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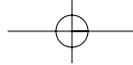
Introduction to Materials Science and Engineering

- 1-4 Steel is often coated with a thin layer of zinc if it is to be used outside. What characteristics do you think the zinc provides to this coated, or galvanized, steel? What precautions should be considered in producing this product? How will the recyclability of the product be affected?

Solution: The zinc provides corrosion resistance to the iron in two ways. If the iron is completely coated with zinc, the zinc provides a barrier between the iron and the surrounding environment, therefore protecting the underlying iron. If the zinc coating is scratched to expose the iron, the zinc continues to protect the iron because the zinc corrodes preferentially to the iron (see Chapter 23). To be effective, the zinc should bond well to the iron so that it does not permit reactions to occur at the interface with the iron and so that the zinc remains intact during any forming of the galvanized material. When the material is recycled, the zinc will be lost by oxidation and vaporization, often producing a “zinc dust” that may pose an environmental hazard. Special equipment may be required to collect and either recycle or dispose of the zinc dust.

- 1-5 We would like to produce a transparent canopy for an aircraft. If we were to use a ceramic (that is, traditional window glass) canopy, rocks or birds might cause it to shatter. Design a material that would minimize damage or at least keep the canopy from breaking into pieces.

Solution: We might sandwich a thin sheet of a transparent polymer between two layers of the glass. This approach, used for windshields of automobiles, will prevent the “safety” glass from completely disintegrating when it



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fails, with the polymer holding the broken pieces of glass together until the canopy can be replaced.

Another approach might be to use a transparent, “glassy” polymer material such as polycarbonate. Some polymers have reasonably good impact properties and may resist failure. The polymers can also be toughened to resist impact by introducing tiny globules of a rubber, or elastomer, into the polymer; these globules improve the energy-absorbing ability of the composite polymer, while being too small to interfere with the optical properties of the material.

- 1-6** Coiled springs ought to be very strong and stiff. Si_3N_4 is a strong, stiff material. Would you select this material for a spring? Explain.

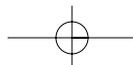
Solution: Springs are intended to resist high elastic forces, where only the atomic bonds are stretched when the force is applied. The silicon nitride would satisfy this requirement. However, we would like to also have good resistance to impact and at least some ductility (in case the spring is overloaded) to assure that the spring will not fail catastrophically. We also would like to be sure that all springs will perform satisfactorily. Ceramic materials such as silicon nitride have virtually no ductility, poor impact properties, and often are difficult to manufacture without introducing at least some small flaws that cause to fail even for relatively low forces. The silicon nitride is NOT recommended.

- 1-7** Temperature indicators are sometimes produced from a coiled metal strip that uncoils a specific amount when the temperature increases. How does this work; from what kind of material would the indicator be made; and what are the important properties that the material in the indicator must possess?

Solution: Bimetallic materials are produced by bonding two materials having different coefficients of thermal expansion to one another, forming a laminar composite. When the temperature changes, one of the materials will expand or contract more than the other material. This difference in expansion or contraction causes the bimetallic material to change shape; if the original shape is that of a coil, then the device will coil or uncoil, depending on the direction of the temperature change. In order for the material to perform well, the two materials must have very different coefficients of thermal expansion and should have high enough modulus of elasticity so that no permanent deformation of the material occurs.

- 1-8** You would like to design an aircraft that can be flown by human power nonstop for a distance of 30 km. What types of material properties would you recommend? What materials might be appropriate?

Solution: Such an aircraft must possess enough strength and stiffness to resist its own weight, the weight of the human “power source”, and any aerodynamic forces imposed on it. On the other hand, it must be as light as possible to assure that the human can generate enough work to operate the aircraft. Composite materials, particularly those based on a polymer matrix, might comprise the bulk of the aircraft. The polymers have a light weight (with densities of less than half that of aluminum) and can be strengthened by introducing strong, stiff fibers made of glass, carbon, or other polymers. Composites having the strength and stiffness



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of steel, but with only a fraction of the weight, can be produced in this manner.

- 1-9** You would like to place a three-foot diameter microsatellite into orbit. The satellite will contain delicate electronic equipment that will send and receive radio signals from earth. Design the outer shell within which the electronic equipment is contained. What properties will be required, and what kind of materials might be considered?

Solution: The shell of the microsatellite must satisfy several criteria. The material should have a low density, minimizing the satellite weight so that it can be lifted economically into its orbit; the material must be strong, hard, and impact resistant in order to assure that any “space dust” that might strike the satellite does not penetrate and damage the electronic equipment; the material must be transparent to the radio signals that provide communication between the satellite and earth; and the material must provide some thermal insulation to assure that solar heating does not damage the electronics.

One approach might be to use a composite shell of several materials. The outside surface might be a very thin reflective metal coating that would help reflect solar heat. The main body of the shell might be a light weight fiber-reinforced composite that would provide impact resistance (preventing penetration by dust particles) but would be transparent to radio signals.

- 1-10** What properties should the head of a carpenter’s hammer possess? How would you manufacture a hammer head?

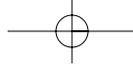
Solution: The head for a carpenter’s hammer is produced by forging, a metal-working process; a simple steel shape is heated and formed in several steps while hot into the required shape. The head is then heat treated to produce the required mechanical and physical properties.

The striking face and claws of the hammer should be hard—the metal should not dent or deform when driving or removing nails. Yet these portions must also possess some impact resistance, particularly so that chips do not flake off the striking face and cause injuries.

- 1-11** The hull of the space shuttle consists of ceramic tiles bonded to an aluminum skin. Discuss the design requirements of the shuttle hull that led to the use of this combination of materials. What problems in producing the hull might the designers and manufacturers have faced?

Solution: The space shuttle experiences extreme temperatures during re-entry into earth’s atmosphere; consequently a thermal protection system must be used to prevent damage to the structure of the shuttle (not to mention its contents!). The skin must therefore be composed of a material that has an exceptionally low thermal conductivity. The material must be capable of being firmly attached to the skin of the shuttle and to be easily repaired when damage occurs.

The tiles used on the space shuttle are composed of silica fibers bonded together to produce a very low density ceramic. The thermal conductivity is so low that a person can hold on to one side of the tile while the opposite surface is red hot. The tiles are attached to the shuttle

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skin using a rubbery polymer that helps assure that the forces do not break the tile loose, which would then expose the underlying skin to high temperatures.

- 1-12** You would like to select a material for the electrical contacts in an electrical switching device which opens and closes frequently and forcefully. What properties should the contact material possess? What type of material might you recommend? Would Al_2O_3 be a good choice? Explain.

Solution: The material must have a high electrical conductivity to assure that no electrical heating or arcing occurs when the switch is closed. High purity (and therefore very soft) metals such as copper, aluminum, silver or gold provide the high conductivity. However, the device must also have good wear resistance, requiring that the material be hard. Most hard, wear resistant materials have poor electrical conductivity.

One solution to this problem is to produce a particulate composite material composed of hard ceramic particles embedded in a continuous matrix of the electrical conductor. For example, silicon carbide particles could be introduced into pure aluminum; the silicon carbide particles provide wear resistance while aluminum provides conductivity. Other examples of these materials are described in Chapter 17.

Al_2O_3 by itself would not be a good choice—alumina is a ceramic material and is an electrical insulator. However, alumina particles dispersed into a copper matrix might provide wear resistance to the composite.

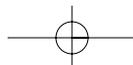
- 1-13** Aluminum has a density of 2.7 g/cm^3 . Suppose you would like to produce a composite material based on aluminum having a density of 1.5 g/cm^3 . Design a material that would have this density. Would introducing beads of polyethylene, with a density of 0.95 g/cm^3 , into the aluminum be a likely possibility? Explain.

Solution: In order to produce an aluminum-matrix composite material with a density of 1.5 g/cm^3 , we would need to select a material having a density considerably less than 1.5 g/cm^3 . While polyethylene's density would make it a possibility, the polyethylene has a very low melting point compared to aluminum; this would make it very difficult to introduce the polyethylene into a solid aluminum matrix—processes such as casting or powder metallurgy would destroy the polyethylene. Therefore polyethylene would NOT be a likely possibility.

One approach, however, might be to introduce *hollow* glass beads. Although ceramic glasses have densities comparable to that of aluminum, a hollow bead will have a very low density. The glass also has a high melting temperature and could be introduced into liquid aluminum for processing as a casting.

- 1-14** You would like to be able to identify different materials without resorting to chemical analysis or lengthy testing procedures. Describe some possible testing and sorting techniques you might be able to use based on the physical properties of materials.

Solution: Some typical methods might include: measuring the density of the material (may help in separating metal groups such as aluminum, copper, steel, magnesium, etc.), determining the electrical conductivity



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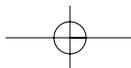
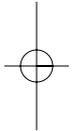
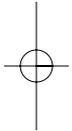
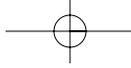
of the material (may help in separating ceramics and polymers from metallic alloys), measuring the hardness of the material (perhaps even just using a file), and determining whether the material is magnetic or nonmagnetic (may help separate iron from other metallic alloys).

- 1–15** You would like to be able to physically separate different materials in a scrap recycling plant. Describe some possible methods that might be used to separate materials such as polymers, aluminum alloys, and steels from one another.

Solution: Steels can be magnetically separated from the other materials; steel (or carbon-containing iron alloys) are ferromagnetic and will be attracted by magnets. Density differences could be used—polymers have a density near that of water; the specific gravity of aluminum alloys is around 2.7; that of steels is between 7.5 and 8. Electrical conductivity measurements could be used—polymers are insulators, aluminum has a particularly high electrical conductivity.

- 1–16** Some pistons for automobile engines might be produced from a composite material containing small, hard silicon carbide particles in an aluminum alloy matrix. Explain what benefits each material in the composite may provide to the overall part. What problems might the different properties of the two materials cause in producing the part?

Solution: Aluminum provides good heat transfer due to its high thermal conductivity. It has good ductility and toughness, reasonably good strength, and is easy to cast and process. The silicon carbide, a ceramic, is hard and strong, providing good wear resistance, and also has a high melting temperature. It provides good strength to the aluminum, even at elevated temperatures. However there may be problems producing the material—for example, the silicon carbide may not be uniformly distributed in the aluminum matrix if the pistons are produced by casting. We need to assure good bonding between the particles and the aluminum—the surface chemistry must therefore be understood. Differences in expansion and contraction with temperature changes may cause debonding and even cracking in the composite.



2

Atomic Structure

- 2–6 (a)** Aluminum foil used for storing food weighs about 0.3 g per square inch. How many atoms of aluminum are contained in one square inch of foil?

Solution: In a one square inch sample:

$$\text{number} = \frac{(0.3 \text{ g})(6.02 \times 10^{23} \text{ atoms/mol})}{26.981 \text{ g/mol}} = 6.69 \times 10^{21} \text{ atoms}$$

- (b)** Using the densities and atomic weights given in Appendix A, calculate and compare the number of atoms per cubic centimeter in (i) lead and (ii) lithium.

Solution: (i) In lead:

$$\frac{(11.36 \text{ g/cm}^3)(1 \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})}{207.19 \text{ g/mol}} = 3.3 \times 10^{22} \text{ atoms/cm}^3$$

(ii) In lithium:

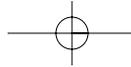
$$\frac{(0.534 \text{ g/cm}^3)(1 \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})}{6.94 \text{ g/mol}} = 4.63 \times 10^{22} \text{ atoms/cm}^3$$

- 2–7 (a)** Using data in Appendix A, calculate the number of iron atoms in one ton (2000 pounds).

Solution:
$$\frac{(2000 \text{ lb})(454 \text{ g/lb})(6.02 \times 10^{23} \text{ atoms/mol})}{55.847 \text{ g/mol}} = 9.79 \times 10^{27} \text{ atoms/ton}$$

- (b)** Using data in Appendix A, calculate the volume in cubic centimeters occupied by one mole of boron.

Solution:
$$\frac{(1 \text{ mol})(10.81 \text{ g/mol})}{2.3 \text{ g/cm}^3} = 4.7 \text{ cm}^3$$



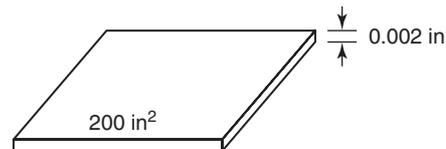
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- 2-8** In order to plate a steel part having a surface area of 200 in.² with a 0.002 in. thick layer of nickel, (a) how many atoms of nickel are required and (b) how many moles of nickel are required?

Solution: Volume = (200 in.²)(0.002 in.)(2.54 cm/in.)³ = 6.555 cm³

(a) $\frac{(6.555 \text{ cm}^3)(8.902 \text{ g/cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})}{58.71 \text{ g/mol}} = 5.98 \times 10^{23} \text{ atoms}$

(b) $\frac{(6.555 \text{ cm}^3)(8.902 \text{ g/cm}^3)}{58.71 \text{ g/mol}} = 0.994 \text{ mol Ni required}$



- 2-9** Suppose an element has a valence of 2 and an atomic number of 27. Based only on the quantum numbers, how many electrons must be present in the 3d energy level?

Solution: We can let x be the number of electrons in the 3d energy level. Then:

$$1s^2 2s^2 2p^6 3s^2 3p^6 3d^x 4s^2 \quad (\text{must be 2 electrons in } 4s \text{ for valence} = 2)$$

Since $27 - (2+2+6+2+6+2) = 7 = x$ there must be 7 electrons in the 3d level.

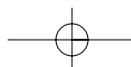
- 2-11** Bonding in the intermetallic compound Ni₃Al is predominantly metallic. Explain why there will be little, if any, ionic bonding component. The electronegativity of nickel is about 1.8.

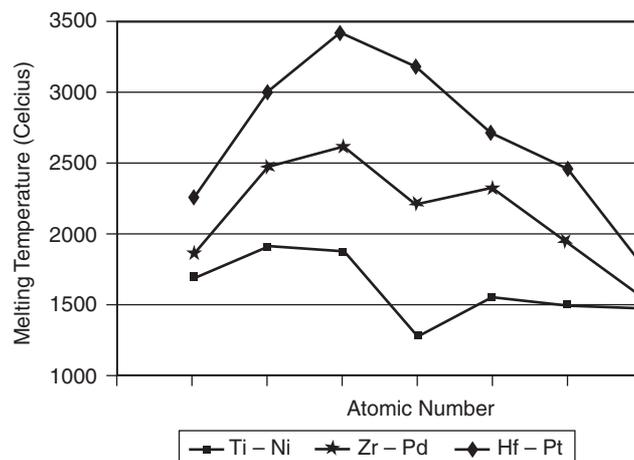
Solution: The electronegativity of Al is 1.5, while that of Ni is 1.8. These values are relatively close, so we wouldn't expect much ionic bonding. Also, both are metals and prefer to give up their electrons rather than share or donate them.

- 2-12** Plot the melting temperatures of elements in the 4A to 8-10 columns of the periodic table versus atomic number (i.e., plot melting temperatures of Ti through Ni, Zr through Pd, and Hf through Pt). Discuss these relationships, based on atomic bonding and binding energy, (a) as the atomic number increases in each row of the periodic table and (b) as the atomic number increases in each column of the periodic table.

Solution:

Ti - 1668	Zr - 1852	Hf - 2227
V - 1900	Nb - 2468	Ta - 2996
Cr - 1875	Mo - 2610	W - 3410
Mn - 1244	Tc - 2200	Re - 3180
Fe - 1538	Ru - 2310	Os - 2700
Co - 1495	Rh - 1963	Ir - 2447
Ni - 1453	Pd - 1552	Pt - 1769



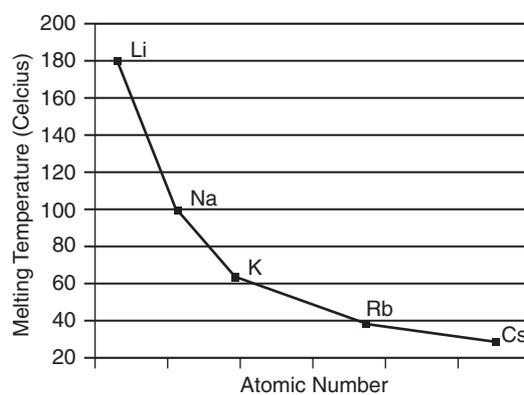


For each row, the melting temperature is highest when the outer “*d*” energy level is partly full. In Cr, there are 5 electrons in the 3*d* shell; in Mo, there are 5 electrons in the 4*d* shell; in W there are 4 electrons in the 5*d* shell. In each column, the melting temperature increases as the atomic number increases—the atom cores contain a larger number of tightly held electrons, making the metals more stable.

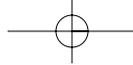
- 2-13** Plot the melting temperature of the elements in the 1A column of the periodic table versus atomic number (i.e., plot melting temperatures of Li through Cs). Discuss this relationship, based on atomic bonding and binding energy.

Solution:

Element	$T(^{\circ}\text{C})$
Li	180.7
Na	97.8
K	63.2
Rb	38.9



As the atomic number increases, the melting temperature decreases, opposite that found in Problem 2-12.



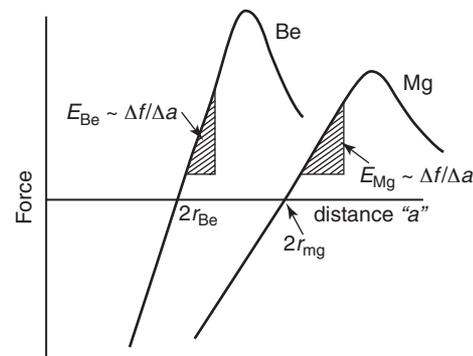
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2-14 Calculate the fraction of bonding of MgO that is ionic.

Solution: $E_{\text{Mg}} = 1.2$ $E_{\text{O}} = 3.5$
 $f_{\text{covalent}} = \exp[(-0.25)(3.5 - 1.2)^2] = \exp(-1.3225) = 0.266$
 $f_{\text{ionic}} = 1 - 0.266 = 0.734 \therefore$ bonding is mostly ionic

2-18 Beryllium and magnesium, both in the 2A column of the periodic table, are light-weight metals. Which would you expect to have the higher modulus of elasticity? Explain, considering binding energy and atom radii and using appropriate sketches of force versus interatomic spacing.

Solution: 4 Be $1s^2 2s^2$ $E = 42 \times 10^6$ psi $r_{\text{Be}} = 1.143 \text{ \AA}$
 12 Mg $1s^2 2s^2 2p^6 3s^2$ $E = 6 \times 10^6$ psi $r_{\text{Mg}} = 1.604 \text{ \AA}$



The smaller Be electrons are held closer to the core \therefore held more tightly, giving a higher binding energy.

2-19 Would you expect MgO or magnesium to have the higher modulus of elasticity? Explain.

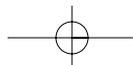
Solution: MgO has ionic bonds, which are strong compared to the metallic bonds in Mg. A higher force will be required to cause the same separation between the ions in MgO compared to the atoms in Mg. Therefore, MgO should have the higher modulus of elasticity. In Mg, $E \approx 6 \times 10^6$ psi; in MgO, $E = 30 \times 10^6$ psi.

2-20 Aluminum and silicon are side-by-side in the periodic table. Which would you expect to have the higher modulus of elasticity (E)? Explain.

Solution: Silicon has covalent bonds; aluminum has metallic bonds. Therefore, Si should have a higher modulus of elasticity.

2-21 Steel is coated with a thin layer of ceramic to help protect against corrosion. What do you expect to happen to the coating when the temperature of the steel is increased significantly? Explain.

Solution: Ceramics are expected to have a low coefficient of thermal expansion due to strong ionic/covalent bonds; steel has a high thermal expansion coefficient. When the structure heats, steel expands more than the coating, which may crack and expose the underlying steel to corrosion.



3

Atomic and Ionic Arrangements

- 3-13** Calculate the atomic radius in cm for the following: (a) BCC metal with $a_0 = 0.3294$ nm and one atom per lattice point; and (b) FCC metal with $a_0 = 4.0862$ Å and one atom per lattice point.

Solution: (a) For BCC metals,

$$r = \frac{(\sqrt{3})a_0}{4} = \frac{(\sqrt{3})(0.3294 \text{ nm})}{4} = 0.1426 \text{ nm} = 1.426 \times 10^{-8} \text{ cm}$$

(b) For FCC metals,

$$r = \frac{(\sqrt{2})a_0}{4} = \frac{(\sqrt{2})(4.0862 \text{ Å})}{4} = 1.4447 \text{ Å} = 1.4447 \times 10^{-8} \text{ cm}$$

- 3-14** Determine the crystal structure for the following: (a) a metal with $a_0 = 4.9489$ Å, $r = 1.75$ Å and one atom per lattice point; and (b) a metal with $a_0 = 0.42906$ nm, $r = 0.1858$ nm and one atom per lattice point.

Solution: We want to determine if “ x ” in the calculations below equals $\sqrt{2}$ (for FCC) or $\sqrt{3}$ (for BCC):

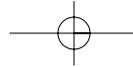
$$(a) (x)(4.9489 \text{ Å}) = (4)(1.75 \text{ Å})$$

$$x = \sqrt{2}, \text{ therefore FCC}$$

$$(b) (x)(0.42906 \text{ nm}) = (4)(0.1858 \text{ nm})$$

$$x = \sqrt{3}, \text{ therefore BCC}$$

- 3-15** The density of potassium, which has the BCC structure and one atom per lattice point, is 0.855 g/cm^3 . The atomic weight of potassium is 39.09 g/mol . Calculate (a) the lattice parameter; and (b) the atomic radius of potassium.



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Solution: (a) Using Equation 3-5:

$$0.855 \text{ g/cm}^3 = \frac{(2 \text{ atoms/cell})(39.09 \text{ g/mol})}{(a_0)^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$a_0^3 = 1.5189 \times 10^{-22} \text{ cm}^3 \text{ or } a_0 = 5.3355 \times 10^{-8} \text{ cm}$$

(b) From the relationship between atomic radius and lattice parameter:

$$r = \frac{(\sqrt{3})(5.3355 \times 10^{-8} \text{ cm})}{4} = 2.3103 \times 10^{-8} \text{ cm}$$

3-16 The density of thorium, which has the FCC structure and one atom per lattice point, is 11.72 g/cm^3 . The atomic weight of thorium is 232 g/mol . Calculate (a) the lattice parameter and (b) the atomic radius of thorium.

Solution: (a) From Equation 3-5:

$$11.72 \text{ g/cm}^3 = \frac{(4 \text{ atoms/cell})(232 \text{ g/mol})}{(a_0)^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$a_0^3 = 1.315297 \times 10^{-22} \text{ cm}^3 \text{ or } a_0 = 5.0856 \times 10^{-8} \text{ cm}$$

(b) From the relationship between atomic radius and lattice parameter:

$$r = \frac{(\sqrt{2})(5.0856 \times 10^{-8} \text{ cm})}{4} = 1.7980 \times 10^{-8} \text{ cm}$$

3-17 A metal having a cubic structure has a density of 2.6 g/cm^3 , an atomic weight of 87.62 g/mol , and a lattice parameter of 6.0849 \AA . One atom is associated with each lattice point. Determine the crystal structure of the metal.

$$\text{Solution: } 2.6 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(87.62 \text{ g/mol})}{(6.0849 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

$x = 4$, therefore FCC

3-18 A metal having a cubic structure has a density of 1.892 g/cm^3 , an atomic weight of 132.91 g/mol , and a lattice parameter of 6.13 \AA . One atom is associated with each lattice point. Determine the crystal structure of the metal.

$$\text{Solution: } 1.892 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(132.91 \text{ g/mol})}{(6.13 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

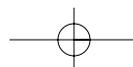
$x = 2$, therefore BCC

3-19 Indium has a tetragonal structure with $a_0 = 0.32517 \text{ nm}$ and $c_0 = 0.49459 \text{ nm}$. The density is 7.286 g/cm^3 and the atomic weight is 114.82 g/mol . Does indium have the simple tetragonal or body-centered tetragonal structure?

Solution:

$$7.286 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(114.82 \text{ g/mol})}{(3.2517 \times 10^{-8} \text{ cm})^2(4.9459 \times 10^{-8} \text{ cm})(6.02 \times 10^{23} \text{ atoms/mol})}$$

$x = 2$, therefore BCT (body-centered tetragonal)



- 3-20** Bismuth has a hexagonal structure, with $a_0 = 0.4546$ nm and $c_0 = 1.186$ nm. The density is 9.808 g/cm³ and the atomic weight is 208.98 g/mol. Determine (a) the volume of the unit cell and (b) the number of atoms in each unit cell.

Solution: (a) The volume of the unit cell is $V = a_0^2 c_0 \cos 30$.

$$V = (0.4546 \text{ nm})^2 (1.186 \text{ nm}) (\cos 30) = 0.21226 \text{ nm}^3 \\ = 2.1226 \times 10^{-22} \text{ cm}^3$$

(b) If “ x ” is the number of atoms per unit cell, then:

$$9.808 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(208.98 \text{ g/mol})}{(2.1226 \times 10^{-22} \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$x = 6 \text{ atoms/cell}$$

- 3-21** Gallium has an orthorhombic structure, with $a_0 = 0.45258$ nm, $b_0 = 0.45186$ nm, and $c_0 = 0.76570$ nm. The atomic radius is 0.1218 nm. The density is 5.904 g/cm³ and the atomic weight is 69.72 g/mol. Determine (a) the number of atoms in each unit cell and (b) the packing factor in the unit cell.

