

Eletrromagnetismo Avançado

1º ciclo

Aula de 20 de agosto

Leis de conservação

1. Carga elétrica

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

Leis de conservação

1. Carga elétrica

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

2. Energia

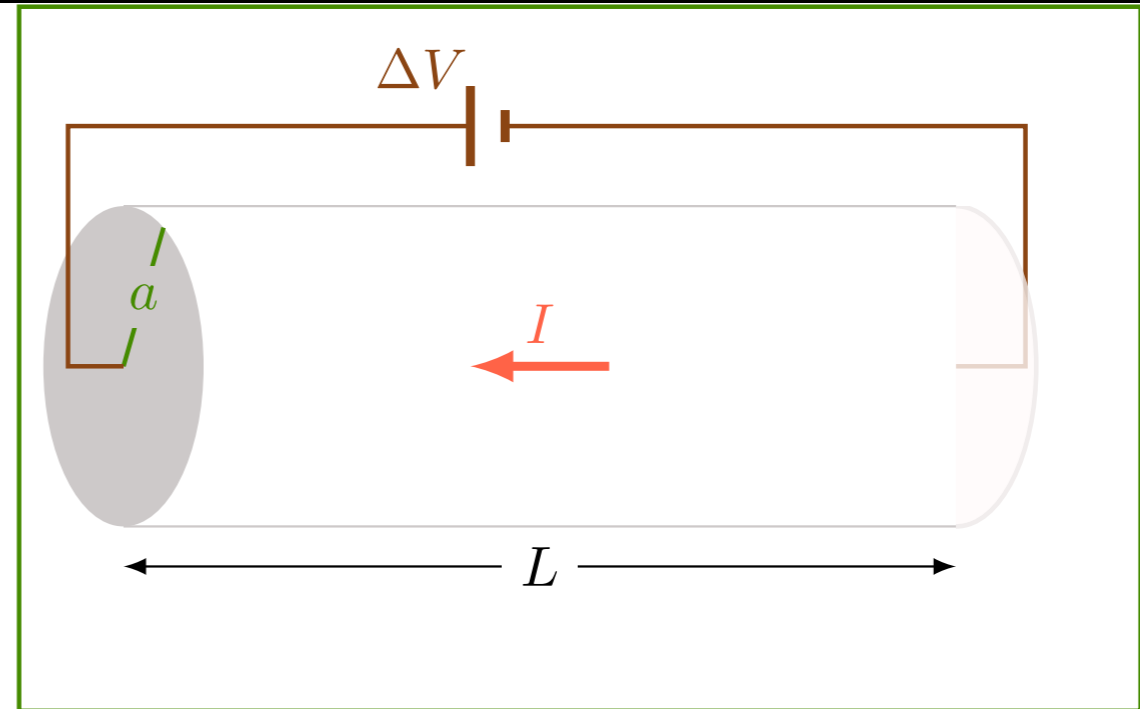
$$\frac{\partial}{\partial t} (u_{mec} + u_{em}) = -\vec{\nabla} \cdot \vec{S}$$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

