Eletromagnetismo Avançado

7600035

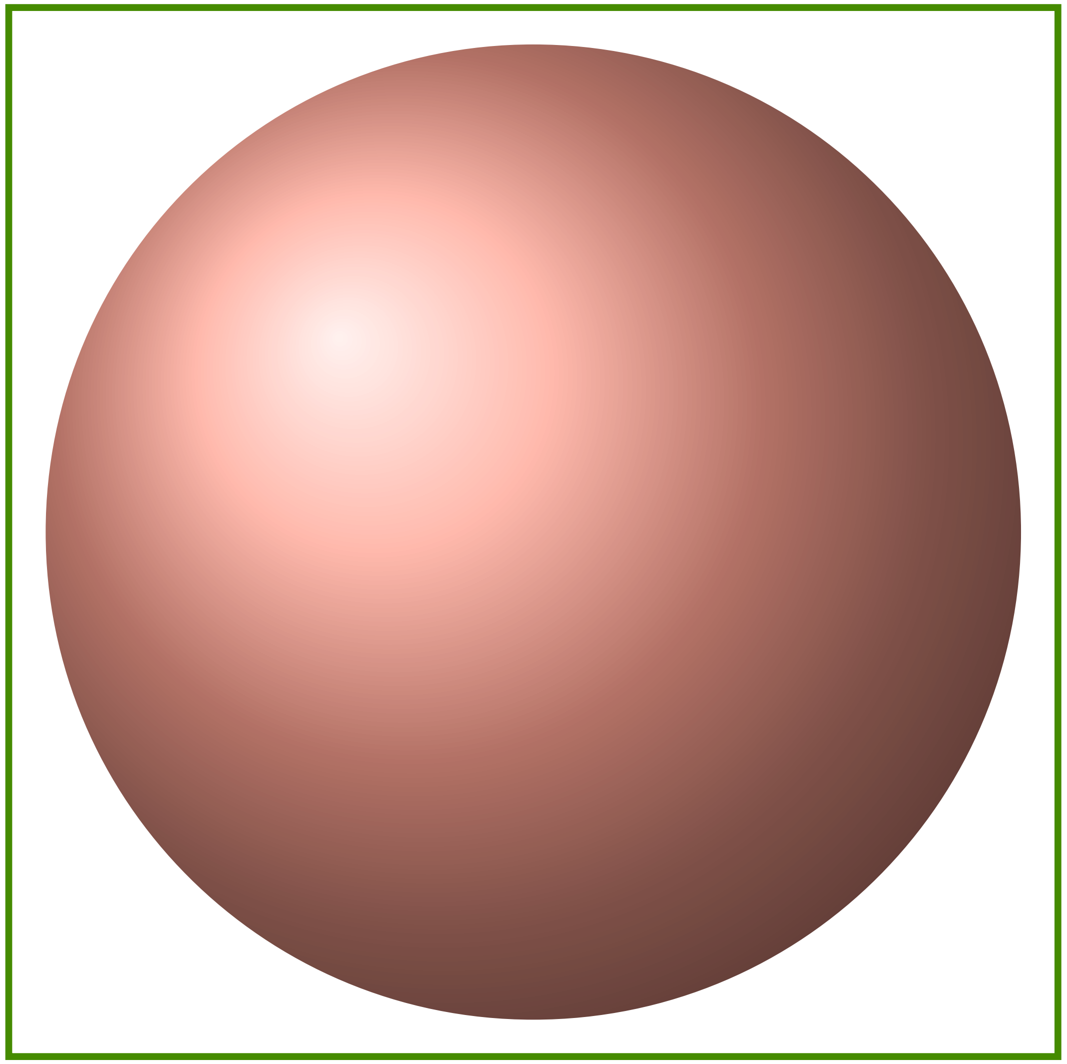
Aula de 20 de agosto

Leis de conservação

1. Carga elétrica

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

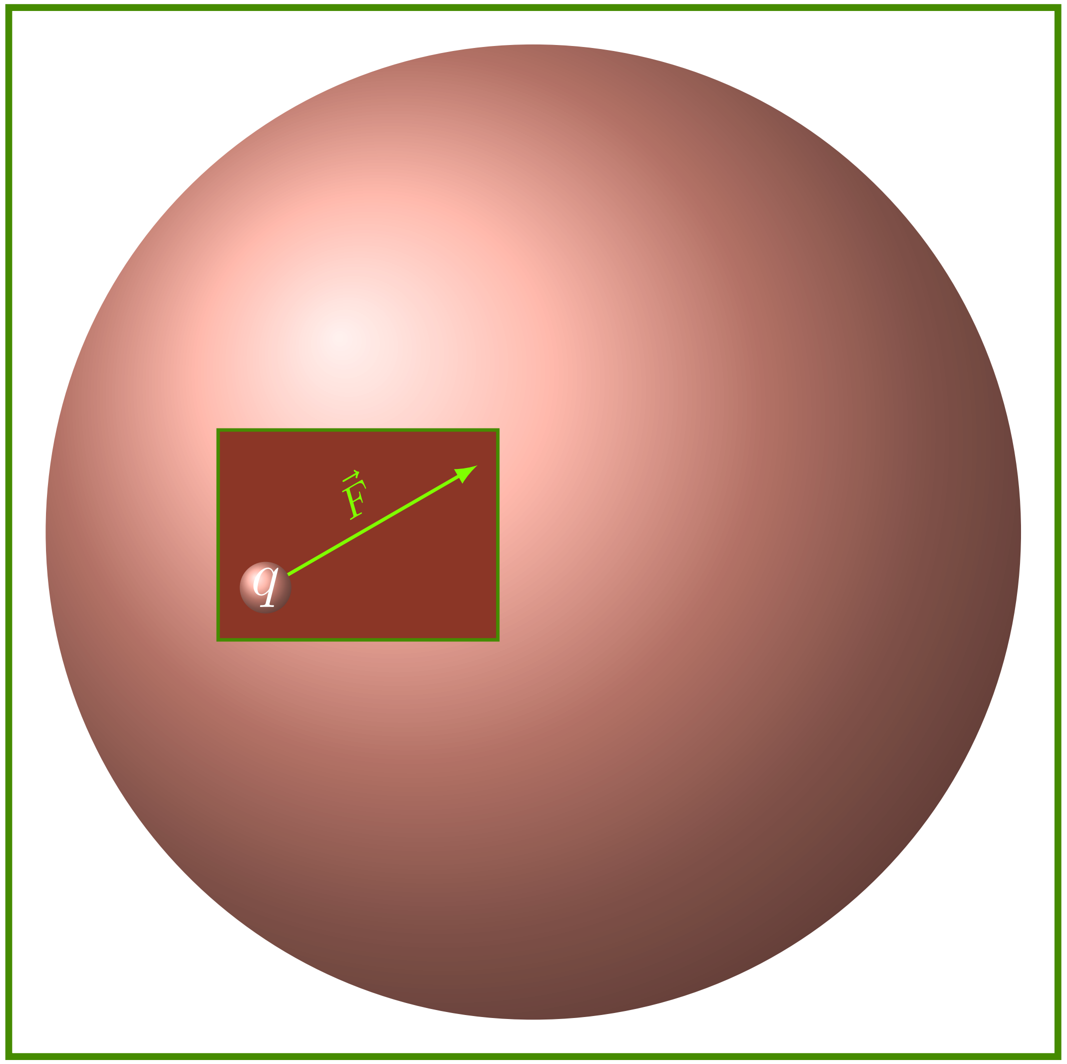
EQ. CONTINUIDADE



Leis de conservação

2. Energia

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

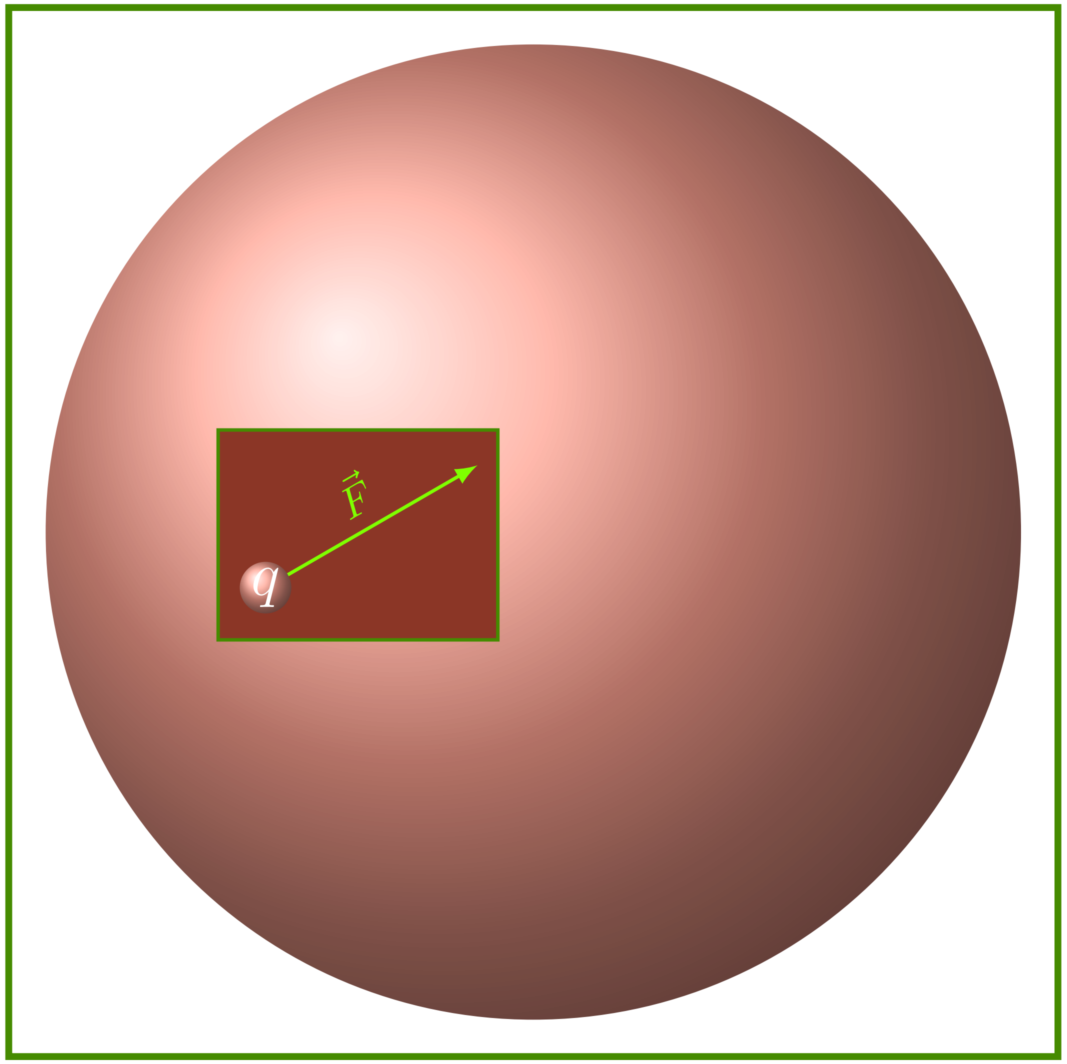


Leis de conservação

2. Energia