Solution: The volume of the unit cell is $V = a_0 b_0 c_0$ or

$$V = (0.45258 \text{ nm})(0.45186 \text{ nm})(0.76570 \text{ nm}) = 0.1566 \text{ nm}^3 \\ = 1.566 \times 10^{-22} \text{ cm}^3$$

(a) From the density equation:

$$5.904 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(69.72 \text{ g/mol})}{(1.566 \times 10^{-22} \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$x = 8 \text{ atoms/cell}$$

(b) From the packing factor (PF) equation:

$$\text{PF} = \frac{(8 \text{ atoms/cell})(4\pi/3)(0.1218 \text{ nm})^3}{0.1566 \text{ nm}^3} = 0.387$$

- 3-22** Beryllium has a hexagonal crystal structure, with $a_0 = 0.22858$ nm and $c_0 = 0.35842$ nm. The atomic radius is 0.1143 nm, the density is 1.848 g/cm³, and the atomic weight is 9.01 g/mol. Determine (a) the number of atoms in each unit cell and (b) the packing factor in the unit cell.

Solution: $V = (0.22858 \text{ nm})^2 (0.35842 \text{ nm}) \cos 30 = 0.01622 \text{ nm}^3 = 16.22 \times 10^{-24} \text{ cm}^3$

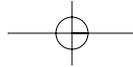
(a) From the density equation:

$$1.848 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(9.01 \text{ g/mol})}{(16.22 \times 10^{-24} \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$x = 2 \text{ atoms/cell}$$

(b) The packing factor (PF) is:

$$\text{PF} = \frac{(2 \text{ atoms/cell})(4\pi/3)(0.1143 \text{ nm})^3}{0.01622 \text{ nm}^3} = 0.77$$


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- 3-23** A typical paper clip weighs 0.59 g and consists of BCC iron. Calculate (a) the number of unit cells and (b) the number of iron atoms in the paper clip. (See Appendix A for required data)

Solution: The lattice parameter for BCC iron is 2.866×10^{-8} cm. Therefore

$$V_{\text{unit cell}} = (2.866 \times 10^{-8} \text{ cm})^3 = 2.354 \times 10^{-23} \text{ cm}^3$$

(a) The density is 7.87 g/cm^3 . The number of unit cells is:

$$\text{number} = \frac{0.59 \text{ g}}{(7.87 \text{ g/cm}^3)(2.354 \times 10^{-23} \text{ cm}^3/\text{cell})} = 3.185 \times 10^{21} \text{ cells}$$

(b) There are 2 atoms/cell in BCC iron. The number of atoms is:

$$\text{number} = (3.185 \times 10^{21} \text{ cells})(2 \text{ atoms/cell}) = 6.37 \times 10^{21} \text{ atoms}$$

- 3-24** Aluminum foil used to package food is approximately 0.001 inch thick. Assume that all of the unit cells of the aluminum are arranged so that a_0 is perpendicular to the foil surface. For a 4 in. \times 4 in. square of the foil, determine (a) the total number of unit cells in the foil and (b) the thickness of the foil in number of unit cells. (See Appendix A.)

Solution: The lattice parameter for aluminum is 4.04958×10^{-8} cm. Therefore:

$$V_{\text{unit cell}} = (4.04958 \times 10^{-8})^3 = 6.6409 \times 10^{-23} \text{ cm}^3$$

The volume of the foil is:

$$V_{\text{foil}} = (4 \text{ in.})(4 \text{ in.})(0.001 \text{ in.}) = 0.016 \text{ in.}^3 = 0.262 \text{ cm}^3$$

(a) The number of unit cells in the foil is:

$$\text{number} = \frac{0.262 \text{ cm}^3}{6.6409 \times 10^{-23} \text{ cm}^3/\text{cell}} = 3.945 \times 10^{21} \text{ cells}$$

(b) The thickness of the foil, in number of unit cells, is:

$$\text{number} = \frac{(0.001 \text{ in.})(2.54 \text{ cm/in.})}{4.04958 \times 10^{-8} \text{ cm}} = 6.27 \times 10^4 \text{ cells}$$

- 3-27** Above 882°C , titanium has a BCC crystal structure, with $a = 0.332$ nm. Below this temperature, titanium has a HCP structure, with $a = 0.2978$ nm and $c = 0.4735$ nm. Determine the percent volume change when BCC titanium transforms to HCP titanium. Is this a contraction or expansion?

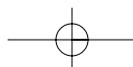
Solution: We can find the volume of each unit cell. Two atoms are present in both BCC and HCP titanium unit cells, so the volumes of the unit cells can be directly compared.

$$V_{\text{BCC}} = (0.332 \text{ nm})^3 = 0.03659 \text{ nm}^3$$

$$V_{\text{HCP}} = (0.2978 \text{ nm})^2(0.4735 \text{ nm})\cos 30^\circ = 0.03637 \text{ nm}^3$$

$$\Delta V = \frac{V_{\text{HCP}} - V_{\text{BCC}}}{V_{\text{BCC}}} \times 100 = \frac{0.03637 \text{ nm}^3 - 0.03659 \text{ nm}^3}{0.03659 \text{ nm}^3} \times 100 = -0.6\%$$

Therefore titanium *contracts* 0.6% during cooling.



- 3-28** α -Mn has a cubic structure with $a_0 = 0.8931$ nm and a density of 7.47 g/cm³. β -Mn has a different cubic structure, with $a_0 = 0.6326$ nm and a density of 7.26 g/cm³. The atomic weight of manganese is 54.938 g/mol and the atomic radius is 0.112 nm. Determine the percent volume change that would occur if α -Mn transforms to β -Mn.

Solution: First we need to find the number of atoms in each unit cell so we can determine the volume change based on equal numbers of atoms. From the density equation, we find for the α -Mn:

$$7.47 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(54.938 \text{ g/mol})}{(8.931 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$x = 58 \text{ atoms/cell} \quad V_{\alpha\text{-Mn}} = (8.931 \times 10^{-8} \text{ cm})^3 = 7.12 \times 10^{-22} \text{ cm}^3$$

For β -Mn:

$$7.26 \text{ g/cm}^3 = \frac{(x \text{ atoms/cell})(54.938 \text{ g/mol})}{(6.326 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$x = 20 \text{ atoms/cell} \quad V_{\beta\text{-Mn}} = (6.326 \times 10^{-8} \text{ cm})^3 = 2.53 \times 10^{-22} \text{ cm}^3$$

The volume of the β -Mn can be adjusted by a factor of $58/20$, to account for the different number of atoms per cell. The volume change is then:

$$\Delta V = \frac{(58/20)V_{\beta\text{-Mn}} - V_{\alpha\text{-Mn}}}{V_{\alpha\text{-Mn}}} \times 100 = \frac{(58/20)(2.53) - 7.12}{7.12} \times 100 = +3.05\%$$

The manganese *expands* by 3.05% during the transformation.

- 3-37** Determine the Miller indices for the directions in the cubic unit cell shown in Figure 3-35.

Solution: A: $0,1,0 - 0,1,1 = 0,0,-1 = [00\bar{1}]$

B: $\frac{1}{2},0,0 - 0,1,0 = \frac{1}{2},-1,0 = [\frac{1}{2}\bar{1}0]$

C: $0,1,1 - 1,0,0 = -1,1,1 = [\bar{1}11]$

D: $1,0,\frac{1}{2} - 0,\frac{1}{2},1 = 1,-\frac{1}{2},-\frac{1}{2} = [2\bar{1}\bar{1}]$

- 3-38** Determine the indices for the directions in the cubic unit cell shown in Figure 3-36.

Solution: A: $0,0,1 - 1,0,0 = -1,0,1 = [\bar{1}01]$

B: $1,0,1 - \frac{1}{2},1,0 = \frac{1}{2},-1,1 = [\frac{1}{2}\bar{1}2]$

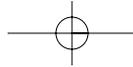
C: $1,0,0 - 0,\frac{3}{4},1 = 1,-\frac{3}{4},-1 = [4\bar{3}\bar{4}]$

D: $0,1,\frac{1}{2} - 0,0,0 = 0,1,\frac{1}{2} = [021]$

- 3-39** Determine the indices for the planes in the cubic unit cell shown in Figure 3-37.

Solution: A: $x = 1 \quad 1/x = 1$
 $y = -1 \quad 1/y = -1 \quad (1\bar{1}1)$
 $z = 1 \quad 1/z = 1$

B: $x = \infty \quad 1/x = 0$
 $y = \frac{1}{3} \quad 1/y = 3 \quad (030)$
 $z = \infty \quad 1/z = 0$


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$$\begin{aligned}
 C: \quad x &= 1 & 1/x &= 1 \\
 y &= \infty & 1/y &= 0 & (10\bar{2}) & \text{(origin at 0,0,1)} \\
 z &= -1/2 & 1/z &= -2
 \end{aligned}$$

3-40 Determine the indices for the planes in the cubic unit cell shown in Figure 3-38.

$$\begin{aligned}
 \text{Solution: } A: \quad x &= -1 & 1/x &= -1 \times 3 = -3 \\
 y &= 1/2 & 1/y &= 2 \times 3 = 6 & (\bar{3}64) & \text{(origin at 1,0,0)} \\
 z &= 3/4 & 1/z &= 4/3 \times 3 = 4
 \end{aligned}$$

$$\begin{aligned}
 B: \quad x &= 1 & 1/x &= 1 \times 3 = 3 \\
 y &= -3/4 & 1/y &= -4/3 \times 3 = -4 & (3\bar{4}0) & \text{(origin at 0,1,0)} \\
 z &= \infty & 1/z &= 0 \times 3 = 0
 \end{aligned}$$

$$\begin{aligned}
 C: \quad x &= 2 & 1/x &= 1/2 \times 6 = 3 \\
 y &= 3/2 & 1/y &= 2/3 \times 6 = 4 & (346) \\
 z &= 1 & 1/z &= 1 \times 6 = 6
 \end{aligned}$$

3-41 Determine the indices for the directions in the hexagonal lattice shown in Figure 3-39, using both the three-digit and four-digit systems.

$$\begin{aligned}
 \text{Solution: } A: \quad 1, -1, 0 - 0, 0, 0 &= 1, -1, 0 = [1\bar{1}0] \\
 h &= 1/3(2 + 1) = 1 \\
 k &= 1/3(-2 - 1) = -1 = [1\bar{1}00] \\
 i &= -1/3(1 - 1) = 0 \\
 l &= 0
 \end{aligned}$$

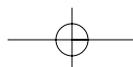
$$\begin{aligned}
 B: \quad 1, 1, 0 - 0, 0, 1 &= 1, 1, -1 = [11\bar{1}] \\
 h &= 1/3(2 - 1) = 1/3 \\
 k &= 1/3(2 - 1) = 1/3 = [11\bar{2}\bar{3}] \\
 i &= -1/3(1 + 1) = -2/3 \\
 l &= -1
 \end{aligned}$$

$$\begin{aligned}
 C: \quad 0, 1, 1 - 0, 0, 0 &= 0, 1, 1 = [011] \\
 h &= 1/3(0 - 1) = -1/3 \\
 k &= 1/3(2 - 0) = 2/3 \\
 i &= -1/3(0 + 1) = -1/3 = [\bar{1}2\bar{1}3] \\
 l &= 1
 \end{aligned}$$

3-42 Determine the indices for the directions in the hexagonal lattice shown in Figure 3-40, using both the three-digit and four-digit systems.

$$\begin{aligned}
 \text{Solution: } A: \quad 0, 1, 1 - 1/2, 1, 0 &= -1/2, 0, 1 = [\bar{1}02] \\
 h &= 1/3(-2 - 0) = -2/3 \\
 k &= 1/3(0 + 1) = 1/3 = [\bar{2}116] \\
 i &= -1/3(-1 + 0) = 1/3 \\
 l &= 2
 \end{aligned}$$

$$\begin{aligned}
 B: \quad 1, 0, 0 - 1, 1, 1 &= 0, -1, -1 = [0\bar{1}\bar{1}] \\
 h &= 1/3(0 + 1) = 1/3 \\
 k &= 1/3(-2 + 0) = -2/3 = [1\bar{2}\bar{1}\bar{3}] \\
 i &= -1/3(0 - 1) = 1/3 \\
 l &= -1
 \end{aligned}$$



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$$\begin{aligned}
 C: 0,0,0 - 1,0,1 &= -1,0,-1 = [\bar{1}0\bar{1}] \\
 h &= \frac{1}{3}(-2 + 0) = -\frac{2}{3} \\
 k &= \frac{1}{3}(0 + 1) = \frac{1}{3} = [\bar{2}11\bar{3}] \\
 i &= -\frac{1}{3}(-1 + 0) = \frac{1}{3} \\
 l &= -1
 \end{aligned}$$

3-43 Determine the indices for the planes in the hexagonal lattice shown in Figure 3-41.

Solution:

A: $a_1 = 1$ $1/a_1 = 1$
 $a_2 = -1$ $1/a_2 = -1$ $(1\bar{1}01)$ (origin at $a_2 = 1$)
 $a_3 = \infty$ $1/a_3 = 0$
 $c = 1$ $1/c = 1$

B: $a_1 = \infty$ $1/a_1 = 0$
 $a_2 = \infty$ $1/a_2 = 0$ (0003)
 $a_3 = \infty$ $1/a_3 = 0$
 $c = \frac{2}{3}$ $1/c = \frac{3}{2}$

C: $a_1 = 1$ $1/a_1 = 1$
 $a_2 = -1$ $1/a_2 = -1$ $(1\bar{1}00)$
 $a_3 = \infty$ $1/a_3 = 0$
 $c = \infty$ $1/c = 0$

3-44 Determine the indices for the planes in the hexagonal lattice shown in Figure 3-42.

Solution:

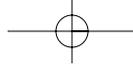
A: $a_1 = 1$ $1/a_1 = 1$
 $a_2 = -1$ $1/a_2 = -1$ $(1\bar{1}02)$
 $a_3 = \infty$ $1/a_3 = 0$
 $c = \frac{1}{2}$ $1/c = 2$

B: $a_1 = \infty$ $1/a_1 = 0$
 $a_2 = 1$ $1/a_2 = 1$ $(01\bar{1}1)$
 $a_3 = -1$ $1/a_3 = -1$
 $c = 1$ $1/c = 1$

C: $a_1 = -1$ $1/a_1 = -1$
 $a_2 = \frac{1}{2}$ $1/a_2 = 2$ $(\bar{1}2\bar{1}0)$
 $a_3 = -1$ $1/a_3 = -1$
 $c = \infty$ $1/c = 0$

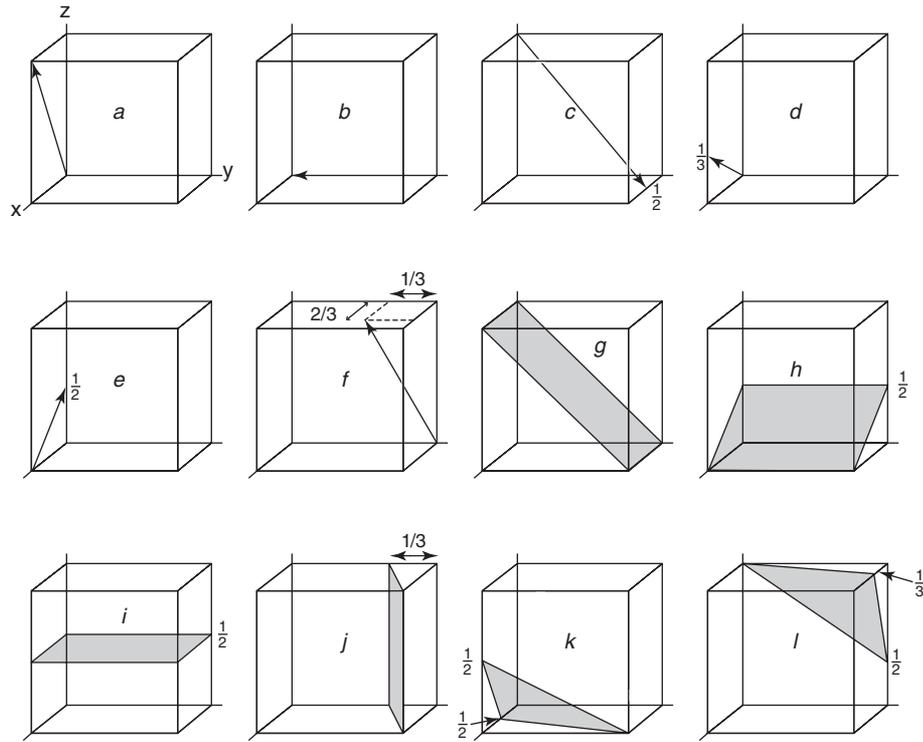
3-45 Sketch the following planes and directions within a cubic unit cell.

- (a) $[101]$ (b) $[0\bar{1}0]$ (c) $[12\bar{2}]$ (d) $[301]$ (e) $[\bar{2}01]$ (f) $[2\bar{1}3]$
 (g) $(0\bar{1}\bar{1})$ (h) (102) (i) (002) (j) $(1\bar{3}0)$ (k) $(\bar{2}12)$ (l) $(3\bar{1}\bar{2})$



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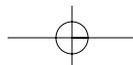
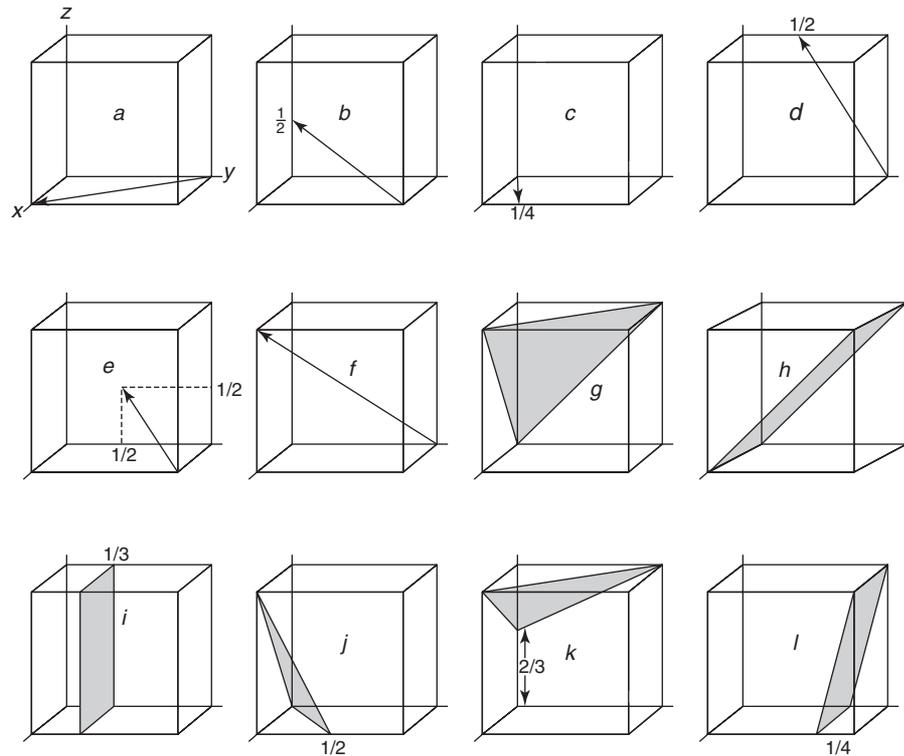
Solution:



3-46 Sketch the following planes and directions within a cubic unit cell.

- (a) $[1\bar{1}0]$ (b) $[\bar{2}\bar{2}1]$ (c) $[410]$ (d) $[0\bar{1}2]$ (e) $[\bar{3}21]$ (f) $[1\bar{1}1]$
 (g) $(11\bar{1})$ (h) $(01\bar{1})$ (i) (030) (j) $(\bar{1}21)$ (k) $(11\bar{3})$ (l) $(0\bar{4}1)$

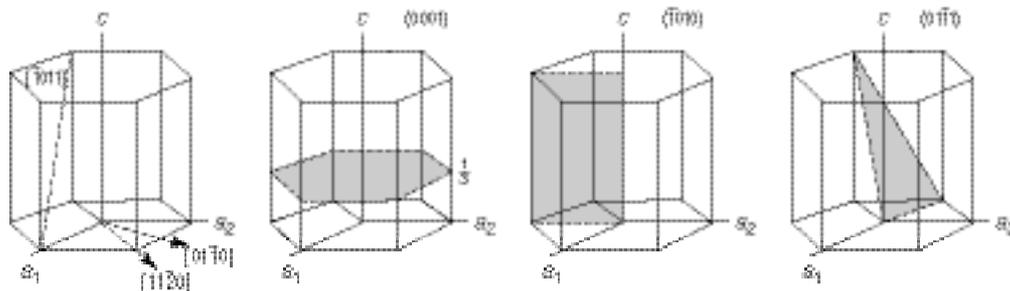
Solution:



3-47 Sketch the following planes and directions within a hexagonal unit cell.

- (a) $[01\bar{1}0]$ (b) $[11\bar{2}0]$ (c) $[\bar{1}011]$ (d) (0003) (e) $(\bar{1}010)$ (f) $(01\bar{1}1)$

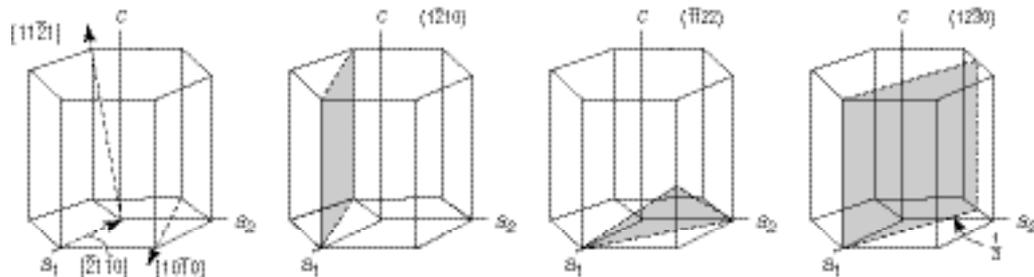
Solution:



3-48 Sketch the following planes and directions within a hexagonal unit cell.

- (a) $[\bar{2}110]$ (b) $[11\bar{2}1]$ (c) $[10\bar{1}0]$ (d) $(1\bar{2}10)$ (e) $(\bar{1}122)$ (f) $(12\bar{3}0)$

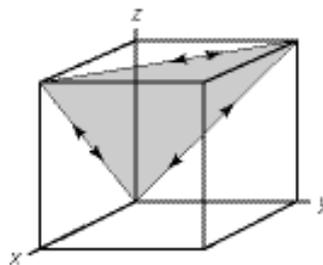
Solution:



3-49 What are the indices of the six directions of the form $\langle 110 \rangle$ that lie in the $(11\bar{1})$ plane of a cubic cell?

Solution:

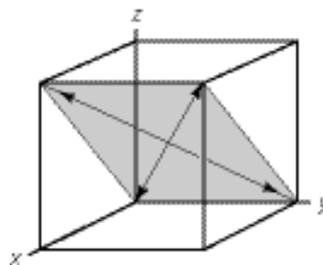
- | | | |
|---------------|---------------------|---------------------|
| $[\bar{1}10]$ | $[101]$ | $[011]$ |
| $[1\bar{1}0]$ | $[\bar{1}0\bar{1}]$ | $[0\bar{1}\bar{1}]$ |

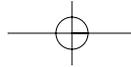


3-50 What are the indices of the four directions of the form $\langle 111 \rangle$ that lie in the $(\bar{1}01)$ plane of a cubic cell?

Solution:

- | | |
|---------------|---------------------------|
| $[111]$ | $[\bar{1}\bar{1}\bar{1}]$ |
| $[1\bar{1}1]$ | $[\bar{1}1\bar{1}]$ |





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- 3-51** Determine the number of directions of the form $\langle 110 \rangle$ in a tetragonal unit cell and compare to the number of directions of the form $\langle 110 \rangle$ in an orthorhombic unit cell.

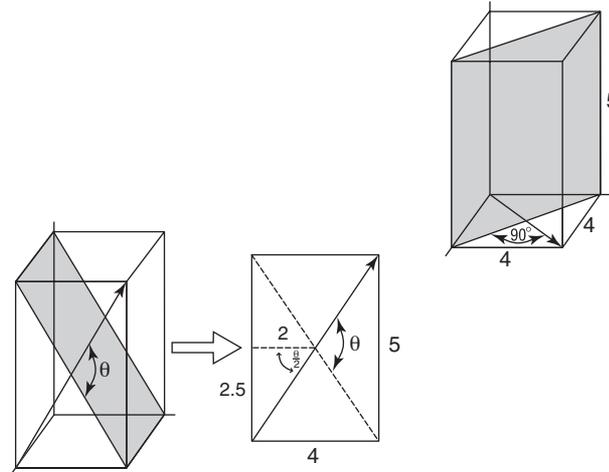
Solution: Tetragonal: $[110], [\bar{1}\bar{1}0], [\bar{1}10], [1\bar{1}0] = 4$

Orthorhombic: $[110], [\bar{1}\bar{1}0] = 2$

Note that in cubic systems, there are 12 directions of the form $\langle 110 \rangle$.

