Eletromagnetismo Avançado

1º ciclo
Aula de 1º setembro

Leis de conservação

1. Carga elétrica

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

Leis de conservação

1. Carga elétrica

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

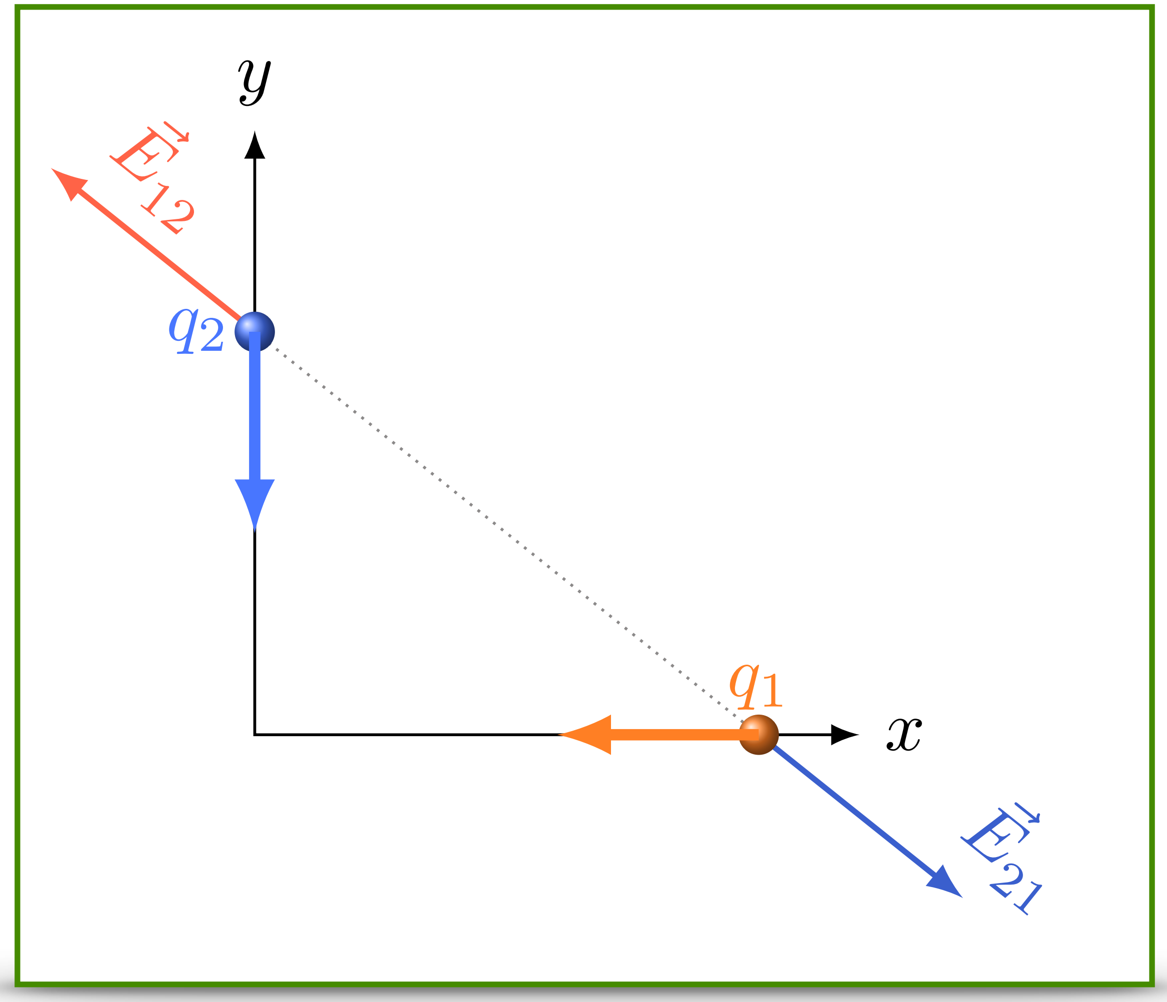
2. Energia

$$\frac{\partial}{\partial t}\left(u_{mec} + u_{em}\right) = -\vec{\nabla} \cdot \vec{S}$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

Leis de conservação

3. Momento



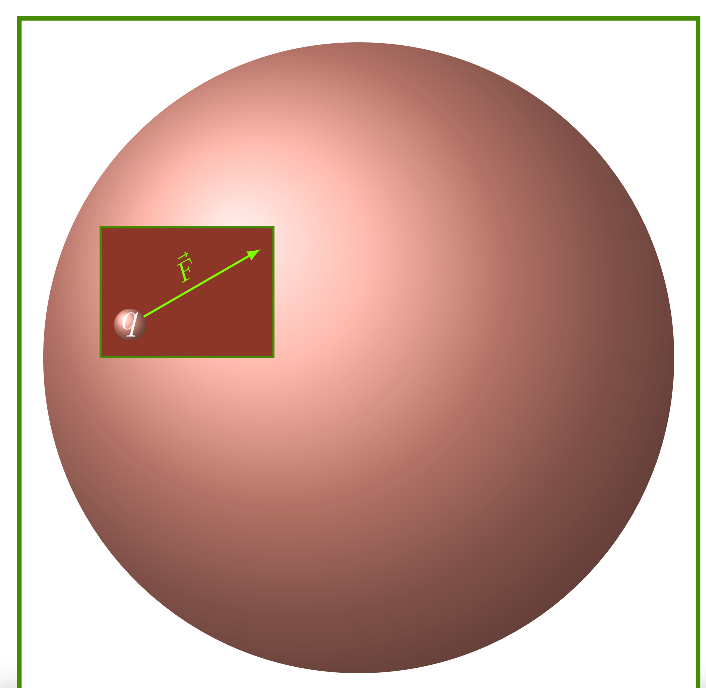
Leis de conservação

3. Momento

$$\vec{\mathbf{f}} = \epsilon_0[(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}]$$
$$- \frac{1}{2}\vec{\boldsymbol{\nabla}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)$$

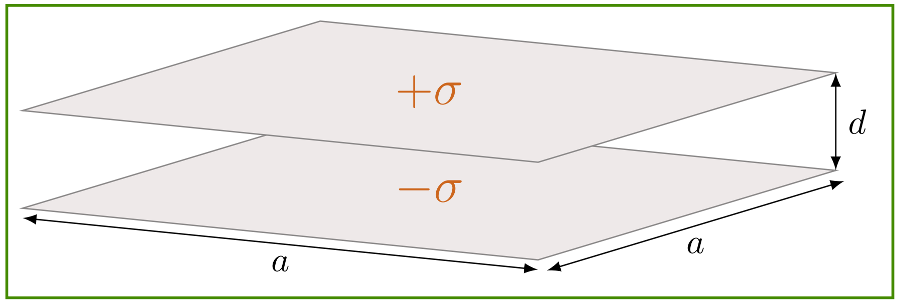
$$T_{ij} = \epsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2}\delta_{ij}(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2)$$

$$F = \int_{\mathcal{A}} \mathbb{T} \cdot \hat{\mathbf{n}} \, \mathrm{d}a - \epsilon_0 \mu_0 \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathcal{V}} \vec{S} \, \mathrm{d}\tau$$



Pratique o que aprendeu

$$F = \int_{\mathcal{A}} \mathbb{T} \cdot \hat{\mathbf{n}} \, \mathrm{d}a - \epsilon_0 \mu_0 \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathcal{V}} \vec{S} \, \mathrm{d}\tau$$