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

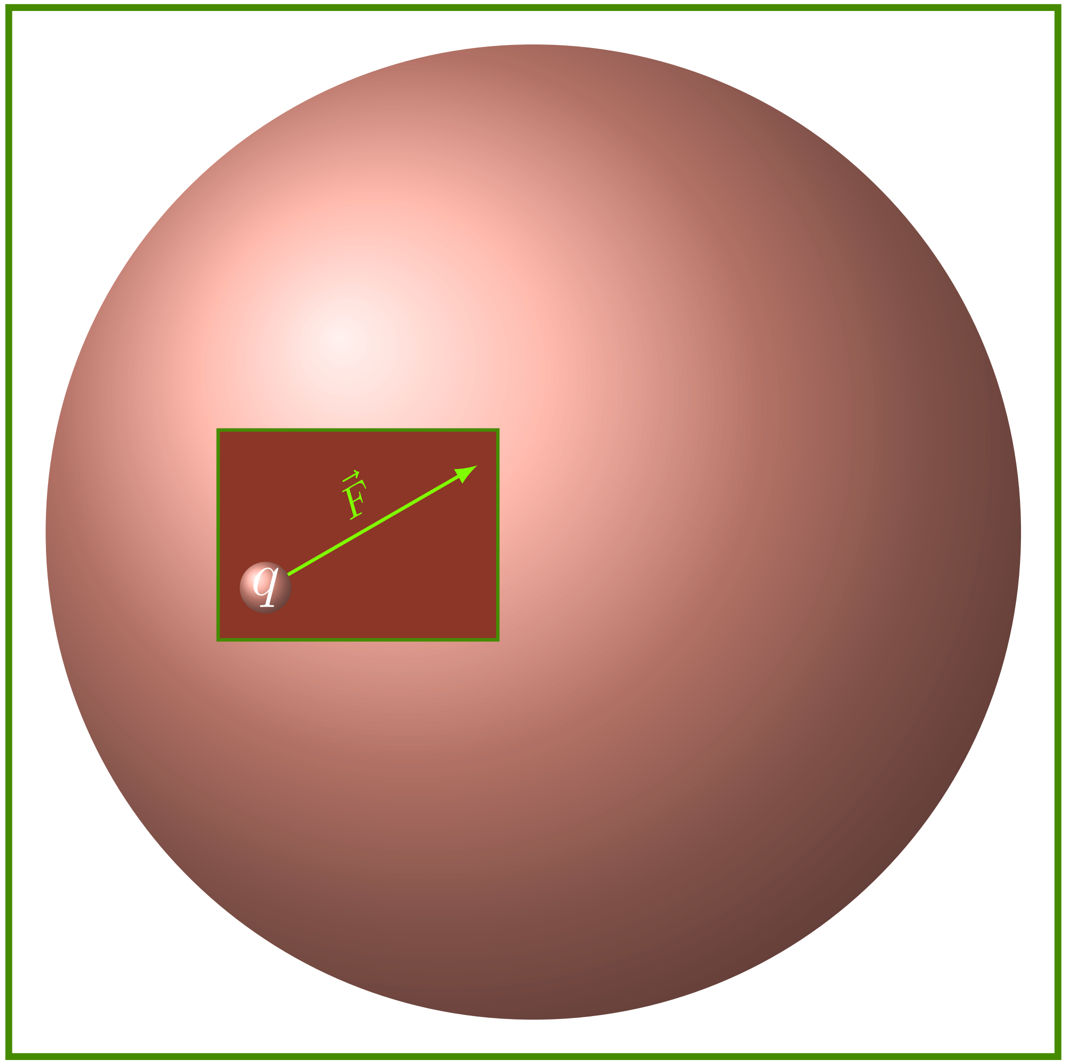
$$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \vec{J} \cdot \vec{E} \, \mathrm{d}\tau$$



Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \vec{J} \cdot \vec{E}\ \mathrm{d}\tau$$



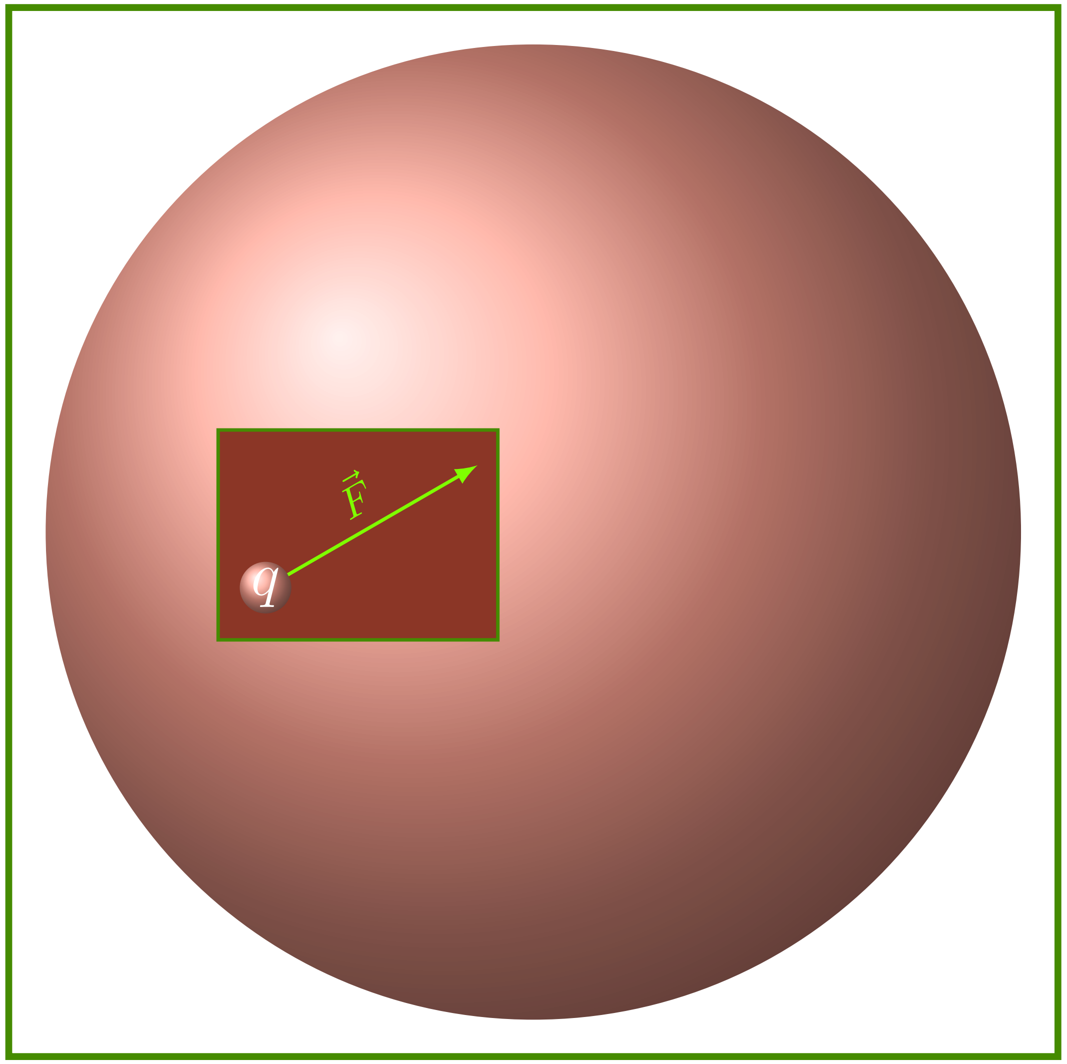
Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \vec{J} \cdot \vec{E} \, \mathrm{d}\tau$$

