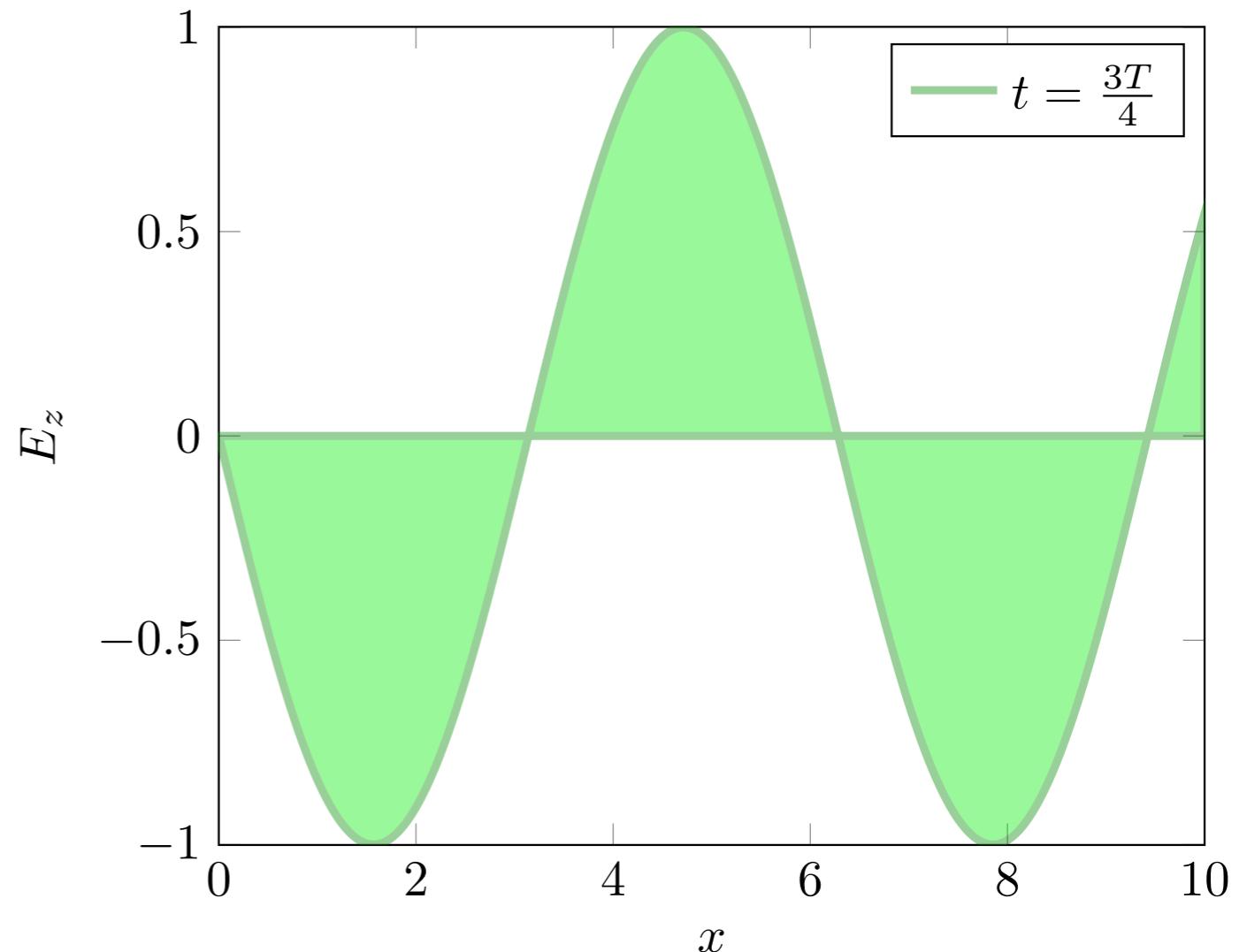
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