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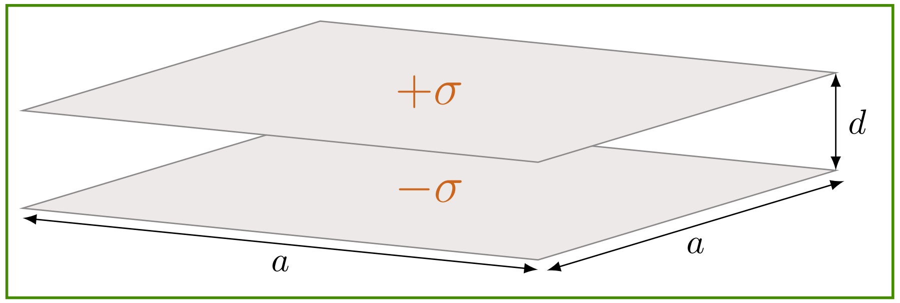
$$\mathbb{T} = \epsilon_0 \begin{bmatrix} -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 & 0 \\ 0 & -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 \\ 0 & 0 & \frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 \end{bmatrix}$$



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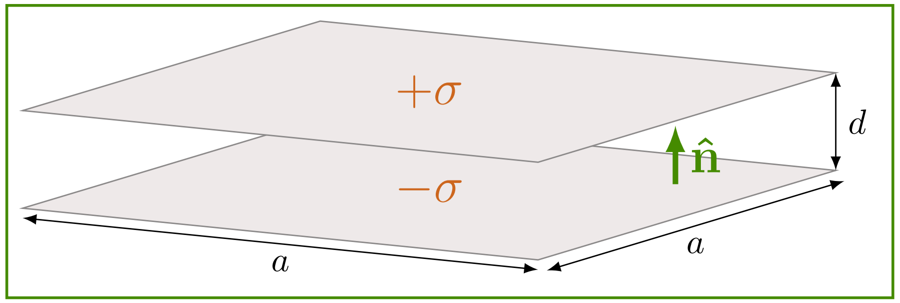

$$\mathbb{T} \cdot \hat{\mathbf{n}} = \epsilon_0 \begin{bmatrix} -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 & 0 \\ 0 & -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 \\ 0 & 0 & \frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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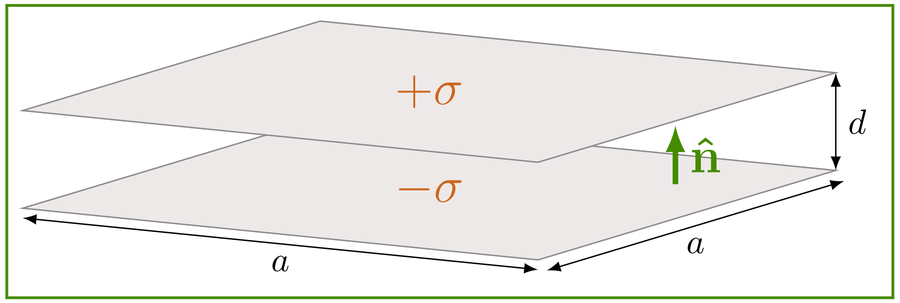

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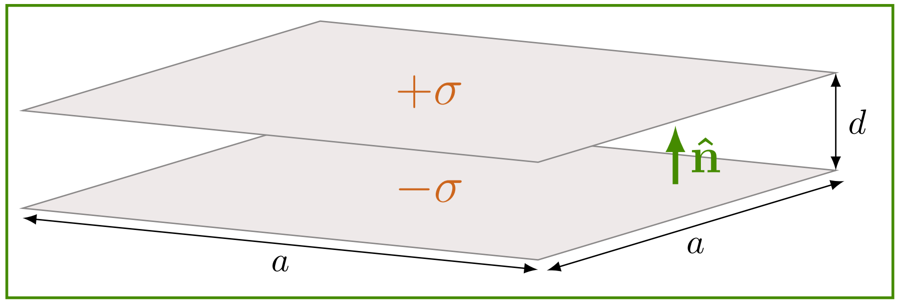
$$\vec{\mathbf{F}} = \frac{\sigma}{2\epsilon_0}\hat{z}$$



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Leis de conservação

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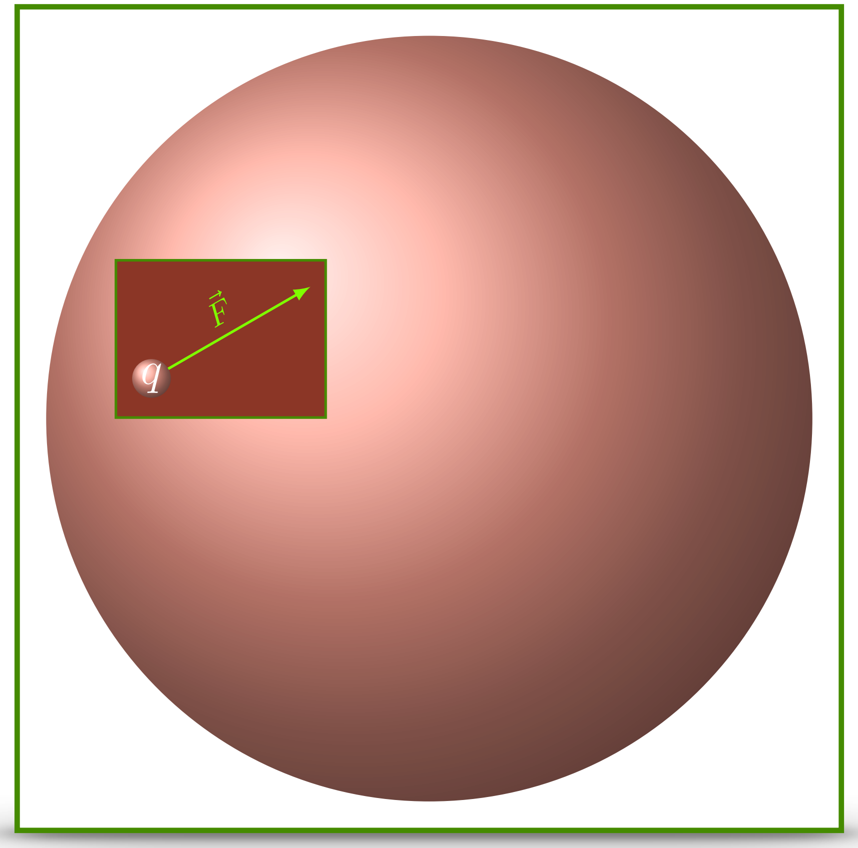
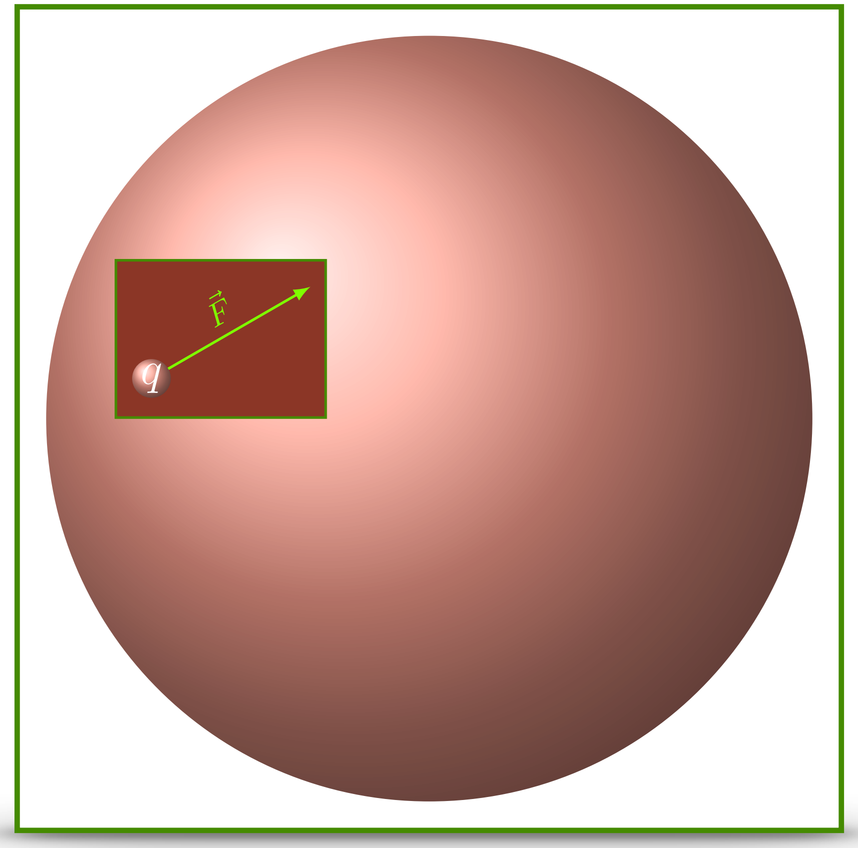
$$F = \int_{\mathcal{A}} \mathbb{T} \cdot \hat{\mathbf{n}} \, \mathrm{d}a - \epsilon_0 \mu_0 \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathcal{V}} \vec{S} \, \mathrm{d}\tau$$



