Eletromagnetismo Avançado

1º ciclo
Aula de 20 de agosto

Leis de conservação

1. Carga elétrica

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

Leis de conservação

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$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

2. Energia

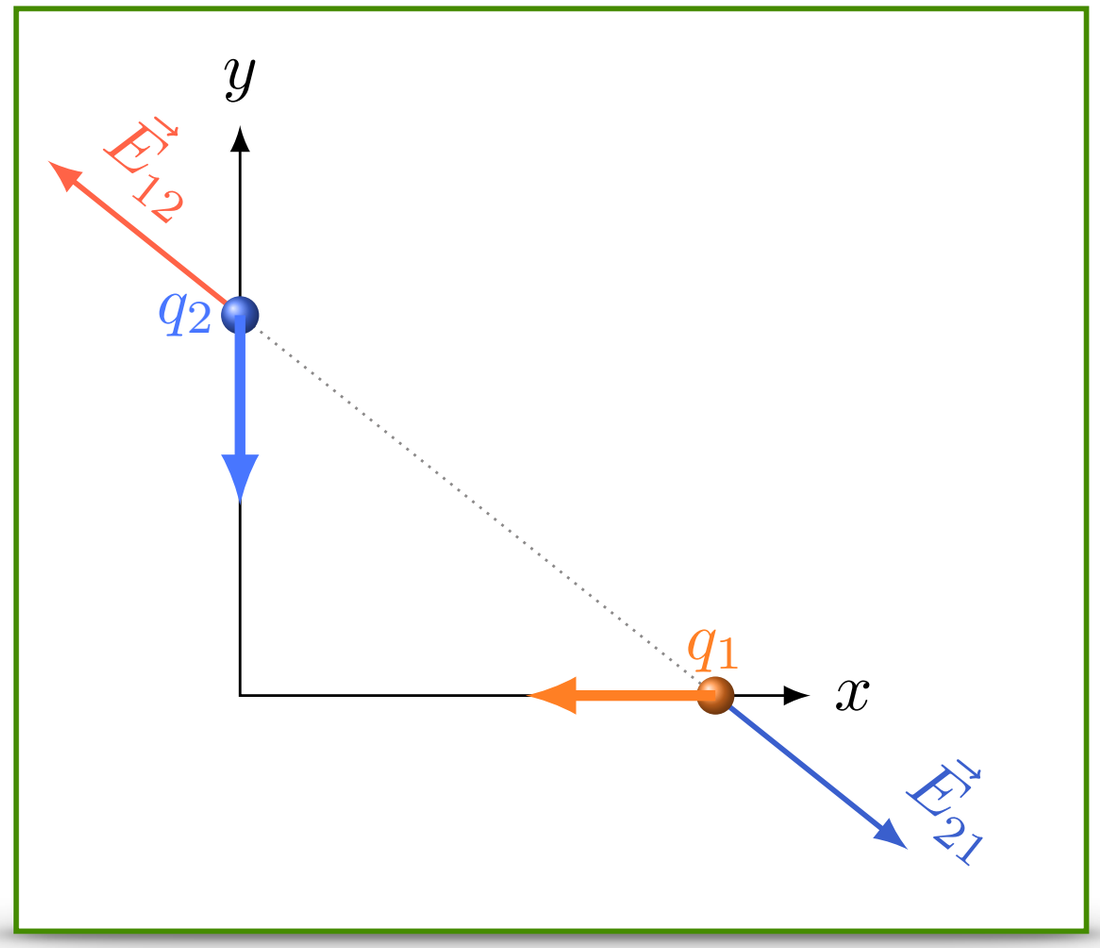
$$\frac{\partial}{\partial t}\left(u_{mec} + u_{em}\right) = -\vec{\nabla} \cdot \vec{S}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

