

# Eletromagnetismo Avançado

1º ciclo  
Aula de 1º setembro

# Leis de conservação

## 1. Carga elétrica

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

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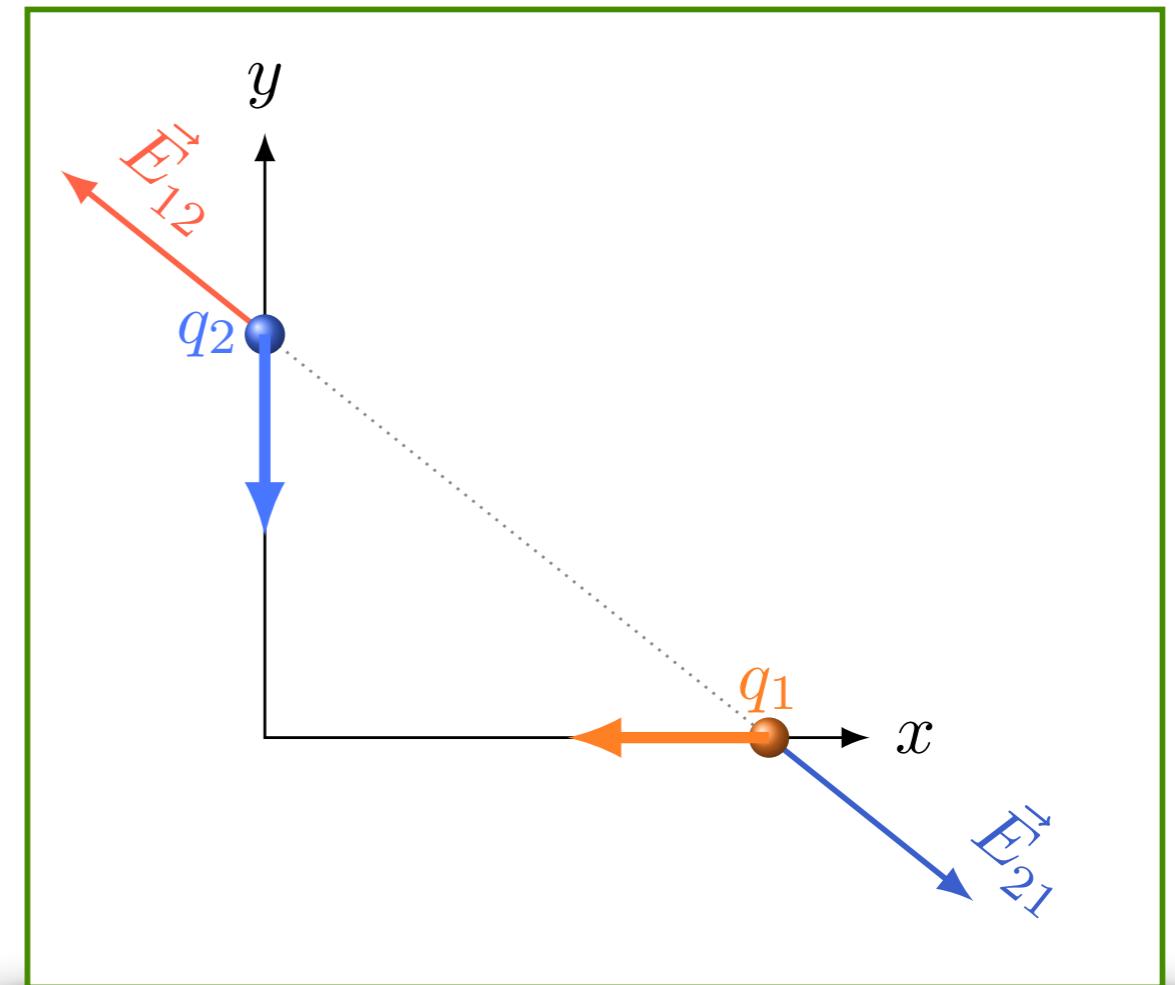
## 2. Energia

$$\frac{\partial}{\partial t} (u_{mec} + u_{em}) = -\vec{\nabla} \cdot \vec{S}$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