AMPÈRE + MAXWELL

$$\vec{\boldsymbol{\nabla}} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$



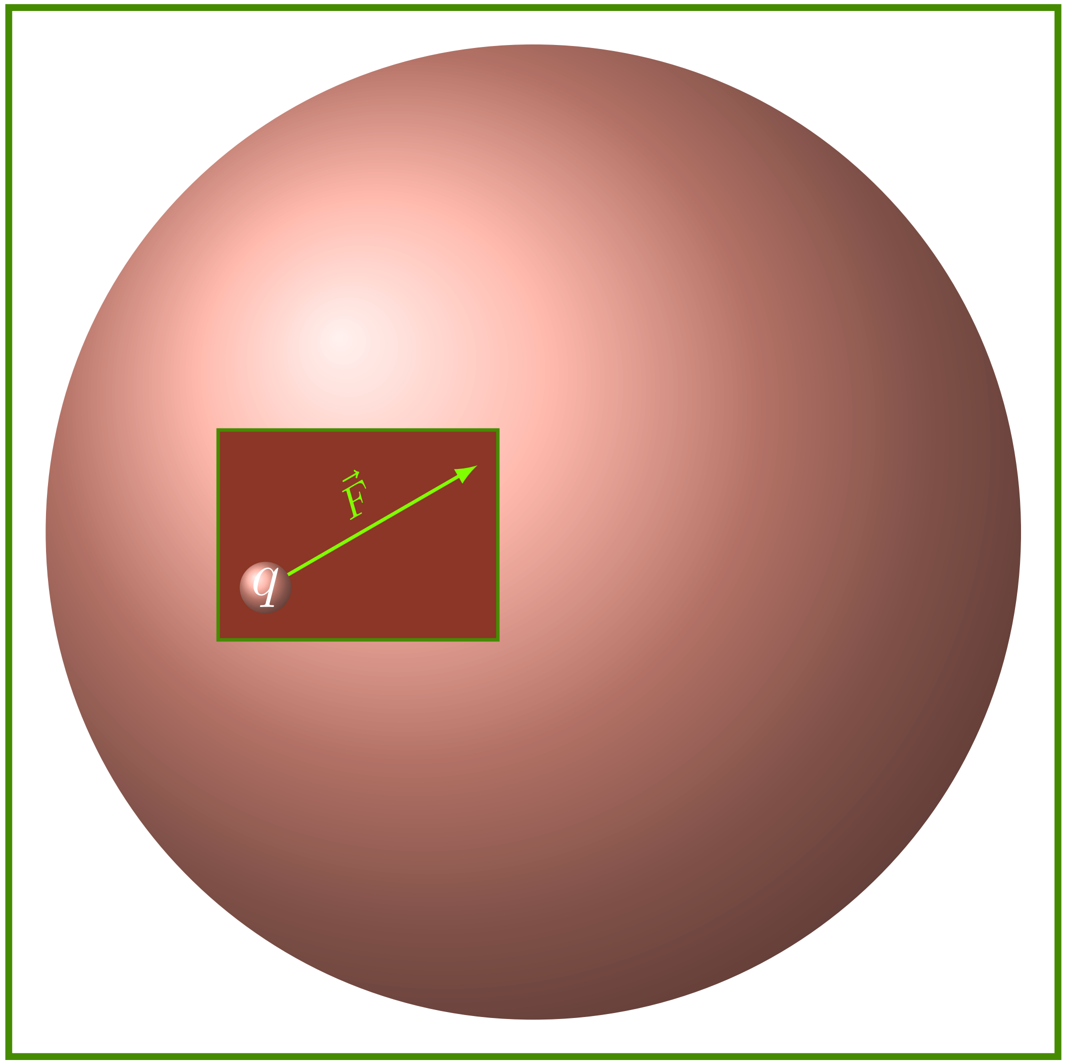
Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \vec{J} \cdot \vec{E} \, \mathrm{d}\tau$$

$$\vec{\boldsymbol{\nabla}} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\vec{J} \cdot \vec{E} = \frac{1}{\mu_0} \vec{E} \cdot \vec{\boldsymbol{\nabla}} \times \vec{B} - \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$



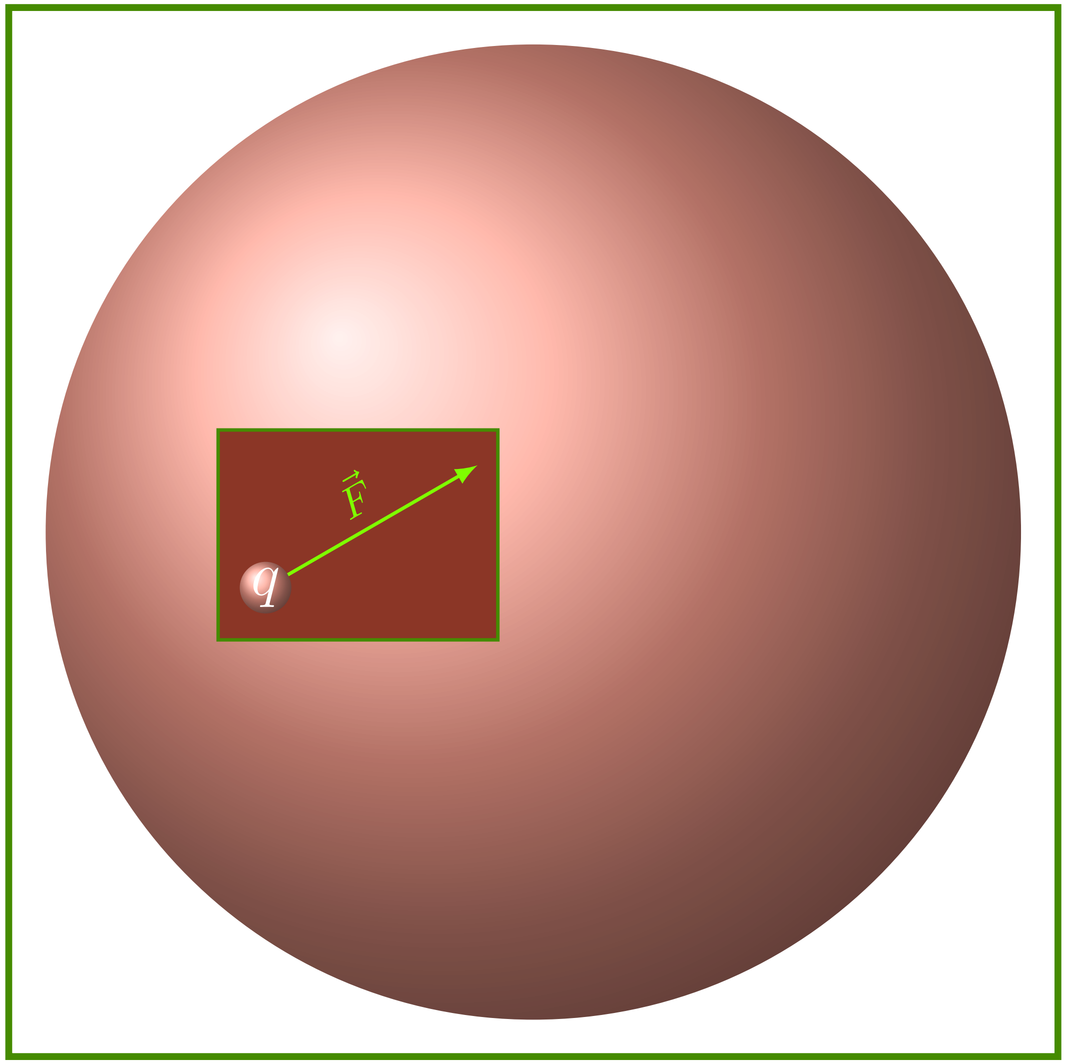
Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \vec{J} \cdot \vec{E} \, \mathrm{d}\tau$$

$$\vec{\boldsymbol{\nabla}} \times \vec{B} = \mu_0 \Big(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \Big)$$

$$\vec{J} \cdot \vec{E} = \frac{1}{\mu_0} \vec{E} \cdot \vec{\boldsymbol{\nabla}} \times \vec{B} - \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$



$$\vec{\boldsymbol{\nabla}} \cdot \vec{E} \times \vec{B} = \vec{B} \cdot \vec{\boldsymbol{\nabla}} \times \vec{E} - \vec{E} \cdot \vec{\boldsymbol{\nabla}} \times \vec{B}$$