3. Momento

$$F = \int_{\mathcal{A}} \mathbb{T} \cdot \hat{\mathbf{n}} \; \mathrm{d}a - \epsilon_0 \mu_0 \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathcal{V}} \vec{S} \; \mathrm{d}\tau$$

$$\frac{\mathrm{d}\vec{\mathbf{p}}_{mec}}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\epsilon_0 \mu_0 \int \vec{S} \, \mathrm{d}\tau\right) = \int \mathbb{T} \cdot \hat{n} \; \mathrm{d}a$$



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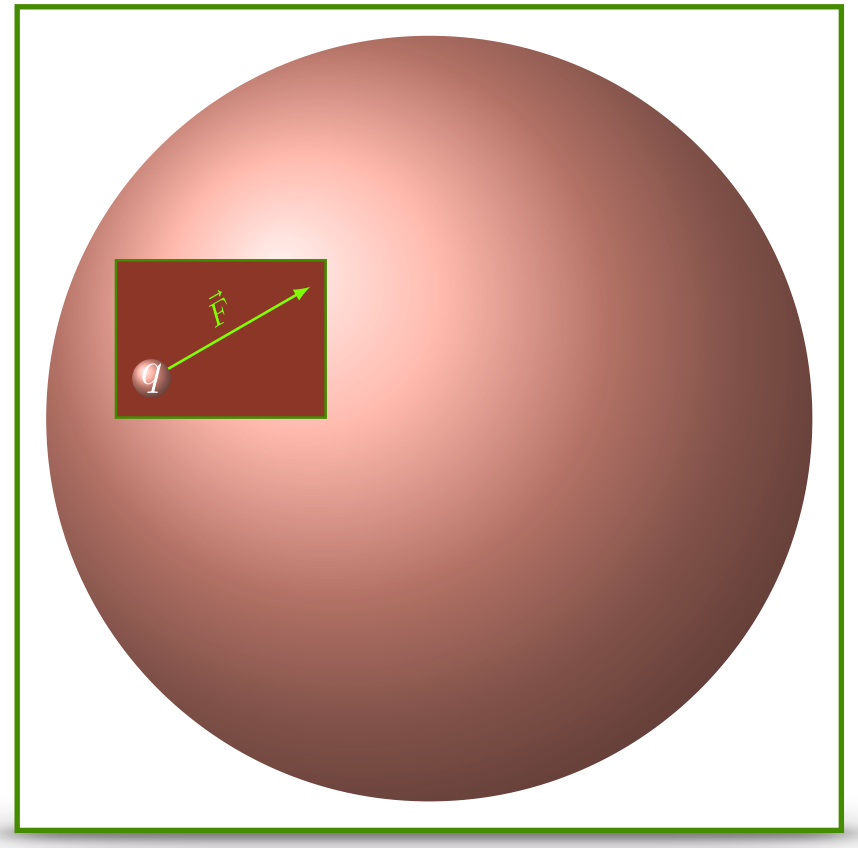
$$\frac{\mathrm{d}\vec{\mathbf{p}}_{mec}}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}t} \underbrace{\left(\epsilon_0 \mu_0 \int \vec{S} \, \mathrm{d}\tau \right)}_{\vec{\mathbf{p}}_{em}} = \int \mathbb{T} \cdot \hat{n} \; \mathrm{d}a$$



Leis de conservação

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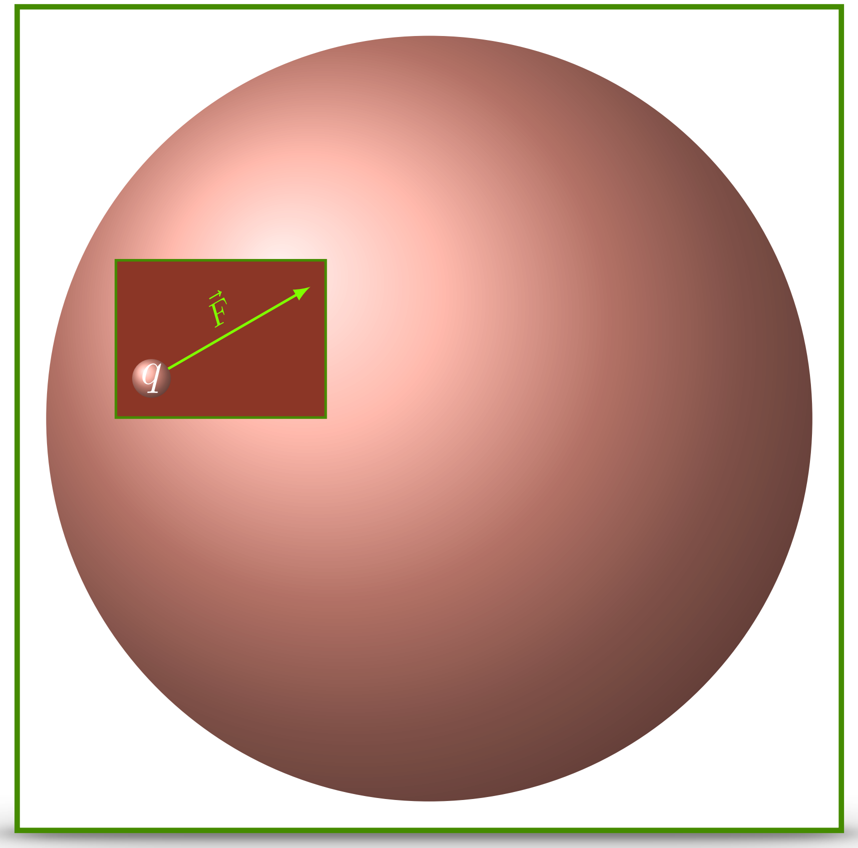
$$\frac{\mathrm{d}\vec{\mathbf{p}}_{mec}}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\epsilon_0 \mu_0 \int \vec{S}\, \mathrm{d}\tau\right) = \int \mathbb{T} \cdot \hat{n} \; \mathrm{d}a$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\vec{\mathbf{p}}_{mec} + \vec{\mathbf{p}}_{em}\right) = \int \mathbb{T} \cdot \hat{n} \; \mathrm{d}a$$