# Leis de conservação

## 3. Momento



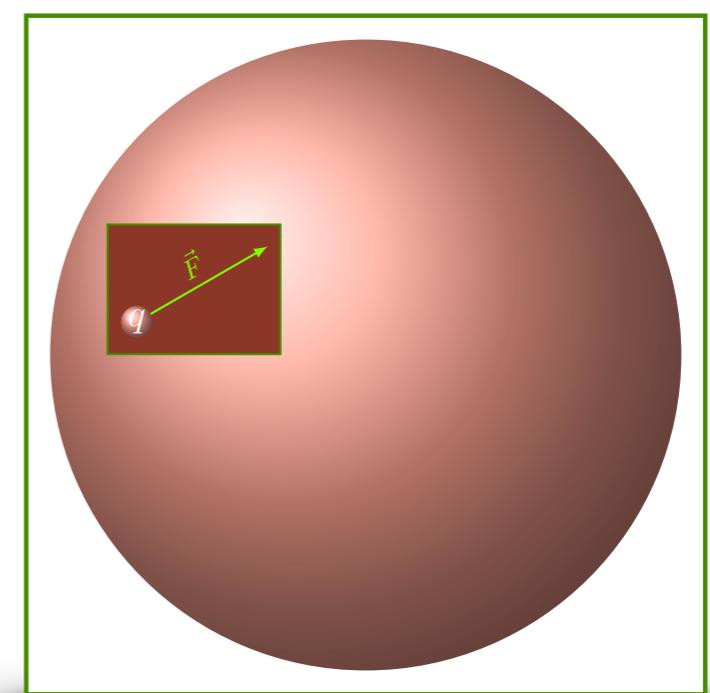
# Leis de conservação

## 3. Momento

$$\vec{f} = \epsilon_0 [(\vec{\nabla} \cdot \vec{E}) \vec{E} + (\vec{E} \cdot \vec{\nabla}) \vec{E}] + \frac{1}{\mu_0} [(\vec{\nabla} \cdot \vec{B}) \vec{B} + (\vec{B} \cdot \vec{\nabla}) \vec{B}] - \frac{1}{2} \vec{\nabla} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) - \epsilon_0 \frac{\partial}{\partial t} (\vec{E} \times \vec{B})$$

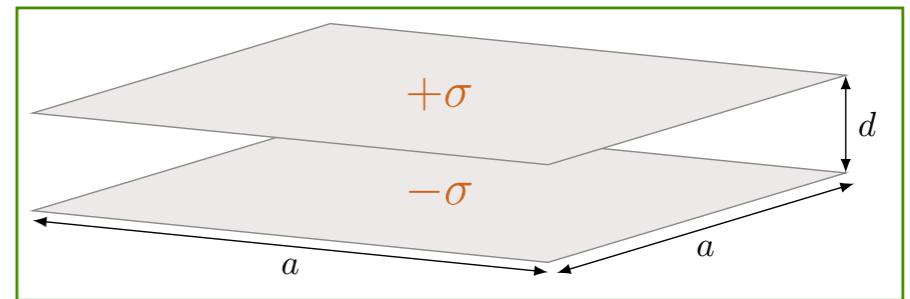
$$T_{ij} = \epsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2} \delta_{ij} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right)$$

$$F = \int_{\mathcal{A}} \mathbb{T} \cdot \hat{\mathbf{n}} \, da - \epsilon_0 \mu_0 \frac{d}{dt} \int_{\mathcal{V}} \vec{S} \, d\tau$$



# Pratique o que aprendeu

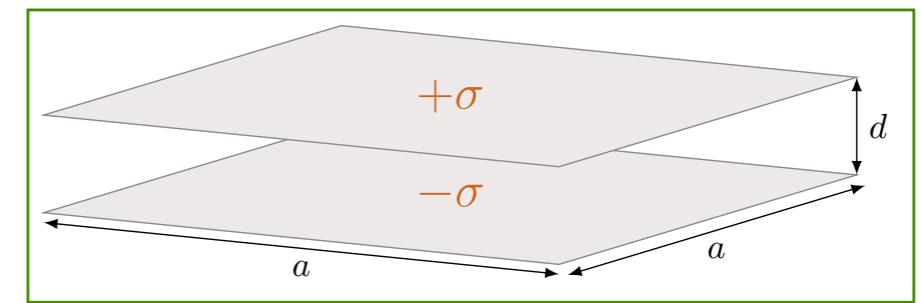
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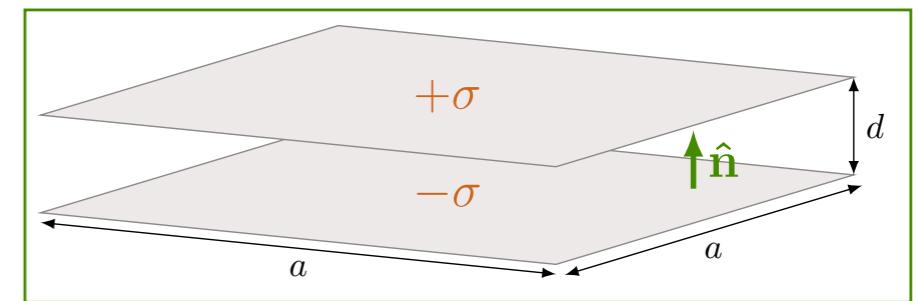
$$\mathbb{T} = \epsilon_0 \begin{bmatrix} -\frac{1}{2} \left( \frac{\sigma}{\epsilon_0} \right)^2 & 0 & 0 \\ 0 & -\frac{1}{2} \left( \frac{\sigma}{\epsilon_0} \right)^2 & 0 \\ 0 & 0 & \frac{1}{2} \left( \frac{\sigma}{\epsilon_0} \right)^2 \end{bmatrix}$$