Leis de conservação

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

3. Momento



Leis de conservação

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

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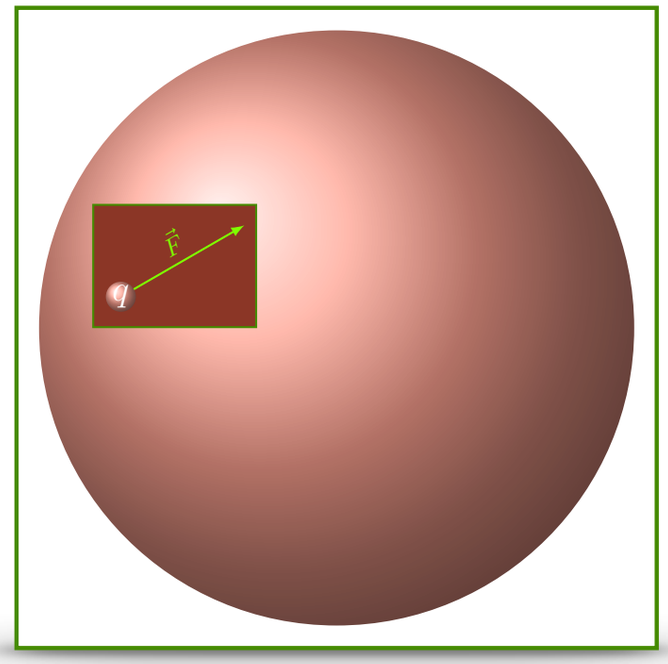
$$\vec{\mathbf{F}} = \int_{\mathcal{V}} \rho\vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}}\ \mathrm{d}\tau$$



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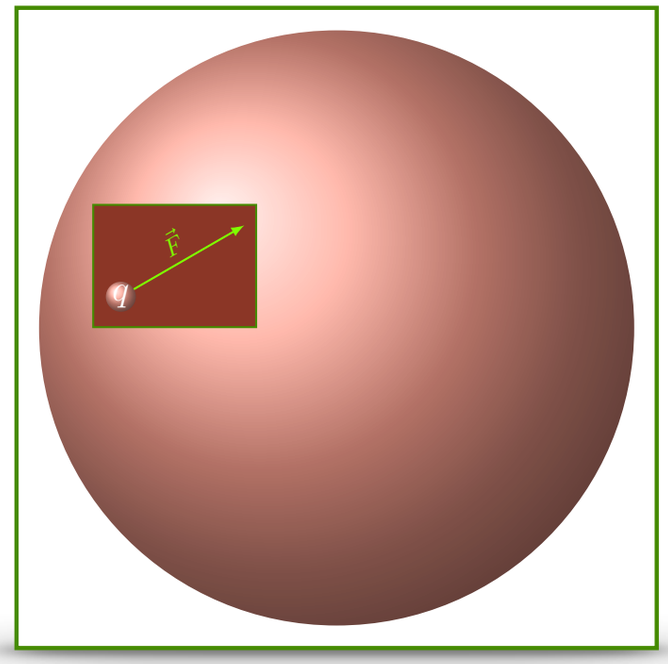

Leis de conservação

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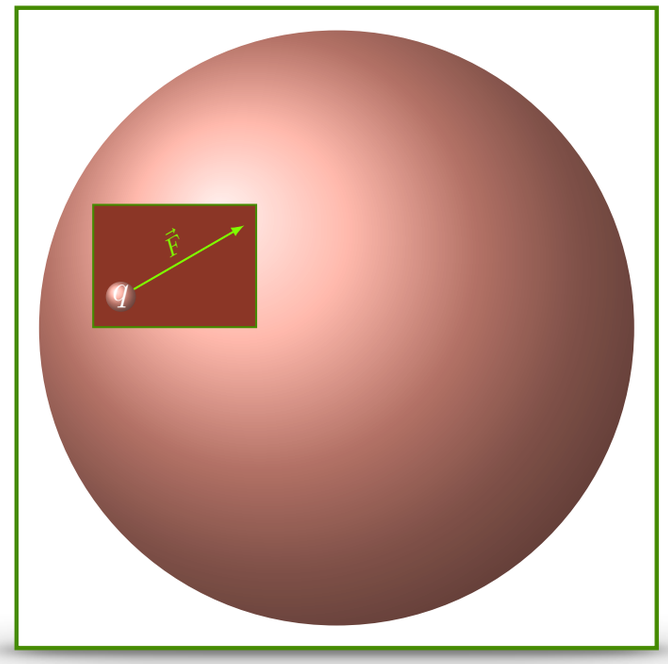