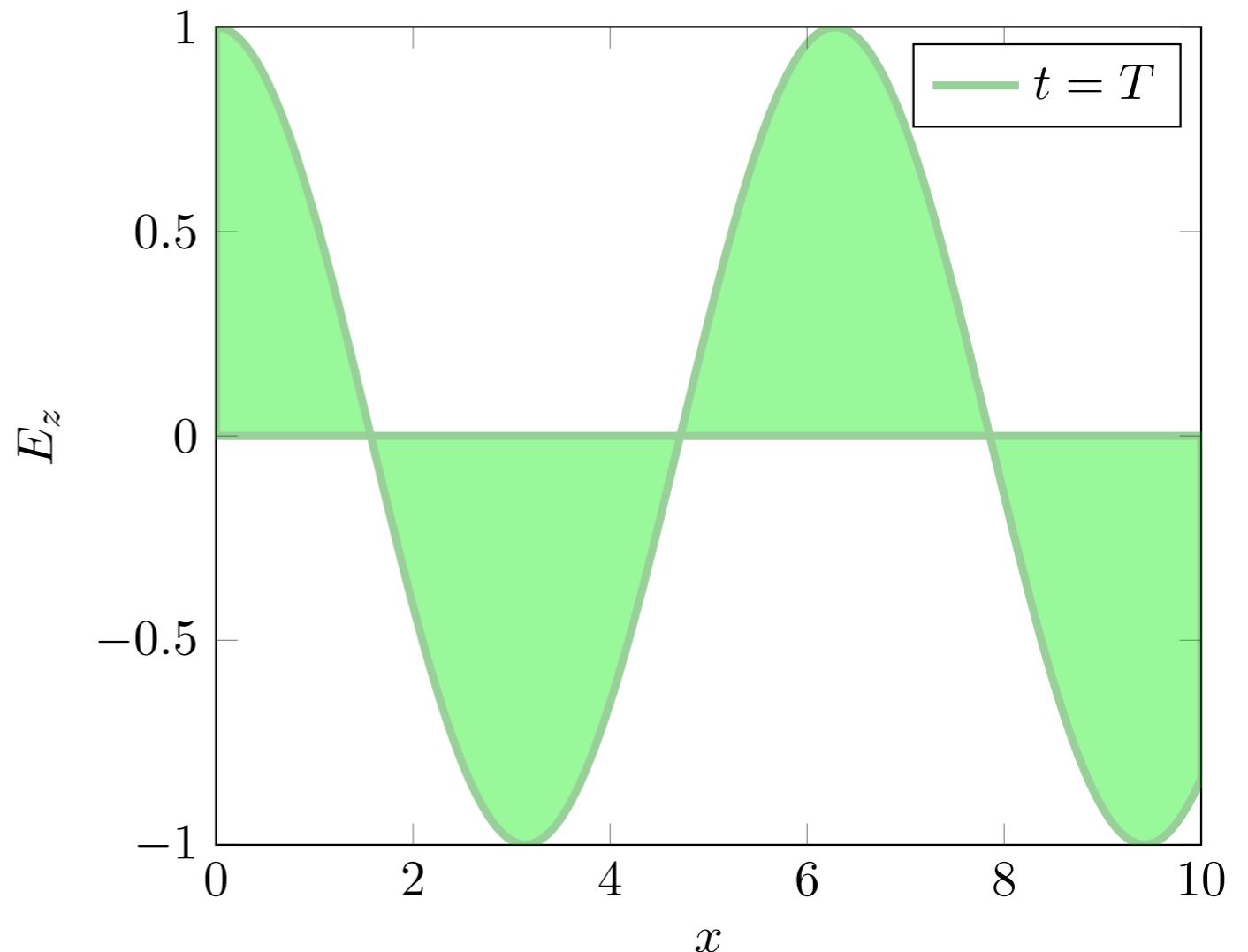
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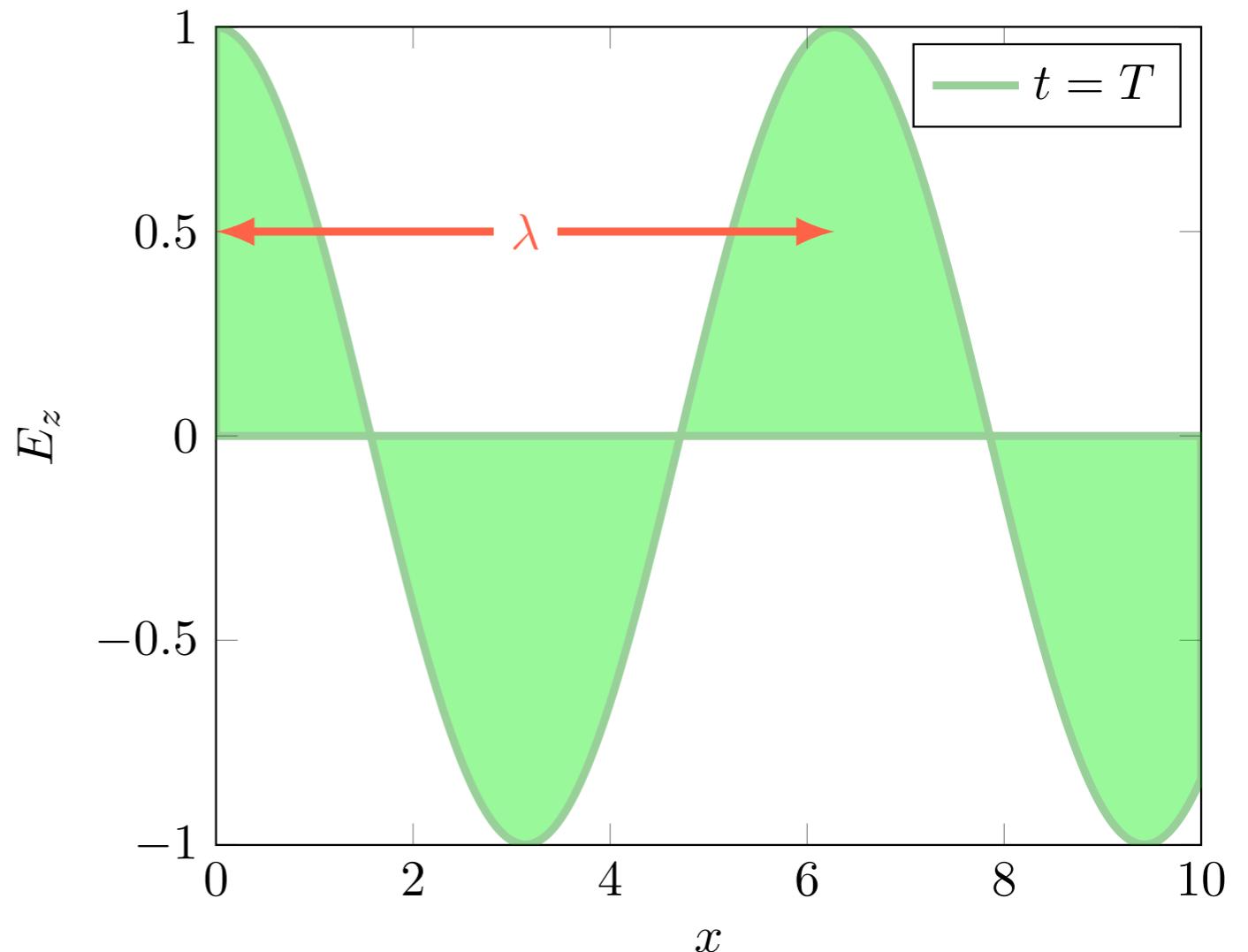
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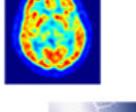