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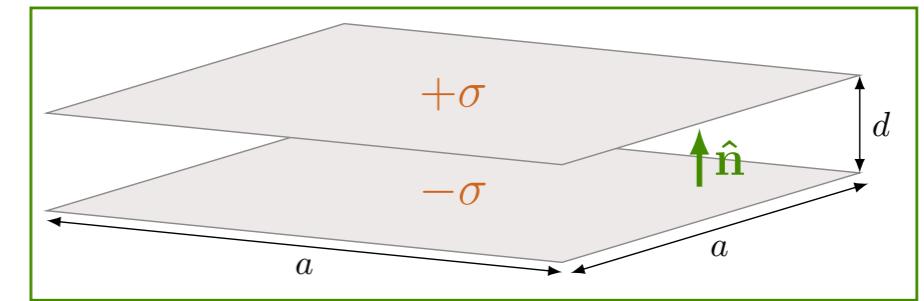


$$\mathbb{T} \cdot \hat{\mathbf{n}} = \epsilon_0 \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{2} \left( \frac{\sigma}{\epsilon_0} \right)^2 & \frac{1}{2} \left( \frac{\sigma}{\epsilon_0} \right)^2 \\ 0 & 0 & \frac{1}{2} \left( \frac{\sigma}{\epsilon_0} \right)^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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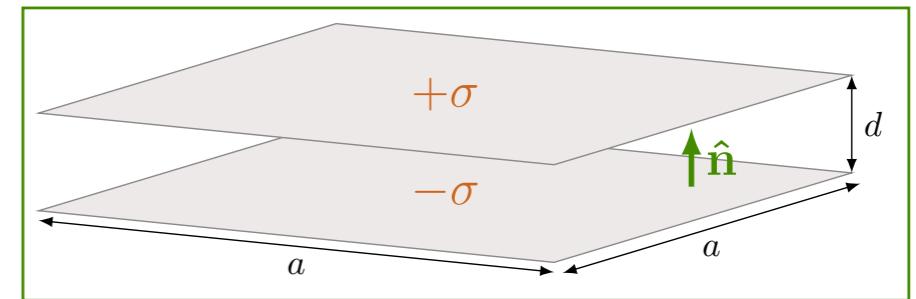


$$\mathbb{T} \cdot \hat{\mathbf{n}} = \epsilon_0 \begin{bmatrix} -\frac{1}{2} \left( \frac{\sigma}{\epsilon_0} \right)^2 & 0 & 0 \\ 0 & -\frac{1}{2} \left( \frac{\sigma}{\epsilon_0} \right)^2 & 0 \\ 0 & 0 & \frac{1}{2} \left( \frac{\sigma}{\epsilon_0} \right)^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{\sigma^2}{2\epsilon_0} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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$$\vec{F} = \frac{\sigma}{2\epsilon_0} \hat{z}$$