- 3-52** Determine the angle between the $[110]$ direction and the (110) plane in a tetragonal unit cell; then determine the angle between the $[011]$ direction and the (011) plane in a tetragonal cell. The lattice parameters are $a_0 = 4 \text{ \AA}$ and $c_0 = 5 \text{ \AA}$. What is responsible for the difference?

Solution: $[110] \perp (110)$



$$\tan(\theta/2) = 2.5 / 2 = 1.25$$

$$\theta/2 = 51.34^\circ$$

$$\theta = 102.68^\circ$$

The lattice parameters in the x and y directions are the same; this allows the angle between $[110]$ and (110) to be 90° . But the lattice parameters in the y and z directions are different!

- 3-53** Determine the Miller indices of the plane that passes through three points having the following coordinates.

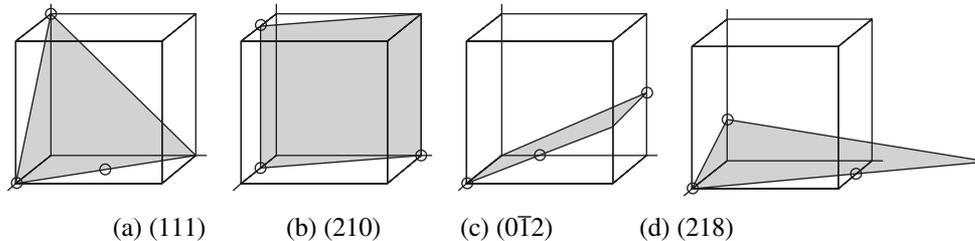
(a) $0,0,1$; $1,0,0$; and $1/2, 1/2, 0$

(b) $1/2, 0, 1$; $1/2, 0, 0$; and $0, 1, 0$

(c) $1, 0, 0$; $0, 1, 1/2$; and $1, 1/2, 1/4$

(d) $1, 0, 0$; $0, 0, 1/4$; and $1/2, 1, 0$

Solution:

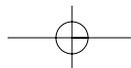


(a) (111)

(b) (210)

(c) $(0\bar{1}2)$

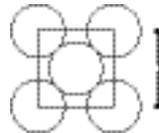
(d) (218)



- 3-54** Determine the repeat distance, linear density, and packing fraction for FCC nickel, which has a lattice parameter of 0.35167 nm, in the [100], [110], and [111] directions. Which of these directions is close packed?

Solution: $r = (\sqrt{2})(0.35167) / 4 = 0.1243 \text{ nm}$

For [100]: repeat distance = $a_o = 0.35167 \text{ nm}$
 linear density = $1/a_o = 2.84 \text{ points/nm}$
 linear packing fraction = $(2)(0.1243)(2.84) = 0.707$



For [110]: repeat distance = $\sqrt{2} a_o / 2 = 0.2487 \text{ nm}$
 linear density = $2/\sqrt{2} a_o = 4.02 \text{ points/nm}$
 linear packing fraction = $(2)(0.1243)(4.02) = 1.0$



For [111]: repeat distance = $\sqrt{3} a_o = 0.6091 \text{ nm}$
 linear density = $1/\sqrt{3} a_o = 1.642 \text{ points/nm}$
 linear packing fraction = $(2)(0.1243)(1.642) = 0.408$

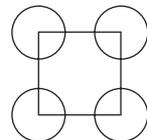


Only the [110] is close packed; it has a linear packing fraction of 1.

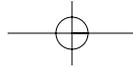
- 3-55** Determine the repeat distance, linear density, and packing fraction for BCC lithium, which has a lattice parameter of 0.35089 nm, in the [100], [110], and [111] directions. Which of these directions is close packed?

Solution: $r = \sqrt{3}(0.35089) / 4 = 0.1519 \text{ nm}$

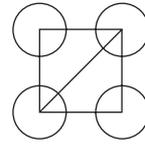
For [100]: repeat distance = $a_o = 0.35089 \text{ nm}$
 linear density = $1/a_o = 2.85 \text{ points/nm}$
 linear packing fraction = $(2)(0.1519)(2.85) = 0.866$



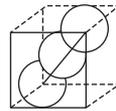
For [110]: repeat distance = $\sqrt{2} a_o = 0.496 \text{ nm}$
 linear density = $1/\sqrt{2} a_o = 2.015 \text{ points/nm}$
 linear packing fraction = $(2)(0.1519)(2.015) = 0.612$



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For [111]: repeat distance = $\sqrt{3} a_o / 2 = 0.3039 \text{ nm}$
 linear density = $2 / \sqrt{3} a_o = 3.291 \text{ points/nm}$
 linear packing fraction = $(2)(0.1519)(3.291) = 1$



The [111] direction is close packed; the linear packing factor is 1.

- 3-56** Determine the repeat distance, linear density, and packing fraction for HCP magnesium in the $[\bar{2}110]$ direction and the $[11\bar{2}0]$ direction. The lattice parameters for HCP magnesium are given in Appendix A.

Solution: $a_o = 3.2087 \text{ \AA}$ $r = 1.604 \text{ \AA}$

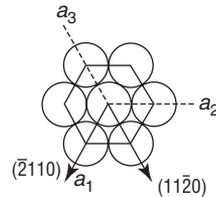
For $[\bar{2}110]$:

repeat distance = $a_o = 3.2087 \text{ \AA}$

linear density = $1/a_o = 0.3116 \text{ points/nm}$

linear packing fraction = $(2)(1.604)(0.3116) = 1$

(Same for $[11\bar{2}0]$)



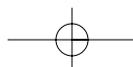
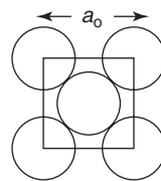
- 3-57** Determine the planar density and packing fraction for FCC nickel in the (100), (110), and (111) planes. Which, if any, of these planes is close packed?

Solution: $a_o = 3.5167 \text{ \AA}$

For (100):

$$\text{planar density} = \frac{2}{(3.5167 \times 10^{-8} \text{ cm})^2} = 0.1617 \times 10^{16} \text{ points/cm}^2$$

$$\text{packing fraction} = \frac{2\pi r^2}{(4r/\sqrt{2})^2} = 0.7854$$

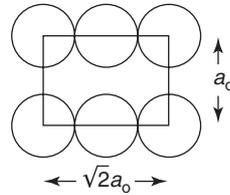


For (110):

$$\text{planar density} = \frac{2 \text{ points}}{(3.5167 \times 10^{-8} \text{ cm})(\sqrt{2})(3.5167 \times 10^{-8} \text{ cm})}$$

$$= 0.1144 \times 10^{-16} \text{ points/cm}^2$$

$$\text{packing fraction} = \frac{2\pi r^2}{\sqrt{2}(4r/\sqrt{2})^2} = 0.555$$



For (111):

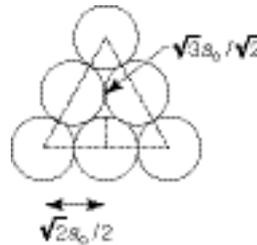
From the sketch, we can determine that the area of the (111) plane is $(\sqrt{2}a_0/2)(\sqrt{3}a_0/\sqrt{2}) = 0.866a_0^2$. There are $(3)(1/2) + (3)(1/6) = 2$ atoms in this area.

$$\text{planar density} = \frac{2 \text{ points}}{0.866(3.5167 \times 10^{-8} \text{ cm})^2}$$

$$= 0.1867 \times 10^{16} \text{ points/cm}^2$$

$$\text{packing fraction} = \frac{2\pi(\sqrt{2}a_0/4)^2}{0.866a_0^2} = 0.907$$

The (111) is close packed.



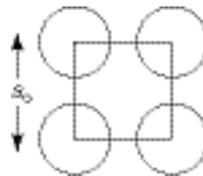
- 3-58** Determine the planar density and packing fraction for BCC lithium in the (100), (110), and (111) planes. Which, if any, of these planes is close packed?

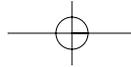
Solution: $a_0 = 3.5089 \text{ \AA}$

For (100):

$$\text{planar density} = \frac{1}{(3.5089 \times 10^{-8} \text{ cm})^2} = 0.0812 \times 10^{16} \text{ points/cm}^2$$

$$\text{packing fraction} = \frac{\pi[\sqrt{3}a_0/4]^2}{a_0^2} = 0.589$$

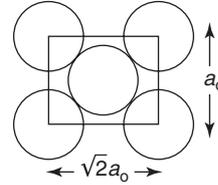



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For (110):

$$\text{planar density} = \frac{2}{\sqrt{2}(3.5089 \times 10^{-8} \text{ cm})^2} = 0.1149 \times 10^{16} \text{ points/cm}^2$$

$$\text{packing fraction} = \frac{2\pi[\sqrt{3}a_0/4]^2}{\sqrt{2}a_0^2} = 0.833$$



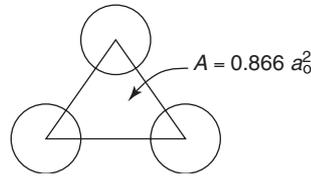
For (111):

There are only $(3)(\frac{1}{6}) = \frac{1}{2}$ points in the plane, which has an area of $0.866a_0^2$.

$$\text{planar density} = \frac{\frac{1}{2}}{0.866(3.5089 \times 10^{-8} \text{ cm})^2} = 0.0469 \times 10^{16} \text{ points/cm}^2$$

$$\text{packing fraction} = \frac{\frac{1}{2} \pi [\sqrt{3}a_0/4]^2}{0.866a_0^2} = 0.34$$

There is no close-packed plane in BCC structures.



- 3-59** Suppose that FCC rhodium is produced as a 1-mm thick sheet, with the (111) plane parallel to the surface of the sheet. How many (111) interplanar spacings d_{111} thick is the sheet? See Appendix A for necessary data.

Solution:
$$d_{111} = \frac{a_0}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{3.796 \text{ \AA}}{\sqrt{3}} = 2.1916 \text{ \AA}$$

$$\text{thickness} = \frac{(1 \text{ mm}/10 \text{ mm/cm})}{2.1916 \times 10^{-8} \text{ cm}} = 4.563 \times 10^6 d_{111} \text{ spacings}$$

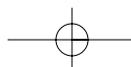
- 3-60** In a FCC unit cell, how many d_{111} are present between the 0,0,0 point and the 1,1,1 point?

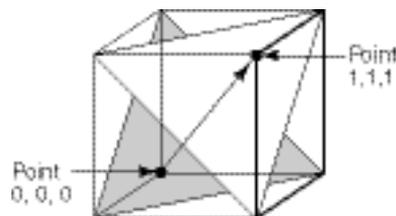
Solution: The distance between the 0,0,0 and 1,1,1 points is $\sqrt{3}a_0$. The interplanar spacing is

$$d_{111} = a_0/\sqrt{1^2 + 1^2 + 1^2} = a_0/\sqrt{3}$$

Therefore the number of interplanar spacings is

$$\text{number of } d_{111} \text{ spacings} = \sqrt{3}a_0/(a_0/\sqrt{3}) = 3$$





- 3-62** Determine the minimum radius of an atom that will just fit into (a) the tetrahedral interstitial site in FCC nickel and (b) the octahedral interstitial site in BCC lithium.

Solution: (a) For the tetrahedral site in FCC nickel ($a_o = 3.5167 \text{ \AA}$):

$$r_{\text{Ni}} = \frac{\sqrt{2}(3.5167 \text{ \AA})}{4} = 1.243 \text{ \AA}$$

$r/r_{\text{Ni}} = 0.225$ for a tetrahedral site. Therefore:

$$r = (1.243 \text{ \AA})(0.225) = 0.2797 \text{ \AA}$$

(b) For the octahedral site in BCC lithium ($a_o = 3.5089 \text{ \AA}$):

$$r_{\text{Li}} = \frac{\sqrt{3}(3.5089)}{4} = 1.519 \text{ \AA}$$

$r/r_{\text{Li}} = 0.414$ for an octahedral site. Therefore:

$$r = (1.519 \text{ \AA})(0.414) = 0.629 \text{ \AA}$$

- 3-64** What is the radius of an atom that will just fit into the octahedral site in FCC copper without disturbing the crystal structure?

Solution: $r_{\text{Cu}} = 1.278 \text{ \AA}$

$r/r_{\text{Cu}} = 0.414$ for an octahedral site. Therefore:

$$r = (1.278 \text{ \AA})(0.414) = 0.529 \text{ \AA}$$

- 3-65** Using the ionic radii given in Appendix B, determine the coordination number expected for the following compounds.

- (a) Y_2O_3 (b) UO_2 (c) BaO (d) Si_3N_4
 (e) GeO_2 (f) MnO (g) MgS (h) KBr

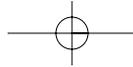
Solution:

$$(a) r_{\text{Y}^{+3}}/r_{\text{O}^{-2}} = \frac{0.89}{1.32} = 0.67 \quad \text{CN} = 6 \quad (e) r_{\text{Ge}^{+4}}/r_{\text{O}^{-2}} = \frac{0.53}{1.32} = 0.40 \quad \text{CN} = 4$$

$$(b) r_{\text{U}^{+4}}/r_{\text{O}^{-2}} = \frac{0.97}{1.32} = 0.73 \quad \text{CN} = 6 \quad (f) r_{\text{Mn}^{+2}}/r_{\text{O}^{-2}} = \frac{0.80}{1.32} = 0.61 \quad \text{CN} = 6$$

$$(c) r_{\text{O}^{-2}}/r_{\text{Ba}^{+2}} = \frac{1.32}{1.34} = 0.99 \quad \text{CN} = 8 \quad (g) r_{\text{Mg}^{+2}}/r_{\text{S}^{-2}} = \frac{0.66}{1.32} = 0.50 \quad \text{CN} = 6$$

$$(d) r_{\text{N}^{-3}}/r_{\text{Si}^{+4}} = \frac{0.15}{0.42} = 0.36 \quad \text{CN} = 4 \quad (h) r_{\text{K}^{+1}}/r_{\text{Br}^{-1}} = \frac{1.33}{1.96} = 0.68 \quad \text{CN} = 6$$


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- 3-66** Would you expect NiO to have the cesium chloride, sodium chloride, or zinc blende structure? Based on your answer, determine (a) the lattice parameter, (b) the density, and (c) the packing factor.

Solution: $r_{\text{Ni}^{+2}} = 0.69 \text{ \AA}$ $r_{\text{O}^{-2}} = 1.32 \text{ \AA}$ $\frac{r_{\text{Ni}^{+2}}}{r_{\text{O}^{-2}}} = 0.52$ CN = 6

A coordination number of 8 is expected for the CsCl structure, and a coordination number of 4 is expected for ZnS. But a coordination number of 6 is consistent with the *NaCl structure*.

(a) $a_0 = 2(0.69) + 2(1.32) = 4.02 \text{ \AA}$

(b) $\rho = \frac{(4 \text{ of each ion/cell})(58.71 + 16 \text{ g/mol})}{(4.02 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ atoms/mol})} = 7.64 \text{ g/cm}^3$

(c) $\text{PF} = \frac{(4\pi/3)(4 \text{ ions/cell})[(0.69)^3 + (1.32)^3]}{(4.02)^3} = 0.678$

- 3-67** Would you expect UO_2 to have the sodium chloride, zinc blende, or fluorite structure? Based on your answer, determine (a) the lattice parameter, (b) the density, and (c) the packing factor.

Solution: $r_{\text{U}^{+4}} = 0.97 \text{ \AA}$ $r_{\text{O}^{-2}} = 1.32 \text{ \AA}$ $\frac{r_{\text{U}^{+4}}}{r_{\text{O}^{-2}}} = 0.97/1.32 = 0.735$

valence of U = +4, valence of O = -2

The radius ratio predicts a coordination number of 8; however there must be twice as many oxygen ions as uranium ions in order to balance the charge. The *fluorite* structure will satisfy these requirements, with:

U = FCC position (4) O = tetrahedral position (8)

(a) $\sqrt{3}a_0 = 4r_{\text{U}} + 4r_{\text{O}} = 4(0.97 + 1.32) = 9.16$ or $a_0 = 5.2885 \text{ \AA}$

(b) $\rho = \frac{4(238.03 \text{ g/mol}) + 8(16 \text{ g/mol})}{(5.2885 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ atoms/mol})} = 12.13 \text{ g/cm}^3$

(c) $\text{PF} = \frac{(4\pi/3)[4(0.97)^3 + 8(1.32)^3]}{(5.2885)^3} = 0.624$

- 3-68** Would you expect BeO to have the sodium chloride, zinc blende, or fluorite structure? Based on your answer, determine (a) the lattice parameter, (b) the density, and (c) the packing factor.

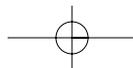
Solution: $r_{\text{Be}^{+2}} = 0.35 \text{ \AA}$ $r_{\text{O}^{-2}} = 1.32 \text{ \AA}$

$r_{\text{Be}^{+2}}/r_{\text{O}^{-2}} = 0.265$ CN = 4 \therefore Zinc Blende

(a) $\sqrt{3}a_0 = 4r_{\text{Be}^{+2}} + 4r_{\text{O}^{-2}} = 4(0.35 + 1.32) = 6.68$ or $a_0 = 3.8567 \text{ \AA}$

(b) $\rho = \frac{4(9.01 + 16 \text{ g/mol})}{(3.8567 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ atoms/mol})} = 2.897 \text{ g/cm}^3$

(c) $\text{PF} = \frac{(4\pi/3)(4)[(0.35)^3 + 8(1.32)^3]}{(3.8567)^3} = 0.684$



- 3-69** Would you expect CsBr to have the sodium chloride, zinc blende, fluorite, or cesium chloride structure? Based on your answer, determine (a) the lattice parameter, (b) the density, and (c) the packing factor.

Solution: $r_{\text{Cs}^+} = 1.67 \text{ \AA}$ $r_{\text{Br}^-} = 1.96 \text{ \AA}$
 $r_{\text{Cs}^+} = 0.852$ $\text{CN} = 8$ $\therefore \text{CsCl}$
 r_{Br^-}

(a) $\sqrt{3} a_0 = 2r_{\text{Cs}^+} + 2r_{\text{Br}^-} = 2(1.96 + 1.67) = 7.26$ or $a_0 = 4.1916 \text{ \AA}$

(b) $\rho = \frac{79.909 + 132.905 \text{ g/mol}}{(4.1916 \times 10^{-8} \text{ cm})^3 (6.02 \times 10^{23} \text{ atoms/mol})} = 4.8 \text{ g/cm}^3$

(c) $\text{PF} = \frac{(4\pi/3)[(1.96)^3 + (1.67)^3]}{(4.1916)^3} = 0.693$

- 3-70** Sketch the ion arrangement on the (110) plane of ZnS (with the zinc blende structure) and compare this arrangement to that on the (110) plane of CaF₂ (with the fluorite structure). Compare the planar packing fraction on the (110) planes for these two materials.

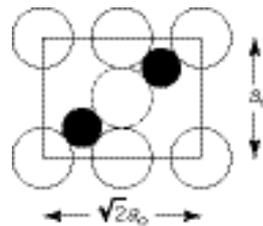
Solution: ZnS:

$$\sqrt{3} a_0 = 4r_{\text{Zn}^{+2}} + 4r_{\text{S}^{-2}}$$

$$\sqrt{3} a_0 = 4(0.074 \text{ nm}) + 4(0.184 \text{ nm})$$

$$a_0 = 0.596 \text{ nm}$$

$$\text{PPF} = \frac{(2)(\pi r_{\text{Zn}^{+2}}^2) + (2)(\pi r_{\text{S}^{-2}}^2)}{(\sqrt{2} a_0) a_0} = \frac{2\pi(0.074)^2 + 2\pi(0.184)^2}{\sqrt{2}(0.596 \text{ nm})^2} = 0.492$$



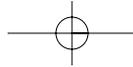
CaF₂:

$$\sqrt{3} a_0 = 4r_{\text{Ca}^{+2}} + 4r_{\text{F}^{-1}}$$

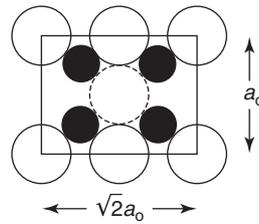
$$\sqrt{3} a_0 = 4(0.099 \text{ nm}) + 4(0.133 \text{ nm})$$

$$a_0 = 0.536 \text{ nm}$$

$$\text{PPF} = \frac{(2)(\pi r_{\text{Ca}^{+2}}^2) + (4)(\pi r_{\text{F}^{-1}}^2)}{(\sqrt{2} a_0) a_0} = \frac{2\pi(0.099)^2 + 4\pi(0.133)^2}{\sqrt{2}(0.536 \text{ nm})^2} = 0.699$$



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- 3-71** MgO, which has the sodium chloride structure, has a lattice parameter of 0.396 nm. Determine the planar density and the planar packing fraction for the (111) and (222) planes of MgO. What ions are present on each plane?

Solution: As described in the answer to Problem 3-57, the area of the (111) plane is $0.866a_o^2$.

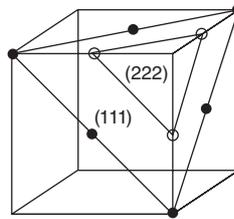
$$a_o = 2r_{\text{Mg}} + 2r_{\text{O}} = 2(0.66 + 1.32) = 3.96 \text{ \AA}$$

$$(111): \text{P.D.} = \frac{2 \text{ Mg}}{(0.866)(3.96 \times 10^{-8} \text{ cm})^2} = 0.1473 \times 10^{16} \text{ points/cm}^2$$

$$\text{PPF} = \frac{2\pi(0.66)^2}{(0.866)(3.96)^2} = 0.202$$

$$(222): \text{P.D.} = 0.1473 \times 10^{16} \text{ points/cm}^2$$

$$\text{PPF} = \frac{2\pi(1.32)^2}{(0.866)(3.96)^2} = 0.806$$



- 3-75** A diffracted x-ray beam is observed from the (220) planes of iron at a 2θ angle of 99.1° when x-rays of 0.15418 nm wavelength are used. Calculate the lattice parameter of the iron.

Solution: $\sin \theta = \lambda/2d_{220}$

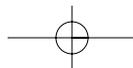
$$\sin(99.1/2) = \frac{0.15418\sqrt{2^2 + 2^2 + 0^2}}{2a_o}$$

$$a_o = \frac{0.15418\sqrt{8}}{2\sin(49.55)} = 0.2865 \text{ nm}$$

- 3-76** A diffracted x-ray beam is observed from the (311) planes of aluminum at a 2θ angle of 78.3° when x-rays of 0.15418 nm wavelength are used. Calculate the lattice parameter of the aluminum.

Solution: $\sin \theta = \lambda/d_{311}$

$$a_o = \frac{0.15418\sqrt{3^2 + 1^2 + 1^2}}{2\sin(78.3/2)} = 0.40497 \text{ nm}$$



CHAPTER 3 Atomic and Ionic Arrangements 29

- 3-77** Figure 3-43 shows the results of an x-ray diffraction experiment in the form of the intensity of the diffracted peak versus the 2θ diffraction angle. If x-rays with a wavelength of 0.15418 nm are used, determine (a) the crystal structure of the metal, (b) the indices of the planes that produce each of the peaks, and (c) the lattice parameter of the metal.

Solution: The 2θ values can be estimated from Figure 3-43:

	2θ	$\sin^2\theta$	$\sin^2\theta/0.0077$	Planar indices	$d = \lambda/2\sin\theta$	$a_o = d\sqrt{h^2 + k^2 + l^2}$
1	17.5	0.023	3	(111)	0.5068	0.8778
2	20.5	0.032	4	(200)	0.4332	0.8664
3	28.5	0.061	8	(220)	0.3132	0.8859
4	33.5	0.083	11	(311)	0.2675	0.8872
5	35.5	0.093	12	(222)	0.2529	0.8761
6	41	0.123	16	(400)	0.2201	0.8804
7	45	0.146	19	(331)	0.2014	0.8779
8	46.5	0.156	20	(420)	0.1953	0.8734

The $\sin^2\theta$ values must be divided by 0.077 (one third the first $\sin^2\theta$ value) in order to produce a possible sequence of numbers)

(a) The 3,4,8,11, . . . sequence means that the material is FCC

(c) The average $a_o = 0.8781$ nm

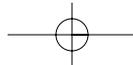
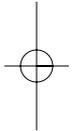
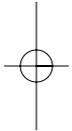
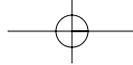
- 3-78** Figure 3-44 shows the results of an x-ray diffraction experiment in the form of the intensity of the diffracted peak versus the 2θ diffraction angle. If x-rays with a wavelength of 0.0717 nm are used, determine (a) the crystal structure of the metal, (b) the indices of the planes that produce each of the peaks, and (c) the lattice parameter of the metal.