Pratique o que aprendeu

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

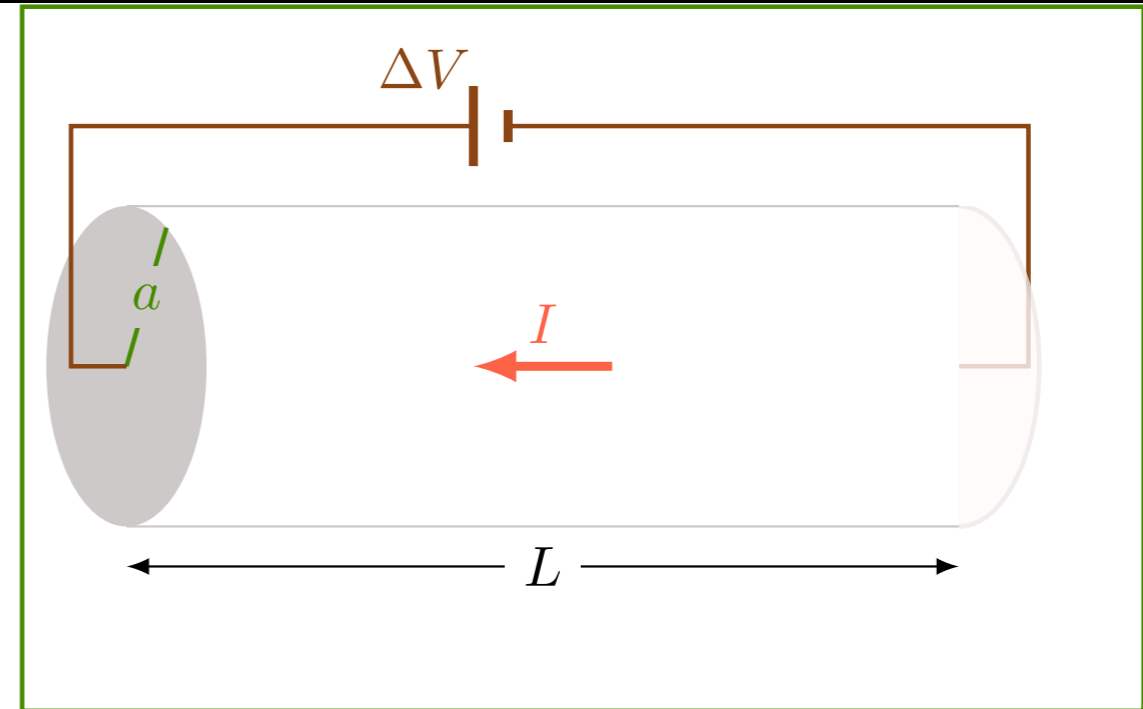
$$\vec{S} = ?$$



Pratique o que aprendeu

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$E = \frac{V}{L}$$

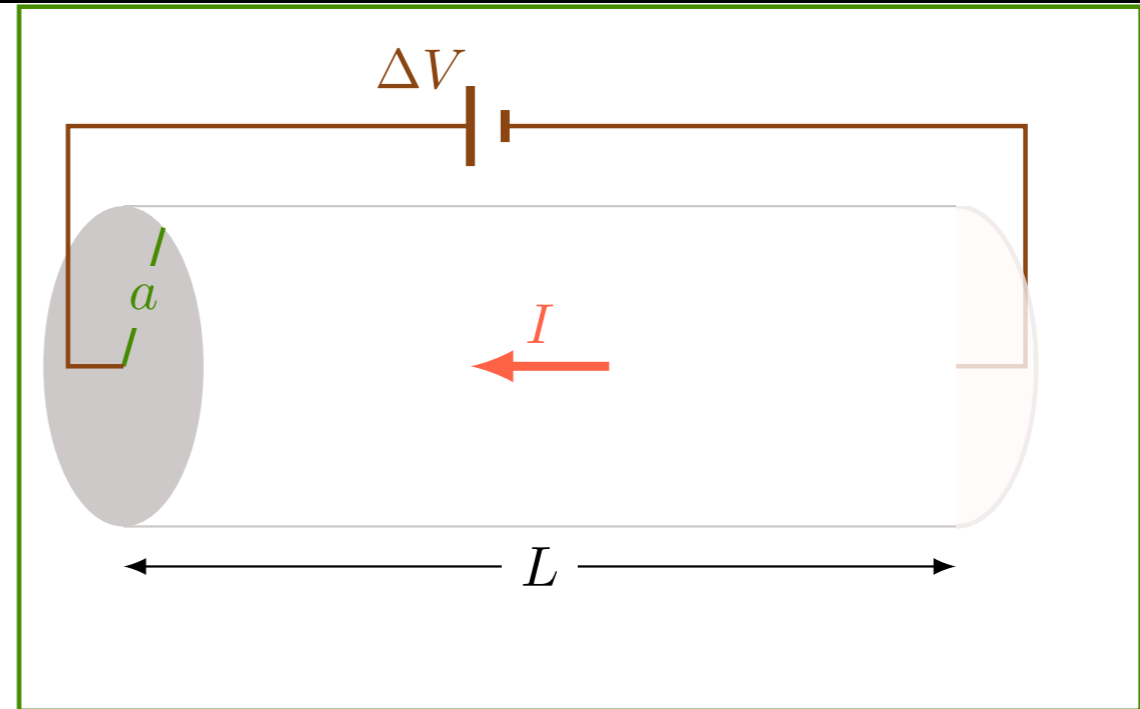


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$$\vec{E} = \frac{V}{L} \hat{z}$$



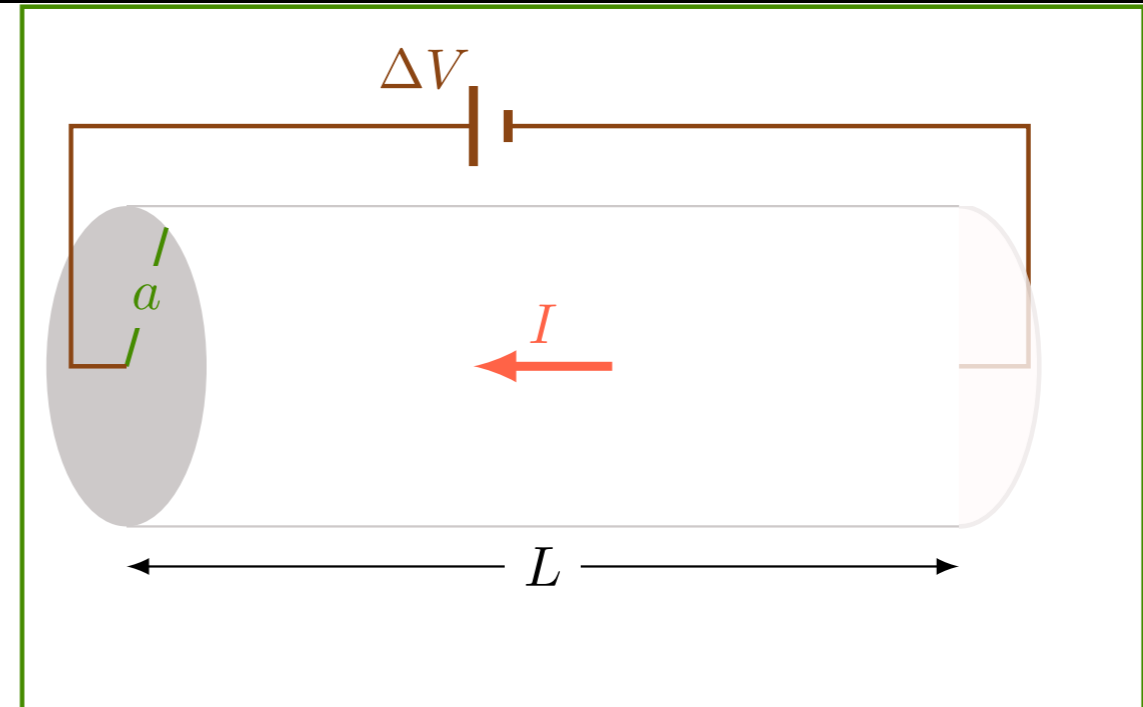
Pratique o que aprendeu

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$$B = \frac{\mu_0}{2\pi a} I$$