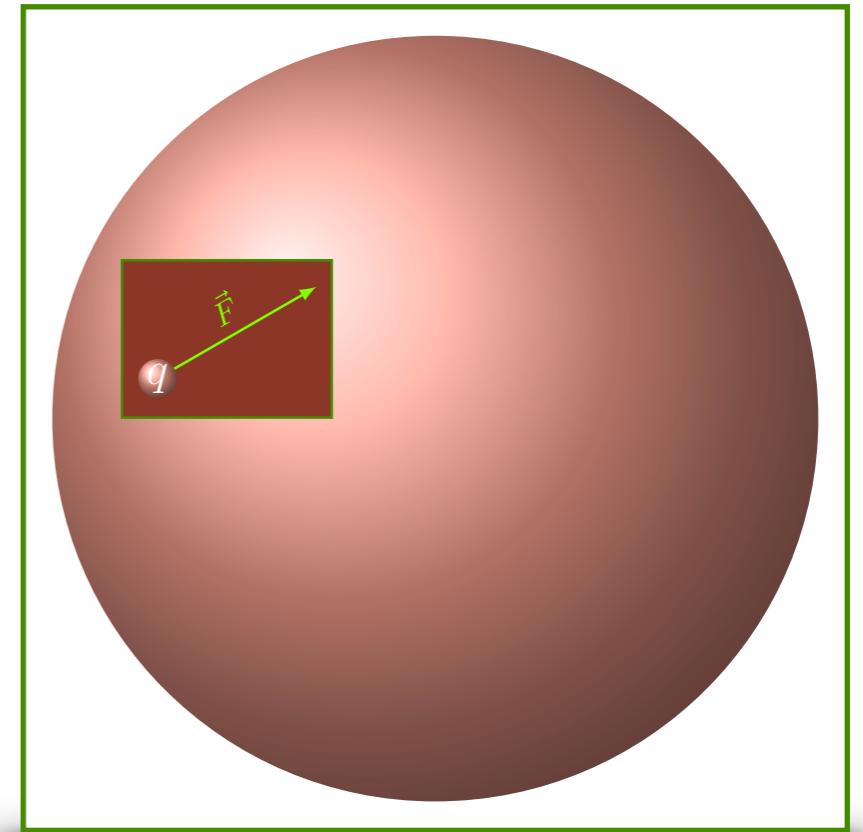


$$\mathbb{T} \cdot \hat{\mathbf{n}} = \epsilon_0 \begin{bmatrix} -\frac{1}{2} \left( \frac{\sigma}{\epsilon_0} \right)^2 & 0 & 0 \\ 0 & -\frac{1}{2} \left( \frac{\sigma}{\epsilon_0} \right)^2 & 0 \\ 0 & 0 & \frac{1}{2} \left( \frac{\sigma}{\epsilon_0} \right)^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{\sigma^2}{2\epsilon_0} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

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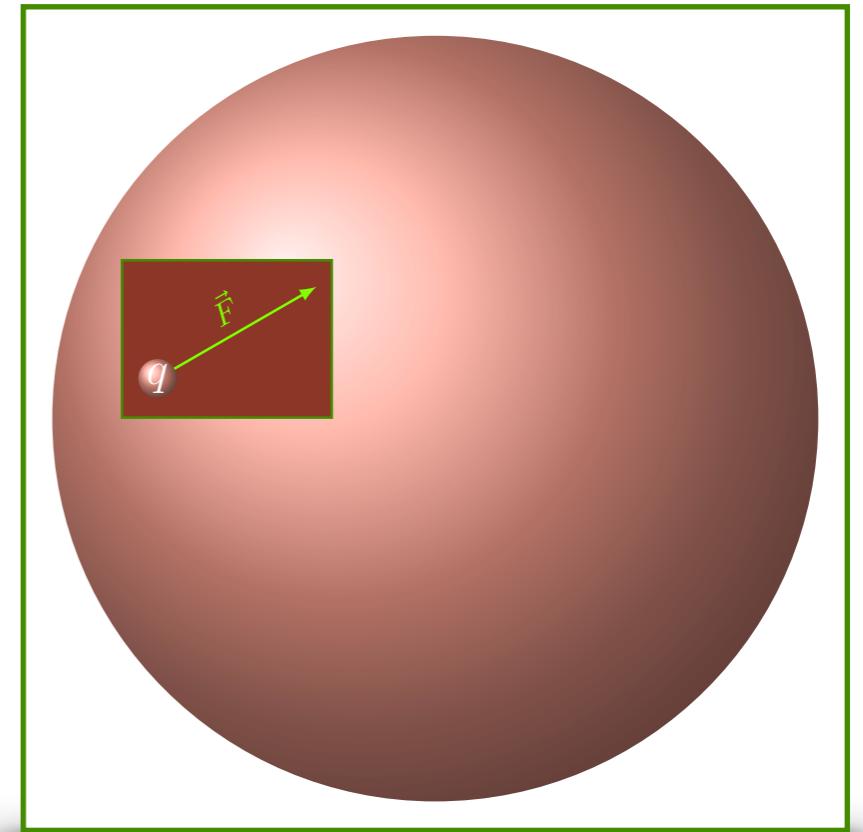