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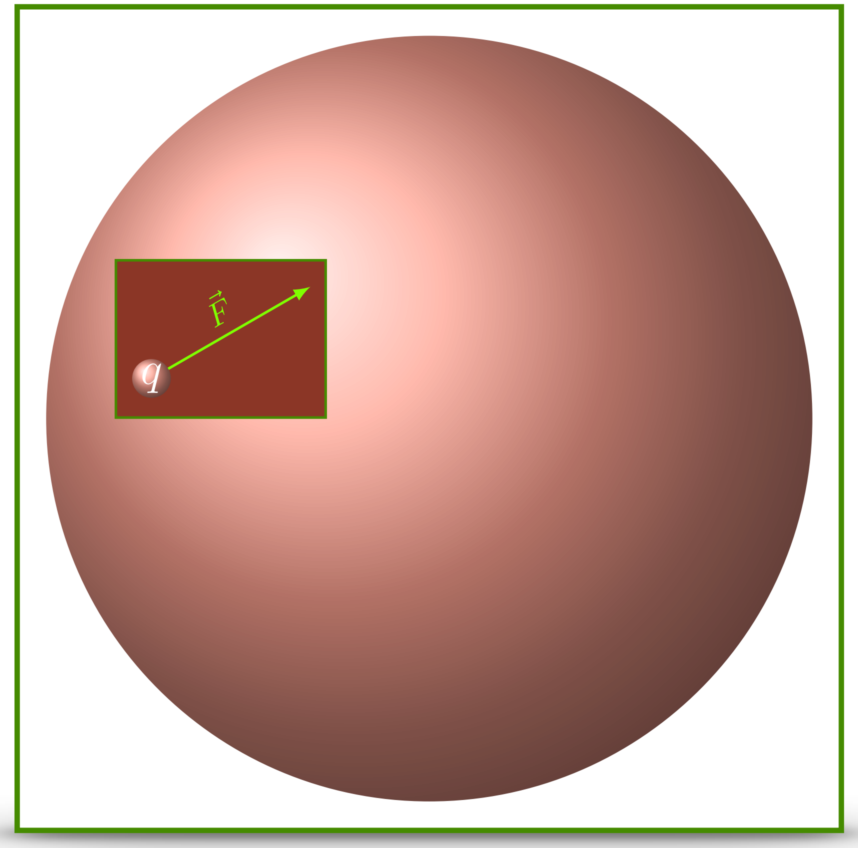
$$\frac{\mathrm{d}\vec{\mathbf{p}}_{mec}}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}t} \left(\epsilon_0 \mu_0 \int \vec{S} \, \mathrm{d}\tau \right) = \int \mathbb{T} \cdot \hat{n} \, \mathrm{d}a$$

$$\frac{\mathrm{d}}{\mathrm{d}t} \left(\vec{\mathbf{p}}_{mec} + \vec{\mathbf{p}}_{em} \right) = \int \mathbb{T} \cdot \hat{n} \, \mathrm{d}a$$

$$\vec{\mathbf{p}}_{em} = \epsilon_0 \mu_0 \int \vec{S} \, \mathrm{d}\tau$$



Eletromagnetismo Avançado

Propagação de ondas em uma dimensão

Ondas eletromagnéticas

$$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

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$$\vec{\boldsymbol{\nabla}} \times \left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}\right) = -\frac{\partial}{\partial t}\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}$$

$$\vec{\boldsymbol{\nabla}}\left(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}}\right) - \nabla^2\vec{\mathbf{E}} = -\frac{\partial}{\partial t}\left(\mu_0\epsilon_0\frac{\partial \vec{\mathbf{E}}}{\partial t}\right)$$

Ondas eletromagnéticas

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$$\nabla^2 \vec{\mathbf{E}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

Ondas eletromagnéticas

$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} = \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$$

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Ondas eletromagnéticas

$$\vec{\nabla} \times \vec{\mathbf{B}} = \mu_0 \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$$

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{\mathbf{B}} \right) = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \vec{\nabla} \times \vec{\mathbf{E}}$$

$$\vec{\nabla} \left(\vec{\nabla} \cdot \vec{\mathbf{B}} \right) - \nabla^2 \vec{\mathbf{B}} = \mu_0 \epsilon_0 \frac{\partial}{\partial t} \left(- \frac{\partial \vec{\mathbf{B}}}{\partial t} \right)$$

$$\nabla^2 \vec{\mathbf{B}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2}$$

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$$\nabla^2 \vec{\mathbf{E}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

$$\nabla^2 \vec{\mathbf{B}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2}$$

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$$\nabla^2 \vec{\mathbf{E}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

$$\nabla^2 \vec{\mathbf{B}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2}$$

$$\nabla^2 \phi = \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2}$$

Ondas eletromagnéticas

$$\nabla^2 \vec{\mathbf{E}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

$$\nabla^2 \vec{\mathbf{B}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2}$$

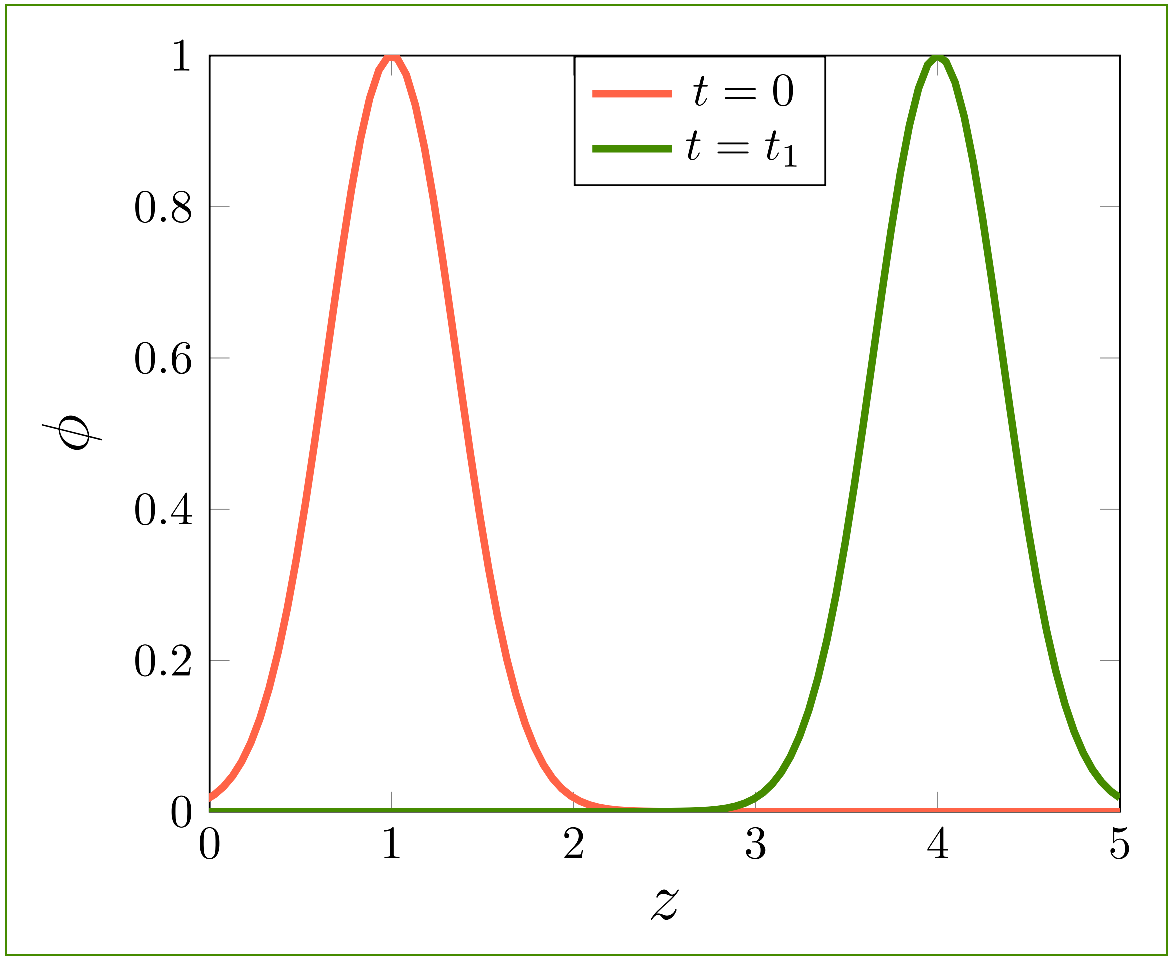
$$\nabla^2 \phi = \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2}$$