Pratique o que aprendeu

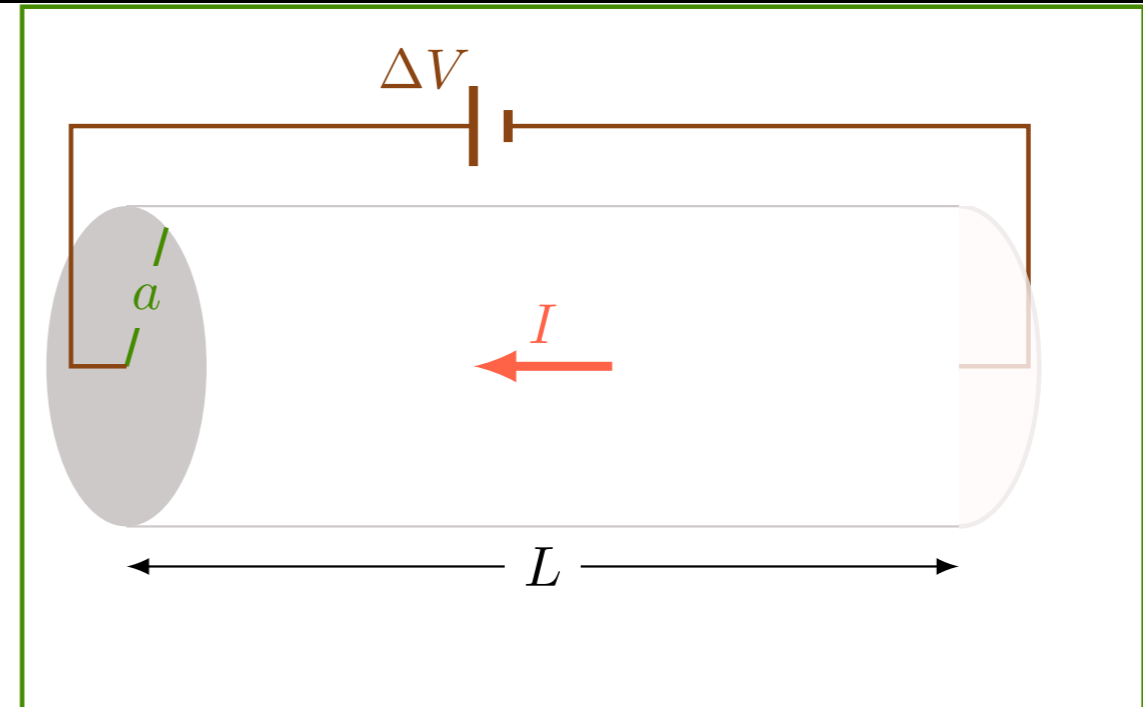
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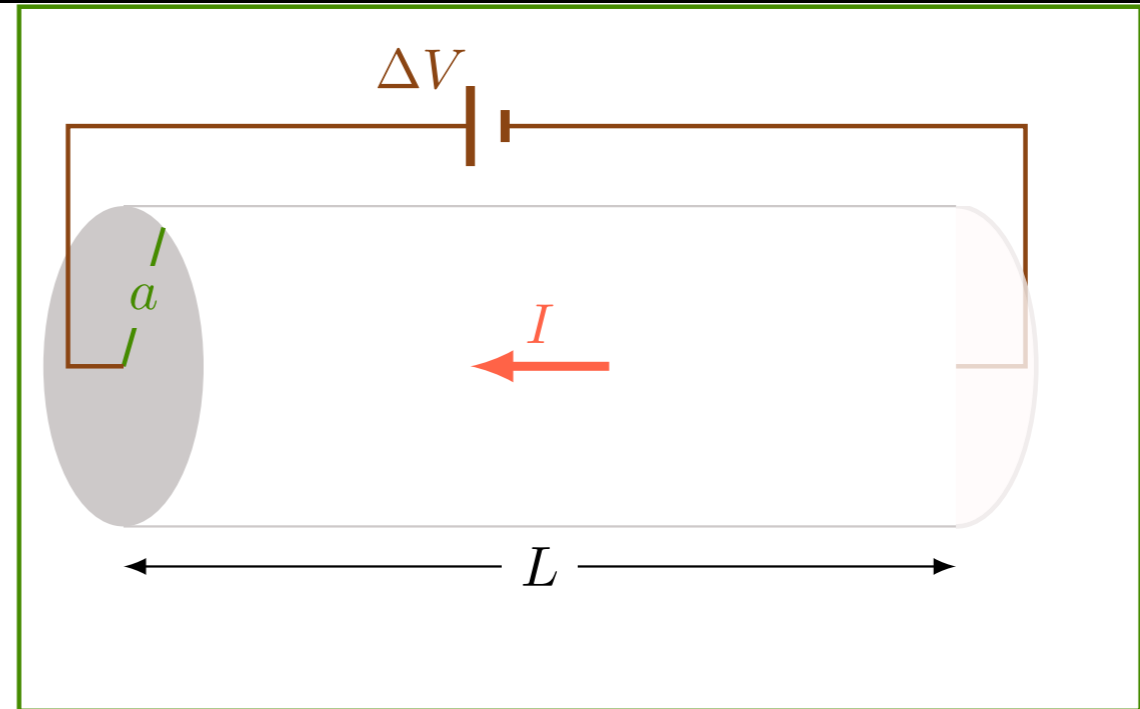
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$$\vec{S} = \frac{1}{\mu_0} \frac{V}{L} \frac{\mu_0}{2\pi a} I \hat{z} \times \hat{\phi}$$



$$\vec{S} = - \frac{VI}{\text{Área}} \hat{s}$$

Pratique o que aprendeu

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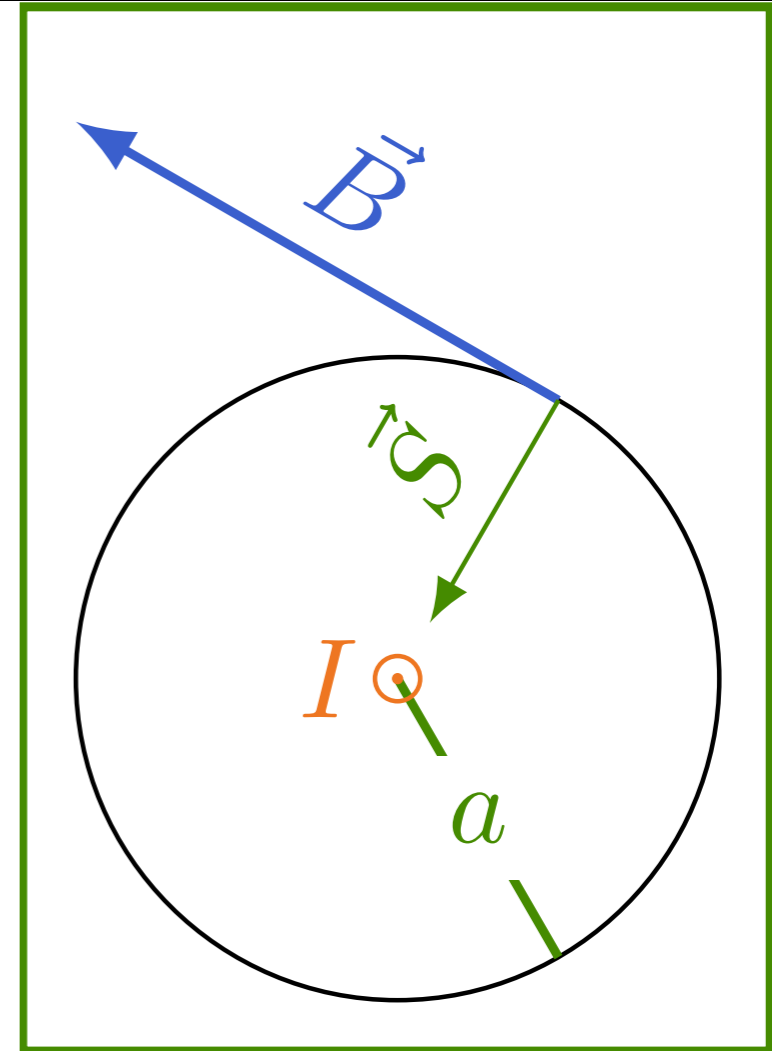
$$\vec{E} = \frac{V}{L} \hat{z}$$