Solution: The 2θ values can be estimated from the figure:

	2θ	$\sin^2\theta$	$\sin^2\theta/0.047$	Planar indices	$d = \lambda/2\sin\theta$	$a_o = d\sqrt{h^2 + k^2 + l^2}$
1	25.5	0.047	1	(111)	0.1610	0.2277
2	36	0.095	2	(200)	0.1150	0.2300
3	44.5	0.143	3	(211)	0.0938	0.2299
4	51.5	0.189	4	(220)	0.0818	0.2313
5	58	0.235	5	(310)	0.0733	0.2318
6	64.5	0.285	6	(222)	0.0666	0.2307
7	70	0.329	7	(321)	0.06195	0.2318
8	75.5	0.375	8	(400)	0.0580	0.2322

(a) The sequence 1,2,3,4,5,6,7,8 (which includes the "7") means that the material is BCC.

(c) The average $a_o = 0.2307$ nm



4

Imperfections in the Atomic and Ionic Arrangements

- 4-1** Calculate the number of vacancies per cm^3 expected in copper at 1080°C (just below the melting temperature). The activation energy for vacancy formation is $20,000$ cal/mol.

Solution:

$$n = \frac{(4 \text{ atoms/u.c.})}{(3.6151 \times 10^{-8} \text{ cm})^3} = 8.47 \times 10^{22} \text{ atoms/cm}^3$$

$$n_v = 8.47 \times 10^{22} \exp[-20,000/(1.987)(1353)]$$

$$= 8.47 \times 10^{22} \exp(-7.4393) = 4.97 \times 10^{19} \text{ vacancies/cm}^3$$

- 4-2** The fraction of lattice points occupied by vacancies in solid aluminum at 660°C is 10^{-3} . What is the activation energy required to create vacancies in aluminum?

Solution:

$$n_v/n = 10^{-3} = \exp[-Q/(1.987)(933)]$$

$$\ln(10^{-3}) = -6.9078 = -Q/(1.987)(933)$$

$$Q = 12,800 \text{ cal/mol}$$

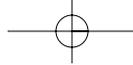
- 4-3** The density of a sample of FCC palladium is 11.98 g/cm^3 and its lattice parameter is 3.8902 \AA . Calculate (a) the fraction of the lattice points that contain vacancies and (b) the total number of vacancies in a cubic centimeter of Pd.

Solution:

$$(a) 11.98 \text{ g/cm}^3 = \frac{(x)(106.4 \text{ g/mol})}{(3.8902 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$x = 3.9905$$

$$\text{fraction} = \frac{4.0 - 3.9905}{4} = 0.002375$$



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$$(b) \text{ number} = \frac{0.0095 \text{ vacancies/u.c.}}{(3.8902 \times 10^{-8} \text{ cm})^3} = 1.61 \times 10^{20} \text{ vacancies/cm}^3$$

- 4-4** The density of a sample of HCP beryllium is 1.844 g/cm^3 and the lattice parameters are $a_0 = 0.22858 \text{ nm}$ and $c_0 = 0.35842 \text{ nm}$. Calculate (a) the fraction of the lattice points that contain vacancies and (b) the total number of vacancies in a cubic centimeter.

Solution: $V_{\text{u.c.}} = (0.22858 \text{ nm})^2(0.35842 \text{ nm})\cos 30 = 0.01622 \text{ nm}^3$
 $= 1.622 \times 10^{-23} \text{ cm}^3$

(a) From the density equation:

$$1.844 \text{ g/cm}^3 = \frac{(x)(9.01 \text{ g/mol})}{(1.622 \times 10^{-23} \text{ cm}^3)(6.02 \times 10^{23} \text{ atoms/mol})} \quad x = 1.9984$$

$$\text{fraction} = \frac{2 - 1.9984}{2} = 0.0008$$

$$(b) \text{ number} = \frac{0.0016 \text{ vacancies/uc}}{1.622 \times 10^{-23} \text{ cm}^3} = 0.986 \times 10^{20} \text{ vacancies/cm}^3$$

- 4-5** BCC lithium has a lattice parameter of $3.5089 \times 10^{-8} \text{ cm}$ and contains one vacancy per 200 unit cells. Calculate (a) the number of vacancies per cubic centimeter and (b) the density of Li.

Solution:

(a) $\frac{1 \text{ vacancy}}{(200)(3.5089 \times 10^{-8} \text{ cm})^3} = 1.157 \times 10^{20} \text{ vacancies/cm}^3$

(b) In 200 unit cells, there are 399 Li atoms. The atoms/cell are $399/200$:

$$\rho = \frac{(399/200)(6.94 \text{ g/mol})}{(3.5089 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})} = 0.532 \text{ g/cm}^3$$

- 4-6** FCC lead has a lattice parameter of 0.4949 nm and contains one vacancy per 500 Pb atoms. Calculate (a) the density and (b) the number of vacancies per gram of Pb.

Solution: (a) The number of atoms/cell = $(499/500)(4 \text{ sites/cell})$

$$\rho = \frac{(499/500)(4)(207.19 \text{ g/mol})}{(4.949 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})} = 11.335 \text{ g/cm}^3$$

(b) The 500 Pb atoms occupy $500 / 4 = 125$ unit cells:

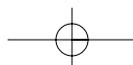
$$\frac{1 \text{ vacancy}}{\left(\frac{125 \text{ cells}}{(4.949 \times 10^{-8} \text{ cm})^3} \right)} \times [(1/11.335 \text{ g/cm}^3)] = 5.82 \times 10^{18} \text{ vacancies/g}$$

- 4-7** A niobium alloy is produced by introducing tungsten substitutional atoms in the BCC structure; eventually an alloy is produced that has a lattice parameter of 0.32554 nm and a density of 11.95 g/cm^3 . Calculate the fraction of the atoms in the alloy that are tungsten.

Solution: $11.95 \text{ g/cm}^3 = \frac{(x_{\text{W}})(183.85 \text{ g/mol}) + (2 - x_{\text{W}})(92.91 \text{ g/mol})}{(3.2554 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$

$$248.186 = 183.85x_{\text{W}} + 185.82 - 92.91x_{\text{W}}$$

$$90.94x_{\text{W}} = 62.366 \quad \text{or} \quad x_{\text{W}} = 0.69 \text{ W atoms/cell}$$



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There are 2 atoms per cell in BCC metals. Thus:

$$f_w = 0.69/2 = 0.345$$

- 4-8** Tin atoms are introduced into a FCC copper crystal, producing an alloy with a lattice parameter of 3.7589×10^{-8} cm and a density of 8.772 g/cm³. Calculate the atomic percentage of tin present in the alloy.

Solution:

$$8.772 \text{ g/cm}^3 = \frac{(x_{\text{Sn}})(118.69 \text{ g/mol}) + (4 - x_{\text{Sn}})(63.54 \text{ g/mol})}{(3.7589 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$$

$$280.5 = 55.15x_{\text{Sn}} + 254.16 \quad \text{or} \quad x_{\text{Sn}} = 0.478 \text{ Sn atoms/cell}$$

There are 4 atoms per cell in FCC metals; therefore the at% Sn is:

$$(0.478/4) = 11.95\%$$

- 4-9** We replace 7.5 atomic percent of the chromium atoms in its BCC crystal with tantalum. X-ray diffraction shows that the lattice parameter is 0.29158 nm. Calculate the density of the alloy.

Solution:

$$\rho = \frac{(2)(0.925)(51.996 \text{ g/mol}) + 2(0.075)(180.95 \text{ g/mol})}{(2.9158 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})} = 8.265 \text{ g/cm}^3$$

- 4-10** Suppose we introduce one carbon atom for every 100 iron atoms in an interstitial position in BCC iron, giving a lattice parameter of 0.2867 nm. For the Fe-C alloy, find (a) the density and (b) the packing factor.

Solution: There is one carbon atom per 100 iron atoms, or 1 C/50 unit cells, or 1/50 C per unit cell:

(a)

$$\rho = \frac{(2)(55.847 \text{ g/mol}) + (1/50)(12 \text{ g/mol})}{(2.867 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})} = 7.89 \text{ g/cm}^3$$

(b)

$$\text{Packing Factor} = \frac{2(4\pi/3)(1.241)^3 + (1/50)(4\pi/3)(0.77)^3}{(2.867)^3} = 0.681$$

- 4-11** The density of BCC iron is 7.882 g/cm³ and the lattice parameter is 0.2866 nm when hydrogen atoms are introduced at interstitial positions. Calculate (a) the atomic fraction of hydrogen atoms and (b) the number of unit cells required on average that contain hydrogen atoms.

Solution:

(a) $7.882 \text{ g/cm}^3 = \frac{2(55.847 \text{ g/mol}) + x(1.00797 \text{ g/mol})}{(2.866 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})}$

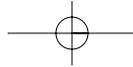
$$x = 0.0081 \text{ H atoms/cell}$$

The total atoms per cell include 2 Fe atoms and 0.0081 H atoms.
Thus:

$$f_{\text{H}} = \frac{0.0081}{2.0081} = 0.004$$

- (b) Since there is 0.0081 H/cell, then the number of cells containing H atoms is:

$$\text{cells} = 1/0.0081 = 123.5 \quad \text{or} \quad 1 \text{ H in } 123.5 \text{ cells}$$


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- 4-12** Suppose one Schottky defect is present in every tenth unit cell of MgO. MgO has the sodium chloride crystal structure and a lattice parameter of 0.396 nm. Calculate (a) the number of anion vacancies per cm^3 and (b) the density of the ceramic.

Solution: In 10 unit cells, we expect 40 Mg + 40 O ions, but due to the defect:

$$40 \text{ Mg} - 1 = 39$$

$$40 \text{ O} - 1 = 39$$

$$(a) \ 1 \text{ vacancy}/(10 \text{ cells})(3.96 \times 10^{-8} \text{ cm})^3 = 1.61 \times 10^{21} \text{ vacancies/cm}^3$$

$$(b) \ \rho = \frac{(39/40)(4)(24.312 \text{ g/mol}) + (39/40)(4)(16 \text{ g/mol})}{(3.96 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ atoms/mol})} = 4.205 \text{ g/cm}^3$$

- 4-13** ZnS has the zinc blende structure. If the density is 3.02 g/cm^3 and the lattice parameter is 0.59583 nm, determine the number of Schottky defects (a) per unit cell and (b) per cubic centimeter.

Solution: Let x be the number of each type of ion in the unit cell. There normally are 4 of each type.

$$(a) \ 3.02 \text{ g/cm}^3 = \frac{x(65.38 \text{ g/mol}) + x(32.064 \text{ g/mol})}{(5.9583 \times 10^{-8} \text{ cm})^3(6.02 \times 10^{23} \text{ ions/mol})} \quad x = 3.9465$$

$$4 - 3.9465 = 0.0535 \text{ defects/u.c.}$$

$$(b) \ \# \text{ of unit cells/cm}^3 = 1/(5.9683 \times 10^{-8} \text{ cm})^3 = 4.704 \times 10^{21}$$

$$\text{Schottky defects per cm}^3 = (4.704 \times 10^{21})(0.0535) = 2.517 \times 10^{20}$$

- 4-14** Suppose we introduce the following point defects. What other changes in each structure might be necessary to maintain a charge balance? Explain.

- Mg²⁺ ions substitute for yttrium atoms in Y₂O₃
- Fe³⁺ ions substitute for magnesium ions in MgO
- Li¹⁺ ions substitute for magnesium ions in MgO
- Fe²⁺ ions replace sodium ions in NaCl

Solution:

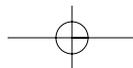
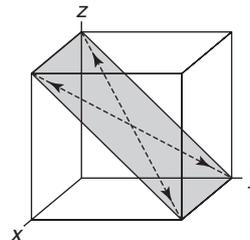
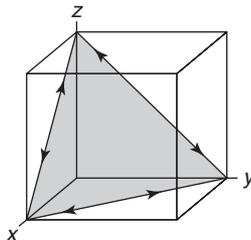
- Remove 2 Y³⁺ and add 3 Mg²⁺ – create cation interstitial.
- Remove 3 Mg²⁺ and add 2 Fe³⁺ – create cation vacancy.
- Remove 1 Mg²⁺ and add 2 Li¹⁺ – create cation interstitial.
- Remove 2 Na⁺ and add 1 Fe²⁺ – create cation vacancy.

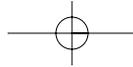
- 4-16** What are the Miller indices of the slip directions

- on the (111) plane in an FCC unit cell
- on the (011) plane in a BCC unit cell?

Solution:

[0 $\bar{1}$ 1], [01 $\bar{1}$]	[1 $\bar{1}$ 1], [$\bar{1}$ 1 $\bar{1}$]
$\bar{1}$ 10], [1 $\bar{1}$ 0]	$\bar{1}$ 11], [11 $\bar{1}$]
$\bar{1}$ 01], [10 $\bar{1}$]	

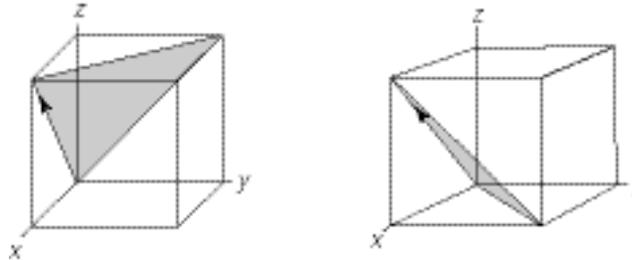




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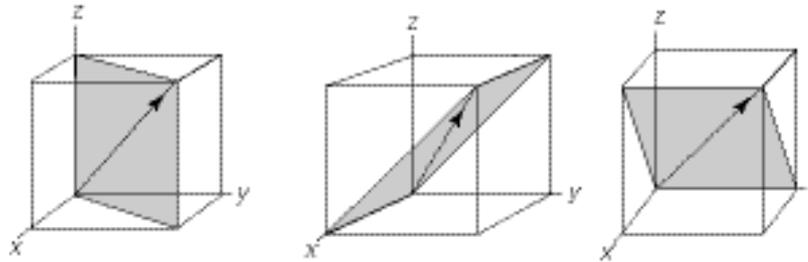
- 4-17 What are the Miller indices of the slip planes in FCC unit cells that include the [101] slip direction?

Solution: $(11\bar{1})$, $(\bar{1}11)$ $(\bar{1}11)$, $(1\bar{1}\bar{1})$



- 4-18 What are the Miller indices of the {110} slip planes in BCC unit cells that include the [111] slip direction?

Solution: $(1\bar{1}0)$, $(\bar{1}10)$ $(0\bar{1}1)$, $(01\bar{1})$ $(10\bar{1})$, $(\bar{1}01)$



- 4-19 Calculate the length of the Burgers vector in the following materials:
(a) BCC niobium (b) FCC silver (c) diamond cubic silicon

Solution: (a) The repeat distance, or Burgers vector, is half the body diagonal, or:

$$b = \text{repeat distance} = (\frac{1}{2}) (\sqrt{3}) (3.294 \text{ \AA}) = 2.853 \text{ \AA}$$

(b) The repeat distance, or Burgers vector, is half of the face diagonal, or:

$$b = (\frac{1}{2}) (\sqrt{2} a_0) = (\frac{1}{2}) (\sqrt{2}) (4.0862 \text{ \AA}) = 2.889 \text{ \AA}$$

(c) The slip direction is [110], where the repeat distance is half of the face diagonal:

$$b = (\frac{1}{2}) (\sqrt{2}) (5.4307 \text{ \AA}) = 3.840 \text{ \AA}$$

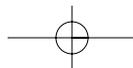
- 4-20 Determine the interplanar spacing and the length of the Burgers vector for slip on the expected slip systems in FCC aluminum. Repeat, assuming that the slip system is a (110) plane and a $[1\bar{1}1]$ direction. What is the ratio between the shear stresses required for slip for the two systems? Assume that $k = 2$ in Equation 4-2.

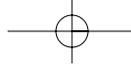
Solution: (a) For $(111)/[110]$,

$$b = (\frac{1}{2}) (\sqrt{2}) (4.04958 \text{ \AA}) = 2.863 \text{ \AA} \quad d_{111} = \frac{4.04958 \text{ \AA}}{\sqrt{1+1+1}} = 2.338 \text{ \AA}$$

(b) If $(110)/[111]$, then:

$$b = \sqrt{3} (4.04958 \text{ \AA}) = 7.014 \text{ \AA} \quad d_{110} = \frac{4.04958 \text{ \AA}}{\sqrt{1^2 + 1^2 + 0^2}} = 2.863 \text{ \AA}$$





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(c) If we assume that $k = 2$ in Equation 4-2, then

$$(d/b)_a = \frac{2.338}{2.863} = 0.8166 \quad (d/b)_b = \frac{2.863}{7.014} = 0.408$$

$$\therefore \frac{\tau_a}{\tau_b} = \frac{\exp(-2(0.8166))}{\exp(-2(0.408))} = 0.44$$

- 4-21** Determine the interplanar spacing and the length of the Burgers vector for slip on the (110)/[1 $\bar{1}$ 1] slip system in BCC tantalum. Repeat, assuming that the slip system is a (111)/[1 $\bar{1}$ 0] system. What is the ratio between the shear stresses required for slip for the two systems? Assume that $k = 2$ in Equation 4-2.

Solution: (a) For (110)/[1 $\bar{1}$ 1]:

$$b = (\frac{1}{2})(\sqrt{3})(3.3026 \text{ \AA}) = 2.860 \text{ \AA} \quad d_{110} = \frac{3.3026 \text{ \AA}}{\sqrt{1^2 + 1^2 + 0^2}} = 2.335 \text{ \AA}$$

(b) If (111)/[1 $\bar{1}$ 0], then:

$$b = \sqrt{2}(3.3026 \text{ \AA}) = 4.671 \text{ \AA} \quad d_{111} = \frac{3.3026 \text{ \AA}}{\sqrt{1^2 + 1^2 + 1^2}} = 1.907 \text{ \AA}$$

(c) If we assume that $k = 2$ in Equation 4-2, then:

$$(d/b)_a = \frac{2.335}{2.86} = 0.8166 \quad (d/b)_b = \frac{1.907}{4.671} = 0.408$$

$$\frac{\tau_a}{\tau_b} = \frac{\exp(-2(0.8166))}{\exp(-2(0.408))} = 0.44$$

- 4-26** How many grams of aluminum, with a dislocation density of 10^{10} cm/cm³, are required to give a total dislocation length that would stretch from New York City to Los Angeles (3000 miles)?

Solution: $(3000 \text{ mi})(5280 \text{ ft/mi})(12 \text{ in./ft})(2.54 \text{ cm/in.}) = 4.828 \times 10^8 \text{ cm}$

$$\frac{(4.828 \times 10^8 \text{ cm})(2.699 \text{ g/cm}^3)}{(10^{10} \text{ cm/cm}^3)} = 0.13 \text{ g}$$

- 4-27** The distance from Earth to the Moon is 240,000 miles. If this were the total length of dislocation in a cubic centimeter of material, what would be the dislocation density?

Solution: $(240,000 \text{ mi})(5280 \text{ ft/mi})(12 \text{ in./ft})(2.54 \text{ cm/in.}) = 3.86 \times 10^{10} \text{ cm/cm}^3$

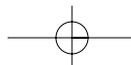
- 4-30** Suppose you would like to introduce an interstitial or large substitutional atom into the crystal near a dislocation. Would the atom fit more easily above or below the dislocation line shown in Figure 4-8(b)? Explain.

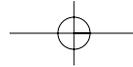
Solution: The atom would fit more easily into the area just below the dislocation due to the atoms being pulled apart; this allows more space into which the atom can fit.

- 4-31** Compare the c/a ratios for the following HCP metals, determine the likely slip processes in each, and estimate the approximate critical resolved shear stress. Explain. (See data in Appendix A)

(a) zinc (b) magnesium (c) titanium
(d) zirconium (e) rhenium (f) beryllium

Solution: We expect metals with $c/a > 1.633$ to have a low τ_{crss} :



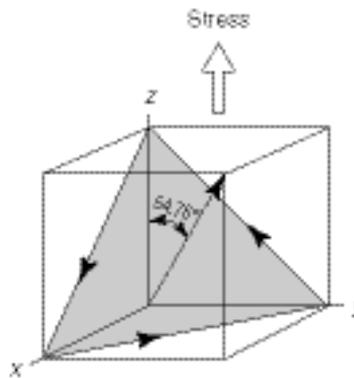

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$$\begin{aligned}
 \text{(a) Zn: } \frac{4.9470}{2.6648} &= 1.856 - \text{low } \tau_{\text{crss}} & \text{(b) Mg: } \frac{5.209}{3.2087} &= 1.62 - \text{medium } \tau_{\text{crss}} \\
 \text{(c) Ti: } \frac{4.6831}{2.9503} &= 1.587 - \text{high } \tau_{\text{crss}} & \text{(d) Zr: } \frac{5.1477}{3.2312} &= 1.593 - \text{high } \tau_{\text{crss}} \\
 \text{(e) Rh: } \frac{4.458}{2.760} &= 1.615 - \text{medium } \tau_{\text{crss}} & \text{(f) Be: } \frac{3.5842}{2.2858} &= 1.568 - \text{high } \tau_{\text{crss}}
 \end{aligned}$$

- 4-32** A single crystal of an FCC metal is oriented so that the [001] direction is parallel to an applied stress of 5000 psi. Calculate the resolved shear stress acting on the (111) slip plane in the $[\bar{1}10]$, $[0\bar{1}1]$, and $[10\bar{1}]$ slip directions. Which slip system(s) will become active first?

Solution:

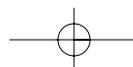
$$\begin{aligned}
 \phi &= 54.76^\circ & \tau &= 5000 \cos 54.76 \cos \lambda \\
 \lambda_{110} &= 90^\circ & \tau &= 0 \\
 \lambda_{011} &= 45^\circ & \tau &= 2040 \text{ psi active} \\
 \lambda_{101} &= 45^\circ & \tau &= 2040 \text{ psi active}
 \end{aligned}$$

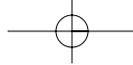


- 4-33** A single crystal of a BCC metal is oriented so that the [001] direction is parallel to the applied stress. If the critical resolved shear stress required for slip is 12,000 psi, calculate the magnitude of the applied stress required to cause slip to begin in the $[\bar{1}\bar{1}1]$ direction on the (110), (011), and (10 $\bar{1}$) slip planes.

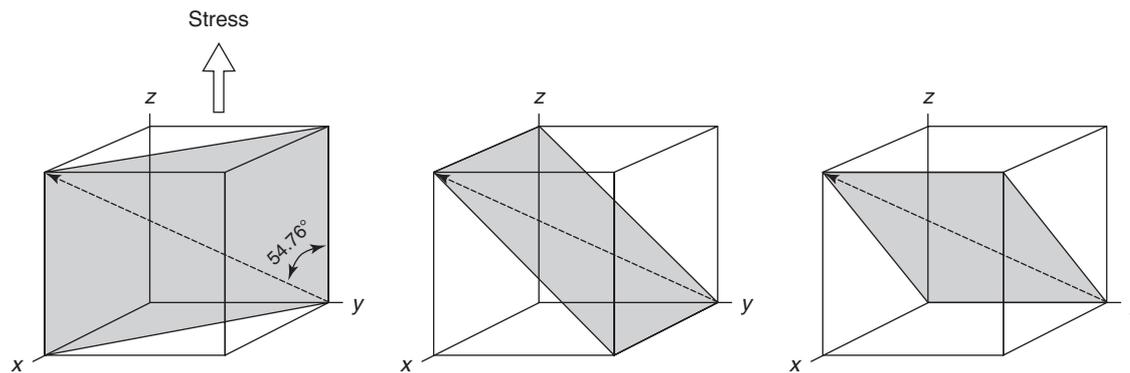
Solution:

$$\begin{aligned}
 \text{CRSS} &= 12,000 \text{ psi} = \sigma \cos \phi \cos \lambda \\
 \lambda = 54.76^\circ & \frac{12,000 \text{ psi}}{\cos \phi \cos \lambda} = \sigma \\
 \phi_{110} &= 90^\circ & \sigma &= \infty \\
 \phi_{011} &= 45^\circ & \sigma &= 29,412 \text{ psi} \\
 \phi_{101} &= 45^\circ & \sigma &= 29,412 \text{ psi}
 \end{aligned}$$





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- 4-34** Our discussion of Schmid's law dealt with single crystals of a metal. Discuss slip and Schmid's law in a polycrystalline material. What might happen as the grain size gets smaller and smaller?

Solution: With smaller grains, the movement of the dislocations is impeded by frequent intersections with the grain boundaries. The strength of metals is not nearly as low as might be predicted from the critical resolved shear stress as a consequence of these interactions.

- 4-38** The strength of titanium is found to be 65,000 psi when the grain size is 17×10^{-6} m and 82,000 psi when the grain size is 0.8×10^{-6} m. Determine (a) the constants in the Hall-Petch equation and (b) the strength of the titanium when the grain size is reduced to 0.2×10^{-6} m.

Solution: $65,000 = \sigma_o + K \frac{1}{\sqrt{17 \times 10^{-6}}} = \sigma_o + 242.5 K$

$$82,000 = \sigma_o + K \frac{1}{\sqrt{0.8 \times 10^{-6}}} = \sigma_o + 1118.0 K$$

- (a) By solving the two simultaneous equations:

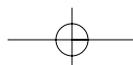
$$K = 19.4 \text{ psi} / \sqrt{d} \quad \sigma_o = 60,290 \text{ psi}$$

(b) $\sigma = 60,290 + 19.4 / \sqrt{0.2 \times 10^{-6}} = 103,670 \text{ psi}$

- 4-39** A copper-zinc alloy has the following properties:

Grain diameter (mm)	Strength (MPa)	$d^{-1/2}$
0.015	170 MPa	8.165
0.025	158 MPa	6.325
0.035	151 MPa	5.345
0.050	145 MPa	4.472

Determine (a) the constants in the Hall-Petch equation and (b) the grain size required to obtain a strength of 200 MPa.