$$\rho = \epsilon_0 \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}}$$

Leis de conservação

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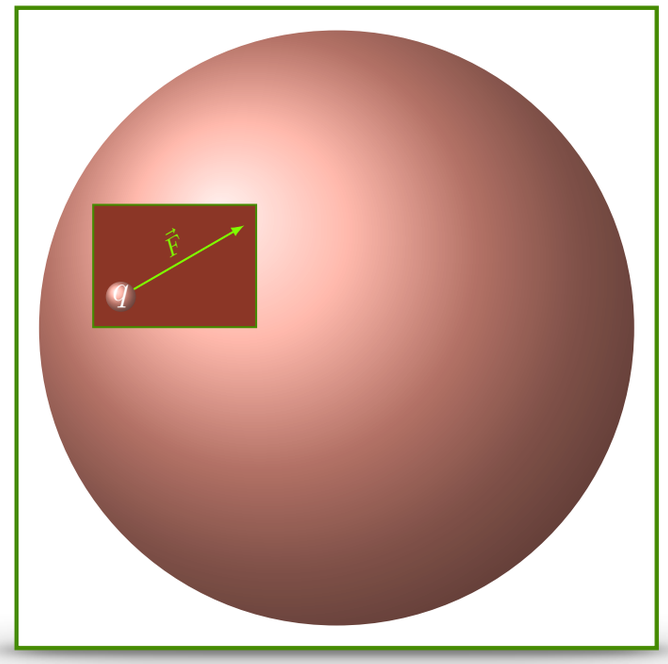

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$$\vec{\mathbf{J}} = \frac{1}{\mu_0}\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} - \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$$

Leis de conservação

3. Momento

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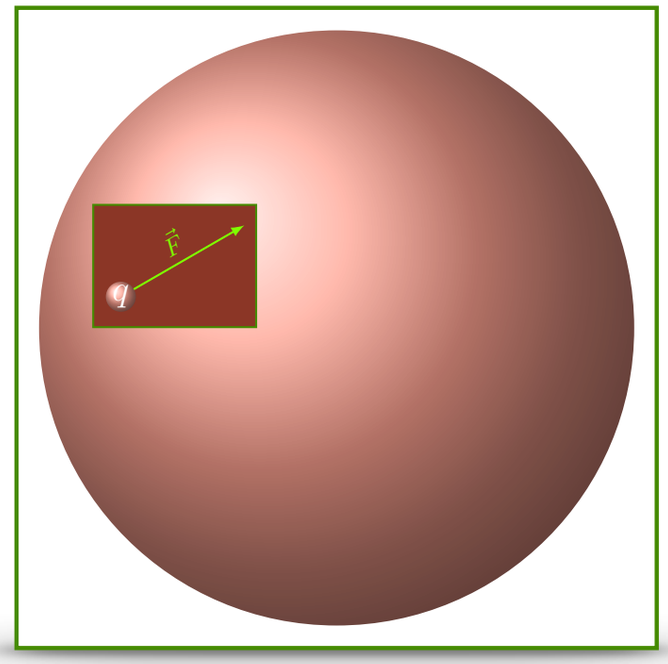

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$$\vec{\mathbf{J}} = \frac{1}{\mu_0}\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} - \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}$$

$$\frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} = \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) + \vec{\mathbf{E}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}})$$

$$\vec{\mathbf{E}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}}) = \frac{1}{2}\vec{\boldsymbol{\nabla}} E^2 - (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}}$$

$$\vec{\mathbf{B}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}) = \frac{1}{2}\vec{\boldsymbol{\nabla}} B^2 - (\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}$$

Leis de conservação

3. Momento

$$\vec{\mathbf{F}} = \int_{\mathcal{V}} \rho\vec{\mathbf{E}} + \vec{\mathbf{J}} \times \vec{\mathbf{B}} \, \mathrm{d}\tau$$

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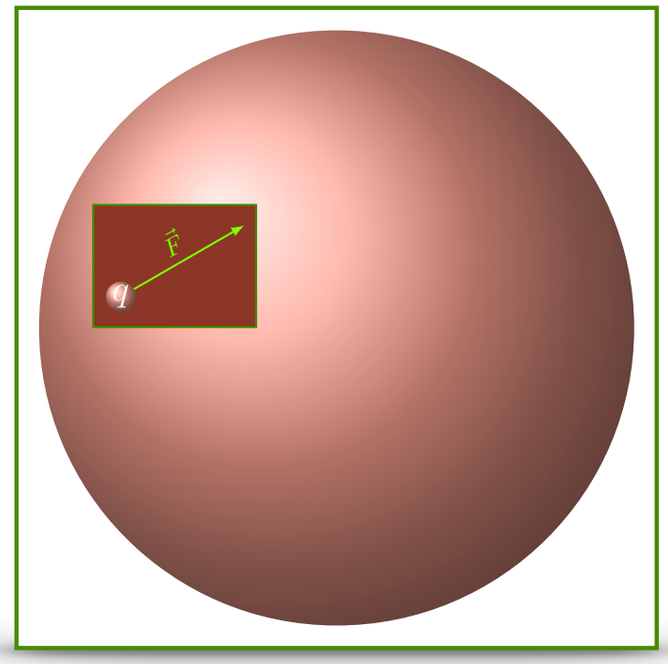