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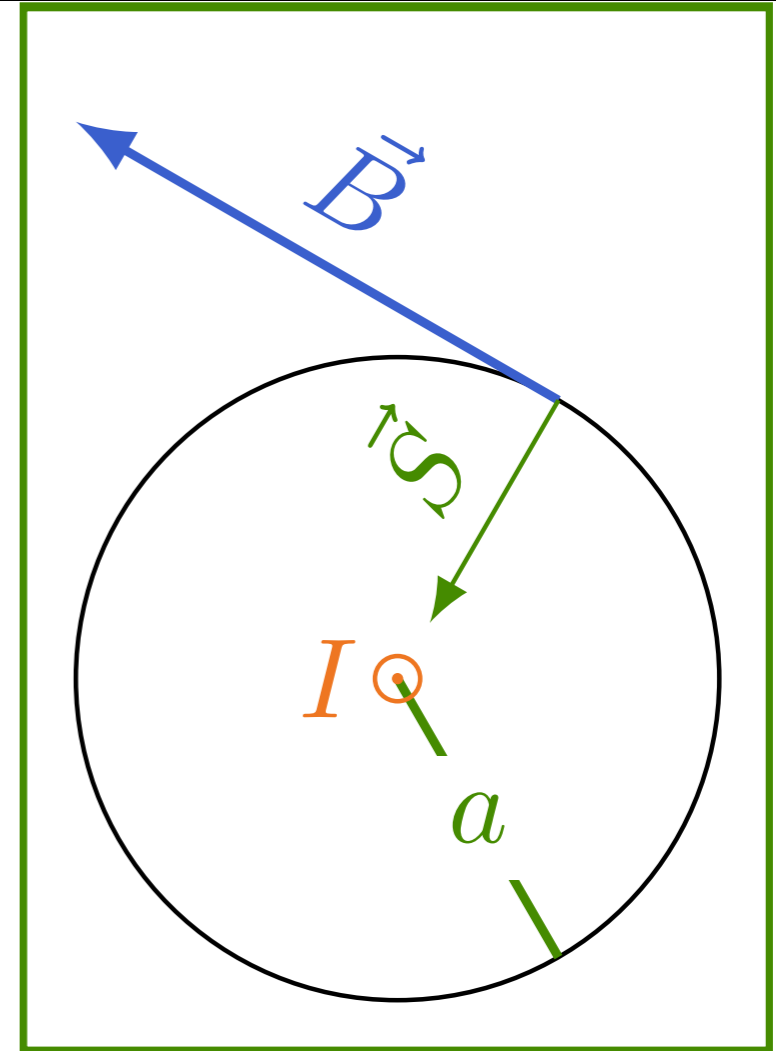


Pratique o que aprendeu

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$\int_A \vec{S} \cdot \hat{n} = VI$$

Um pouco de física

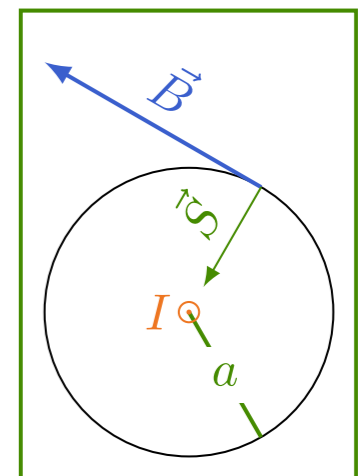
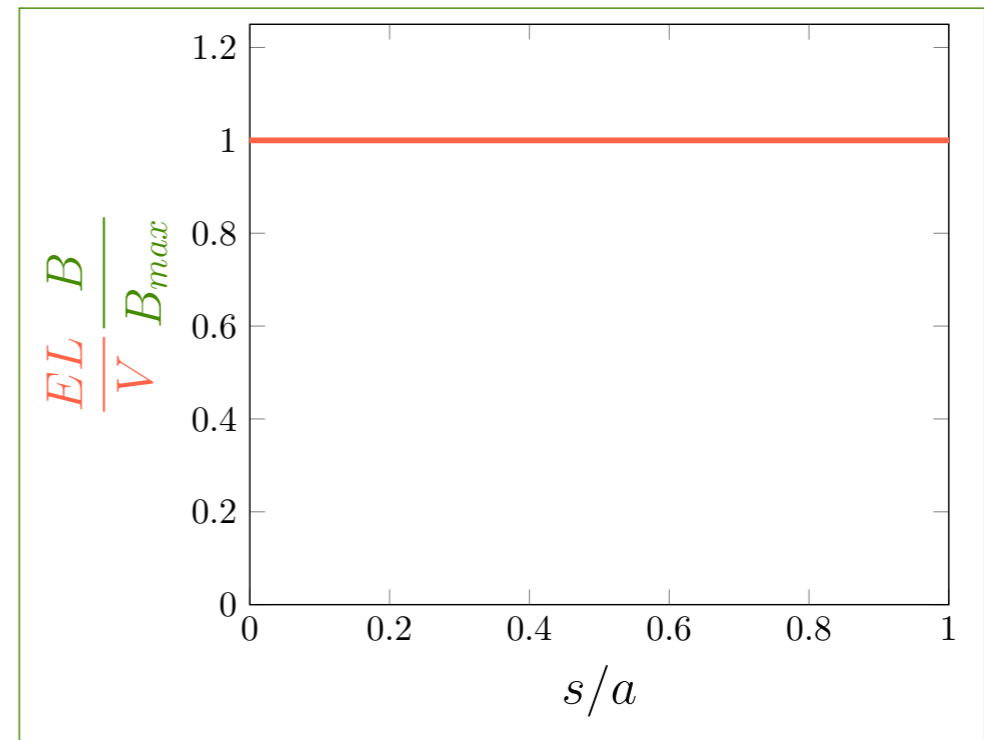