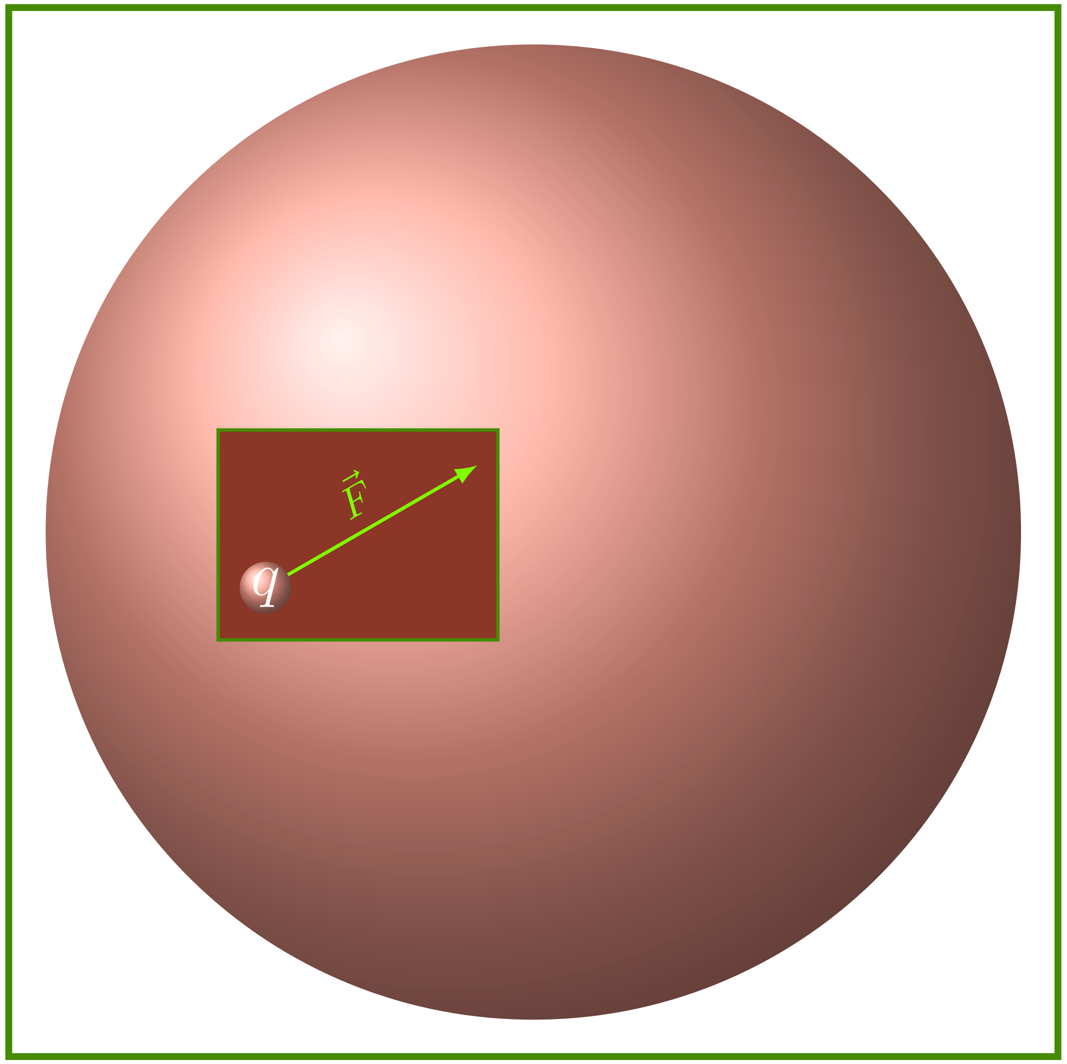
Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \vec{J}\cdot\vec{E}\,\mathrm{d}\tau$$

$$\vec{\nabla}\times\vec{B} = \mu_0\Big(\vec{J} + \epsilon_0\frac{\partial\vec{E}}{\partial t}\Big)$$

$$\vec{J}\cdot\vec{E} = \frac{1}{\mu_0}\vec{E}\cdot\vec{\nabla}\times\vec{B} - \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$



$$\vec{\nabla}\cdot\vec{E}\times\vec{B} = \vec{B}\cdot\vec{\nabla}\times\vec{E} - \vec{E}\cdot\vec{\nabla}\times\vec{B}$$

FARADAY

$$\vec{E}\cdot\vec{\nabla}\times\vec{B} = -\vec{\nabla}\cdot\vec{E}\times\vec{B} - \vec{B}\cdot\frac{\partial\vec{B}}{\partial t}$$

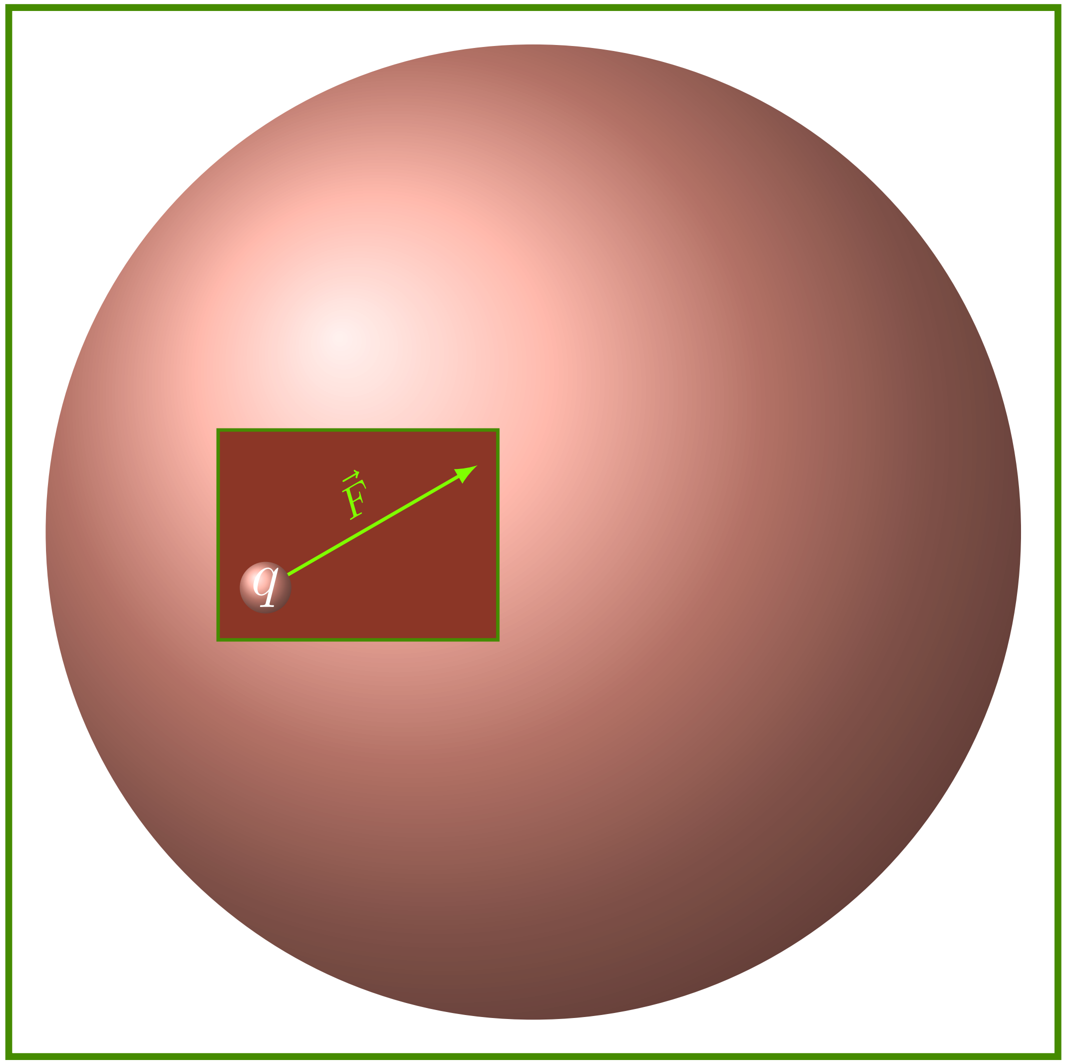
Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \vec{J} \cdot \vec{E} \, \mathrm{d}\tau$$

$$\vec{\boldsymbol{\nabla}} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\vec{J} \cdot \vec{E} = \frac{1}{\mu_0} \vec{E} \cdot \vec{\boldsymbol{\nabla}} \times \vec{B} - \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