Ondas eletromagnéticas

Ondas planas

$$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$$

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| | | |
|-------------|---|-------------------------------|
| Radio |  | AM radio |
| |  | Amateur radio |
| |  | Aircraft communication |
| |  | Microwave oven |
| Infrared |  | TV Remote Control |
| |  | Night vision goggles |
| Visible |  | |
| Ultraviolet |  | UV light from the Sun |
| X-ray |  | Airport security scanner |
| Gamma-ray |  | PET scan |
| |  | Terrestrial gamma-ray flashes |

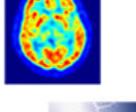
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$$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$$

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$$\vec{\nabla} \cdot \vec{E} = 0$$

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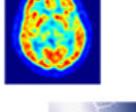
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$$\vec{\nabla} \cdot \vec{E} = 0 \quad \Rightarrow \quad \frac{dE_z}{dz} = 0$$

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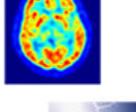
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$$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$$

$$\vec{B} = \vec{B}_0 e^{i(kz - \omega t)}$$

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \Rightarrow \quad \frac{dE_z}{dz} = 0$$

$$\vec{E}_0 = E_{0x} \hat{x} + E_{0y} \hat{y}$$

| | | |
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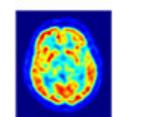
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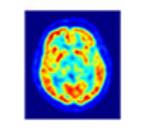
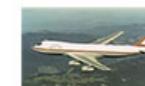
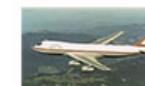
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Em geral

$$\vec{E} = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

| | | | | | | | |
|---|---|---|---|--|---|---|---|
| Gamma-ray | X-ray | Ultraviolet | Visible | Infrared | Microwave | Radio | |
|  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |
| | | | |  |  |  | AM radio Amateur radio |

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$$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$$

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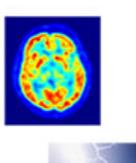
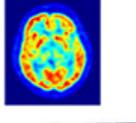
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| | | | | | | | |
|---|---|---|---|---|---|---|---------------|
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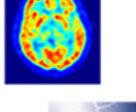
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$$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$$

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$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

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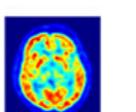
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$$-\frac{\partial E_{0y}}{\partial z} = -\frac{\partial B_{0x}}{\partial t}$$

| | | |
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| | | | | | | | |
|-----------|-------|-------------|---------|----------|-----------|-------|-------------------------------|
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$$-\frac{\partial E_{0y}}{\partial z} = -\frac{\partial B_{0x}}{\partial t} \quad \Rightarrow \quad kE_{0x} = \omega B_{0y}$$

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| | | | | | | | |
|-----------|-------|-------------|---------|----------|-----------|-------|-------------------------------|
| Gamma-ray | X-ray | Ultraviolet | Visible | Infrared | Microwave | Radio | AM radio |
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| | | | | | | | |
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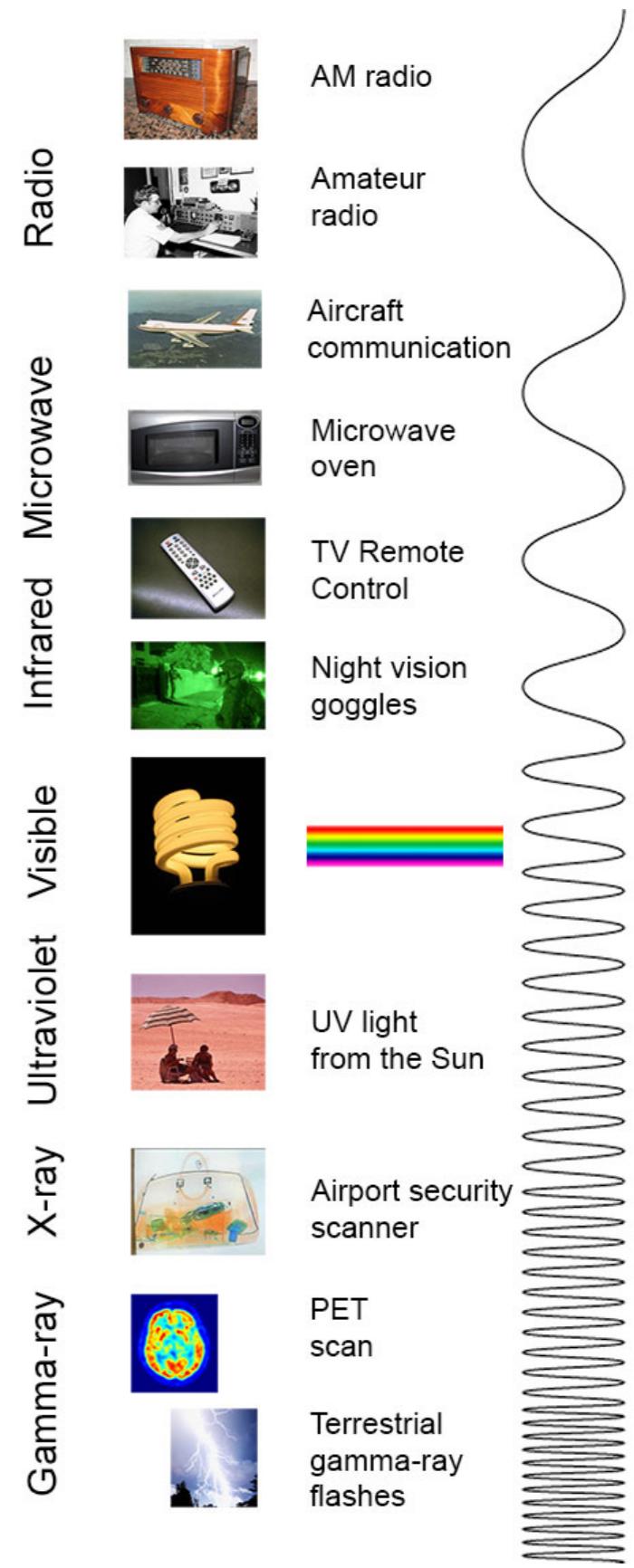
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$$\vec{B}_0 = \frac{k}{\omega} \hat{\mathbf{z}} \times \vec{\mathbf{E}}_0$$