$$\rho = \epsilon_0 \vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}}$$

$$\vec{\mathbf{J}} = \frac{1}{\mu_0}\left(\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}} - \epsilon_0 \frac{\partial \vec{\mathbf{E}}}{\partial t}\right)$$

$$\vec{\mathbf{J}} \times \vec{\mathbf{B}} = -\frac{1}{\mu_0}\vec{\mathbf{B}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}) + \epsilon_0 \vec{\mathbf{B}} \times \frac{\partial \vec{\mathbf{E}}}{\partial t}$$

$$\frac{\partial \vec{\mathbf{E}}}{\partial t} \times \vec{\mathbf{B}} = \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right) + \vec{\mathbf{E}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}})$$

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$$\vec{\mathbf{B}} \times (\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{B}}) = \frac{1}{2}\vec{\boldsymbol{\nabla}} B^2 - (\vec{\mathbf{B}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}$$

Leis de conservação

3. Momento

$$\vec{\mathbf{f}} = \epsilon_0[(\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}}\cdot\vec{\boldsymbol{\nabla}})\vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\vec{\mathbf{B}}\cdot\vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}]$$
$$- \frac{1}{2}\vec{\boldsymbol{\nabla}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \epsilon_0\frac{\partial}{\partial t}\left(\vec{\mathbf{E}}\times\vec{\mathbf{B}}\right)$$

Leis de conservação

3. Momento

$$\vec{\mathbf{f}} = \epsilon_0[(\vec{\boldsymbol{\nabla}}\cdot\vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}}\cdot\vec{\boldsymbol{\nabla}})\vec{\mathbf{E}}] + \frac{1}{\mu_0}[(\vec{\mathbf{B}}\cdot\vec{\boldsymbol{\nabla}})\vec{\mathbf{B}}]$$
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$$- \frac{1}{2}\vec{\boldsymbol{\nabla}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \epsilon_0\frac{\partial}{\partial t}\left(\vec{\mathbf{E}}\times\vec{\mathbf{B}}\right)$$

Leis de conservação

3. Momento

$$(\vec{\mathbf{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\mathbf{\nabla}})\vec{\mathbf{E}} = ?$$

$$\begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{bmatrix} \begin{bmatrix} E_x E_x & E_x E_y & E_x E_z \\ E_y E_x & E_y E_y & E_y E_z \\ E_z E_x & E_z E_y & E_z E_z \end{bmatrix} = \begin{bmatrix} & & \end{bmatrix}$$

Leis de conservação

3. Momento

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Leis de conservação

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$$\vec{\mathbf{f}} = \epsilon_0 [(\vec{\mathbf{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\mathbf{\nabla}})\vec{\mathbf{E}}] + \frac{1}{\mu_0} [(\vec{\mathbf{\nabla}} \cdot \vec{\mathbf{B}})\vec{\mathbf{B}} + (\vec{\mathbf{B}} \cdot \vec{\mathbf{\nabla}})\vec{\mathbf{B}}]$$
$$- \frac{1}{2} \vec{\mathbf{\nabla}} \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) - \epsilon_0 \frac{\partial}{\partial t} \left(\vec{\mathbf{E}} \times \vec{\mathbf{B}} \right)$$

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3. Momento

$$(\vec{\boldsymbol{\nabla}} \cdot \vec{\mathbf{E}})\vec{\mathbf{E}} + (\vec{\mathbf{E}} \cdot \vec{\boldsymbol{\nabla}})\vec{\mathbf{E}} = ?$$