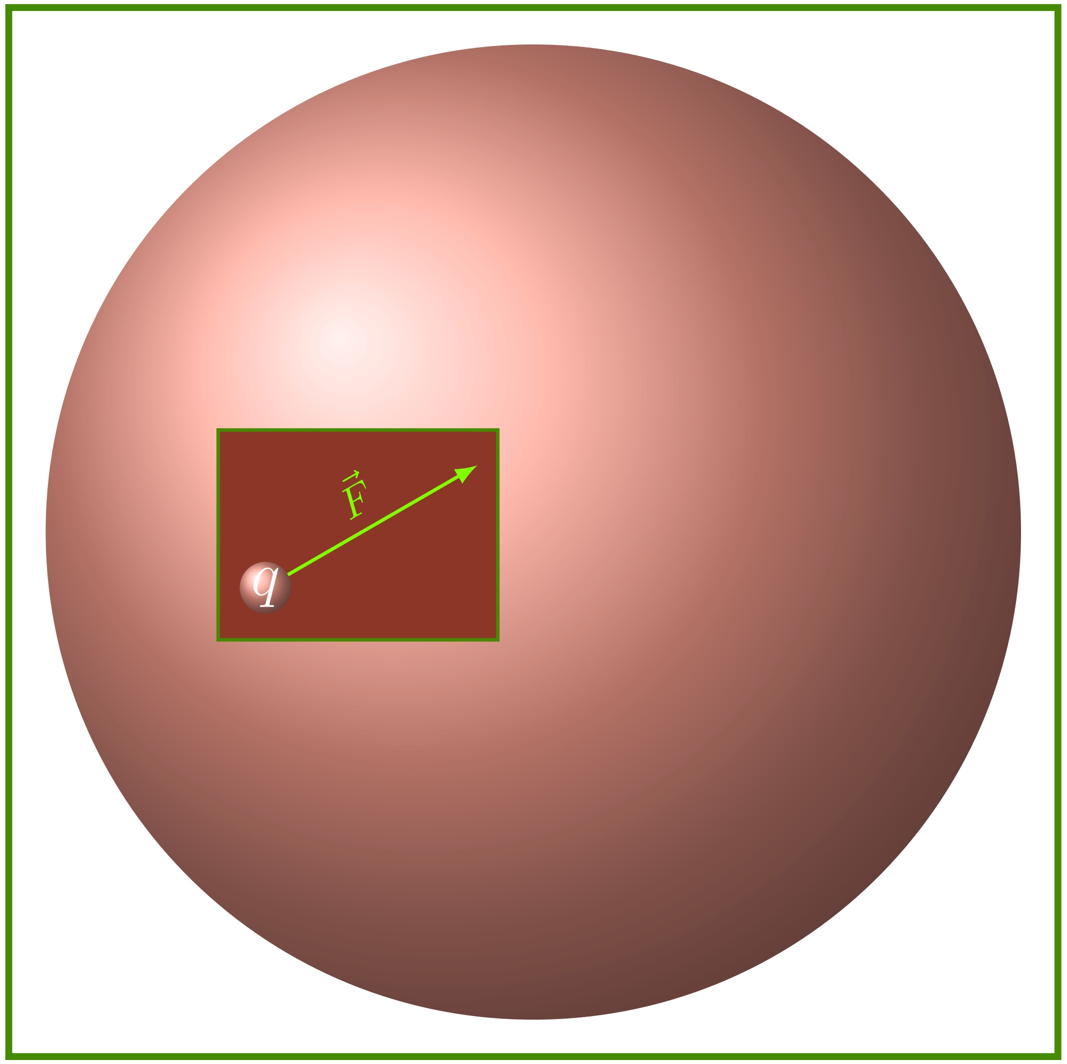


$$\vec{E} \cdot \vec{\boldsymbol{\nabla}} \times \vec{B} = -\vec{\boldsymbol{\nabla}} \cdot \vec{E} \times \vec{B} - \vec{B} \cdot \frac{\partial \vec{B}}{\partial t}$$

Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \vec{J}\cdot\vec{E}\;\mathrm{d}\tau$$



$$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J} + \epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{J}\cdot\vec{E} = \frac{1}{\mu_0}\vec{E}\cdot\vec{\nabla}\times\vec{B} - \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

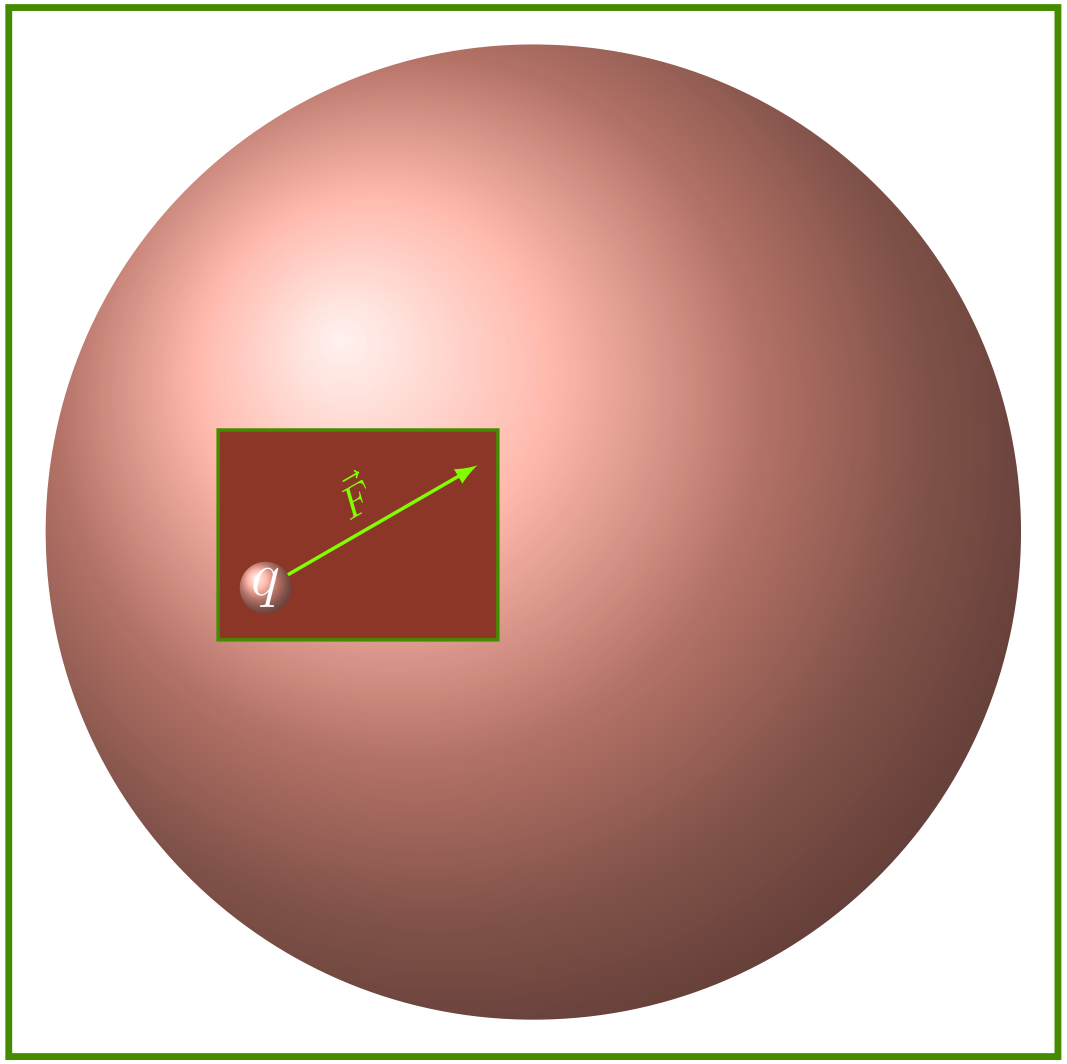
$$\vec{J}\cdot\vec{E} = \frac{-1}{\mu_0}\left(\vec{\nabla}\cdot\vec{E}\times\vec{B} + \vec{B}\cdot\frac{\partial\vec{B}}{\partial t}\right) - \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$\vec{E}\cdot\vec{\nabla}\times\vec{B} = -\vec{\nabla}\cdot\vec{E}\times\vec{B} - \vec{B}\cdot\frac{\partial\vec{B}}{\partial t}$$

Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \vec{J} \cdot \vec{E}\, \mathrm{d}\tau$$



$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\vec{J} \cdot \vec{E} = \frac{1}{\mu_0} \vec{E} \cdot \vec{\nabla} \times \vec{B} - \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