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Solution: The values of $d^{-1/2}$ are included in the table; the graph shows the relationship. We can determine K and σ_o either from the graph or by using two of the data points.

$$(a) \quad 170 = \sigma_o + K(8.165)$$

$$145 = \sigma_o + K(4.472)$$

$$25 = 3.693K$$

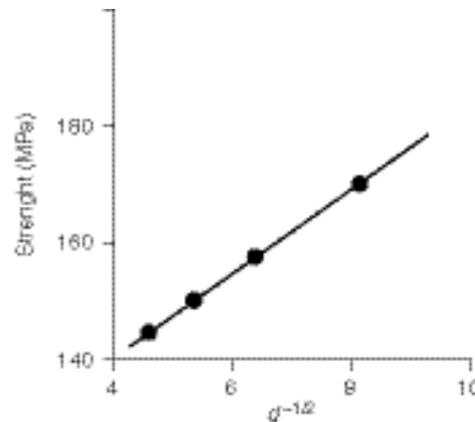
$$K = 6.77 \text{ MPa} / \sqrt{\text{mm}} \quad \sigma_o = 114.7 \text{ MPa}$$

(b) To obtain a strength of 200 MPa:

$$200 = 114.7 + 6.77 / \sqrt{d}$$

$$85.3 = 6.77 / \sqrt{d}$$

$$d = 0.0063 \text{ mm}$$



4-40 For an ASTM grain size number of 8, calculate the number of grains per square inch (a) at a magnification of 100 and (b) with no magnification.

Solution: (a) $N = 2^{n-1}$ $N = 2^{8-1} = 2^7 = 128 \text{ grains/in.}^2$

(b) No magnification means that the magnification is "1":

$$(2^7)(100/1)^2 = 1.28 \times 10^6 \text{ grains/in.}^2$$

4-41 Determine the ASTM grain size number if 20 grains/square inch are observed at a magnification of 400.

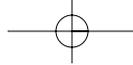
Solution: $(20)(400/100)^2 = 2^{n-1}$ $\log(320) = (n-1)\log(2)$

$$2.505 = (n-1)(0.301) \text{ or } n = 9.3$$

4-42 Determine the ASTM grain size number if 25 grains/square inch are observed at a magnification of 50.

Solution: $25(50/100)^2 = 2^{n-1}$ $\log(6.25) = (n-1)\log(2)$

$$0.796 = (n-1)(0.301) \text{ or } n = 3.6$$


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4-43 Determine the ASTM grain size number for the materials in

(a) Figure 4-17 (b) Figure 4-21

Solution: (a) There are about 26 grains in the photomicrograph, which has the dimensions 2.375 in. \times 2 in. The magnification is 100, thus:

$$\frac{26}{(2.375)(2)} = 2^{n-1} \quad \log(5.47) = 0.738 = (n-1)\log(2) \quad n = 3.5$$

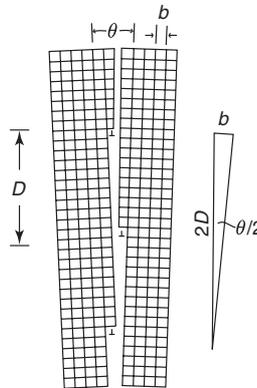
 (b) There are about 59 grains in the photomicrograph, which has the dimensions 2.25 in. \times 2 in. The magnification is 500, thus:

$$\frac{59(500/100)^2}{(2.25)(2)} = 2^{n-1} \quad \log(328) = 2.516 = (n-1)\log(2) \quad n = 9.4$$

 There are about 28 grains in the photomicrograph, which has the dimensions 2 in. \times 2.25 in. The magnification is 200, thus:

$$\frac{28(200/100)^2}{(2.25)(2)} = 2^{n-1} \quad \log(24.889) = 1.396 = (n-1)\log(2) \quad n = 5.6$$

4-46 The angle θ of a tilt boundary is given by $\sin(\theta/2) = b/2D$ (see Figure 4-18). Verify the correctness of this equation.

Solution: From the figure, we note that the grains are offset one Burgers vector, b , only for two spacings D . Then it is apparent that $\sin(\theta/2)$ must be b divided by two D .

4-47 Calculate the angle θ of a small-angle grain boundary in FCC aluminum when the dislocations are 5000 Å apart. (See Figure 4-18 and equation in Problem 4-46.)

Solution: $b = (\frac{1}{2})(\sqrt{2})(4.04958) = 2.8635 \text{ \AA}$ and $D = 5000 \text{ \AA}$

$$\sin(\theta/2) = \frac{2.8635}{(2)(5000)} = 0.000286$$

$$\theta/2 = 0.0164$$

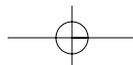
$$\theta = 0.0328^\circ$$

4-48 For BCC iron, calculate the average distance between dislocations in a small-angle grain boundary tilted 0.50°. (See Figure 4-18.)

Solution: $\sin(0.5/2) = \frac{\frac{1}{2}(\sqrt{3})(2.866)}{2D}$

$$0.004364 = 1.241/D$$

$$D = 284 \text{ \AA}$$



5

Atom and Ion Movements in Materials

- 5-8** Atoms are found to move from one lattice position to another at the rate of 5×10^5 jumps/s at 400°C when the activation energy for their movement is $30,000$ cal/mol. Calculate the jump rate at 750°C .

Solution:

$$\text{Rate} = \frac{5 \times 10^5}{x} = \frac{c_0 \exp[-30,000/(1.987)(673)]}{c_0 \exp[-30,000/(1.987)(1023)]} = \exp(-22.434 + 14.759)$$

$$\frac{5 \times 10^5}{x} = \exp(-7.675) = 4.64 \times 10^{-4}$$

$$x = \frac{5 \times 10^5}{4.64 \times 10^{-4}} = 1.08 \times 10^9 \text{ jumps/s}$$

- 5-9** The number of vacancies in a material is related to temperature by an Arrhenius equation. If the fraction of lattice points containing vacancies is 8×10^{-5} at 600°C , determine the fraction at 1000°C .

Solution: $8 \times 10^{-5} = \exp[-Q/(1.987)(873)]$ $Q = 16,364$ cal/mol

$$f = n_v/n = \exp[-16,364/(1.987)(1273)] = 0.00155$$

- 5-15** The diffusion coefficient for Cr^{+3} in Cr_2O_3 is 6×10^{-15} cm^2/s at 727°C and is 1×10^{-9} cm^2/s at 1400°C . Calculate (a) the activation energy and (b) the constant D_0 .

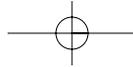
Solution: (a) $\frac{6 \times 10^{-15}}{1 \times 10^{-9}} = \frac{D_0 \exp[-Q/(1.987)(1000)]}{D_0 \exp[-Q/(1.987)(1673)]}$

$$6 \times 10^{-6} = \exp[-Q(0.000503 - 0.00030)] = \exp[-0.000203 Q]$$

$$-12.024 = -0.000203 Q \quad \text{or} \quad Q = 59,230 \text{ cal/mol}$$

(b) $1 \times 10^{-9} = D_0 \exp[-59,230/(1.987)(1673)] = D_0 \exp(-17.818)$

$$1 \times 10^{-9} = 1.828 \times 10^{-8} D_0 \quad \text{or} \quad D_0 = 0.055 \text{ cm}^2/\text{s}$$


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- 5-16** The diffusion coefficient for O^{-2} in Cr_2O_3 is $4 \times 10^{-15} \text{ cm}^2/\text{s}$ at 1150°C and $6 \times 10^{-11} \text{ cm}^2/\text{s}$ at 1715°C . Calculate (a) the activation energy and (b) the constant D_0 .

Solution:

$$\frac{4 \times 10^{-15}}{6 \times 10^{-11}} = \frac{D_0 \exp[-Q/(1.987)(1423)]}{D_0 \exp[-Q/(1.987)(1988)]}$$

$$6.67 \times 10^{-5} = \exp[-0.0001005 Q]$$

$$-9.615 = -0.0001005 Q \quad \text{or} \quad Q = 95,700 \text{ cal/mol}$$

$$4 \times 10^{-15} = D_0 \exp[-95,700/(1.987)(1423)] = D_0(2.02 \times 10^{-15})$$

$$D_0 = 1.98 \text{ cm}^2/\text{s}$$

- 5-23** A 0.2-mm thick wafer of silicon is treated so that a uniform concentration gradient of antimony is produced. One surface contains 1 Sb atom per 10^8 Si atoms and the other surface contains 500 Sb atoms per 10^8 Si atoms. The lattice parameter for Si is 5.407 \AA (Appendix A). Calculate the concentration gradient in (a) atomic percent Sb per cm and (b) Sb atoms/ $\text{cm}^3 \cdot \text{cm}$.

Solution:

$$\Delta c/\Delta x = \frac{(1/10^8 - 500/10^8)}{0.02 \text{ cm}} \times 100\% = -0.02495 \text{ at\%/cm}$$

$$a_0 = 5.4307 \text{ \AA} \quad V_{\text{unit cell}} = 160.16 \times 10^{-24} \text{ cm}^3$$

$$c_1 = \frac{(8 \text{ Si atoms/u.c.})(1 \text{ Sb}/10^8 \text{ Si})}{160.16 \times 10^{-24} \text{ cm}^3/\text{u.c.}} = 0.04995 \times 10^{16} \text{ Sb atoms/cm}^3$$

$$c_2 = \frac{(8 \text{ Si atoms/u.c.})(500 \text{ Sb}/10^8 \text{ Si})}{160.16 \times 10^{-24} \text{ cm}^3/\text{u.c.}} = 24.975 \times 10^{16} \text{ Sb atoms/cm}^3$$

$$\Delta c/\Delta x = \frac{(0.04995 - 24.975) \times 10^{16}}{0.02 \text{ cm}} = -1.246 \times 10^{19} \text{ Sb atoms/cm}^3 \cdot \text{cm}$$

- 5-24** When a Cu-Zn alloy solidifies, one portion of the structure contains 25 atomic percent zinc and another portion 0.025 mm away contains 20 atomic percent zinc. The lattice parameter for the FCC alloy is $3.63 \times 10^{-8} \text{ cm}$. Determine the concentration gradient in (a) atomic percent Zn per cm, (b) weight percent Zn per cm, and (c) Zn atoms/ $\text{cm}^3 \cdot \text{cm}$.

Solution: (a) $\Delta c/\Delta x = \frac{20\% - 25\%}{(0.025 \text{ mm})(0.1 \text{ cm/mm})} = -2000 \text{ at\% Zn/cm}$

(b) We now need to determine the wt% of zinc in each portion:

$$\text{wt\% Zn} = \frac{(20)(65.38 \text{ g/mol})}{(20)(65.38) + (80)(63.54)} \times 100 = 20.46$$

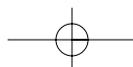
$$\text{wt\% Zn} = \frac{(25)(65.38 \text{ g/mol})}{(25)(65.38) + (75)(63.54)} \times 100 = 25.54$$

$$\Delta c/\Delta x = \frac{20.46\% - 25.54\%}{0.0025 \text{ cm}} = -2032 \text{ wt\% Zn/cm}$$

(c) Now find the number of atoms per cm^3 :

$$c_1 = \frac{(4 \text{ atoms/cell})(0.2 \text{ Zn fraction})}{(3.63 \times 10^{-8} \text{ cm})^3} = 0.0167 \times 10^{24} \text{ Zn atoms/cm}^3$$

$$c_2 = \frac{(4 \text{ atoms/cell})(0.25 \text{ Zn fraction})}{(3.63 \times 10^{-8} \text{ cm})^3} = 0.0209 \times 10^{24} \text{ Zn atoms/cm}^3$$



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$$\Delta c/\Delta x = \frac{0.0167 \times 10^{24} - 0.0209 \times 10^{24}}{0.0025 \text{ cm}} = -1.68 \text{ Zn atoms/cm}^3 \cdot \text{cm}$$

- 5-25** A 0.001-in. BCC iron foil is used to separate a high hydrogen gas from a low hydrogen gas at 650°C. 5×10^8 H atoms/cm³ are in equilibrium with the hot side of the foil, while 2×10^3 H atoms/cm³ are in equilibrium with the cold side. Determine (a) the concentration gradient of hydrogen and (b) the flux of hydrogen through the foil.

Solution: (a) $\Delta c/\Delta x = \frac{2 \times 10^3 - 5 \times 10^8}{(0.001 \text{ in.})(2.54 \text{ cm/in.})} = -1969 \times 10^8 \text{ H atoms/cm}^3 \cdot \text{cm}$

(b) $J = -D(\Delta c/\Delta x) = -0.0012 \exp[-3600/(1.987)(923)](-1969 \times 10^8)$
 $J = 0.33 \times 10^8 \text{ H atoms/cm}^2 \cdot \text{s}$

- 5-26** A 1-mm sheet of FCC iron is used to contain nitrogen in a heat exchanger at 1200°C. The concentration of N at one surface is 0.04 atomic percent and the concentration at the second surface is 0.005 atomic percent. Determine the flux of nitrogen through the foil in N atoms/cm² · s.

Solution: (a) $\Delta c/\Delta x = \frac{(0.00005 - 0.0004)(4 \text{ atoms per cell})/(3.589 \times 10^{-8} \text{ cm})^3}{(1 \text{ mm})(0.1 \text{ cm/mm})}$
 $= -3.03 \times 10^{20} \text{ N atoms/cm}^3 \cdot \text{cm}$

(b) $J = -D(\Delta c/\Delta x) = -0.0034 \exp[-34,600/(1.987)(1473)](-3.03 \times 10^{20})$
 $= 7.57 \times 10^{12} \text{ N atoms/cm}^2 \cdot \text{s}$

- 5-27** A 4-cm-diameter, 0.5-mm-thick spherical container made of BCC iron holds nitrogen at 700°C. The concentration at the inner surface is 0.05 atomic percent and at the outer surface is 0.002 atomic percent. Calculate the number of grams of nitrogen that are lost from the container per hour.

Solution: $\Delta c/\Delta x = \frac{[0.00002 - 0.0005](2 \text{ atoms/cell})/(2.866 \times 10^{-8} \text{ cm})^3}{(0.5 \text{ mm})(0.1 \text{ cm/mm})}$
 $= -8.16 \times 10^{20} \text{ N/cm}^3 \cdot \text{cm}$

$$J = -0.0047 \exp[-18,300/(1.987)(973)](-8.16 \times 10^{20}) = 2.97 \times 10^{14} \text{ N/cm}^2 \cdot \text{s}$$

$$A_{\text{sphere}} = 4\pi r^2 = 4\pi(2 \text{ cm})^2 = 50.27 \text{ cm}^2 \quad t = 3600 \text{ s/h}$$

$$\text{N atoms/h} = (2.97 \times 10^{14})(50.27)(3600) = 5.37 \times 10^{19} \text{ N atoms/h}$$

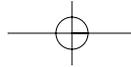
$$\text{N loss} = \frac{(5.37 \times 10^{19} \text{ atoms})(14.007 \text{ g/mol})}{(6.02 \times 10^{23} \text{ atoms/mol})} = 1.245 \times 10^{-3} \text{ g/h}$$

- 5-28** A BCC iron structure is to be manufactured that will allow no more than 50 g of hydrogen to be lost per year through each square centimeter of the iron at 400°C. If the concentration of hydrogen at one surface is 0.05 H atom per unit cell and is 0.001 H atom per unit cell at the second surface, determine the minimum thickness of the iron.

Solution: $c_1 = 0.05 \text{ H}/(2.866 \times 10^{-8} \text{ cm})^3 = 212.4 \times 10^{19} \text{ H atoms/cm}^3$

$$c_2 = 0.001 \text{ H}/(2.866 \times 10^{-8} \text{ cm})^3 = 4.25 \times 10^{19} \text{ H atoms/cm}^3$$

$$\Delta c/\Delta x = \frac{4.25 \times 10^{19} - 212.4 \times 10^{19}}{\Delta x} = \frac{-2.08 \times 10^{21}}{\Delta x}$$


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$$J = \frac{(50 \text{ g/cm}^2 \text{ y})(6.02 \times 10^{23} \text{ atoms/mol})}{(1.00797 \text{ g/mol})(31.536 \times 10^6 \text{ s/y})} = 9.47 \times 10^{17} \text{ H atoms/cm}^2 \cdot \text{s}$$

$$J = 9.47 \times 10^{17} \text{ H atoms/cm}^2 \cdot \text{s}$$

$$= (-2.08 \times 10^{21}/\Delta x)(0.0012)\exp[-3600/((1.987)(673))]$$

$$\Delta x = 0.179 \text{ cm}$$

- 5-29** Determine the maximum allowable temperature that will produce a flux of less than $2000 \text{ H atoms/cm}^2 \cdot \text{s}$ through a BCC iron foil when the concentration gradient is $-5 \times 10^{16} \text{ atoms/cm}^3 \cdot \text{cm}$. (Note the negative sign for the flux.)

Solution:

$$2000 \text{ H atoms/cm}^2 \cdot \text{s} = -0.0012 \exp[-3600/1.987T] [-5 \times 10^{16} \text{ atoms/cm}^3 \cdot \text{cm}]$$

$$\ln(3.33 \times 10^{-11}) = -3600/1.987T$$

$$T = -3600/((-24.12)(1.987)) = 75 \text{ K} = -198^\circ\text{C}$$

- 5-35** Compare the rate at which oxygen ions diffuse in Al_2O_3 with the rate at which aluminum ions diffuse in Al_2O_3 at 1500°C . Explain the difference.

Solution: $D_{\text{O}}^{-2} = 1900 \exp[-152,000/(1.987)(1773)] = 3.47 \times 10^{-16} \text{ cm}^2/\text{s}$

$$D_{\text{Al}}^{+3} = 28 \exp[-114,000/(1.987)(1773)] = 2.48 \times 10^{-13} \text{ cm}^2/\text{s}$$

The ionic radius of the oxygen ion is 1.32 \AA , compared with the aluminum ionic radius of 0.51 \AA ; consequently it is much easier for the smaller aluminum ion to diffuse in the ceramic.

- 5-36** Compare the diffusion coefficients of carbon in BCC and FCC iron at the allotropic transformation temperature of 912°C and explain the difference.

Solution: $D_{\text{BCC}} = 0.011 \exp[-20,900/(1.987)(1185)] = 1.51 \times 10^{-6} \text{ cm}^2/\text{s}$

$$D_{\text{FCC}} = 0.23 \exp[-32,900/(1.987)(1185)] = 1.92 \times 10^{-7} \text{ cm}^2/\text{s}$$

Packing factor of the BCC lattice (0.68) is less than that of the FCC lattice; consequently atoms are expected to be able to diffuse more rapidly in the BCC iron.

- 5-37** Compare the diffusion coefficients for hydrogen and nitrogen in FCC iron at 1000°C and explain the difference in their values.

Solution: $D_{\text{H in BCC}} = 0.0063 \exp[-10,300/(1.987)(1273)] = 1.074 \times 10^{-4} \text{ cm}^2/\text{s}$

$$D_{\text{N in FCC}} = 0.0034 \exp[-34,600/(1.987)(1273)] = 3.898 \times 10^{-9} \text{ cm}^2/\text{s}$$

Nitrogen atoms have a larger atoms radius (0.71 \AA) compared with that of hydrogen atoms (0.46 \AA); the smaller hydrogen ions are expected to diffuse more rapidly.

- 5-41** A carburizing process is carried out on a 0.10% C steel by introducing 1.0% C at the surface at 980°C , where the iron is FCC. Calculate the carbon content at 0.01 cm, 0.05 cm, and 0.10 cm beneath the surface after 1 h.

Solution: $D = 0.23 \exp[-32,900/(1.987)(1253)] = 42 \times 10^{-8} \text{ cm}^2/\text{s}$

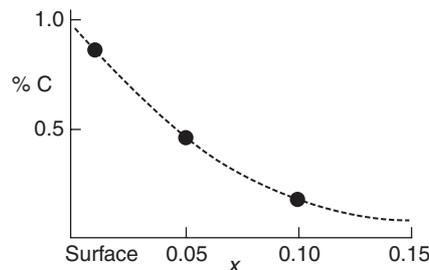
$$\frac{1 - c_x}{1 - 0.1} = \text{erf}[x / (2\sqrt{(42 \times 10^{-8})(3600)})] = \text{erf}[x / 0.0778]$$

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$$x = 0.01: \operatorname{erf}[0.01/0.0778] = \operatorname{erf}(0.1285) = \frac{(1 - c_x)}{0.9} = 0.144 \quad c_x = 0.87\% \text{ C}$$

$$x = 0.05: \operatorname{erf}[0.05/0.0778] = \operatorname{erf}(0.643) = \frac{(1 - c_x)}{0.9} = 0.636 \quad c_x = 0.43\% \text{ C}$$

$$x = 0.10: \operatorname{erf}[0.10/0.0778] = \operatorname{erf}(1.285) = \frac{(1 - c_x)}{0.9} = 0.914 \quad c_x = 0.18\% \text{ C}$$



- 5-42** Iron containing 0.05% C is heated to 912°C in an atmosphere that produces 1.20% C at the surface and is held for 24 h. Calculate the carbon content at 0.05 cm beneath the surface if (a) the iron is BCC and (b) the iron is FCC. Explain the difference.

Solution: $t = (24 \text{ h})(3600 \text{ s/h}) = 86,400 \text{ s}$

$$D_{\text{BCC}} = 0.011 \exp[-20,900/(1.987)(1185)] = 1.54 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$D_{\text{FCC}} = 0.23 \exp[-32,900/(1.987)(1185)] = 1.97 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$\text{BCC: } \frac{1.2 - c_x}{1.2 - 0.05} = \operatorname{erf}\left[0.05 / (2\sqrt{(1.54 \times 10^{-6})(86,400)})\right] = \operatorname{erf}[0.0685] = 0.077$$

$$c_x = 1.11\% \text{ C}$$

$$\text{FCC: } \frac{1.2 - c_x}{1.2 - 0.05} = \operatorname{erf}\left[0.05 / (2\sqrt{(1.97 \times 10^{-7})(86,400)})\right] = \operatorname{erf}[0.192] = 0.2139$$

$$c_x = 0.95\% \text{ C}$$

Faster diffusion occurs in the looser packed BCC structure, leading to the higher carbon content at point “x”.

- 5-43** What temperature is required to obtain 0.50% C at a distance of 0.5 mm beneath the surface of a 0.20% C steel in 2 h, when 1.10% C is present at the surface? Assume that the iron is FCC.

Solution: $\frac{1.1 - 0.5}{1.1 - 0.2} = 0.667 = \operatorname{erf}[0.05 / 2\sqrt{Dt}]$

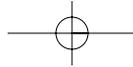
$$0.05 / 2\sqrt{Dt} = 0.685 \quad \text{or} \quad \sqrt{Dt} = 0.0365 \quad \text{or} \quad Dt = 0.00133$$

$$t = (2 \text{ h})(3600 \text{ s/h}) = 7200 \text{ s}$$

$$D = 0.00133/7200 = 1.85 \times 10^{-7} = 0.23 \exp[-32,900/1.987T]$$

$$\exp(-16,558/T) = 8.043 \times 10^{-7}$$

$$T = 1180\text{K} = 907^\circ\text{C}$$


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- 5-44** A 0.15% C steel is to be carburized at 1100°C, giving 0.35% C at a distance of 1 mm beneath the surface. If the surface composition is maintained at 0.90% C, what time is required?

Solution:
$$\frac{0.9 - 0.35}{0.9 - 0.15} = 0.733 = \text{erf}[0.1/2\sqrt{Dt}]$$

$$0.1/2\sqrt{Dt} = 0.786 \quad \text{or} \quad \sqrt{Dt} = 0.0636 \quad \text{or} \quad Dt = 0.00405$$

$$D = 0.23 \exp[-32,900/(1.987)(1373)] = 1.332 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$t = 0.00405/1.332 \times 10^{-6} = 3040 \text{ s} = 51 \text{ min}$$

- 5-45** A 0.02% C steel is to be carburized at 1200°C in 4 h, with a point 0.6 mm beneath the surface reaching 0.45% C. Calculate the carbon content required at the surface of the steel.

Solution:
$$\frac{c_s - 0.45}{c_s - 0.02} = \text{erf}[0.06/2\sqrt{Dt}]$$

$$D = 0.23 \exp[-32,900/(1.987)(1473)] = 3.019 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$t = (4 \text{ h})(3600) = 14,400 \text{ s}$$

$$\sqrt{Dt} = \sqrt{(3.019 \times 10^{-6})(14,400)} = 0.2085$$

$$\text{erf}[0.06/(2)(0.2085)] = \text{erf}(0.144) = 0.161$$

$$\frac{c_s - 0.45}{c_s - 0.02} = 0.161 \quad \text{or} \quad c_s = 0.53\% \text{ C}$$

- 5-46** A 1.2% C tool steel held at 1150°C is exposed to oxygen for 48 h. The carbon content at the steel surface is zero. To what depth will the steel be decarburized to less than 0.20% C?