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$$- \frac{1}{2}\vec{\boldsymbol{\nabla}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) - \epsilon_0 \frac{\partial}{\partial t}\left(\vec{\mathbf{E}} \times \vec{\mathbf{B}}\right)$$

$$\mathbb{T} \equiv \epsilon_0 \begin{bmatrix} E_x E_x & E_x E_y & E_x E_z \\ E_y E_x & E_y E_y & E_y E_z \\ E_z E_x & E_z E_y & E_z E_z \end{bmatrix} + \frac{1}{\mu_0} \begin{bmatrix} B_x B_x & B_x B_y & B_x B_z \\ B_y B_x & B_y B_y & B_y B_z \\ B_z B_x & B_z B_y & B_z B_z \end{bmatrix}$$
$$- \frac{1}{2}(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Leis de conservação

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$$T_{ij} = \epsilon_0 E_i\, E_j + \frac{1}{\mu_0}B_i\, B_j - \frac{1}{2}\delta_{ij}(\epsilon_0 E_{ii}^2 + \frac{1}{\mu_0}B_{ii}^2)$$

Leis de conservação

3. Momento

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$$\vec{\mathbf{f}} = \vec{\boldsymbol{\nabla}}\cdot\mathbb{T} - \epsilon_0\mu_0\frac{\partial\vec{S}}{\partial t}$$

$$F = \int_{\mathcal{V}} \vec{\boldsymbol{\nabla}}\cdot\mathbb{T}\ \mathrm{d}\tau - \epsilon_0\mu_0\int_{\mathcal{V}}\frac{\partial\vec{S}}{\partial t}\ \mathrm{d}\tau$$

Leis de conservação

3. Momento

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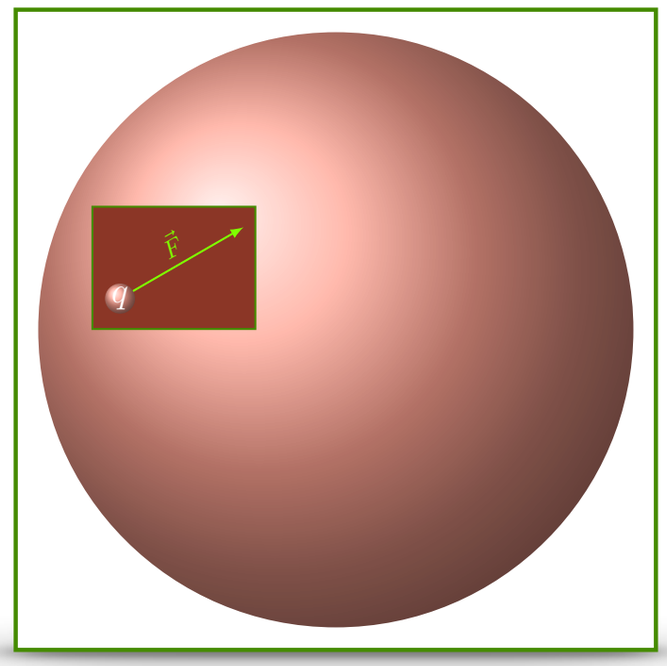
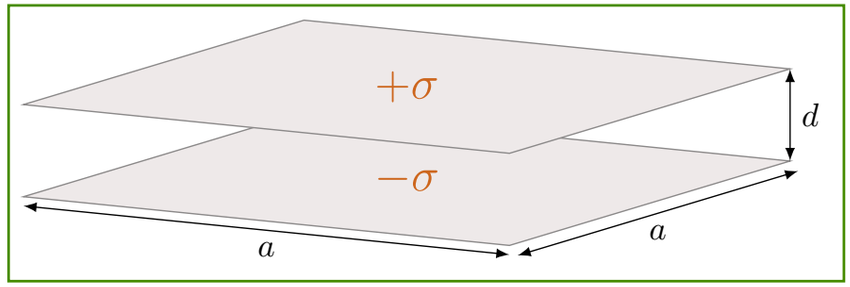
$$F = \int_{\mathcal{V}} \vec{\boldsymbol{\nabla}}\cdot\mathbb{T}\ \mathrm{d}\tau - \epsilon_0\mu_0\int_{\mathcal{V}}\frac{\partial\vec{S}}{\partial t}\ \mathrm{d}\tau$$

$$F = \int_{\mathcal{A}} \mathbb{T}\cdot\hat{\mathbf{n}}\ \mathrm{d}a - \epsilon_0\mu_0\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{V}}\vec{S}\ \mathrm{d}\tau$$