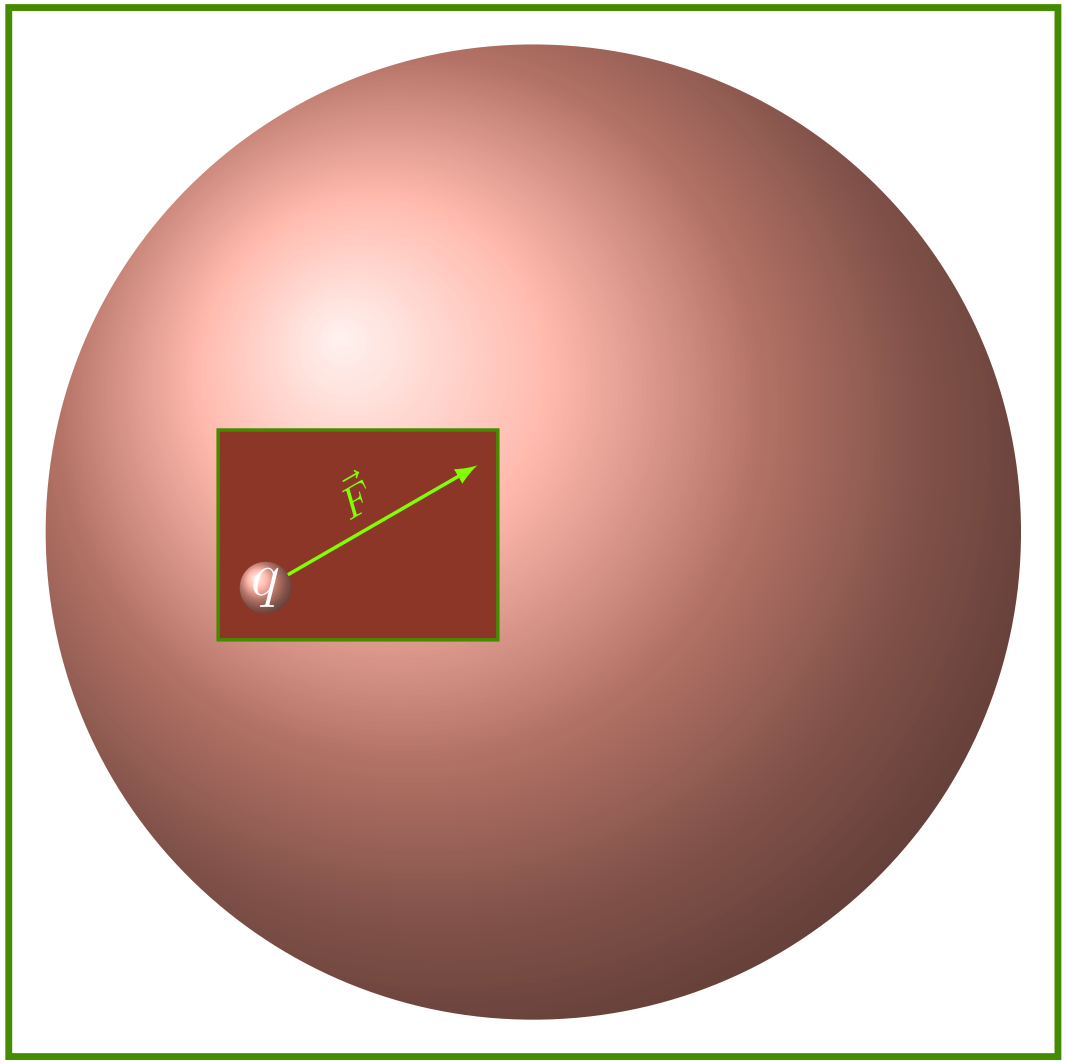
$$\vec{J} \cdot \vec{E} = \frac{-1}{\mu_0} \left(\vec{\nabla} \cdot \vec{E} \times \vec{B} + \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} \right) - \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}$$

$$\vec{J} \cdot \vec{E} = -\frac{1}{\mu_0} \vec{\nabla} \cdot \vec{E} \times \vec{B} - \frac{1}{2\mu_0} \frac{\partial B^2}{\partial t} - \frac{\epsilon_0}{2} \frac{\partial E^2}{\partial t}$$

Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \vec{J}\cdot\vec{E}\,\mathrm{d}\tau$$



$$\vec{\nabla}\times\vec{B} = \mu_0\Big(\vec{J} + \epsilon_0\frac{\partial\vec{E}}{\partial t}\Big)$$

$$\vec{J}\cdot\vec{E} = \frac{1}{\mu_0}\vec{E}\cdot\vec{\nabla}\times\vec{B} - \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$\vec{J}\cdot\vec{E} = \frac{-1}{\mu_0}\Big(\vec{\nabla}\cdot\vec{E}\times\vec{B} + \vec{B}\cdot\frac{\partial\vec{B}}{\partial t}\Big) - \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

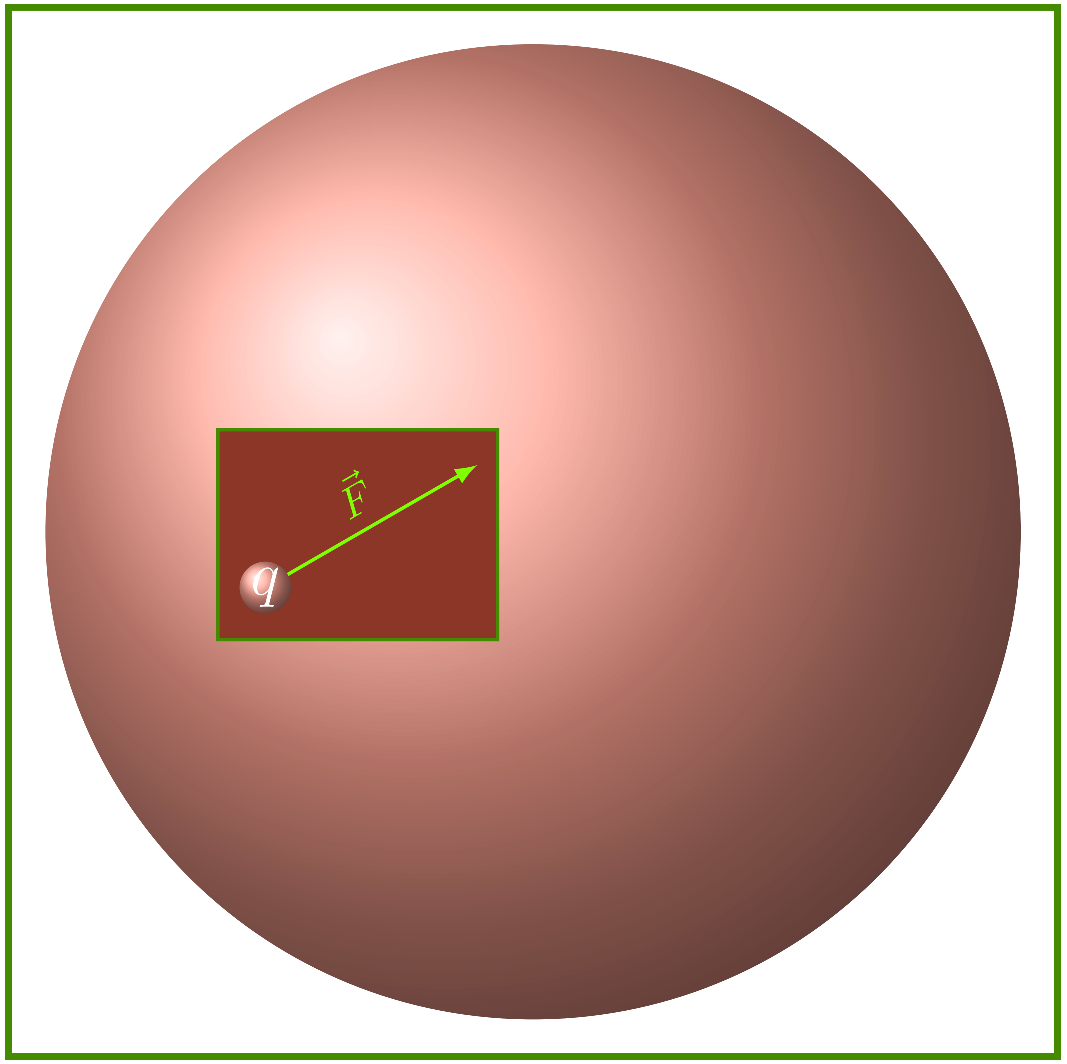
$$\vec{J}\cdot\vec{E} = -\frac{1}{\mu_0}\vec{\nabla}\cdot\vec{E}\times\vec{B} - \frac{1}{2\mu_0}\frac{\partial B^2}{\partial t} - \frac{\epsilon_0}{2}\frac{\partial E^2}{\partial t}$$

Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \vec{J} \cdot \vec{E}\, \mathrm{d}\tau$$

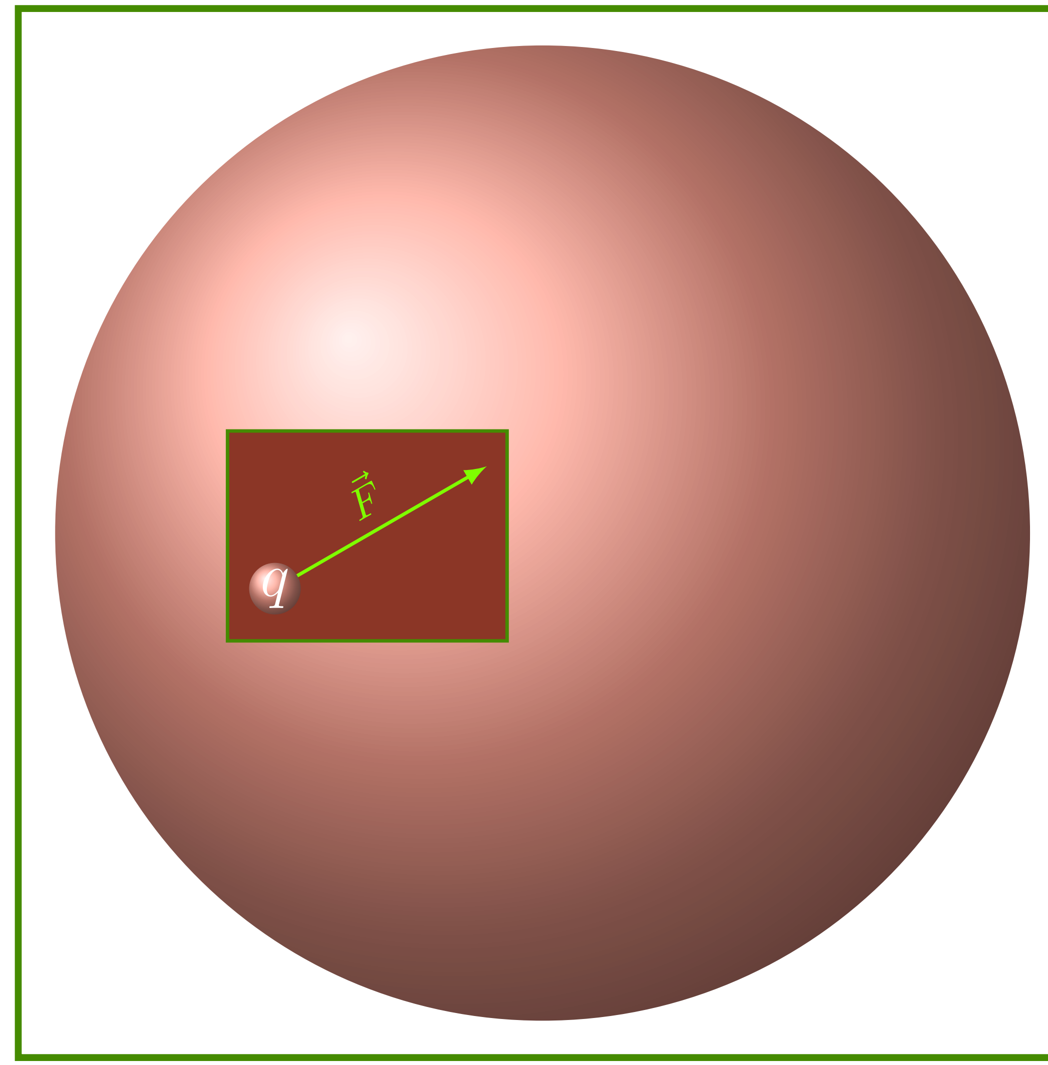
$$\vec{J} \cdot \vec{E} = -\frac{1}{\mu_0} \boldsymbol{\vec{\nabla}} \cdot \vec{E} \times \vec{B} - \frac{1}{2\mu_0} \frac{\partial B^2}{\partial t} - \frac{\epsilon_0}{2} \frac{\partial E^2}{\partial t}$$



Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \vec{J} \cdot \vec{E} \; \mathrm{d}\tau$$



$$\vec{J} \cdot \vec{E} = -\frac{1}{\mu_0} \vec{\boldsymbol{\nabla}} \cdot \vec{E} \times \vec{B} - \frac{1}{2\mu_0} \frac{\partial B^2}{\partial t} - \frac{\epsilon_0}{2} \frac{\partial E^2}{\partial t}$$

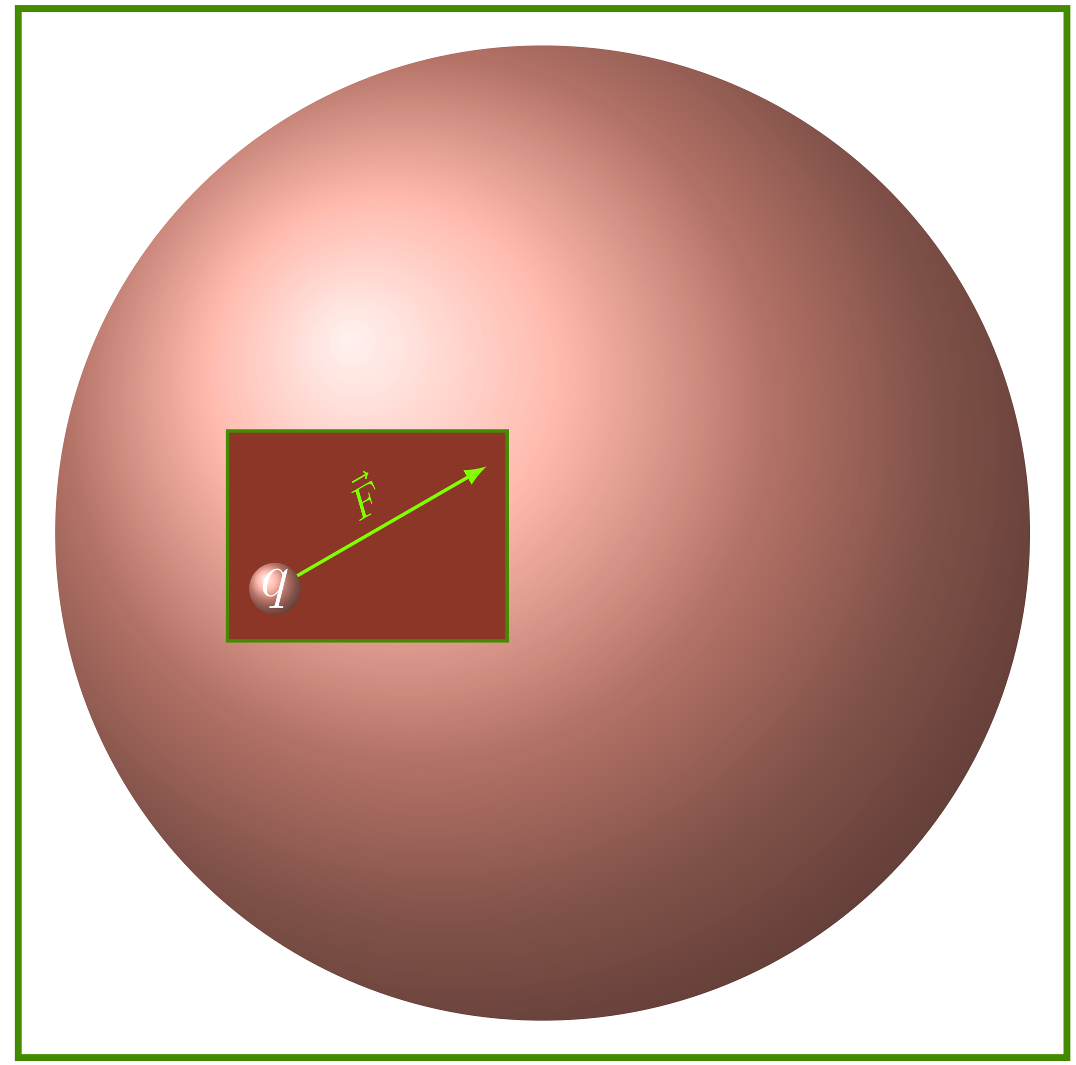
$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\frac{1}{\mu_0} \int_{\mathcal{A}} \vec{E} \times \vec{B} \cdot \hat{n} \; \mathrm{d}a - \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathcal{V}} \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) \mathrm{d}\tau$$

DENSIDADE DE ENERGIA ELETROMAGNÉTICA

Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = \int \vec{J} \cdot \vec{E} \, \mathrm{d}\tau$$



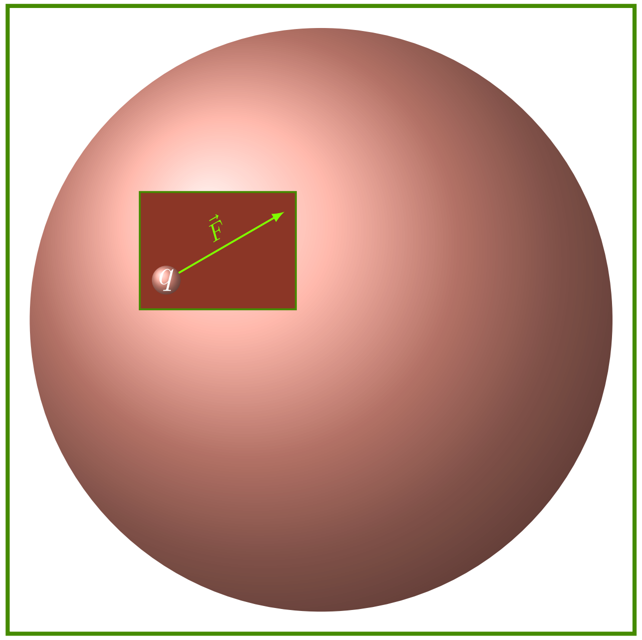
$$\vec{J} \cdot \vec{E} = -\frac{1}{\mu_0} \vec{\nabla} \cdot \vec{E} \times \vec{B} - \frac{1}{2\mu_0} \frac{\partial B^2}{\partial t} - \frac{\epsilon_0}{2} \frac{\partial E^2}{\partial t}$$

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\frac{1}{\mu_0} \int_{\mathcal{A}} \vec{E} \times \vec{B} \cdot \hat{n} \, \mathrm{d}a - \frac{1}{2} \frac{\mathrm{d}}{\mathrm{d}t} \int_{\mathcal{V}} \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) \mathrm{d}\tau$$

Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\frac{1}{\mu_0}\int_{\mathcal{A}} \vec{E}\times\vec{B}\cdot\hat{n}\ \mathrm{d}a - \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{V}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)\mathrm{d}\tau$$