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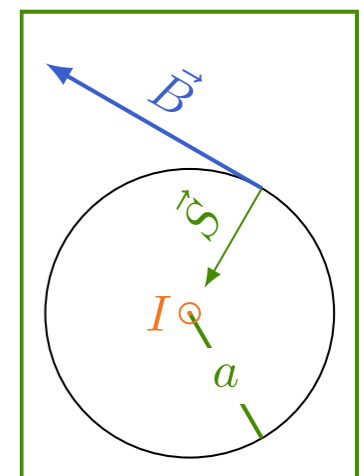
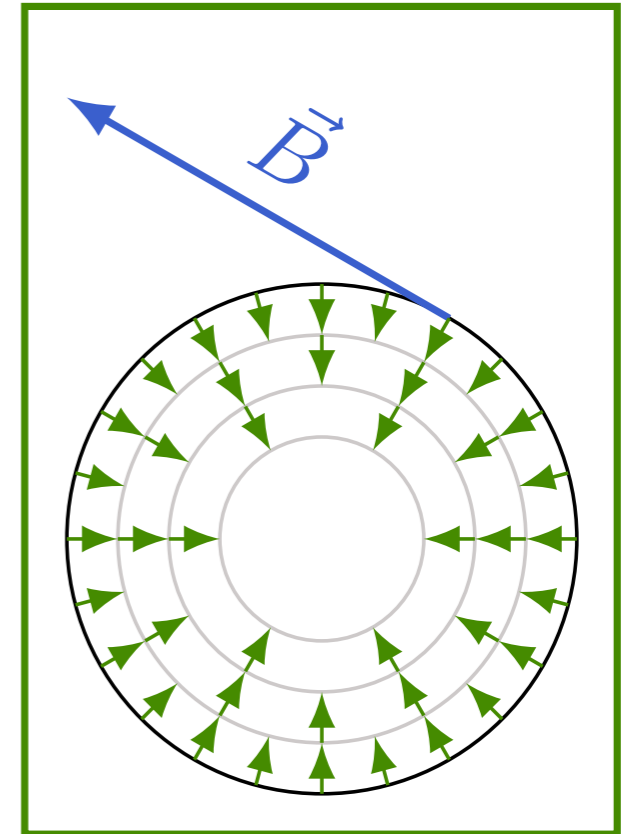


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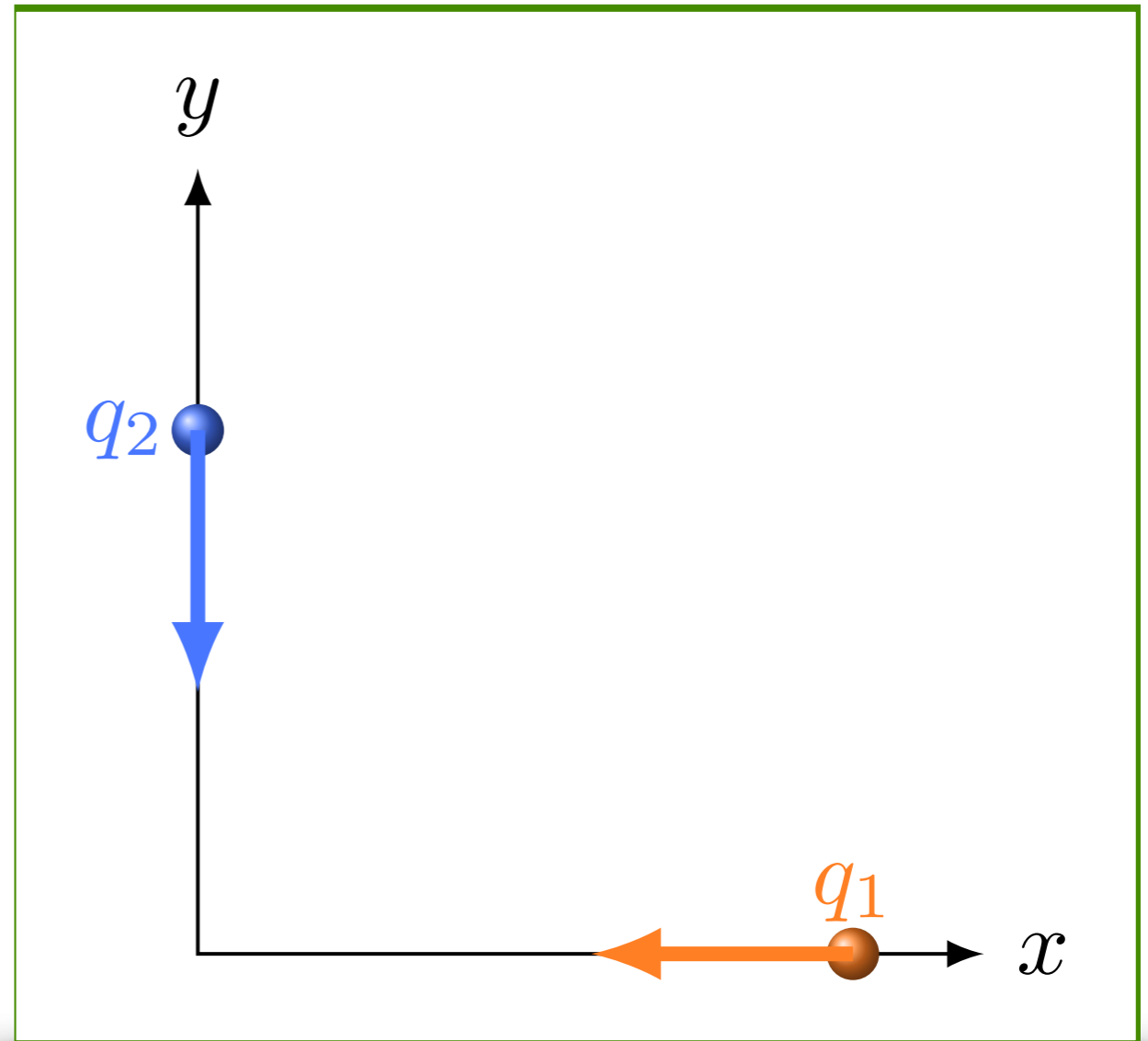
Um pouco de física