Pratique o que aprendeu

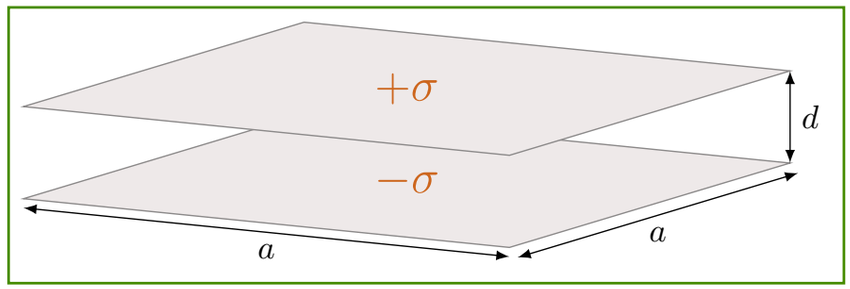
$$F = \int_{\mathcal{A}} \mathbb{T} \cdot \hat{\mathbf{n}} \, \mathrm{d}a - \epsilon_0 \mu_0 \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathcal{V}} \vec{S} \, \mathrm{d}\tau$$



Pratique o que aprendeu

$$F = \int_{\mathcal{A}} \mathbb{T} \cdot \hat{\mathbf{n}} \, \mathrm{d}a - \epsilon_0 \mu_0 \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathcal{V}} \vec{S} \, \mathrm{d}\tau$$

$$\mathbb{T} = \epsilon_0 \begin{bmatrix} -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 & 0 \\ 0 & -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 \\ 0 & 0 & \frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 \end{bmatrix}$$



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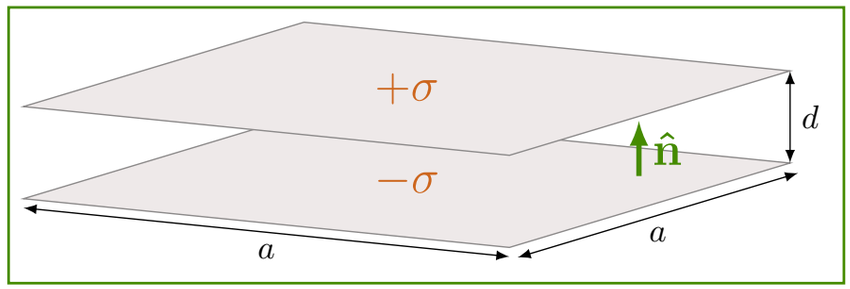
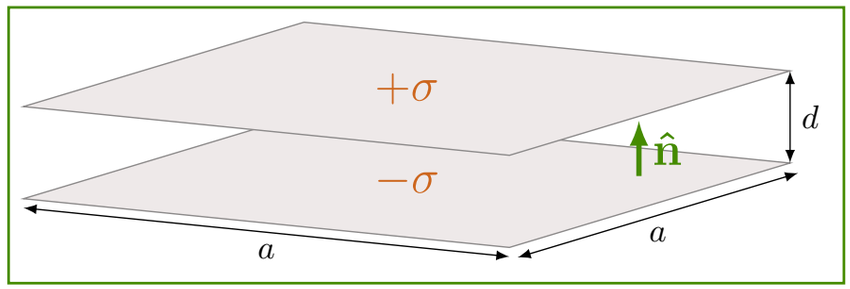


$$\mathbb{T} \cdot \hat{\mathbf{n}} = \epsilon_0 \begin{bmatrix} -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 & 0 \\ 0 & -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 \\ 0 & 0 & \frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Pratique o que aprendeu

$$F = \int_{\mathcal{A}} \mathbb{T} \cdot \hat{\mathbf{n}} \, \mathrm{d}a - \epsilon_0 \mu_0 \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathcal{V}} \vec{S} \, \mathrm{d}\tau$$

$$\mathbb{T} = \epsilon_0 \begin{bmatrix} -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 & 0 \\ 0 & -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 \\ 0 & 0 & \frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 \end{bmatrix}$$



$$\mathbb{T} \cdot \hat{\mathbf{n}} = \epsilon_0 \begin{bmatrix} -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 & 0 \\ 0 & -\frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 & 0 \\ 0 & 0 & \frac{1}{2}\left(\frac{\sigma}{\epsilon_0}\right)^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \frac{\sigma^2}{2\epsilon_0} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$