Solution:
$$\frac{0 - 0.2}{0 - 1.2} = 0.1667 \quad \therefore x/2\sqrt{Dt} = 0.149$$

$$D = 0.23 \exp[-32,900/(1.987)(1423)] = 2.034 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$t = (48 \text{ h})(3600 \text{ s/h}) = 17.28 \times 10^4 \text{ s}$$

$$\sqrt{Dt} = 0.5929$$

Then from above, $x = (0.149)(2)(0.5929) = 0.177 \text{ cm}$

- 5-47** A 0.80% C steel must operate at 950°C in an oxidizing environment, where the carbon content at the steel surface is zero. Only the outermost 0.02 cm of the steel part can fall below 0.75% C. What is the maximum time that the steel part can operate?

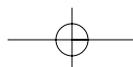
Solution:
$$\frac{0 - 0.75}{0 - 0.8} = 0.9375 = \text{erf}[x/2\sqrt{Dt}] \quad \therefore x/2\sqrt{Dt} = 1.384$$

$$0.02/2\sqrt{Dt} = 1.384 \quad \text{or} \quad \sqrt{Dt} = 0.007226 \quad \text{or} \quad Dt = 5.22 \times 10^{-5}$$

$$D = 0.23 \exp[-32,900/(1.987)(1223)] = 3.03 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$t = 5.22 \times 10^{-5} / 3.03 \times 10^{-7} = 172 \text{ s} = 2.9 \text{ min}$$

- 5-48** A steel with BCC crystal structure containing 0.001% N is nitrided at 550°C for 5 h. If the nitrogen content at the steel surface is 0.08%, determine the nitrogen content at 0.25 mm from the surface.



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$$\text{Solution: } \frac{0.08 - c_s}{0.08 - 0.001} = \text{erf}[0.025 / 2\sqrt{Dt}] \quad t = (5 \text{ h})(3600 \text{ s/h}) = 1.8 \times 10^4 \text{ s}$$

$$D = 0.0047 \exp[-18,300/(1.987)(823)]$$

$$= 6.488 \times 10^{-8} \text{ cm}^2/\text{s}$$

$$\sqrt{Dt} = 0.0342$$

$$\text{erf}[0.025/(2)(0.0342)] = \text{erf}(0.3655) = 0.394$$

$$\frac{0.08 - c_s}{0.079} = 0.394 \quad \text{or } c_s = 0.049\% \text{ N}$$

- 5-49** What time is required to nitride a 0.002 N steel to obtain 0.12% N at a distance of 0.002 in. beneath the surface at 625°C? The nitrogen content at the surface is 0.15%.

$$\text{Solution: } \frac{0.15 - 0.12}{0.15 - 0.002} = 0.2027 = \text{erf}[x / 2\sqrt{Dt}] \quad \therefore x / 2\sqrt{Dt} = 0.2256$$

$$D = 0.0047 \exp[-18,300/(1.987)(898)] = 1.65 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$x = 0.002 \text{ in.} = 0.00508 \text{ cm}$$

$$\frac{0.00508}{2\sqrt{(1.65 \times 10^{-7})t}} = 0.2256$$

$$Dt = 1.267 \times 10^{-4} \quad \text{or } t = 1.267 \times 10^{-4} / 1.65 \times 10^{-7} = 768 \text{ s} = 12.8 \text{ min}$$

- 5-50** We currently can successfully perform a carburizing heat treatment at 1200°C in 1 h. In an effort to reduce the cost of the brick lining in our furnace, we propose to reduce the carburizing temperature to 950°C. What time will be required to give us a similar carburizing treatment?

$$\text{Solution: } D_{1200} = 0.23 \exp[-32,900/(1.987)(1473)] = 3.019 \times 10^{-6} \text{ cm}^2/\text{s}$$

$$D_{950} = 0.23 \exp[-32,900/(1.987)(1223)] = 3.034 \times 10^{-7} \text{ cm}^2/\text{s}$$

$$t_{1200} = 1 \text{ h}$$

$$t_{950} = D_{1200} t_{1200} / D_{950} = \frac{(3.019 \times 10^{-6})(1)}{3.034 \times 10^{-7}} = 9.95 \text{ h}$$

- 5-56** During freezing of a Cu-Zn alloy, we find that the composition is nonuniform. By heating the alloy to 600°C for 3 hours, diffusion of zinc helps to make the composition more uniform. What temperature would be required if we wished to perform this *homogenization* treatment in 30 minutes?

$$\text{Solution: } D_{600} = 0.78 \exp[-43,900/(1.987)(873)] = 7.9636 \times 10^{-12} \quad t_{600} = 3 \text{ h}$$

$$t_x = 0.5 \text{ h}$$

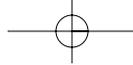
$$D_x = D_{600} t_{600} / t_x = (7.9636 \times 10^{-12})(3) / 0.5$$

$$D_x = 4.778 \times 10^{-11} = 0.78 \exp[-43,900/1.987T]$$

$$\ln(6.1258 \times 10^{-11}) = -23.516 = -43,900/1.987 T$$

$$T = 940 \text{ K} = 667^\circ\text{C}$$

- 5-57** A ceramic part made of MgO is sintered successfully at 1700°C in 90 minutes. To minimize thermal stresses during the process, we plan to reduce the temperature to


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1500°C. Which will limit the rate at which sintering can be done: diffusion of magnesium ions or diffusion of oxygen ions? What time will be required at the lower temperature?

Solution: Diffusion of oxygen is the slower of the two, due to the larger ionic radius of the oxygen.

$$D_{1700} = 0.000043 \exp[-82,100/(1.987)(1973)] = 3.455 \times 10^{-14} \text{ cm}^2/\text{s}$$

$$D_{1500} = 0.000043 \exp[-82,100/(1.987)(1773)] = 3.255 \times 10^{-15} \text{ cm}^2/\text{s}$$

$$t_{1500} = D_{1700} t_{1700} / D_{1500} = \frac{(3.455 \times 10^{-14})(90)}{3.255 \times 10^{-15}} = 955 \text{ min} = 15.9 \text{ h}$$

5-58 A Cu-Zn alloy has an initial grain diameter of 0.01 mm. The alloy is then heated to various temperatures, permitting grain growth to occur. The times required for the grains to grow to a diameter of 0.30 mm are

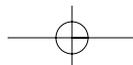
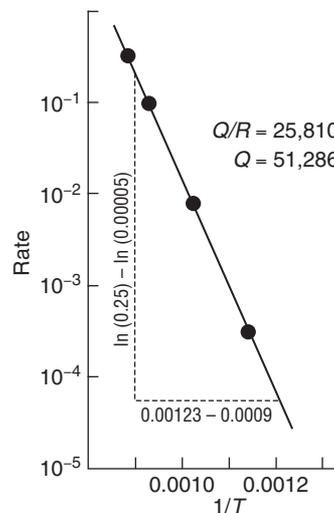
Temperature (°C)	Time (min)
500	80,000
600	3,000
700	120
800	10
850	3

Determine the activation energy for grain growth. Does this correlate with the diffusion of zinc in copper? (*Hint:* Note that rate is the reciprocal of time.)

Solution:

Temperature (°C)	Temperature (K)	1/T (K ⁻¹)	Time (min)	Rate (min ⁻¹)
500	773	0.00129	80,000	1.25 × 10 ⁻⁵
600	873	0.00115	3,000	3.33 × 10 ⁻⁴
700	973	0.001028	120	8.33 × 10 ⁻³
800	1073	0.000932	10	0.100
850	1123	0.000890	3	0.333

From the graph, we find that $Q = 51,286$ cal/mol, which does correlate with the activation energy for diffusion of zinc in copper.



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- 5-60** A sheet of gold is diffusion-bonded to a sheet of silver in 1 h at 700°C. At 500°C, 440 h are required to obtain the same degree of bonding, and at 300°C, bonding requires 1530 years. What is the activation energy for the diffusion bonding process? Does it appear that diffusion of gold or diffusion of silver controls the bonding rate? (*Hint*: Note that rate is the reciprocal of time.)

Solution:

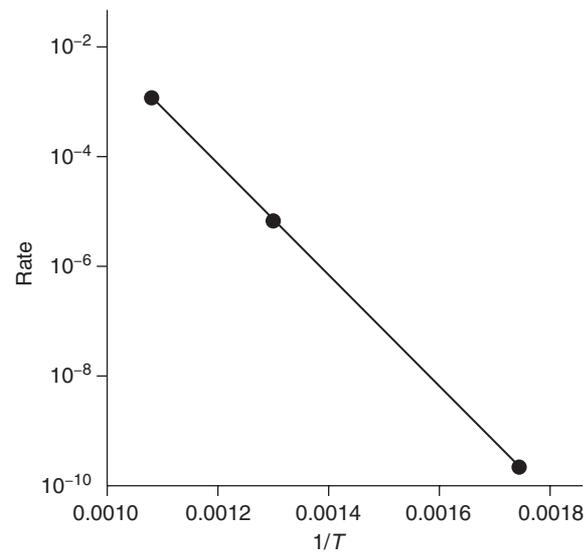
Temperature (°C)	Temperature (K)	1/T (K ⁻¹)	Time (s)	Rate (sec ⁻¹)
700	973	0.001007	3600	0.278×10^{-3}
500	773	0.001294	1.584×10^6	0.631×10^{-6}
300	573	0.001745	4.825×10^{10}	0.207×10^{-10}

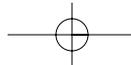
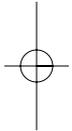
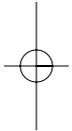
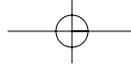
$$\frac{0.278 \times 10^{-3}}{0.207 \times 10^{-10}} = \frac{\exp[-Q/(1.987)(973)]}{\exp[-Q/(1.987)(573)]} = \frac{\exp[-0.0005172Q]}{\exp[-0.0008783Q]}$$

$$\ln(1.343 \times 10^7) = 16.413 = 0.0003611 Q$$

$$Q = 45,450 \text{ cal/mol.}$$

The activation energy for the diffusion of gold in silver is 45,500 cal/mole; thus the diffusion of gold appears to control the bonding rate.





6

Mechanical Properties and Behavior

- 6-24** A 850-lb force is applied to a 0.15-in.-diameter nickel wire having a yield strength of 45,000 psi and a tensile strength of 55,000 psi. Determine (a) whether the wire will plastically deform and (b) whether the wire will experience necking.

Solution: (a) First determine the stress acting on the wire:

$$\sigma = F/A = 850 \text{ lb} / (\pi/4)(0.15 \text{ in.})^2 = 48,100 \text{ psi}$$

Because σ is greater than the yield strength of 45,000 psi, the wire will plastically deform.

- (b) Because σ is less than the tensile strength of 55,000 psi, no necking will occur.

- 6-25 (a)** A force of 100,000 N is applied to a 10 mm \times 20 mm iron bar having a yield strength of 400 MPa and a tensile strength of 480 MPa. Determine whether the bar will plastically deform and whether the bar will experience necking.

Solution: First determine the stress acting on the wire:

$$\sigma = F/A = 100,000 \text{ N} / (10 \text{ mm})(20 \text{ mm}) = 500 \text{ N/mm}^2 = 500 \text{ MPa}$$

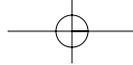
Because σ is greater than the yield strength of 400 MPa, the wire will plastically deform.

Because σ is greater than the tensile strength of 480 MPa, the wire will also neck.

- (b)** Calculate the maximum force that a 0.2-in. diameter rod of Al_2O_3 , having a yield strength of 35,000 psi, can withstand with no plastic deformation. Express your answer in pounds and Newtons.

Solution: $F = \sigma A = (35,000 \text{ psi})(\pi/4)(0.2 \text{ in.})^2 = 1100 \text{ lb}$

$$F = (1100 \text{ lb})(4.448 \text{ N/lb}) = 4891 \text{ N}$$


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- 6-26** A force of 20,000 N will cause a 1 cm × 1 cm bar of magnesium to stretch from 10 cm to 10.045 cm. Calculate the modulus of elasticity, both in GPa and psi.

Solution: The strain ε is $\varepsilon = (10.045 \text{ cm} - 10 \text{ cm}) / 10 \text{ cm} = 0.0045 \text{ cm/cm}$
 The stress σ is $\sigma = 20,000 \text{ N} / (10 \text{ mm})(10 \text{ mm}) = 200 \text{ N/mm}^2$
 $= 200 \text{ MPa}$
 $E = \sigma / \varepsilon = 200 \text{ MPa} / 0.0045 \text{ cm/cm} = 44,444 \text{ MPa} = 44.4 \text{ GPa}$
 $E = (44,444 \text{ MPa})(145 \text{ psi/MPa}) = 6.44 \times 10^6 \text{ psi}$

- 6-27** A polymer bar's dimensions are 1 in. × 2 in. × 15 in. The polymer has a modulus of elasticity of 600,000 psi. What force is required to stretch the bar elastically to 15.25 in.?

Solution: The strain ε is $\varepsilon = (15.25 \text{ in.} - 15 \text{ in.}) / (15 \text{ in.}) = 0.01667 \text{ in./in.}$
 The stress σ is $\sigma = E\varepsilon = (600,000 \text{ psi})(0.01667 \text{ in./in.}) = 10,000 \text{ psi}$
 The force is then $F = \sigma A = (10,000 \text{ psi})(1 \text{ in.})(2 \text{ in.}) = 20,000 \text{ lb}$

- 6-28** An aluminum plate 0.5 cm thick is to withstand a force of 50,000 N with no permanent deformation. If the aluminum has a yield strength of 125 MPa, what is the minimum width of the plate?

Solution: The area is $A = F / \sigma = 50,000 \text{ N} / 125 \text{ N/mm}^2 = 400 \text{ mm}^2$
 The minimum width is $w = A / t = (400 \text{ mm}^2)(0.1 \text{ cm/mm})^2 / 0.5 \text{ cm}$
 $= 8 \text{ cm}$

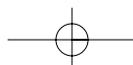
- 6-29 (a)** A 3-in.-diameter rod of copper is to be reduced to a 2-in.-diameter rod by being pushed through an opening. To account for the elastic strain, what should be the diameter of the opening? The modulus of elasticity for the copper is $17 \times 10^6 \text{ psi}$ and the yield strength is 40,000 psi.

Solution: (a) The strain is $\varepsilon = \sigma / E = 40,000 \text{ psi} / 17 \times 10^6 \text{ psi} = 0.00235 \text{ in./in.}$
 The strain is also $\varepsilon = (2 \text{ in.} - d_o) / d_o = 0.00235 \text{ in./in.}$
 $2 - d_o = 0.00235 d_o$
 $d_o = 2 / 1.00235 = 1.995 \text{ in.}$
 The opening in the die must be smaller than the final diameter.

- 6-30** A steel cable 1.25 in. in diameter and 50 ft long is to lift a 20-ton load. What is the length of the cable during lifting? The modulus of elasticity of the steel is $30 \times 10^6 \text{ psi}$.

Solution: The stress is $\sigma = F / A = \frac{(20 \text{ ton})(2000 \text{ lb/ton})}{(\pi/4)(1.25 \text{ in.})^2} = 32,595 \text{ psi}$
 The strain is $\varepsilon = \sigma / E = 32,595 \text{ psi} / 30 \times 10^6 \text{ psi} = 0.0010865 \text{ in./in.}$
 $\varepsilon = (\ell_f - 50 \text{ ft}) / 50 \text{ ft} = 0.0010865 \text{ ft/ft}$
 $\ell_f = 50.0543 \text{ ft}$

- 6-33** The following data were collected from a standard 0.505-in.-diameter test specimen of a copper alloy (initial length $(\ell_o) = 2.0 \text{ in.}$):

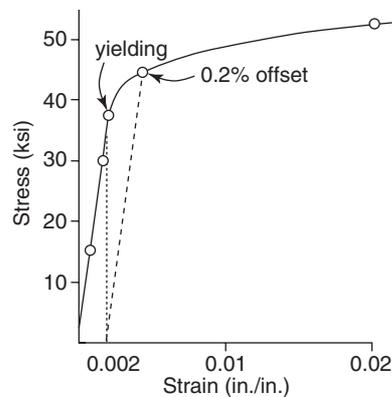


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Load (lb)	Gage Length (in.)	Stress (psi)	Strain (in./in.)
0	2.00000	0	0.0
3,000	2.00167	15,000	0.000835
6,000	2.00333	30,000	0.001665
7,500	2.00417	37,500	0.002085
9,000	2.0090	45,000	0.0045
10,500	2.040	52,500	0.02
12,000	2.26	60,000	0.13
12,400	2.50 (max load)	62,000	0.25
11,400	3.02 (fracture)	57,000	0.51

After fracture, the gage length is 3.014 in. and the diameter is 0.374 in. Plot the data and calculate the 0.2% offset yield strength along with (a) the tensile strength, (b) the modulus of elasticity, (c) the % elongation, (d) the % reduction in area, (e) the engineering stress at fracture, (f) the true stress at fracture, and (g) the modulus of resilience.

Solution: $\sigma = F / (\pi/4)(0.505)^2 = F/0.2$
 $\epsilon = (\ell - 2) / 2$



0.2% offset yield strength = 45,000 psi

(a) tensile strength = 62,000 psi

(b) $E = (30,000 - 0) / (0.001665 - 0) = 18 \times 10^6$ psi

(c) % elongation = $\frac{(3.014 - 2)}{2} \times 100 = 50.7\%$

(d) % reduction in area = $\frac{(\pi/4)(0.505)^2 - (\pi/4)(0.374)^2}{(\pi/4)(0.505)^2} \times 100 = 45.2\%$

(e) engineering stress at fracture = 57,000 psi

(f) true stress at fracture = $11,400 \text{ lb} / (\pi/4)(0.374)^2 = 103,770$ psi

(g) From the graph, yielding begins at about 37,500 psi. Thus:

$$\frac{1}{2}(\text{yield strength})(\text{strain at yield}) = \frac{1}{2}(37,500)(0.002085) = 39.1 \text{ psi}$$

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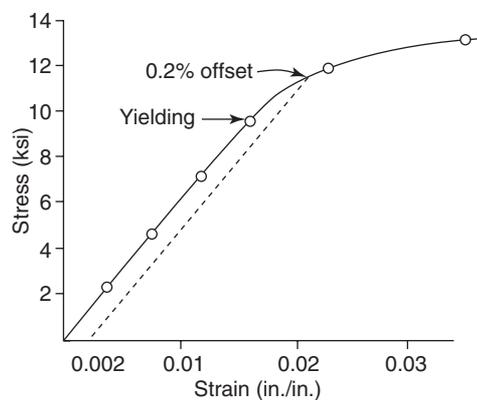
6-34 The following data were collected from a 0.4-in. diameter test specimen of polyvinyl chloride ($\ell_0 = 2.0$ in.):

Load (lb)	Gage Length (in.)	Stress (psi)	Strain (in./in.)
0	2.00000	0	0.0
300	2.00746	2,387	0.00373
600	2.01496	4,773	0.00748
900	2.02374	7,160	0.01187
1200	2.032	9,547	0.016
1500	2.046	11,933	0.023
1660	2.070 (max load)	13,206	0.035
1600	2.094	12,729	0.047
1420	2.12 (fracture)	11,297	0.06

After fracture, the gage length is 2.09 in. and the diameter is 0.393 in. Plot the data and calculate (a) the 0.2% offset yield strength, (b) the tensile strength, (c) the modulus of elasticity, (d) the % elongation, (e) the % reduction in area, (f) the engineering stress at fracture, (g) the true stress at fracture, and (h) the modulus of resilience.

Solution: $\sigma = F / (\pi/4)(0.4)^2 = F/0.1257$

$$\varepsilon = (\ell - 2) / 2$$



(a) 0.2% offset yield strength = 11,600 psi

(b) tensile strength = 12,729 psi

(c) $E = (7160 - 0) / (0.01187 - 0) = 603,000$ psi

(d) % elongation = $\frac{(2.09 - 2)}{2} \times 100 = 4.5\%$

(e) % reduction in area = $\frac{(\pi/4)(0.4)^2 - (\pi/4)(0.393)^2}{(\pi/4)(0.4)^2} \times 100 = 3.5\%$

(f) engineering stress at fracture = 11,297 psi

(g) true stress at fracture = $1420 \text{ lb} / (\pi/4)(0.393)^2 = 11,706$ psi

(h) From the figure, yielding begins near 9550 psi. Thus:

$$\frac{1}{2}(\text{yield strength})(\text{strain at yield}) = \frac{1}{2}(9550)(0.016) = 76.4 \text{ psi}$$

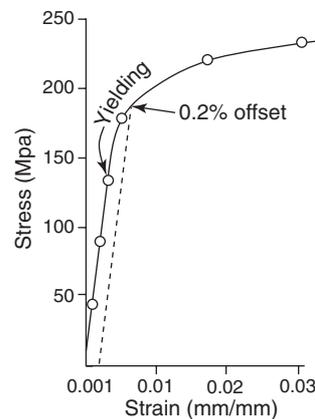
6-35 The following data were collected from a 12-mm-diameter test specimen of magnesium ($\ell_o = 30.00$ mm):

Load (N)	Gage Length (mm)	Stress (MPa)	Strain (mm/mm)
0	30.0000	0	0.0
5,000	30.0296	44.2	0.000987
10,000	30.0592	88.4	0.001973
15,000	30.0888	132.6	0.00296
20,000	30.15	176.8	0.005
25,000	30.51	221.0	0.017
26,500	30.90	234.3	0.030
27,000	31.50 (max load)	238.7	0.050
26,500	32.10	234.3	0.070
25,000	32.79 (fracture)	221.0	0.093

After fracture, the gage length is 32.61 mm and the diameter is 11.74 mm. Plot the data and calculate (a) the 0.2% offset yield strength, (b) the tensile strength, (c) the modulus of elasticity, (d) the % elongation, (e) the % reduction in area, (f) the engineering stress at fracture, (g) the true stress at fracture, and (h) the modulus of resilience.

Solution: $\sigma = F / (\pi/4)(12 \text{ mm})^2 = F/113.1$

$$\varepsilon = (\ell - 30)/30$$



(a) 0.2% offset yield strength = 186 MPa

(b) tensile strength = 238.7 MPa

(c) $E = (132.6 - 0) / (0.00296 - 0) = 44,800 \text{ MPa} = 44.8 \text{ GPa}$

(d) % elongation = $\frac{(32.61 - 30)}{30} \times 100 = 8.7\%$

(e) % reduction in area = $\frac{(\pi/4)(12)^2 - (\pi/4)(11.74)^2}{(\pi/4)(12)^2} \times 100 = 4.3\%$

(f) engineering stress at fracture = 221 MPa

(g) true stress at fracture = $25,000 \text{ N} / (\pi/4)(11.74)^2 = 231 \text{ MPa}$

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(h) From the figure, yielding begins near 138 MPa psi. Thus:

$$\frac{1}{2}(\text{yield strength})(\text{strain at yield}) = \frac{1}{2}(138)(0.00296) = 0.2 \text{ MPa}$$

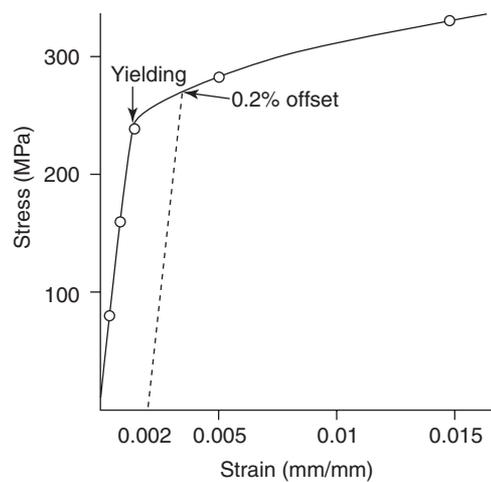
6-36 The following data were collected from a 20 mm diameter test specimen of a ductile cast iron ($\ell_o = 40.00 \text{ mm}$):

Load (N)	Gage Length (mm)	Stress (MPa)	Strain (mm/mm)
0	40.0000	0	0.0
25,000	40.0185	79.6	0.00046
50,000	40.0370	159.2	0.000925
75,000	40.0555	238.7	0.001388
90,000	40.20	286.5	0.005
105,000	40.60	334.2	0.015
120,000	41.56	382.0	0.039
131,000	44.00 (max load)	417.0	0.010
125,000	47.52 (fracture)	397.9	0.188

After fracture, the total length is 47.42 mm and the diameter is 18.35 mm. Plot the data and calculate (a) the 0.2% offset yield strength, (b) the tensile strength, (c) the modulus of elasticity, (d) the % elongation, (e) the % reduction in area, (f) the engineering stress at fracture, (g) the true stress at fracture, and (h) the modulus of resilience.