$$\phi_{\vec{\mathbf{k}}}(\vec{\mathbf{r}}, t) = f(\vec{\mathbf{k}} \cdot \vec{r} - \omega t) \qquad (\omega = kv)$$

Ondas eletromagnéticas

$$\nabla^2 \vec{\mathbf{E}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{E}}}{\partial t^2}$$

$$\nabla^2 \vec{\mathbf{B}} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{\mathbf{B}}}{\partial t^2}$$



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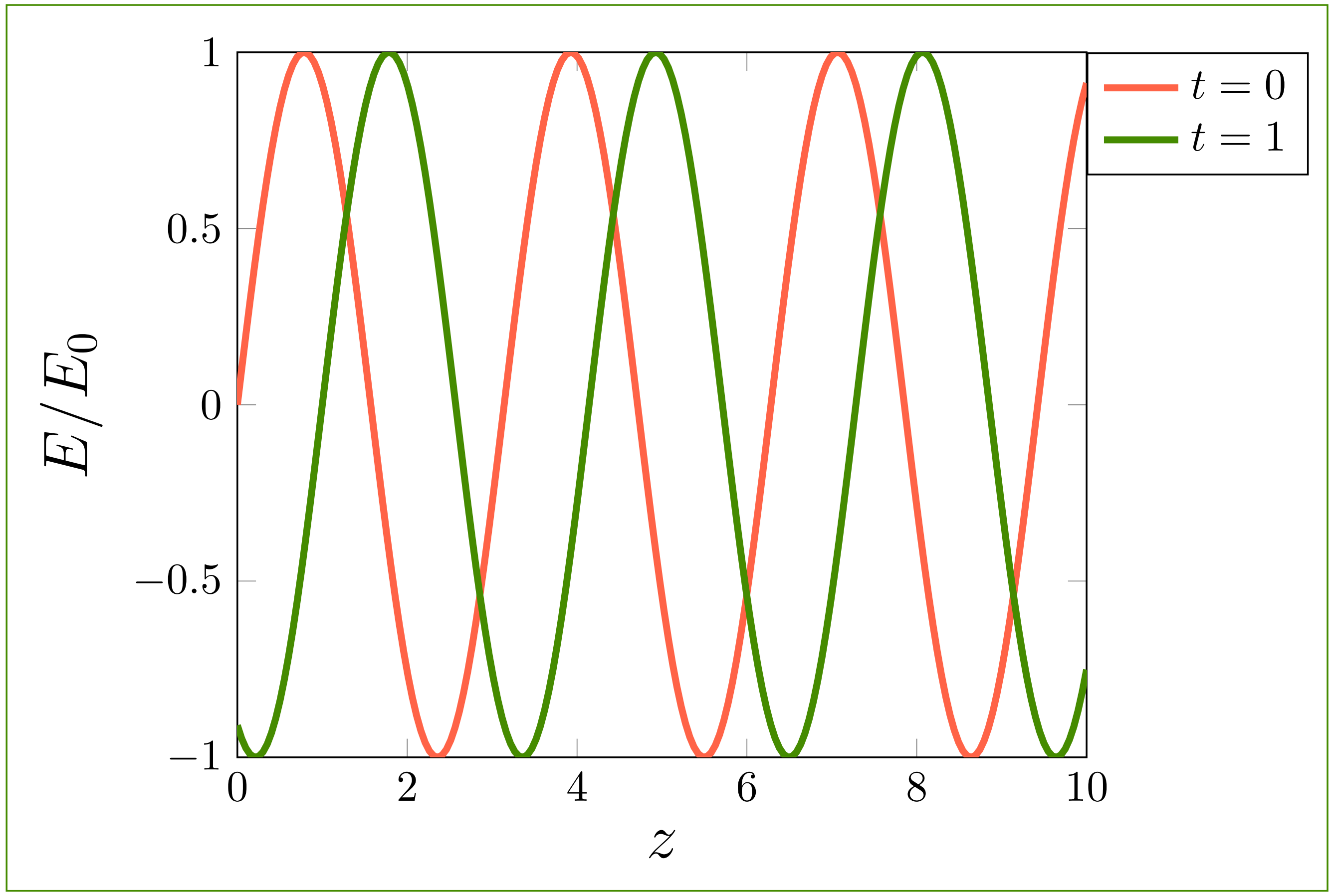

Ondas planas

$$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$$

$$\vec{B} = \vec{B}_0 e^{i(kz - \omega t)}$$

Ondas eletromagnéticas

Ondas planas

$$\vec{E} = \vec{E}_0 e^{i(kz-\omega t)}$$

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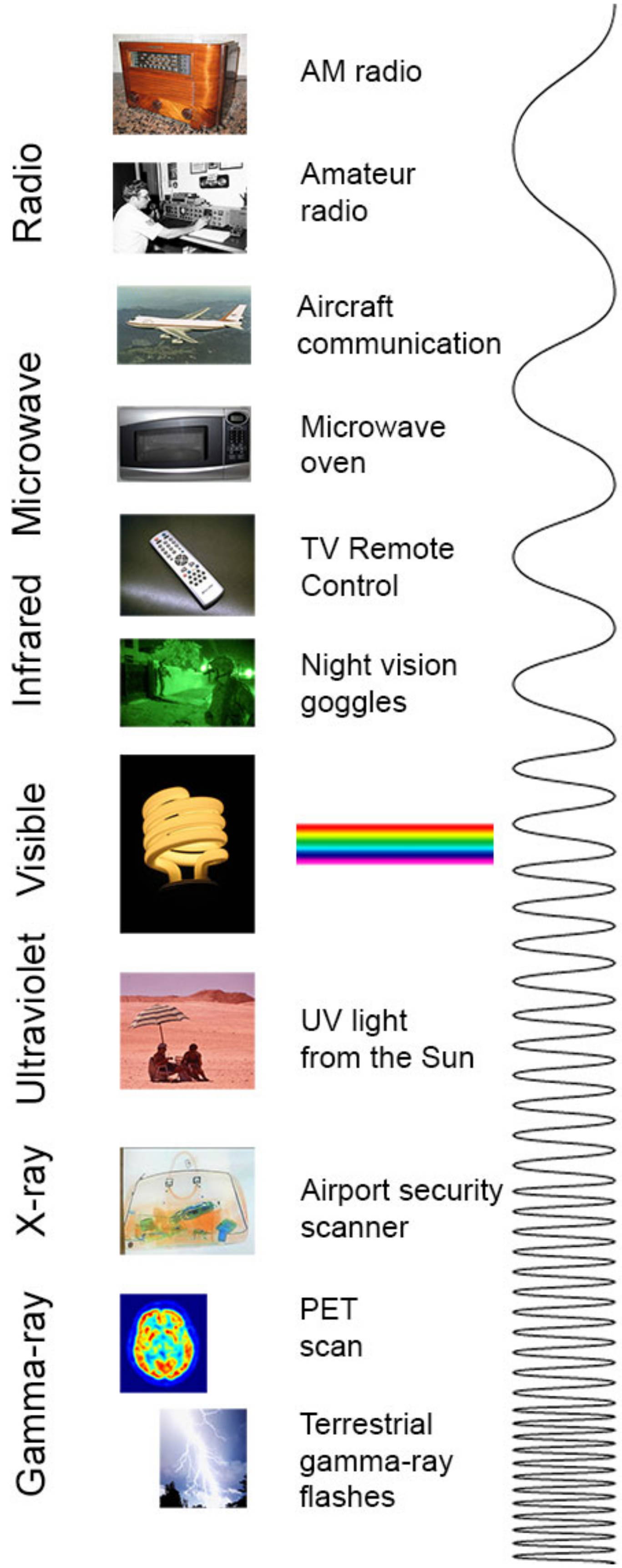