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$$\frac{d\vec{p}_{mec}}{dt} + \frac{d}{dt} \left( \epsilon_0 \mu_0 \int \vec{S} \, d\tau \right) = \int \mathbb{T} \cdot \hat{n} \, da$$

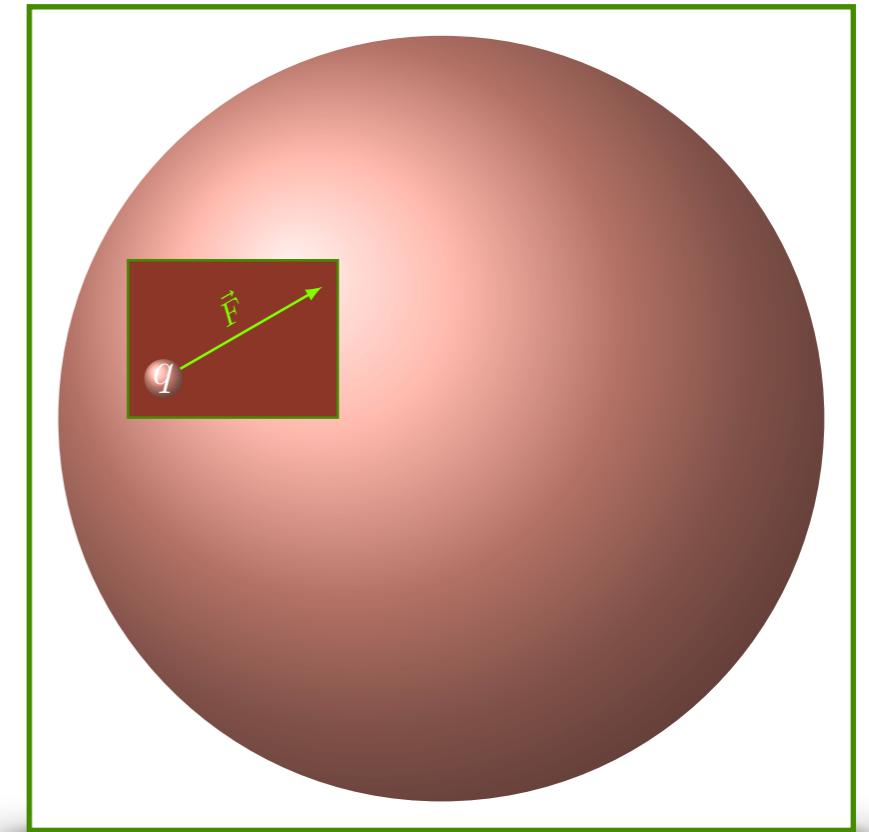


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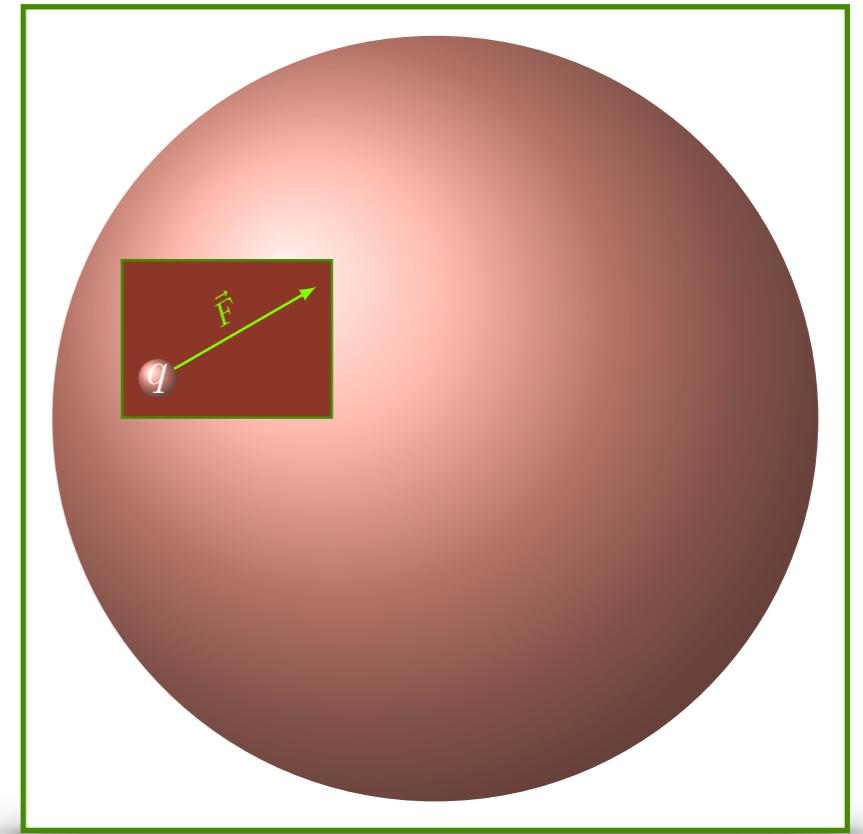
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$$\frac{d}{dt} (\vec{\mathbf{p}}_{mec} + \vec{\mathbf{p}}_{em}) = \int \mathbb{T} \cdot \hat{n} \, da$$