Solution: $\sigma = F/(\pi/4)(20 \text{ mm})^2 = F/314.2$

$$\varepsilon = (\ell - 40)/40$$



(a) 0.2% offset yield strength = 274 MPa

(b) tensile strength = 417 MPa

(c) $E = (238.7 - 0) / (0.001388 - 0) = 172,000 \text{ MPa} = 172 \text{ GPa}$

(d) % elongation = $\frac{(47.42 - 40)}{40} \times 100 = 18.55\%$

(e) % reduction in area = $\frac{(\pi/4)(20)^2 - (\pi/4)(18.35)^2}{(\pi/4)(20)^2} \times 100 = 15.8\%$

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(f) engineering stress at fracture = 397.9 MPa

(g) true stress at fracture = $125,000 \text{ N} / (\pi/4)(18.35)^2 = 473 \text{ MPa}$

(h) From the figure, yielding begins near 240 MPa. Thus:

$$\frac{1}{2}(\text{yield strength})(\text{strain at yield}) = \frac{1}{2}(240)(0.001388) = 0.17 \text{ MPa}$$

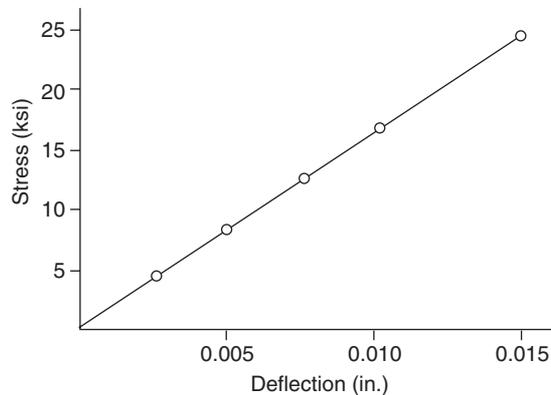
- 6-39** A bar of Al_2O_3 that is 0.25 in. thick, 0.5 in. wide, and 9 in. long is tested in a three-point bending apparatus, with the supports located 6 in. apart. The deflection of the center of the bar is measured as a function of the applied load. The data are shown below. Determine the flexural strength and the flexural modulus.

Force (lb)	Deflection (in.)	Stress (psi)
14.5	0.0025	4,176
28.9	0.0050	8,323
43.4	0.0075	12,499
57.9	0.0100	16,675
86.0	0.0149 (fracture)	24,768

Solution: $\text{stress} = 3LF/2wh^2$ (6-15)

$$= (3)(6 \text{ in.})F / (2)(0.5 \text{ in.})(0.25 \text{ in.})^2$$

$$= 288F$$



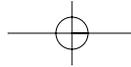
The flexural strength is the stress at fracture, or 24,768 psi.

The flexural modulus can be calculated from the linear curve; picking the first point as an example:

$$\text{FM} = \frac{FL^3}{4wh^3\delta} = \frac{(14.5 \text{ lb})(6 \text{ in.})^3}{(4)(0.5 \text{ in.})(0.25 \text{ in.})^3(0.0025 \text{ in.})} = 40 \times 10^6 \text{ psi}$$

- 6-40 (a)** A 0.4-in. diameter, 12-in.-long titanium bar has a yield strength of 50,000 psi, a modulus of elasticity of 16×10^6 psi, and Poisson's ratio of 0.30. Determine the length and diameter of the bar when a 500-lb load is applied.

Solution: The stress is $\sigma = F/A = 500 \text{ lb}/(\pi/4)(0.4 \text{ in.})^2 = 3,979 \text{ psi}$



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The applied stress is much less than the yield strength; therefore Hooke's law can be used.

The strain is $\varepsilon = \sigma/E = 3,979 \text{ psi} / (16 \times 10^6 \text{ psi}) = 0.00024868 \text{ in./in.}$

$$\frac{\ell_f - \ell_o}{\ell_o} = \frac{\ell_f - 12 \text{ in.}}{12 \text{ in.}} = 0.00024868 \text{ in./in.}$$

$$\ell_f = 12.00298 \text{ in.}$$

From Poisson's ratio, $\mu = -\varepsilon_{\text{lat}} / \varepsilon_{\text{long}} = 0.3$

$$\varepsilon_{\text{lat}} = - (0.3)(0.00024868) = - 0.0000746 \text{ in./in.}$$

$$\frac{d_f - d_o}{d_f} = \frac{d_f - 0.4 \text{ in.}}{0.4} = - 0.0000746 \text{ in./in.}$$

$$d_f = 0.39997 \text{ in.}$$

- (b) When a tensile load is applied to a 1.5-cm diameter copper bar, the diameter is reduced to 1.498-cm diameter. Determine the applied load, using the data in Table 6-3.

Solution: From Table 6-3, $\mu = -\varepsilon_{\text{lat}} / \varepsilon_{\text{long}} = 0.36$

$$\varepsilon_{\text{lat}} = \frac{1.498 - 1.5}{1.5} = - 0.001333$$

$$\varepsilon_{\text{long}} = -\varepsilon_{\text{lat}} / \mu = -(-0.001333) / 0.36 = 0.0037 \text{ in./in.}$$

$$\sigma = E\varepsilon = (124.8 \text{ GPa})(1000 \text{ MPa/GPa})(0.0037 \text{ in./in.}) = 462 \text{ MPa}$$

$$F = \sigma A = (462 \text{ MPa})(\pi/4)(15 \text{ mm})^2 = 81,640 \text{ N}$$

- 6-41** A three-point bend test is performed on a block of ZrO_2 that is 8 in. long, 0.50 in. wide, and 0.25 in. thick and is resting on two supports 4 in. apart. When a force of 400 lb is applied, the specimen deflects 0.037 in. and breaks. Calculate (a) the flexural strength and (b) the flexural modulus, assuming that no plastic deformation occurs.

Solution: (a) flexural strength = $3FL/2wh^2 = \frac{(3)(400 \text{ lb})(4 \text{ in.})}{(2)(0.5 \text{ in.})(0.25 \text{ in.})^2} = 76,800 \text{ psi}$

(b) flexural modulus = $FL^3/4wh^3\delta$

$$= \frac{(400 \text{ lb})(4 \text{ in.})^3}{(4)(0.5 \text{ in.})(0.25 \text{ in.})^3(0.037 \text{ in.})}$$

$$= 22.14 \times 10^6 \text{ psi}$$

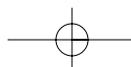
- 6-42** A three-point bend test is performed on a block of silicon carbide that is 10 cm long, 1.5 cm wide, and 0.6 cm thick and is resting on two supports 7.5 cm apart. The sample breaks when a deflection of 0.09 mm is recorded. Calculate (a) the force that caused the fracture and (b) the flexural strength. The flexural modulus for silicon carbide is 480 GPa. Assume that no plastic deformation occurs.

Solution: (a) The force F required to produce a deflection of 0.09 mm is

$$F = (\text{flexural modulus})(4wh^3\delta)/L^3$$

$$F = (480,000 \text{ MPa})(4)(15 \text{ mm})(6 \text{ mm})^3(0.09 \text{ mm}) / (75 \text{ mm})^3$$

$$F = 1327 \text{ N}$$



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$$\begin{aligned} \text{(b) flexural strength} &= 3FL/2wh^2 = (3)(1327 \text{ N})(75 \text{ mm})/(2)(15 \text{ mm})(6 \text{ mm})^2 \\ &= 276 \text{ MPa} \end{aligned}$$

- 6-43 (a)** A thermosetting polymer containing glass beads is required to deflect 0.5 mm when a force of 500 N is applied. The polymer part is 2 cm wide, 0.5 cm thick, and 10 cm long. If the flexural modulus is 6.9 GPa, determine the minimum distance between the supports. Will the polymer fracture if its flexural strength is 85 MPa? Assume that no plastic deformation occurs.

Solution: (a) The minimum distance L between the supports can be calculated from the flexural modulus.

$$L^3 = 4wh^3\delta(\text{flexural modulus})/F$$

$$L^3 = (4)(20 \text{ mm})(5 \text{ mm})^3(0.5 \text{ mm})(6.9 \text{ GPa})(1000 \text{ MPa/GPa}) / 500 \text{ N}$$

$$L^3 = 69,000 \text{ mm}^3 \quad \text{or} \quad L = 41 \text{ mm}$$

The stress acting on the bar when a deflection of 0.5 mm is obtained is

$$\begin{aligned} \sigma &= 3FL/2wh^2 = (3)(500 \text{ N})(41 \text{ mm}) / (2)(20 \text{ mm})(5 \text{ mm})^2 \\ &= 61.5 \text{ MPa} \end{aligned}$$

The applied stress is less than the flexural strength of 85 MPa; the polymer is not expected to fracture.

- (b)** The flexural modulus of alumina is 45×10^6 psi and its flexural strength is 46,000 psi. A bar of alumina 0.3 in. thick, 1.0 in. wide, and 10 in. long is placed on supports 7 in. apart. Determine the amount of deflection at the moment the bar breaks, assuming that no plastic deformation occurs.

Solution: (b) The force required to break the bar is

$$F = 2wh^2(\text{flexural strength})/3L$$

$$F = (2)(1 \text{ in.})(0.3 \text{ in.})^2(46,000 \text{ psi}) / (3)(7 \text{ in.}) = 394 \text{ lb}$$

The deflection just prior to fracture is

$$\delta = FL^3/4wh^3(\text{flexural modulus})$$

$$\delta = (394 \text{ lb})(7 \text{ in.})^3/(4)(1 \text{ in.})(0.3 \text{ in.})^3(45 \times 10^6 \text{ psi}) = 0.0278 \text{ in.}$$

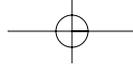
- 6-52** A Brinell hardness measurement, using a 10-mm-diameter indenter and a 500-kg load, produces an indentation of 4.5 mm on an aluminum plate. Determine the Brinell hardness number (HB) of the metal.

Solution:
$$HB = \frac{500 \text{ kg}}{(\pi/2)(10 \text{ mm})[10 - \sqrt{10^2 - 4.5^2}]} = 29.8$$

- 6-53** When a 3000-kg load is applied to a 10-mm-diameter ball in a Brinell test of a steel, an indentation of 3.1 mm is produced. Estimate the tensile strength of the steel.

Solution:
$$HB = \frac{3000 \text{ kg}}{(\pi/2)(10 \text{ mm})[10 - \sqrt{10^2 - 3.1^2}]} = 388$$

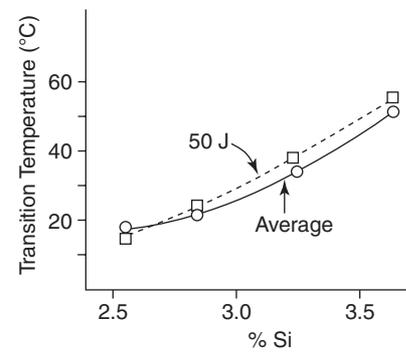
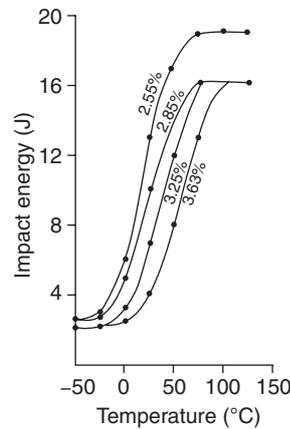
$$\text{Tensile strength} = 500 \text{ HB} = (500)(388) = 194,000 \text{ psi}$$


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- 6-57** The following data were obtained from a series of Charpy impact tests performed on four ductile cast irons, each having a different silicon content. Plot the data and determine (a) the transition temperature (defined by the mean of the absorbed energies in the ductile and brittle regions) and (b) the transition temperature (defined as the temperature that provides 10 J absorbed energy). Plot the transition temperature versus silicon content and discuss the effect of silicon on the toughness of the cast iron. What would be the maximum silicon allowed in the cast iron if a part is to be used at 25°C?

Test temperature °C	Impact energy (J)			
	2.55% Si	2.85% Si	3.25% Si	3.63% Si
-50	2.5	2.5	2	2
-5	3	2.5	2	2
0	6	5	3	2.5
25	13	10	7	4
50	17	14	12	8
75	19	16	16	13
100	19	16	16	16
125	19	16	16	16

Solution:



- (a) Transition temperatures defined by the mean of the absorbed energies are:

$$2.55\% \text{ Si: mean energy} = 2.5 + (19 + 2.5)/2 = 13.2 \text{ J}; T = 26^\circ\text{C}$$

$$2.85\% \text{ Si: mean energy} = 2.5 + (16 + 2.5)/2 = 11.8 \text{ J}; T = 35^\circ\text{C}$$

$$3.25\% \text{ Si: mean energy} = 2 + (16 + 2)/2 = 11 \text{ J}; T = 45^\circ\text{C}$$

$$3.63\% \text{ Si: mean energy} = 2 + (16 + 2)/2 = 11 \text{ J}; T = 65^\circ\text{C}$$

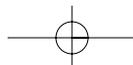
- (b) Transition temperatures defined by 10 J are:

$$2.55\% \text{ Si: } T = 15^\circ\text{C}$$

$$2.85\% \text{ Si: } T = 25^\circ\text{C}$$

$$3.25\% \text{ Si: } T = 38^\circ\text{C}$$

$$3.63\% \text{ Si: } T = 56^\circ\text{C}$$



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Increasing the silicon decreases the toughness and increases the transition temperature; silicon therefore reduces the impact properties of the cast iron.

If the part is to be used at 25°C, we would want a maximum of about 2.9% Si in the cast iron.

- 6-58** FCC metals are often recommended for use at low temperatures, particularly when any sudden loading of the part is expected. Explain.

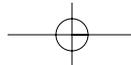
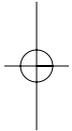
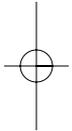
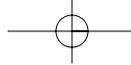
Solution: FCC metals do not normally display a transition temperature; instead the impact energies decrease slowly with decreasing temperature and, in at least some cases (such as some aluminum alloys), the energies even increase at low temperatures. The FCC metals can obtain large ductilities, giving large areas beneath the true stress-strain curve.

- 6-59** A steel part can be made by powder metallurgy (compacting iron powder particles and sintering to produce a solid) or by machining from a solid steel block. Which part is expected to have the higher toughness? Explain.

Solution: Parts produced by powder metallurgy often contain considerable amounts of porosity due to incomplete sintering; the porosity provides sites at which cracks might easily nucleate. Parts machined from solid steel are less likely to contain flaws that would nucleate cracks, therefore improving toughness.

- 6-62** A number of aluminum-silicon alloys have a structure that includes sharp-edged plates of brittle silicon in the softer, more ductile aluminum matrix. Would you expect these alloys to be notch-sensitive in an impact test? Would you expect these alloys to have good toughness? Explain your answers.

Solution: The sharp-edged plates of the brittle silicon may act as stress-raisers, or notches, thus giving poor toughness to the alloy. The presence of additional notches, such as machining marks, will not have a significant effect, since there are already very large numbers of “notches” due to the microstructure. Consequently this type of alloy is expected to have poor toughness but is not expected to be notch sensitive.



7

Fracture Mechanics, Fatigue, and Creep Behaviour

- 7-1** Alumina (Al_2O_3) is a brittle ceramic with low toughness. Suppose that fibers of silicon carbide (SiC), another brittle ceramic with low toughness, could be embedded within the alumina. Would doing this affect the toughness of the ceramic matrix composite? Explain. (These materials are discussed in later chapters.)

Solution: The SiC fibers may improve the toughness of the alumina matrix. The fibers may do so by several mechanisms. By introducing an interface (between the fibers and the matrix), a crack may be blocked; to continue growing, the crack may have to pass around the fiber, thus increasing the total energy of the crack and thus the energy that can be absorbed by the material. Or extra energy may be required to force the crack through the interface in an effort to continue propagating. In addition, the fibers may begin to pull out of the matrix, particularly if bonding is poor; the fiber pull-out requires energy, thus improving toughness. Finally, the fibers may bridge across the crack, helping to hold the material together and requiring more energy to propagate the crack.

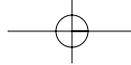
- 7-2** A ceramic matrix composite contains internal flaws as large as 0.001 cm in length. The plane strain fracture toughness of the composite is $45 \text{ MPa}\sqrt{\text{m}}$ and the tensile strength is 550 MPa. Will the flaw cause the composite to fail before the tensile strength is reached? Assume that $f = 1$.

Solution: Since the crack is internal, $2a = 0.001 \text{ cm} = 0.00001 \text{ m}$. Therefore

$$a = 0.000005 \text{ m}$$

$$K_{Ic} = f\sigma\sqrt{\pi a} \quad \text{or} \quad \sigma = K_{Ic} / f\sqrt{\pi a}$$

$$\sigma = (45 \text{ MPa}\sqrt{\text{m}}) / (1)\sqrt{\pi(0.000005 \text{ m})} = 11,354 \text{ MPa}$$


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The applied stress required for the crack to cause failure is much larger than the tensile strength of 550 MPa. Any failure of the ceramic should be expected due to the massive overload, not because of the presence of the flaws.

- 7-3** An aluminum alloy that has a plane strain fracture toughness of 25,000 psi $\sqrt{\text{in}}$. fails when a stress of 42,000 psi is applied. Observation of the fracture surface indicates that fracture began at the surface of the part. Estimate the size of the flaw that initiated fracture. Assume that $f = 1.1$.

Solution: $K_{Ic} = f\sigma\sqrt{\pi a}$ or $a = (1/\pi)[K_{Ic}/f\sigma]^2$

$$a = (1/\pi)[25,000 \text{ psi}\sqrt{\text{in.}} / (1.1)(42,000 \text{ psi})]^2 = 0.093 \text{ in.}$$

- 7-4** A polymer that contains internal flaws 1 mm in length fails at a stress of 25 MPa. Determine the plane strain fracture toughness of the polymer. Assume that $f = 1$.

Solution: Since the flaws are internal, $2a = 1 \text{ mm} = 0.001 \text{ m}$; thus $a = 0.0005 \text{ m}$

$$K_{Ic} = f\sigma\sqrt{\pi a} = (1)(25 \text{ MPa})\sqrt{\pi(0.0005 \text{ m})} = 0.99 \text{ MPa}\sqrt{\text{m}}$$

- 7-5** A ceramic part for a jet engine has a yield strength of 75,000 psi and a plane strain fracture toughness of 5,000 psi $\sqrt{\text{in}}$. To be sure that the part does not fail, we plan to assure that the maximum applied stress is only one third the yield strength. We use a nondestructive test that will detect any internal flaws greater than 0.05 in. long. Assuming that $f = 1.4$, does our nondestructive test have the required sensitivity? Explain.

Solution: The applied stress is $\sigma = (1/3)(75,000 \text{ psi}) = 25,000 \text{ psi}$

$$a = (1/\pi)[K_{Ic}/f\sigma]^2 = (1/\pi)[5,000 \text{ psi}\sqrt{\text{in.}} / (1.4)(25,000 \text{ psi})]^2$$

$$a = 0.0065 \text{ in.}$$

The length of internal flaws is $2a = 0.013 \text{ in.}$

Our nondestructive test can detect flaws as small as 0.05 in. long, which is not smaller than the critical flaw size required for failure. Thus our NDT test is not satisfactory.

- 7-22** A cylindrical tool steel specimen that is 6 in. long and 0.25 in. in diameter rotates as a cantilever beam and is to be designed so that failure never occurs. Assuming that the maximum tensile and compressive stresses are equal, determine the maximum load that can be applied to the end of the beam. (See Figure 7-19.)

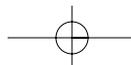
Solution: The stress must be less than the endurance limit, 60,000 psi.

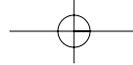
$$\sigma = 10.18LF/d^3 \text{ or } F = (\text{endurance limit})d^3/10.18L$$

$$F = (60,000 \text{ psi})(0.25 \text{ in.})^3 / (10.18)(6 \text{ in.}) = 15.35 \text{ lb}$$

- 7-23** A 2-cm-diameter, 20-cm-long bar of an acetal polymer (Figure 7-29) is loaded on one end and is expected to survive one million cycles of loading, with equal maximum tensile and compressive stresses, during its lifetime. What is the maximum permissible load that can be applied?

Solution: From the figure, we find that the fatigue strength must be 22 MPa in order for the polymer to survive one million cycles. Thus, the maximum load is




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$$F = (\text{fatigue strength})d^3/10.18L$$

$$F = (22 \text{ MPa})(20 \text{ mm})^3 / (10.18)(200 \text{ mm}) = 86.4 \text{ N}$$

- 7-24** A cyclical load of 1500 lb is to be exerted at the end of a 10-in.-long aluminum beam (Figure 7-19). The bar must survive for at least 10^6 cycles. What is the minimum diameter of the bar?

Solution: From the figure, we find that the fatigue strength must be 35,000 psi in order for the aluminum to survive 10^6 cycles. Thus, the minimum diameter of the bar is

$$d = \sqrt[3]{10.18LF / \text{fatigue strength}}$$

$$d = \sqrt[3]{(10.18)(10 \text{ in.})(1500 \text{ lb}) / 35,000 \text{ psi}} = 1.634 \text{ in.}$$

- 7-25** A cylindrical acetal polymer bar 20 cm long and 1.5 cm in diameter is subjected to a vibrational load at a frequency of 500 vibrations per minute with a load of 50 N. How many hours will the part survive before breaking? (See Figure 7-29)

Solution: The stress acting on the polymer is

$$\sigma = 10.18LF/d^3 = (10.18)(200 \text{ mm})(50 \text{ N}) / (15 \text{ mm})^3 = 30.16 \text{ MPa}$$

From the figure, the fatigue life at 30.16 MPa is about 2×10^5 cycles.

Based on 500 cycles per minute, the life of the part is

$$\text{life} = 2 \times 10^5 \text{ cycles} / (500 \text{ cycles/min})(60 \text{ min/h}) = 6.7 \text{ h}$$

- 7-26** Suppose that we would like a part produced from the acetal polymer shown in Figure 7-29 to survive for one million cycles under conditions that provide for equal compressive and tensile stresses. What is the fatigue strength, or maximum stress amplitude, required? What are the maximum stress, the minimum stress, and the mean stress on the part during its use? What effect would the frequency of the stress application have on your answers? Explain.

Solution: From the figure, the fatigue strength at one million cycles is 22 MPa.

The maximum stress is +22 MPa, the minimum stress is -22 MPa, and the mean stress is 0 MPa.

A high frequency will cause heating of the polymer. As the temperature of the polymer increases, the fatigue strength will decrease. If the applied stress is not reduced, then the polymer will fail in a shorter time.

- 7-27** The high-strength steel in Figure 7-21 is subjected to a stress alternating at 200 revolutions per minute between 600 MPa and 200 MPa (both tension). Calculate the growth rate of a surface crack when it reaches a length of 0.2 mm in both m/cycle and m/s. Assume that $f = 1.0$.

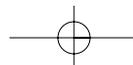
Solution: For the steel, $C = 1.62 \times 10^{-12}$ and $n = 3.2$. The change in the stress intensity factor ΔK is

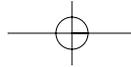
$$\Delta K = f\Delta\sigma\sqrt{\pi a} = (1.2)(600 \text{ MPa} - 200 \text{ MPa})\sqrt{\pi(0.0002 \text{ m})} = 12.03 \text{ MPa}\sqrt{\text{m}}$$

The crack growth rate is

$$da/dN = 1.62 \times 10^{-12}(\Delta K)^{3.2}$$

$$da/dN = 1.62 \times 10^{-12}(12.03)^{3.2} = 4.638 \times 10^{-9} \text{ m/cycle}$$





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$$da/dt = (4.638 \times 10^{-9} \text{ m/cycle})(200 \text{ cycles/min}) / 60 \text{ s/min}$$

$$da/dt = 1.55 \times 10^{-8} \text{ m/s}$$

- 7-28** The high-strength steel in Figure 7-21, which has a critical fracture toughness of $80 \text{ MPa}\sqrt{\text{m}}$, is subjected to an alternating stress varying from -900 MPa (compression) to $+900 \text{ MPa}$ (tension). It is to survive for 10^5 cycles before failure occurs. Calculate (a) the size of a surface crack required for failure to occur and (b) the largest initial surface crack size that will permit this to happen. Assume that $f = 1$.

Solution: (a) Only the tensile portion of the applied stress is considered in $\Delta\sigma$. Based on the applied stress of 900 MPa and the fracture toughness of $80 \text{ MPa}\sqrt{\text{m}}$, the size of a surface crack required for failure to occur is

$$K = f\sigma\sqrt{\pi a_c} \quad \text{or} \quad a_c = (1/\pi)[K/f\sigma]^2$$

$$a_c = (1/\pi)[80 \text{ MPa}\sqrt{\text{m}} / (1)(900 \text{ MPa})]^2 = 0.0025 \text{ m} = 2.5 \text{ mm}$$

(b) The largest initial surface crack tolerable to prevent failure within 10^5 cycles is

$$N = 10^5 \text{ cycles} = \frac{2[(0.0025 \text{ m})^{(2-3.2)/2} - a_i^{(2-3.2)/2}]}{(2-3.2)(1.62 \times 10^{-12})(1)^{3.2}(900)^{3.2}(\pi)^{3.2/2}}$$

$$10^5 = \frac{2[36.41 - (a_i)^{-0.60}]}{(-1.2)(1.62 \times 10^{-12})(1)(2.84 \times 10^9)(6.244)}$$

$$(a_i)^{-0.6} = 1760$$

$$a_i = 3.9 \times 10^{-6} \text{ m} = 0.0039 \text{ mm}$$

- 7-29** The acrylic polymer from which Figure 7-30 was obtained has a critical fracture toughness of $2 \text{ MPa}\sqrt{\text{m}}$. It is subjected to a stress alternating between -10 and $+10 \text{ MPa}$. Calculate the growth rate of a surface crack when it reaches a length of $5 \times 10^{-6} \text{ m}$ if $f = 1.0$.