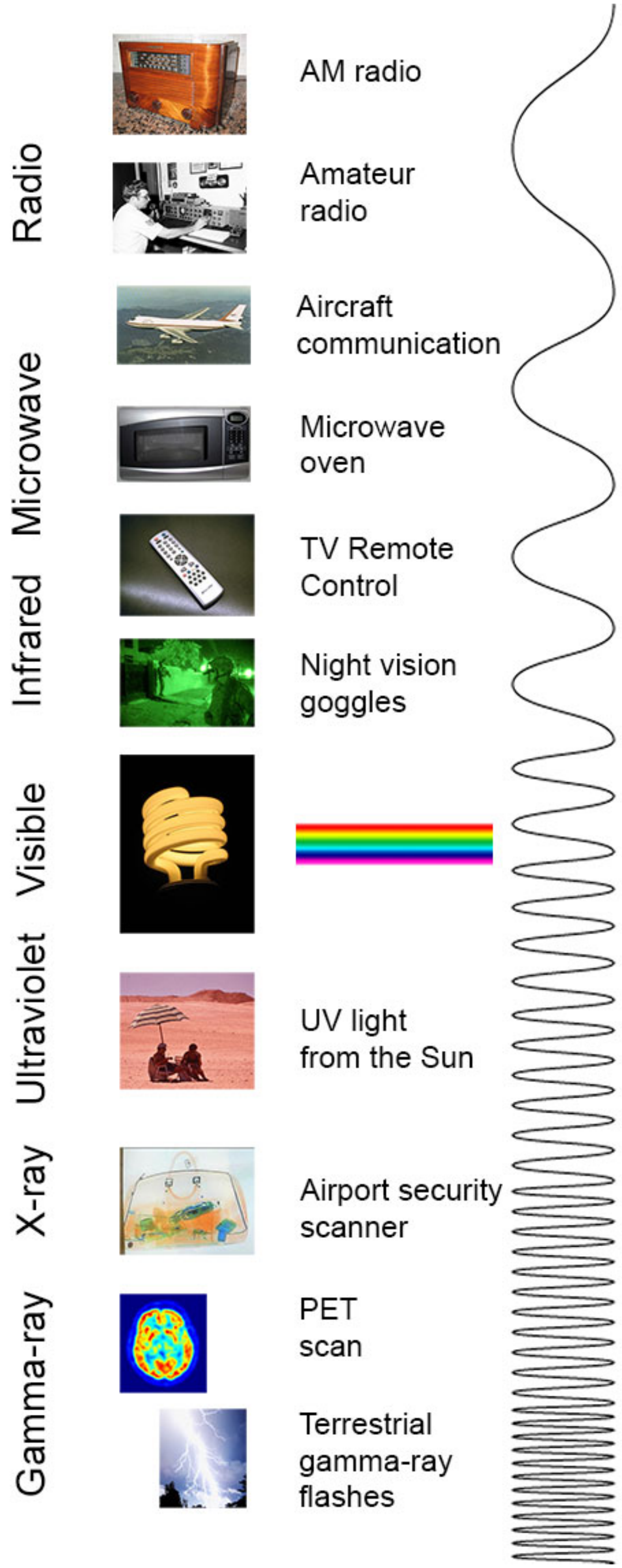
Ondas eletromagnéticas

Ondas planas

$$\vec{E} = \vec{E}_0 e^{i(kz-\omega t)}$$

$$\vec{B} = \vec{B}_0 e^{i(kz-\omega t)}$$

$$\vec{\nabla} \cdot \vec{\mathbf{E}} = 0$$



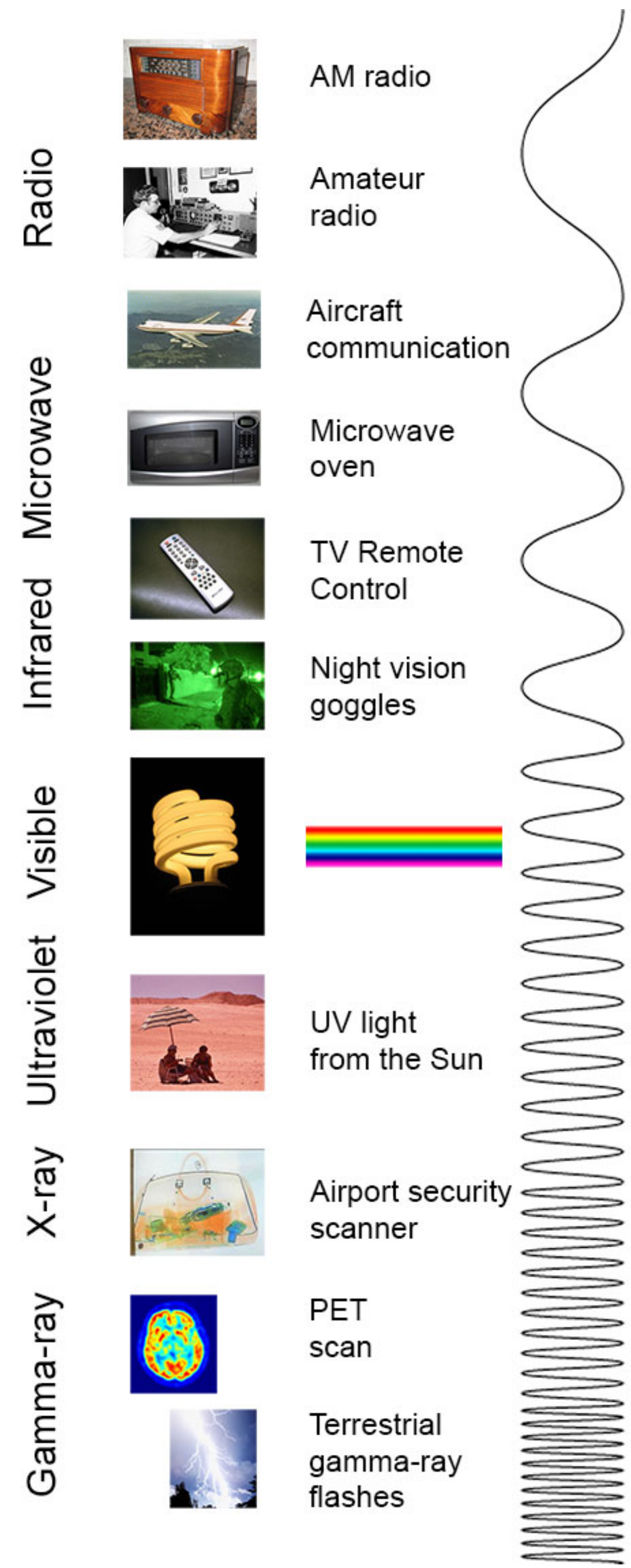
Ondas eletromagnéticas

Ondas planas

$$\vec{E} = \vec{E}_0 e^{i(kz-\omega t)}$$

$$\vec{B} = \vec{B}_0 e^{i(kz-\omega t)}$$

$$\boldsymbol{\vec{\nabla}} \cdot \mathbf{\vec{E}} = 0 \quad \Rightarrow \quad \frac{\mathrm{d}E_z}{\mathrm{d}z} = 0$$