# Leis de conservação

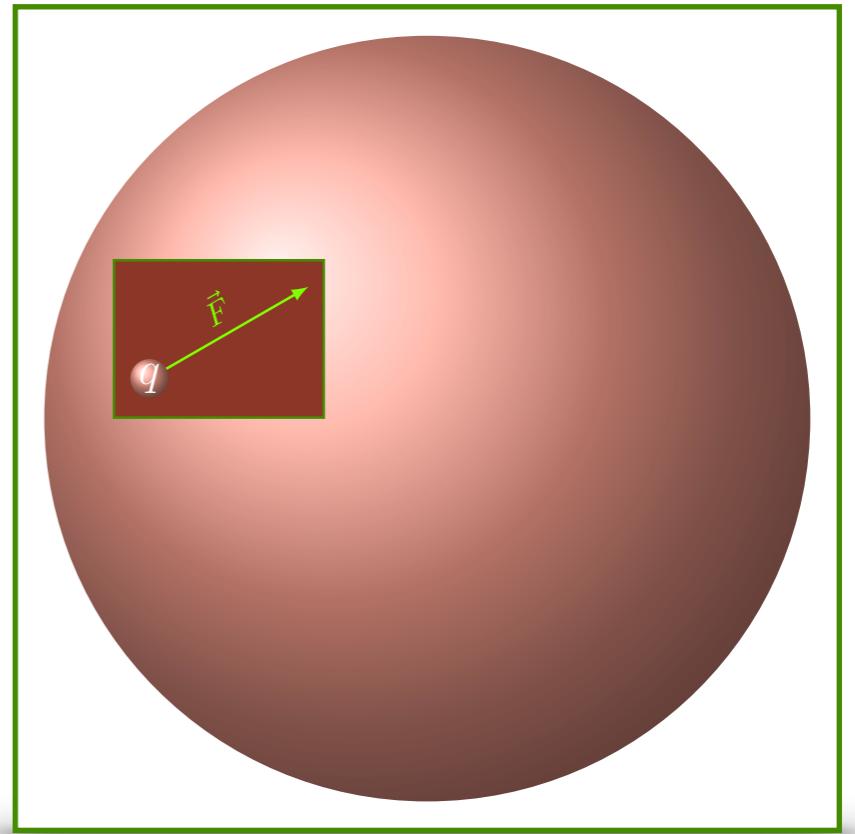
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$$\vec{\mathbf{p}}_{em} = \epsilon_0 \mu_0 \int \vec{S} \, d\tau$$



# Eletromagnetismo Avançado

Propagação de ondas em uma dimensão

# Ondas eletromagnéticas

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

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$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t} \left( \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

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$$\nabla^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

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$$\nabla^2 \phi = \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2}$$

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$$\nabla^2 \phi = \frac{1}{v^2} \frac{\partial^2 \phi}{\partial t^2}$$

$$\phi_{\vec{\mathbf{k}}}(\vec{\mathbf{r}}, t) = f(\vec{\mathbf{k}} \cdot \vec{\mathbf{r}} - \omega t) \quad (\omega = kv)$$