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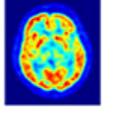
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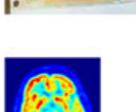
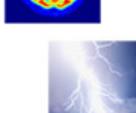
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Em geral

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$$\vec{B}_0 = B_{0x} \hat{x} + B_{0y} \hat{y}$$

$$\vec{\nabla} \times \vec{E} = i \vec{k} \times \vec{E}$$

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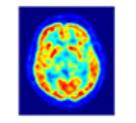
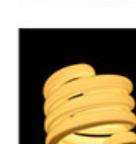
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Em geral

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_0 e^{i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)}$$

$$\vec{\mathbf{B}} = \frac{1}{c} \hat{\mathbf{k}} \times \vec{\mathbf{E}}$$

$$\vec{\nabla} \times \vec{\mathbf{E}} = i \hat{\mathbf{k}} \times \vec{\mathbf{E}}$$

| | | | | | | | |
|---|---|---|---|---|---|---|--|
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$$\vec{E}_0 = E_{0x} \hat{\mathbf{x}} + E_{0y} \hat{\mathbf{y}}$$

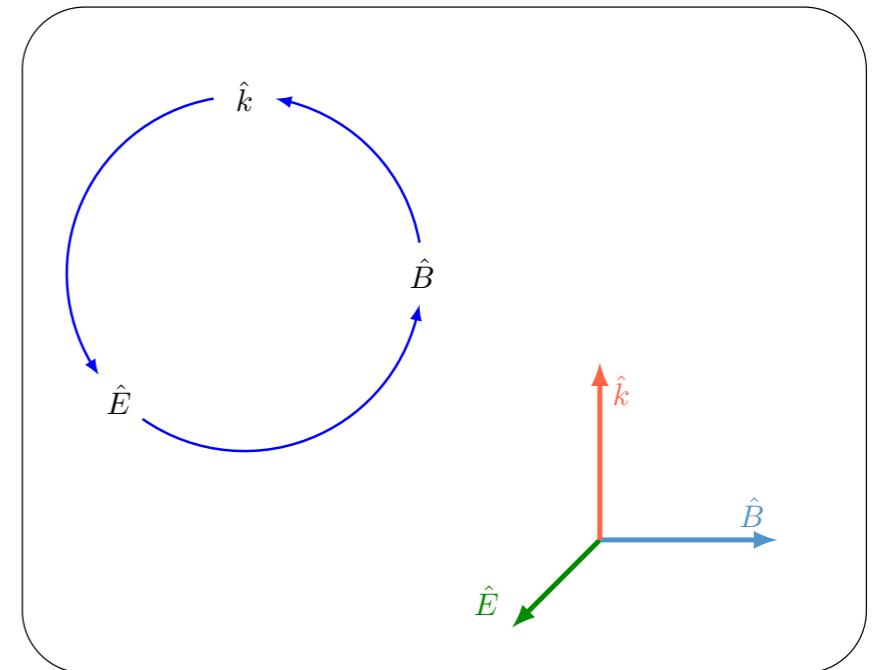
$$\vec{B}_0 = B_{0x} \hat{\mathbf{x}} + B_{0y} \hat{\mathbf{y}}$$

Em geral

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_0 e^{i(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t)}$$

$$\vec{\nabla} \times \vec{\mathbf{E}} = i \vec{\mathbf{k}} \times \vec{\mathbf{E}}$$