Ondas eletromagnéticas

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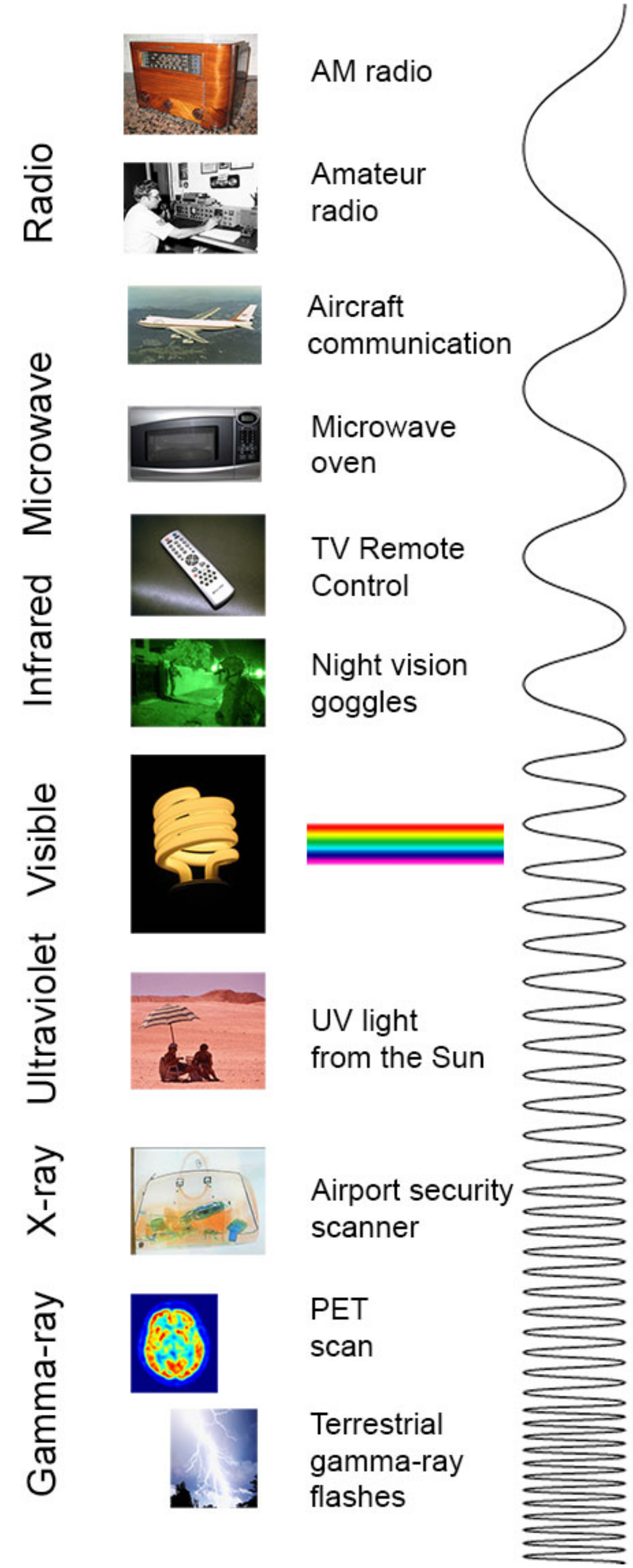
$$\vec{E}_0 = E_{0x}\hat{\mathbf{x}} + E_{0y}\hat{\mathbf{y}}$$



Ondas eletromagnéticas

Ondas planas

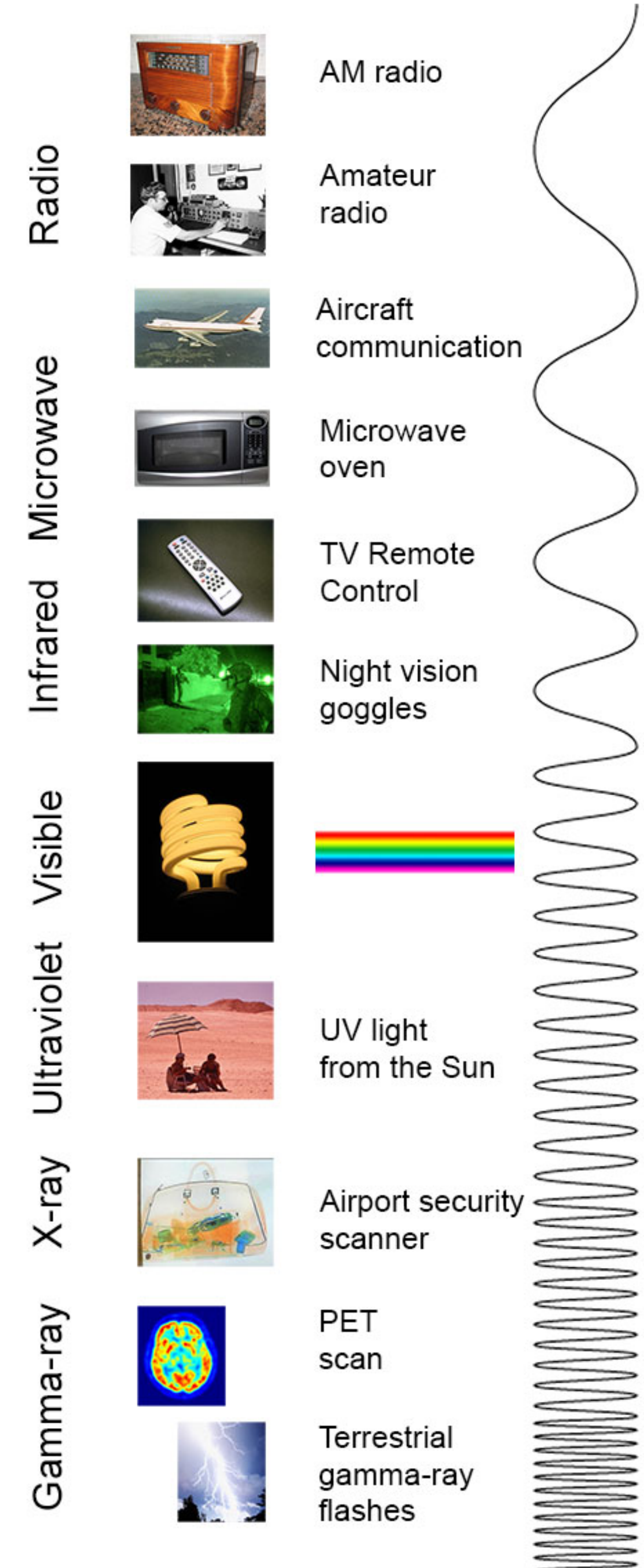
$$\vec{E} = \vec{E}_0 e^{i(kz-\omega t)}$$

$$\vec{B} = \vec{B}_0 e^{i(kz-\omega t)}$$

$$\vec{\nabla} \cdot \vec{\mathbf{E}} = 0 \quad \Rightarrow \quad \frac{dE_z}{dz} = 0$$

$$\vec{\nabla} \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$$

$$\vec{E}_0 = E_{0x}\hat{\mathbf{x}} + E_{0y}\hat{\mathbf{y}} \qquad \vec{B}_0 = B_{0x}\hat{\mathbf{x}} + B_{0y}\hat{\mathbf{y}}$$