Leis de conservação

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

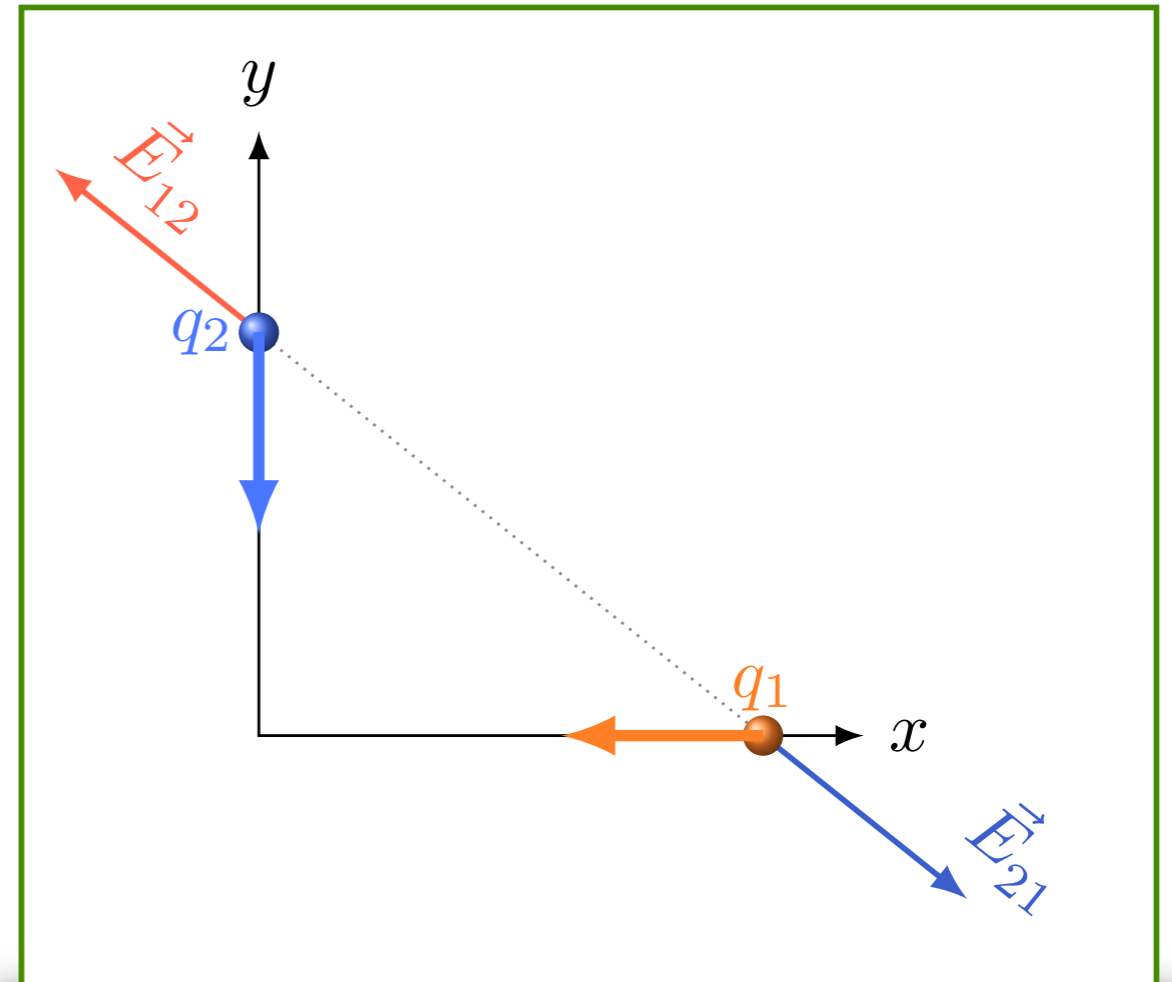
3. Momento



Leis de conservação

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

3. Momento

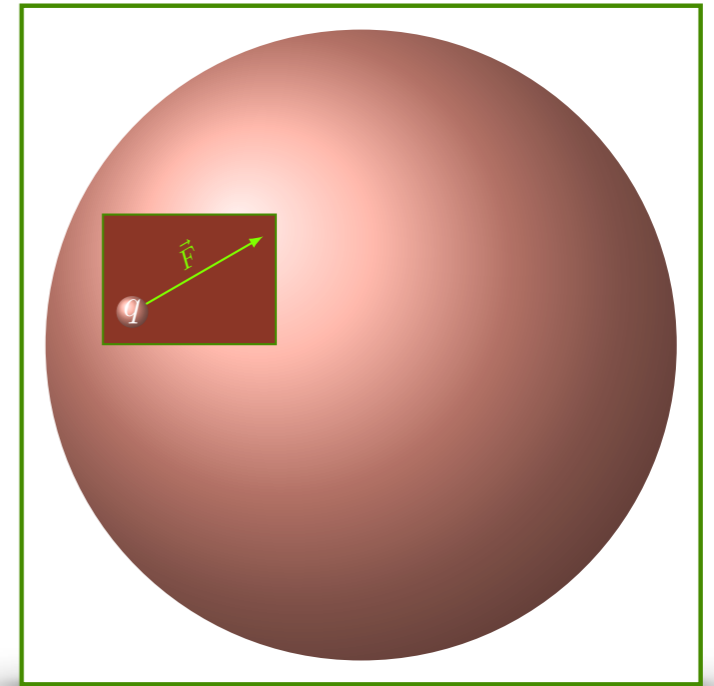


Leis de conservação

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

3. Momento

$$\vec{F} = \int_V \rho \vec{E} + \vec{J} \times \vec{B} \, d\tau$$



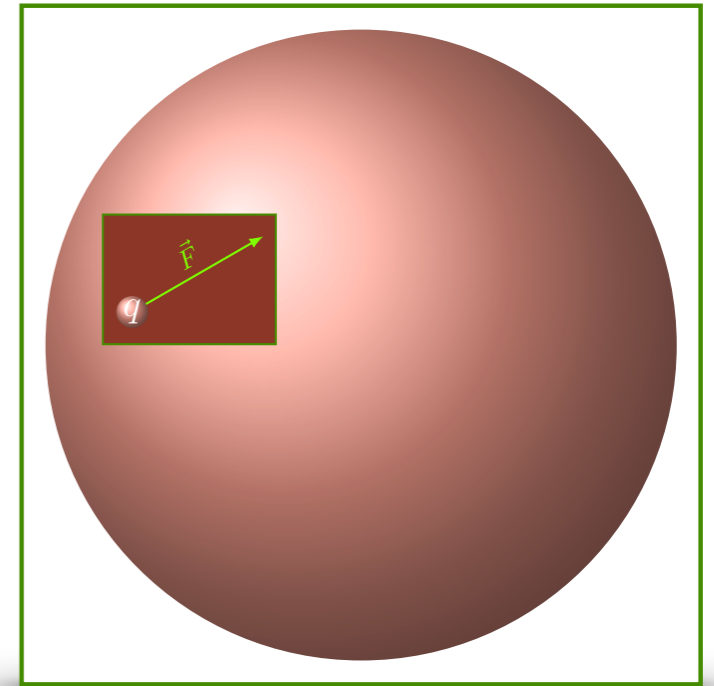
Leis de conservação

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3. Momento

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Leis de conservação

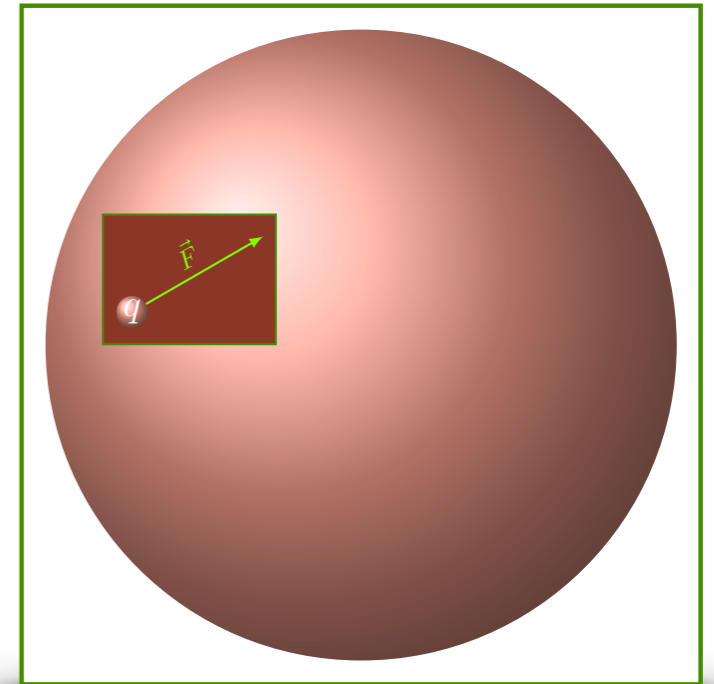
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$$\vec{f} = \rho \vec{E} + \vec{J} \times \vec{B}$$

$$\rho = \epsilon_0 \vec{\nabla} \cdot \vec{E}$$



Leis de conservação

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

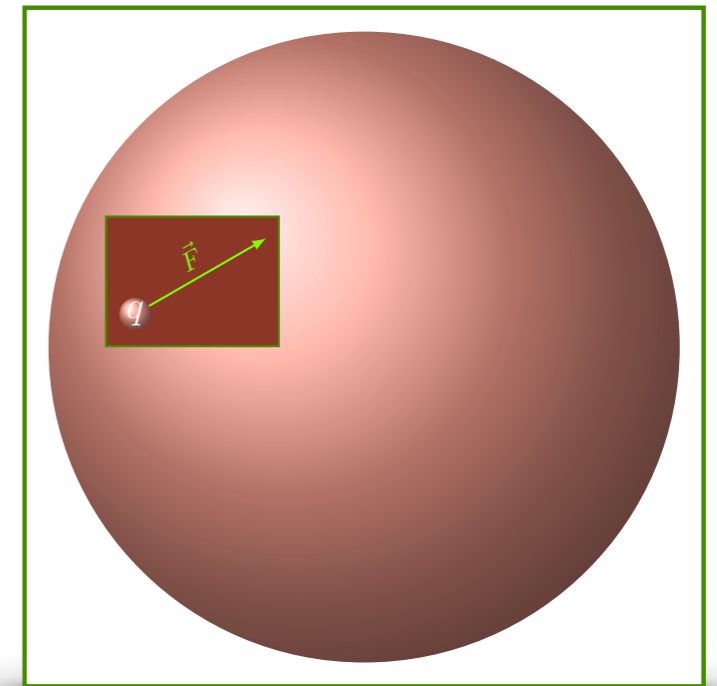
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$$\vec{f} = \rho \vec{E} + \vec{J} \times \vec{B}$$

$$\rho = \epsilon_0 \vec{\nabla} \cdot \vec{E}$$

$$\vec{J} = \frac{1}{\mu_0} \vec{\nabla} \times \vec{B} - \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$



$$\frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} = ?$$

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$$\frac{\partial}{\partial t} (\vec{\mathbf{E}} \times \vec{\mathbf{B}}) = \frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} + \vec{\mathbf{E}} \times \frac{\partial \vec{\mathbf{B}}}{\partial t}$$

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$$\vec{\mathbf{E}} \times \vec{\nabla} \times \vec{\mathbf{E}} = \frac{1}{2} \vec{\nabla} E^2 - (\vec{\mathbf{E}} \cdot \vec{\nabla}) \vec{\mathbf{E}}$$

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$$\vec{\mathbf{B}} \times (\vec{\nabla} \times \vec{\mathbf{B}}) = \frac{1}{2} \vec{\nabla} B^2 - (\vec{\mathbf{B}} \cdot \vec{\nabla}) \vec{\mathbf{B}}$$