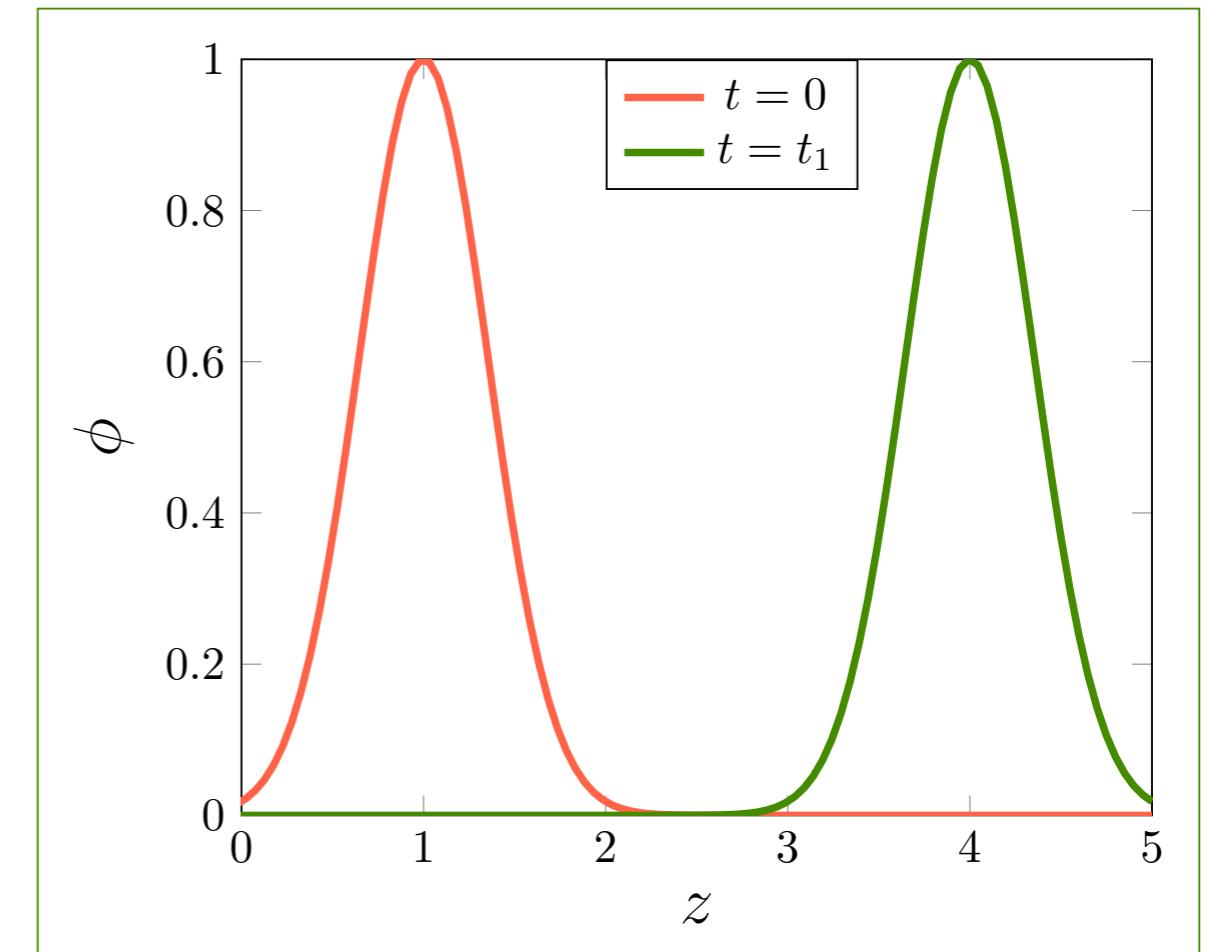
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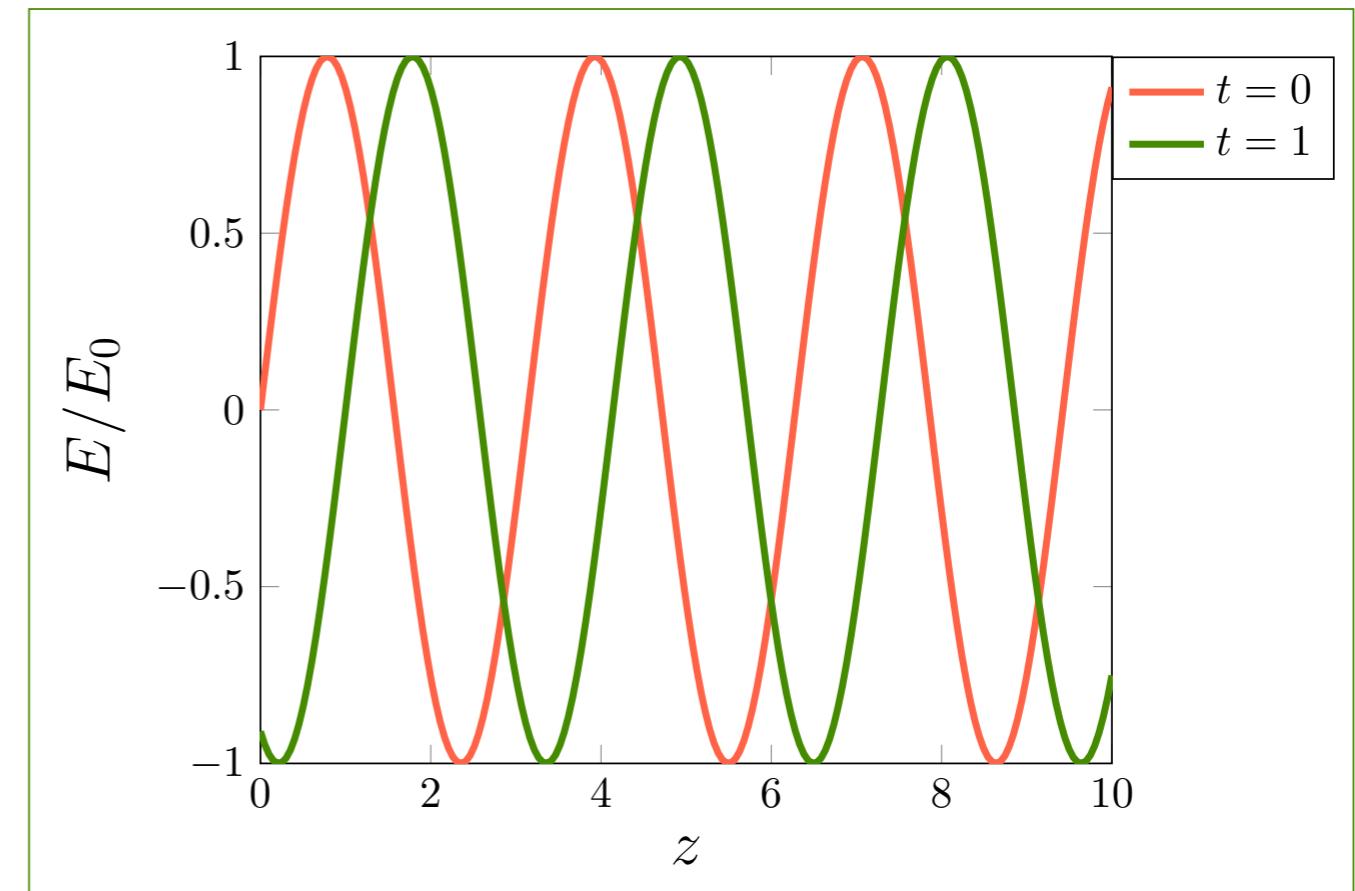
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Ondas planas