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Ondas eletromagnéticas

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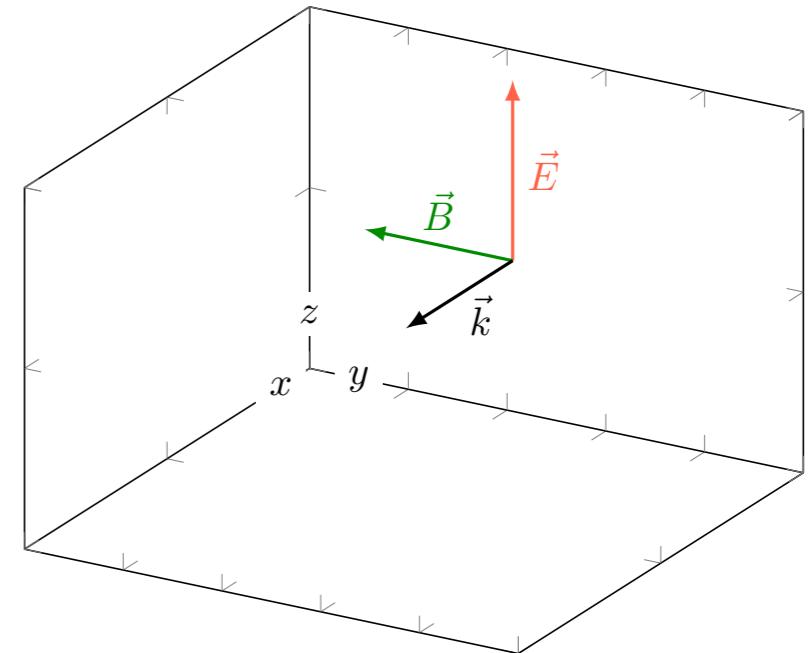
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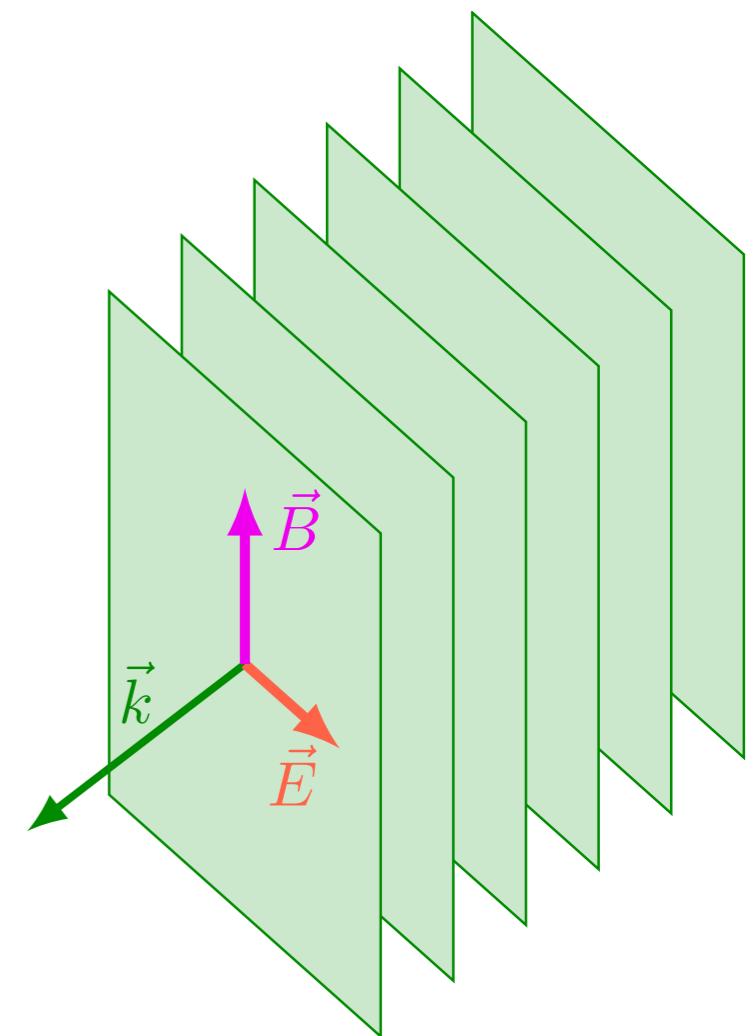
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