Leis de conservação

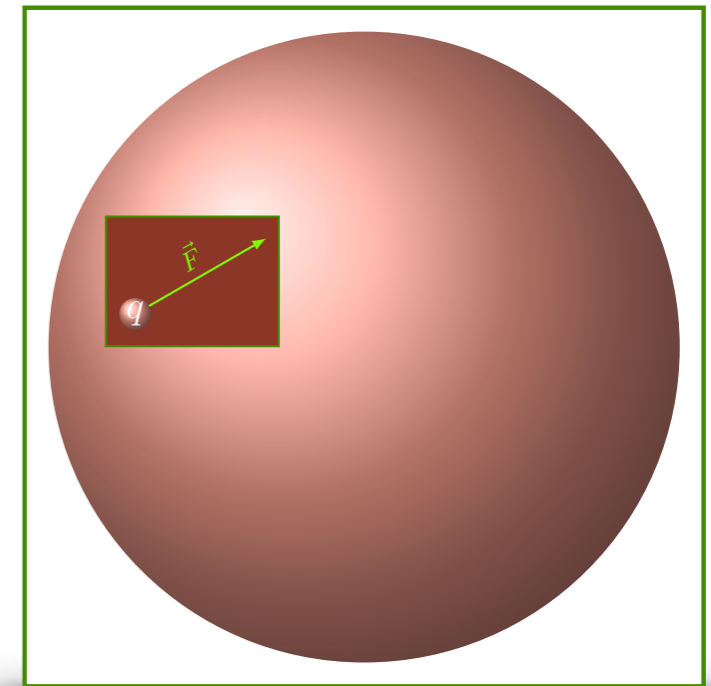
3. Momento

$$\vec{F} = \int_V \rho \vec{E} + \vec{J} \times \vec{B} \, d\tau$$

$$\vec{f} = \rho \vec{E} + \vec{J} \times \vec{B}$$

$$\rho = \epsilon_0 \vec{\nabla} \cdot \vec{E}$$

$$\vec{J} = \frac{1}{\mu_0} \vec{\nabla} \times \vec{B} - \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$



$$\frac{\partial \vec{E}}{\partial t} \times \vec{B} = \frac{\partial}{\partial t} (\vec{E} \times \vec{B}) + \vec{E} \times (\vec{\nabla} \times \vec{E})$$

$$\vec{E} \times (\vec{\nabla} \times \vec{E}) = \frac{1}{2} \vec{\nabla} E^2 - (\vec{E} \cdot \vec{\nabla}) \vec{E}$$

$$\vec{B} \times (\vec{\nabla} \times \vec{B}) = \frac{1}{2} \vec{\nabla} B^2 - (\vec{B} \cdot \vec{\nabla}) \vec{B}$$

Leis de conservação

3. Momento

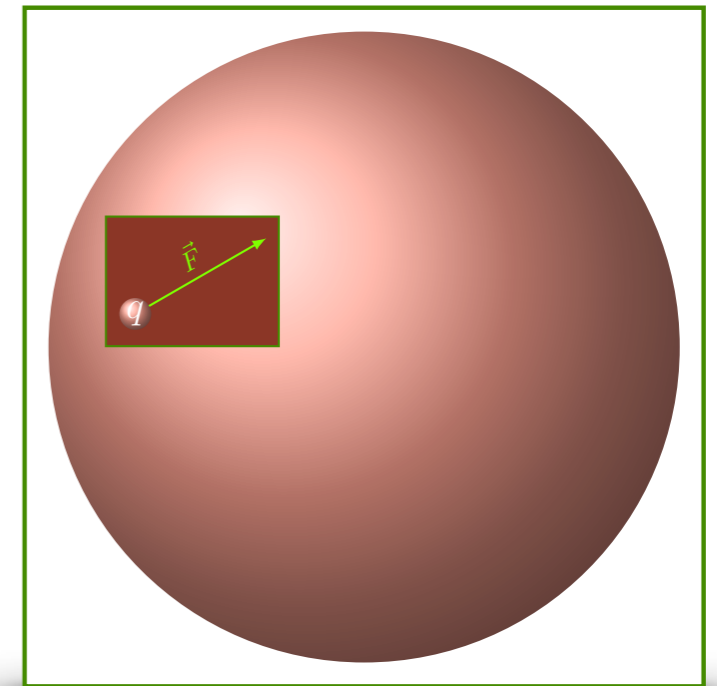
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$$\vec{J} = \frac{1}{\mu_0} \vec{\nabla} \times \vec{B} - \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{J} \times \vec{B} = -\frac{1}{\mu_0} \vec{B} \times (\vec{\nabla} \times \vec{B}) + \epsilon_0 \vec{B} \times \frac{\partial \vec{E}}{\partial t}$$



$$\frac{\partial \vec{E}}{\partial t} \times \vec{B} = \frac{\partial}{\partial t} (\vec{E} \times \vec{B}) + \vec{E} \times (\vec{\nabla} \times \vec{E})$$

$$\vec{E} \times (\vec{\nabla} \times \vec{E}) = \frac{1}{2} \vec{\nabla} E^2 - (\vec{E} \cdot \vec{\nabla}) \vec{E}$$

$$\vec{B} \times (\vec{\nabla} \times \vec{B}) = \frac{1}{2} \vec{\nabla} B^2 - (\vec{B} \cdot \vec{\nabla}) \vec{B}$$

Leis de conservação

3. Momento

$$\vec{f} = \epsilon_0 [(\vec{\nabla} \cdot \vec{E})\vec{E} + (\vec{E} \cdot \vec{\nabla})\vec{E}] + \frac{1}{\mu_0} [(\vec{B} \cdot \vec{\nabla})\vec{B}] - \frac{1}{2} \vec{\nabla} \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) - \epsilon_0 \frac{\partial}{\partial t} (\vec{E} \times \vec{B})$$

Leis de conservação

3. Momento

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Leis de conservação

3. Momento

$$(\vec{\nabla} \cdot \vec{E})\vec{E} + (\vec{E} \cdot \vec{\nabla})\vec{E} = ?$$

Leis de conservação

3. Momento

$$(\vec{\nabla} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\nabla})\vec{\mathbf{E}} = ?$$

$$\left[\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z} \right] \begin{bmatrix} E_x E_x & E_x E_y & E_x E_z \\ E_y E_x & E_y E_y & E_y E_z \\ E_z E_x & E_z E_y & E_z E_z \end{bmatrix} = \left[(\vec{\nabla} \cdot \vec{\mathbf{E}})E_x + \vec{\mathbf{E}} \cdot \vec{\nabla} E_x \quad \{\} E_y \quad \{\} E_z \right]$$

Leis de conservação

Noether