$$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$$

$$\vec{B} = \vec{B}_0 e^{i(kz - \omega t)}$$

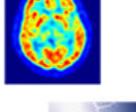


# Ondas eletromagnéticas

Ondas planas

$$\vec{E} = \vec{E}_0 e^{i(kz - \omega t)}$$

$$\vec{B} = \vec{B}_0 e^{i(kz - \omega t)}$$

Radio		AM radio
		Amateur radio
		Aircraft communication
		Microwave oven
Infrared		TV Remote Control
		Night vision goggles
Visible		
Ultraviolet		UV light from the Sun
X-ray		Airport security scanner
Gamma-ray		PET scan
		Terrestrial gamma-ray flashes

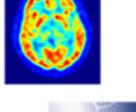
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$$\vec{B} = \vec{B}_0 e^{i(kz - \omega t)}$$

$$\vec{\nabla} \cdot \vec{E} = 0$$

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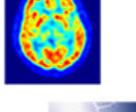
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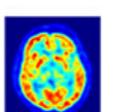
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$$\vec{E}_0 = E_{0x} \hat{x} + E_{0y} \hat{y}$$

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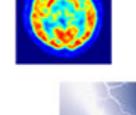
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Infrared		
		
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Gamma-ray		
X-ray		

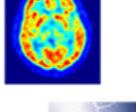
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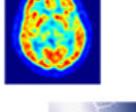
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# Radiação cósmica de fundo



Wilson



Penzias