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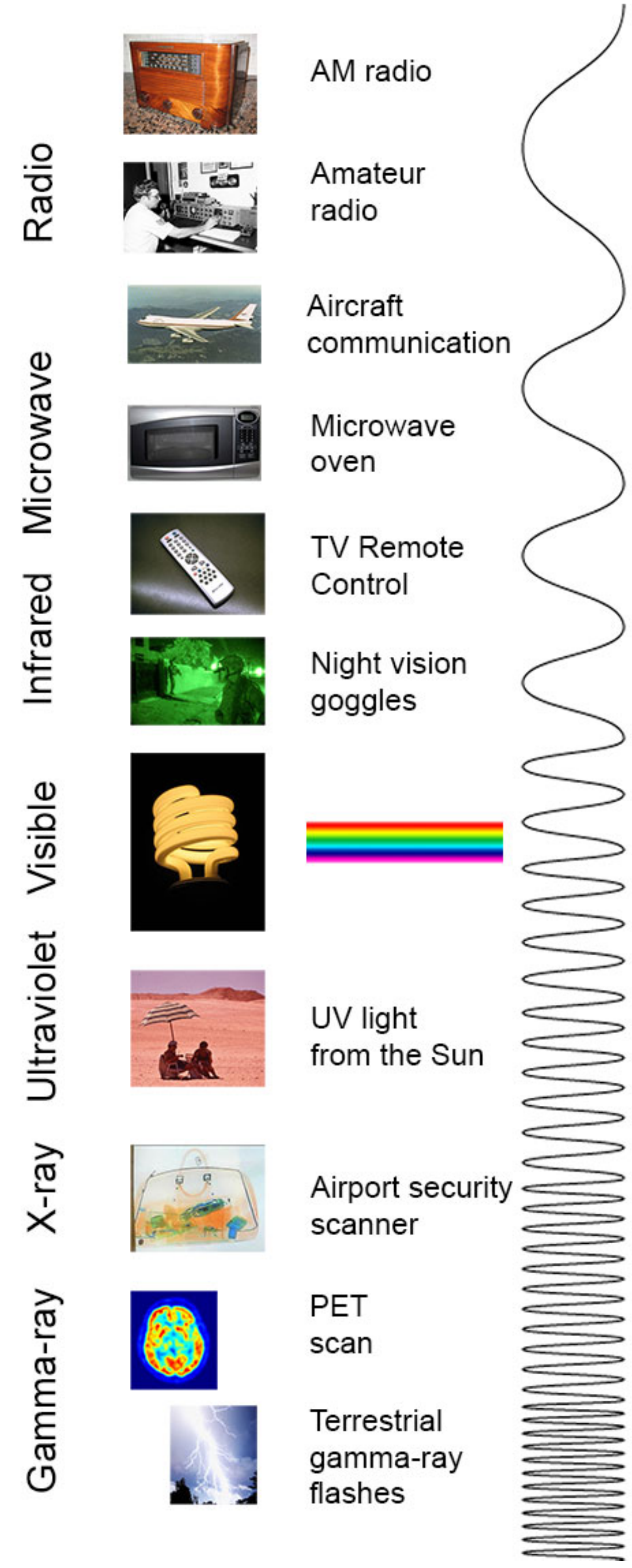

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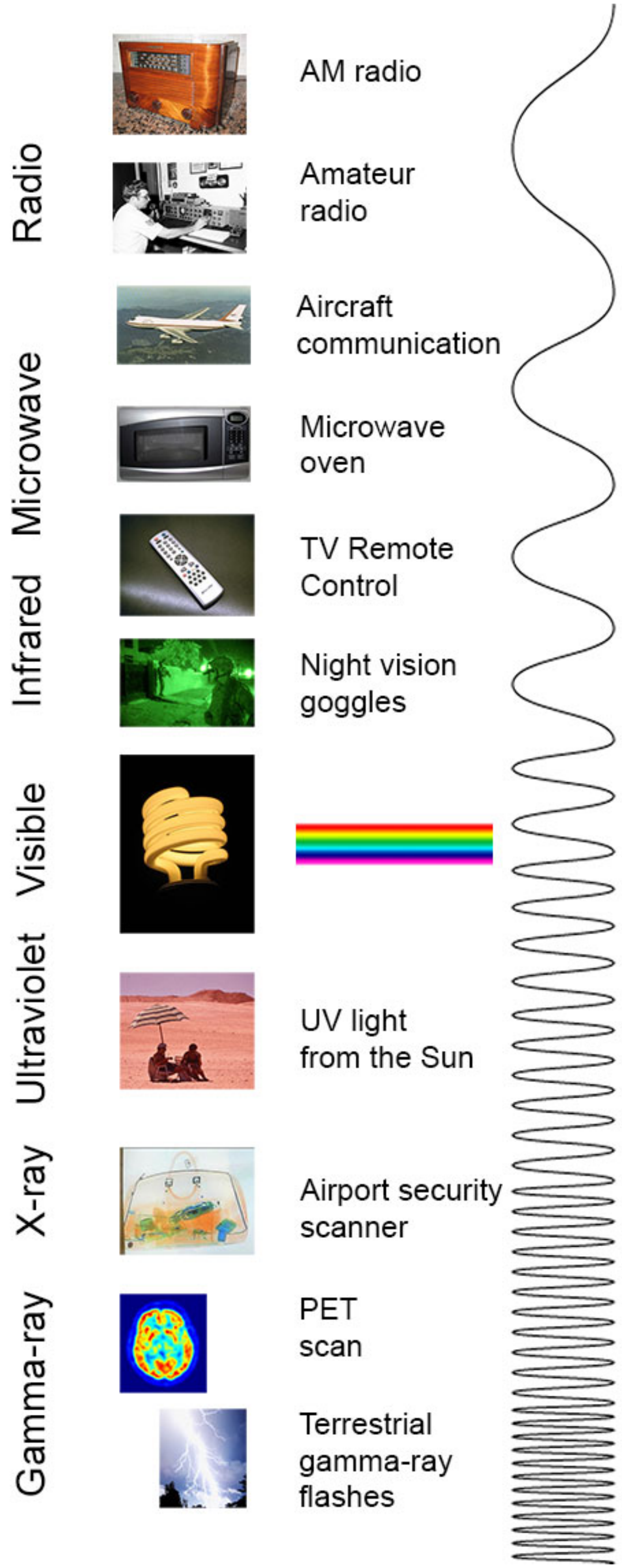
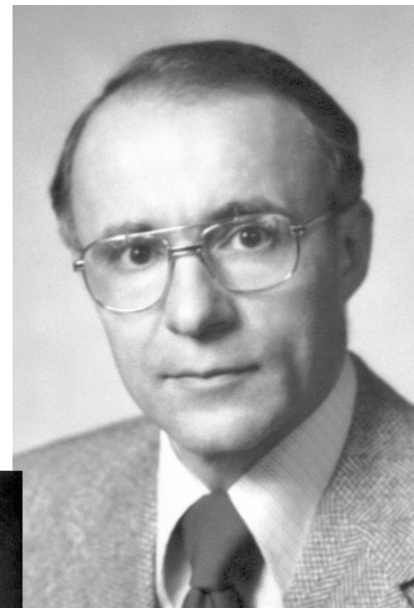


Radiação cósmica de fundo



Penzias

Wilson