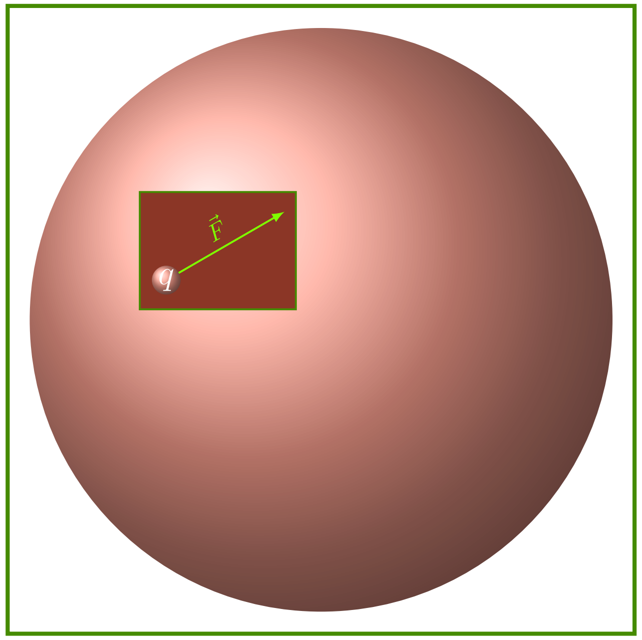


Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\frac{1}{\mu_0}\int_{\mathcal{A}} \vec{E}\times\vec{B}\cdot\hat{n}\,\mathrm{d}a - \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{V}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)\mathrm{d}\tau$$

POTÊNCIA DISSIPADA PELA FORÇA ELÉTRICA CONVERTE-SE EM ENERGIA MECÂNICA DAS CARGAS

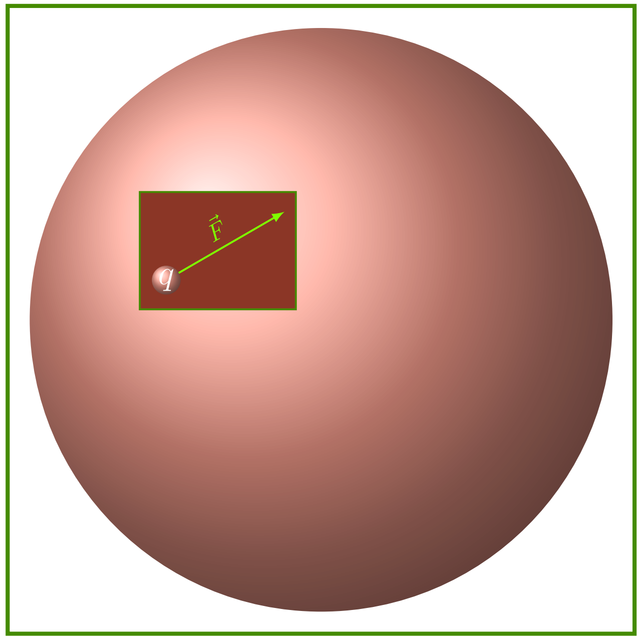


Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\frac{1}{\mu_0}\int_{\mathcal{A}} \vec{E}\times\vec{B}\cdot\hat{n}\ \mathrm{d}a - \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{V}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)\mathrm{d}\tau$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(U_{mec} + U_{em}) = -\frac{1}{\mu_0}\int_{\mathcal{A}} \vec{E}\times\vec{B}\cdot\hat{n}\ \mathrm{d}a$$



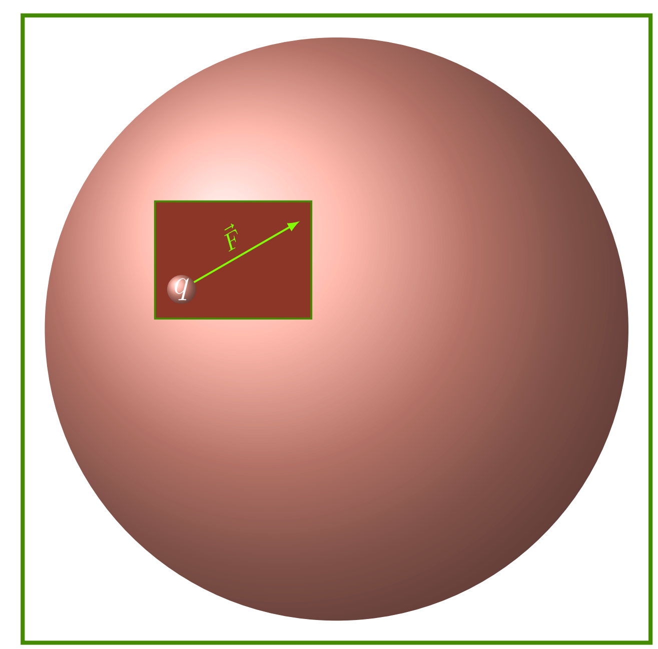
Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\frac{1}{\mu_0}\int_{\mathcal{A}} \vec{E}\times\vec{B}\cdot\hat{n}\ \mathrm{d}a - \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{V}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)\mathrm{d}\tau$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(U_{mec} + U_{em}\right) = -\frac{1}{\mu_0}\int_{\mathcal{A}} \vec{E}\times\vec{B}\cdot\hat{n}\ \mathrm{d}a$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$ => VETOR DE POYNTING



Leis de conservação

2. Energia

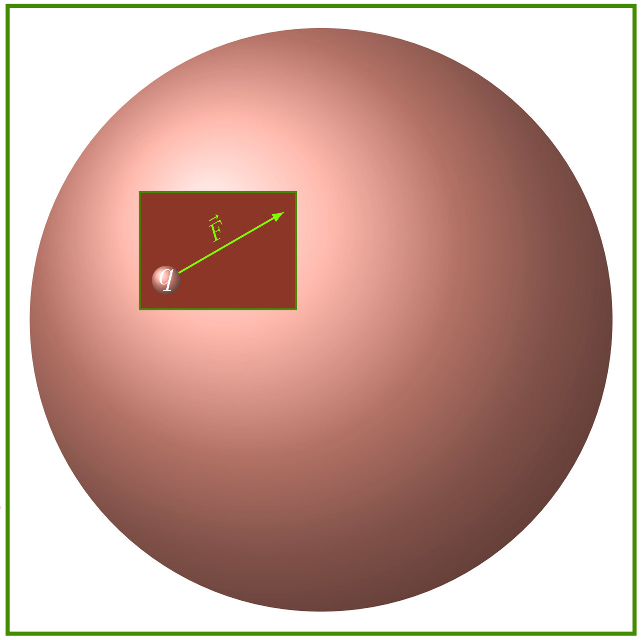
$$\frac{dW}{dt} = -\frac{1}{\mu_0}\int_{\mathcal{A}} \vec{E}\times\vec{B}\cdot\hat{n}\ da - \frac{1}{2}\frac{d}{dt}\int_{\mathcal{V}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) d\tau$$

$$\frac{d}{dt}\left(U_{mec} + U_{em}\right) = -\frac{1}{\mu_0}\int_{\mathcal{A}} \vec{E}\times\vec{B}\cdot\hat{n}\ da$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$\frac{d}{dt}\left(U_{mec} + U_{em}\right) = -\int_{\mathcal{A}} \vec{S}\cdot\hat{n}\ da$$

FLUXO DO VETOR DE POYNTING



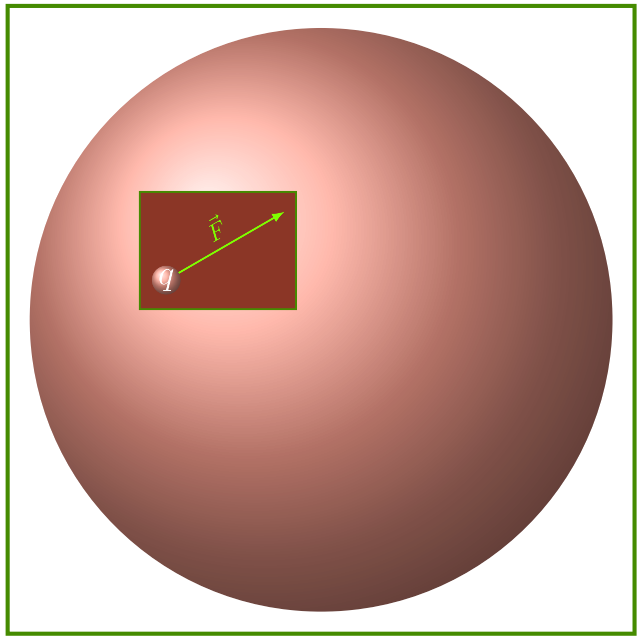
Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\frac{1}{\mu_0}\int_{\mathcal{A}} \vec{E}\times\vec{B}\cdot\hat{n}\ \mathrm{d}a - \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{V}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)\mathrm{d}\tau$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(U_{mec} + U_{em}) = -\frac{1}{\mu_0}\int_{\mathcal{A}} \vec{E}\times\vec{B}\cdot\hat{n}\ \mathrm{d}a$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$



$$\frac{\mathrm{d}}{\mathrm{d}t}(U_{mec} + U_{em}) = -\int_{\mathcal{A}} \vec{S}\cdot\hat{n}\ \mathrm{d}a$$

TEOREMA GAUSS

$$\frac{\partial}{\partial t}(u_{mec} + u_{em}) = -\vec{\nabla}\cdot\vec{S}$$

Leis de conservação

2. Energia

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\frac{1}{\mu_0}\int_{\mathcal{A}} \vec{E}\times\vec{B}\cdot\hat{n}\ \mathrm{d}a - \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\mathcal{V}}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)\mathrm{d}\tau$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$\frac{\partial}{\partial t}\left(u_{mec} + u_{em}\right) = -\vec{\nabla}\cdot\vec{S}$$

OUTRA EQUAÇÃO DE CONTINUIDADE