Solution: $\Delta\sigma = 10 \text{ MPa} - 0 = 10 \text{ MPa}$, since the crack doesn't propagate for compressive loads.

$$\Delta K = f\Delta\sigma\sqrt{\pi a} = (1.3)(10 \text{ MPa})\sqrt{\pi(5 \times 10^{-6} \text{ m})} = 0.0515 \text{ MPa}\sqrt{\text{m}}$$

From the graph, $da/dN = 3 \times 10^{-7} \text{ m/cycle}$

- 7-30** Calculate the constants " C " and " n " in the Equation 7-18 for the crack growth rate of an acrylic polymer. (See Figure 7-30.)

Solution: Let's pick two points on the graph:

$$da/dN = 2 \times 10^{-6} \text{ m/cycle when } \Delta K = 0.1 \text{ MPa}\sqrt{\text{m}}$$

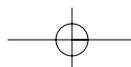
$$da/dN = 1 \times 10^{-7} \text{ m/cycle when } \Delta K = 0.037 \text{ MPa}\sqrt{\text{m}}$$

$$\frac{2 \times 10^{-6}}{1 \times 10^{-7}} = \frac{C(0.1)^n}{C(0.037)^n}$$

$$20 = (0.1/0.037)^n = (2.703)^n$$

$$\ln(20) = n \ln(2.703) \quad 2.9957 = 0.994n \quad n = 3.01$$

$$2 \times 10^{-6} = C(0.1)^{3.01} = 0.000977C \quad C = 2.047 \times 10^{-3}$$



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- 7-31** The acrylic polymer from which Figure 7-30 was obtained is subjected to an alternating stress between 15 MPa and 0 MPa. The largest surface cracks initially detected by nondestructive testing are 0.001 mm in length. If the critical fracture toughness of the polymer is 2 MPa $\sqrt{\text{m}}$, calculate the number of cycles required before failure occurs. Let $f = 1.0$. (*Hint:* Use the results of Problem 7-30.)

Solution: From Problem 7-30, $C = 2.047 \times 10^{-3}$ and $n = 3.01$

The critical flaw size a_c is

$$a_c = (1/\pi)[K_{Ic} / f\sigma]^2 = (1/\pi)[(2 \text{ MPa}\sqrt{\text{m}}) / (1.2)(15 \text{ MPa})]^2$$

$$a_c = 0.00393 \text{ m} = 3.93 \text{ mm}$$

Then

$$N = \frac{2[(0.00393 \text{ m})^{(2-3.01)/2} - (0.000001 \text{ m})^{(2-3.01)/2}]}{(2-3.01)(2.047 \times 10^{-3})(1.2)^{3.01}(15 \text{ mPa})^{3.01}(\pi)^{3.01/2}}$$

$$N = \frac{2(16.3995 - 1071.52)}{(-1.01)(2.047 \times 10^{-3})(1.7312)(3467.65)(5.6)} = 30.36 \text{ cycles}$$

- 7-33** Verify that integration of $da/dN = C(\Delta K)^n$ will give Equation 7-20.

Solution: $dN = (1/cf^n \Delta \sigma^n \pi^{n/2})(da/a^{n/2})$ or $N = (1/cf^n \Delta \sigma^n \pi^{n/2}) \int (da/a^{n/2})$

$$\text{since } \int a^p da = [1/(p+1)](a^{p+1})$$

$$\text{then if } p = -n/2, \int da/a^{n/2} = \frac{1}{1-n/2} [a^{-n/2+1}]_{a_i}^{a_c} = (2/2-n)[a_c^{(2-n)/2} - a_i^{(2-n)/2}]$$

$$\text{thus } N = \frac{2[a_c^{(2-n)/2} - a_i^{(2-n)/2}]}{(2-n)cf^n \Delta \sigma^n \pi^{n/2}}$$

- 7-38** The activation energy for self-diffusion in copper is 49,300 cal/mol. A copper specimen creeps at 0.002 in./in. · h when a stress of 15,000 psi is applied at 600°C. If the creep rate of copper is dependent on self-diffusion, determine the creep rate if the temperature is 800°C.

Solution: The creep rate is governed by an Arrhenius relationship of the form rate = $A \exp(-Q/RT)$. From the information given,

$$\frac{x}{0.002 \text{ in./in.} \cdot \text{h}} = \frac{A \exp[-49,300/(1.987)(800+273)]}{A \exp[-49,300/(1.987)(600+273)]} = \frac{9.07 \times 10^{-11}}{4.54 \times 10^{-13}}$$

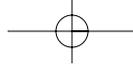
$$x = (0.002)(9.07 \times 10^{-11} / 4.54 \times 10^{-13}) = 0.4 \text{ in./in.} \cdot \text{h}$$

- 7-39** When a stress of 20,000 psi is applied to a material heated to 900°C, rupture occurs in 25,000 h. If the activation energy for rupture is 35,000 cal/mol, determine the rupture time if the temperature is reduced to 800°C.

Solution: The rupture time is related to temperature by an Arrhenius relationship of the form $t_r = A \exp(+Q/RT)$; the argument of the exponential is positive because the rupture time is inversely related to the rate. From the information given

$$\frac{t_r}{25,000 \text{ h}} = \frac{A \exp[35,000/(1.987)(800+273)]}{A \exp[35,000/(1.987)(900+273)]} = \frac{1.35 \times 10^7}{3.32 \times 10^6}$$

$$t_r = (25,000)(1.35 \times 10^7 / 3.32 \times 10^6) = 101,660 \text{ h}$$



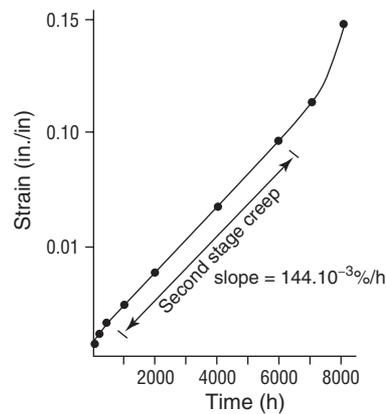
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- 7-40 The following data were obtained from a creep test for a specimen having an initial gage length of 2.0 in. and an initial diameter of 0.6 in. The initial stress applied to the material is 10,000 psi. The diameter of the specimen after fracture is 0.52 in.

Length Between Gage Marks (in.)	Time (h)	Strain (in./in.)
2.004	0	0.002
2.01	100	0.005
2.02	200	0.010
2.03	400	0.015
2.045	1000	0.0225
2.075	2000	0.0375
2.135	4000	0.0675
2.193	6000	0.0965
2.23	7000	0.115
2.30	8000 (fracture)	0.15

Determine (a) the load applied to the specimen during the test, (b) the approximate length of time during which linear creep occurs, (c) the creep rate in in./in. · h and in %/h, and (d) the true stress acting on the specimen at the time of rupture.

Solution:

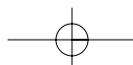


- (a) The load is $F = \sigma A = (10,000 \text{ psi})(\pi/4)(0.6 \text{ in.})^2 = 2827 \text{ lb}$
- (b) The plot of strain versus time is linear between approximately 500 and 6000 hours, or a total of 5500 hours.
- (c) From the graph, the strain rate is the slope of the linear portion of the curve.

$$\Delta \varepsilon / \Delta t = \frac{0.095 - 0.03}{6000 - 1500} = 1.44 \times 10^{-5} \text{ in./in.} \cdot \text{h} = 1.44 \times 10^{-3} \text{ \%/h}$$

- (d) At the time of rupture, the force is still 2827 lb, but the diameter is reduced to 0.52 in. The true stress is therefore

$$\sigma_t = F/A = 2827 \text{ lb} / (\pi/4)(0.52 \text{ in.})^2 = 13,312 \text{ psi}$$



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- 7-41 A stainless steel is held at 705°C under different loads. The following data are obtained:

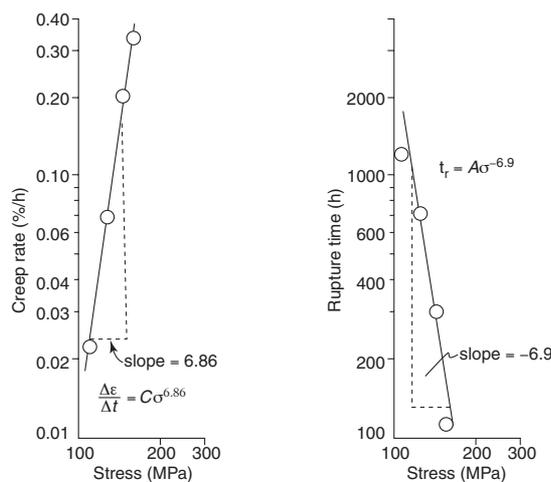
Applied Stress (MPa)	Rupture Time (h)	Creep Rate (%/h)
106.9	1200	0.022
128.2	710	0.068
147.5	300	0.201
160.0	110	0.332

Determine the exponents “ n ” and “ m ” in Equations 7-22 and 7-23 that describe the dependence of creep rate and rupture time on applied stress.

Solution: Plots describing the effect of applied stress on creep rate and on rupture time are shown below. In the first plot, the creep rate is given by $\Delta\epsilon/\Delta t = C\sigma^n$ and the graph is a log-log plot. In the second plot, rupture time is given by $t_r = A\sigma^m$, another log-log plot.

The exponents “ n ” and “ m ” are the slopes of the two graphs. In this case,

$$n = 6.86 \quad m = -6.9$$



- 7-42 Using the data in Figure 7-27 for an iron-chromium-nickel alloy, determine the activation energy Q_r and the constant “ m ” for rupture in the temperature range 980 to 1090°C.

Solution: The appropriate equation is $t_r = K\sigma^m \exp(Q_r/RT)$.

From Figure 7-27(a), we can determine the rupture time versus temperature for a fixed stress, say $\sigma = 1000$ psi:

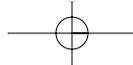
$$t_r = 2,400 \text{ h at } 1090^\circ\text{C} = 1363 \text{ K}$$

$$t_r = 14,000 \text{ h at } 1040^\circ\text{C} = 1313 \text{ K}$$

$$t_r = 100,000 \text{ h at } 980^\circ\text{C} = 1253 \text{ K}$$

From this data, the equation becomes $t_r = K'\exp(Q_r/RT)$ and we can find Q_r by simultaneous equations or graphically.

$$Q_r = 117,000 \text{ cal/mol}$$



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We can also determine the rupture time versus applied stress for a constant temperature, say 1090°C:

$$t_r = 10^5 \text{ h} \quad \text{for } \sigma = 450 \text{ psi}$$

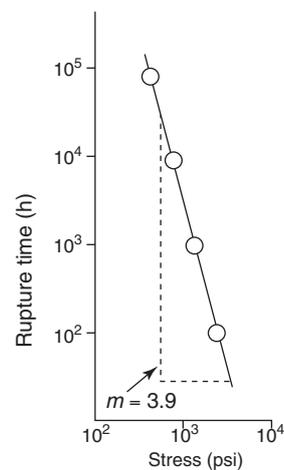
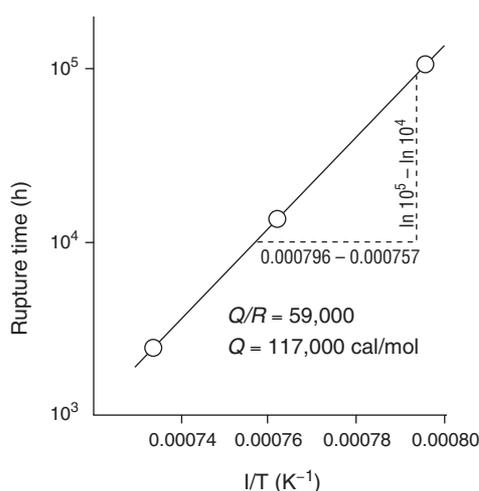
$$t_r = 10^4 \text{ h} \quad \text{for } \sigma = 800 \text{ psi}$$

$$t_r = 10^3 \text{ h} \quad \text{for } \sigma = 1200 \text{ psi}$$

$$t_r = 10^2 \text{ h} \quad \text{for } \sigma = 2100 \text{ psi}$$

With this approach, the equation becomes $t_r = K''\sigma^m$, where “ m ” is obtained graphically or by simultaneous equations:

$$m = 3.9$$



- 7-43** A 1-in.-diameter bar of an iron-chromium-nickel alloy is subjected to a load of 2500 lb. How many days will the bar survive without rupturing at 980°C? [See Figure 7-27(a).]

Solution: The stress is $\sigma = F/A = 2500 \text{ lb} / (\pi/4)(1 \text{ in.})^2 = 3183 \text{ psi}$

From the graph, the rupture time is 700 h / 24 h/day = 29 days

- 7-44** A 5 mm × 20 mm bar of an iron-chromium-nickel alloy is to operate at 1040°C for 10 years without rupturing. What is the maximum load that can be applied? [See Figure 7-27(a).]

Solution: The operating time is (10 years)(365 days/year)(24 h/day) = 87,600 h

From the graph, the stress must be less than 500 psi. The load is then

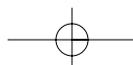
$$F = \sigma A = (500 \text{ psi})(5 \text{ mm}/25.4 \text{ mm/in.})(20 \text{ mm}/25.4 \text{ mm/in.}) = 77.5 \text{ lb}$$

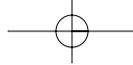
- 7-45** An iron-chromium-nickel alloy is to withstand a load of 1500 lb at 760°C for 6 years. Calculate the minimum diameter of the bar. [See Figure 7-27(a).]

Solution: The operating time is (6 years)(365 days/year)(24 h/day) = 52,560 h

From the graph, the stress must be less than 7000 psi. The minimum diameter of the bar is then

$$d = \sqrt{(4/\pi)(F/\sigma)} = \sqrt{(4/\pi)(1500 \text{ lb}/7000 \text{ psi})} = 0.52 \text{ in.}$$





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- 7-46** A 1.2-in.-diameter bar of an iron-chromium-nickel alloy is to operate for 5 years under a load of 4000 lb. What is the maximum operating temperature? [See Figure 7-27(a).]

Solution: The operating time is $(5 \text{ years})(365 \text{ days/year})(24 \text{ h/day}) = 43,800 \text{ h}$

The stress is $\sigma = F/A = 4000 \text{ lb} / (\pi/4)(1.2 \text{ in.})^2 = 3537 \text{ psi}$

From the figure, the temperature must be below 850°C in order for the bar to survive five years at 3537 psi.

- 7-47** A 1 in. \times 2 in. ductile cast iron bar must operate for 9 years at 650°C . What is the maximum load that can be applied? [See Figure 7-27(b).]

Solution: The operating time is $(9 \text{ year})(365 \text{ days/year})(24 \text{ h/day}) = 78,840 \text{ h}$.

The temperature is $650 + 273 = 923 \text{ K}$

$LM = (923/1000)[36 + 0.78 \ln(78,840)] = 41.35$

From the graph, the stress must be no more than about 1000 psi. The load is then

$F = \sigma A = (1000 \text{ psi})(2 \text{ in.}^2) = 2000 \text{ lb}$

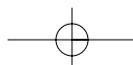
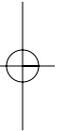
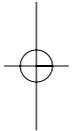
- 7-48** A ductile cast iron bar is to operate at a stress of 6000 psi for 1 year. What is the maximum allowable temperature? [See Figure 7-27(b).]

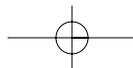
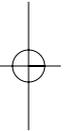
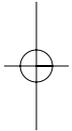
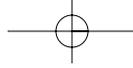
Solution: The operating time is $(1 \text{ year})(365 \text{ days/year})(24 \text{ h/day}) = 8760 \text{ h}$

From the graph, the Larson-Miller parameter must be 34.4 at a stress of 6000 psi. Thus

$34.4 = (T / 1000)[36 + 0.78 \ln(8760)] = 0.043T$

$T = 800 \text{ K} = 527^\circ\text{C}$





8

Strain Hardening and Annealing

- 8-5 A 0.505-in.-diameter metal bar with a 2-in. gage length l_0 is subjected to a tensile test. The following measurements are made in the plastic region:

Force (lb)	Change in Gage length (in.) (Δl)	Diameter (in.)
27,500	0.2103	0.4800
27,000	0.4428	0.4566
25,700	0.6997	0.4343

Determine the strain-hardening exponent for the metal. Is the metal most likely to be FCC, BCC, or HCP? Explain.

Solution:

Force (lb)	Gage length (in.)	Diameter (in.)	True stress (psi)	True strain (in./in.)
27,500	2.2103	0.4800	151,970	0.100
27,000	2.4428	0.4566	164,893	0.200
25,700	2.6997	0.4343	173,486	0.300

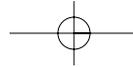
$$\sigma_t = K \varepsilon_t^n \quad \text{or} \quad \ln \sigma = \ln K + n \ln \varepsilon$$

$$\ln(151,970) = \ln K + n \ln(0.1) \quad 11.9314 = \ln K - n (2.3026)$$

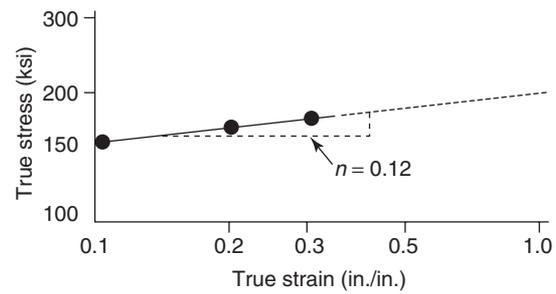
$$\ln(173,486) = \ln K + n \ln(0.3) \quad 12.0639 = \ln K - n (1.2040)$$

$$\underline{\hspace{10em}} \quad -0.1325 = -1.0986 n$$

$$n = 0.12 \quad \text{which is in the range of BCC metals}$$



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- 8-7 A 1.5-cm-diameter metal bar with a 3-cm gage length (l_0), is subjected to a tensile test. The following measurements are made.

Force (N)	Change in Gage length (cm) (Δl)	Diameter (cm)
16,240	0.6642	1.2028
19,066	1.4754	1.0884
19,273	2.4663	0.9848

Determine the strain-hardening coefficient for the metal. Is the metal most likely to be FCC, BCC, or HCP? Explain.

Solution:

Force (N)	Gage length (cm)	Diameter (mm)	True stress (MPa)	True strain (cm/cm)
16,240	3.6642	12.028	143	0.200
19,066	4.4754	10.884	205	0.400
19,273	5.4663	9.848	249	0.600

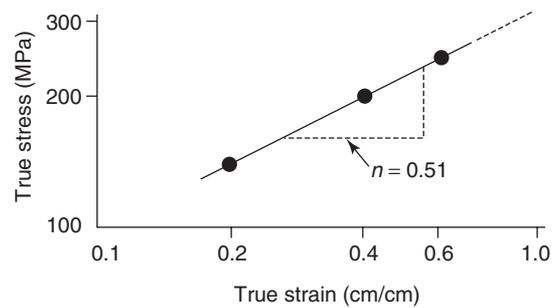
$$\sigma_t = K\varepsilon_t^n \quad \ln 143 = \ln K + n \ln 0.2$$

$$\ln 249 = \ln K + n \ln 0.6$$

$$(4.962 - 5.517) = n(-1.609 + 0.511)$$

$$n = 0.51$$

A strain hardening coefficient of 0.51 is typical of FCC metals.



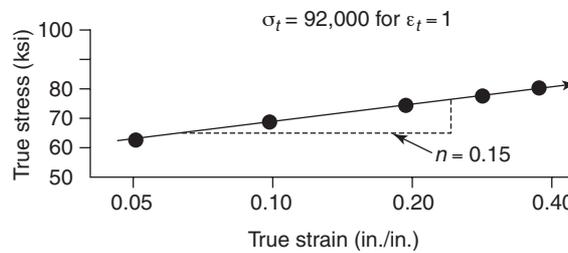
- 8-9** A true stress-true strain curve is shown in Figure 8-21. Determine the strain hardening exponent for the metal.

Solution: $\sigma_t = K\varepsilon_t^n$

ε_t	σ_t
0.05 in./in.	60,000 psi
0.10 in./in.	66,000 psi
0.20 in./in.	74,000 psi
0.30 in./in.	76,000 psi
0.40 in./in.	81,000 psi

From graph: $K = 92,000$ psi

$n = 0.15$



- 8-10** A Cu-30% Zn alloy tensile bar has a strain-hardening coefficient of 0.50. The bar, which has an initial diameter of 1 cm and an initial gage length of 3 cm, fails at an engineering stress of 120 MPa. After fracture, the gage length is 3.5 cm and the diameter is 0.926 cm. No necking occurred. Calculate the true stress when the true strain is 0.05 cm/cm.

Solution: $\varepsilon_t = \ln(\ell_f/\ell_o) = \ln(3.5/3.0) = 0.154$

$$\sigma_E = 120 \text{ MPa} = \frac{F}{(\pi/4)(10\text{mm})^2}$$

$$F = 9425 \text{ N}$$

$$\sigma_t = \frac{9425 \text{ N}}{(\pi/4)(9.26 \text{ mm})^2} = 139.95 \text{ MPa}$$

$$\sigma_t = K(0.154)^{0.5} = 139.95 \text{ MPa} \quad \text{or} \quad K = 356.6$$

The true stress at $\varepsilon_t = 0.05$ cm/cm is:

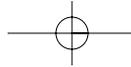
$$\sigma_t = 356.6 (0.05)^{0.5} \quad \text{or} \quad \sigma_t = 79.7 \text{ MPa}$$

- 8-18** A 0.25-in.-thick copper plate is to be cold worked 63%. Find the final thickness.

Solution: (See Figure 8-7.) $63 = \frac{0.25 - t_f}{0.25} \times 100\%$ or $t_f = 0.0925$ in.

- 8-19** A 0.25-in.-diameter copper bar is to be cold worked 63%. Find the final diameter.

Solution: $63 = \frac{(0.25)^2 - d_f^2}{(0.25)^2} \times 100\%$ or $d_f^2 = 0.023$ or $d_f = 0.152$ in.


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- 8-20** A 2-in.-diameter copper rod is reduced to 1.5 in. diameter, then reduced again to a final diameter of 1 in. In a second case, the 2-in.-diameter rod is reduced in one step from 2 in. to a 1 in. diameter. Calculate the % CW for both cases.

Solution: $\% \text{ CW} = \frac{(2)^2 - (1)^2}{(2)^2} \times 100 = 75\%$ in both cases

- 8-21** A 3105 aluminum plate is reduced from 1.75 in. to 1.15 in. Determine the final properties of the plate. (See Figure 8-22.)

Solution: $\% \text{ CW} = \frac{1.75 - 1.15}{1.75} \times 100\% = 34.3\%$

TS = 24 ksi YS = 22 ksi % elongation = 5%

- 8-22** A Cu-30% Zn brass bar is reduced from 1-in. diameter to a 0.45-in. diameter. Determine the final properties of the bar. (See Figure 8-23.)

Solution: $\% \text{ CW} = \frac{(1)^2 - (0.45)^2}{(1)^2} \times 100 = 79.75\%$

TS = 105 ksi YS = 68 ksi % elongation = 1%

- 8-23** A 3105 aluminum bar is reduced from a 1-in. diameter, to a 0.8-in. diameter, to a 0.6-in. diameter, to a final 0.4-in. diameter. Determine the % CW and the properties after each step of the process. Calculate the total percent cold work. (See Figure 8-22.)

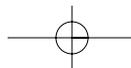
Solution: If we calculated the percent deformation in each step separately, we would find that 36% deformation is required to go from 1 in. to 0.8 in. The deformation from 0.8 in. to 0.6 in. (using 0.8 in. as the initial diameter) is 43.75%, and the deformation from 0.6 in. to 0.4 in. (using 0.6 in. as the initial diameter) is 55.6%. If we added these three deformations, the total would be 135.35%. This would not be correct. Instead, we must always use the original 1 in. diameter as our starting point. The following table summarizes the actual deformation and properties after each step.

	TS	YS	%
	ksi	ksi	elongation
$\frac{(1)^2 - (0.8)^2}{(1)^2} \times 100 = 36\%$	26	23	6
$\frac{(1)^2 - (0.6)^2}{(1)^2} \times 100 = 64\%$	30	27	3
$\frac{(1)^2 - (0.4)^2}{(1)^2} \times 100 = 84\%$	32	29	2

The total percent cold work is actually 84%, not the 135.35%.

- 8-24** We want a copper bar to have a tensile strength of at least 70,000 psi and a final diameter of 0.375 in. What is the minimum diameter of the original bar? (See Figure 8-7.)

Solution: $\% \text{ CW} \geq 50\%$ to achieve the minimum tensile strength



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$$50 = \frac{d_o^2 - (0.375)^2}{d_o^2} \times 100$$

$$0.5 d_o^2 = 0.140625 \quad \text{or} \quad d_o = 0.53 \text{ in.}$$

- 8-25** We want a Cu-30% Zn brass plate originally 1.2-in. thick to have a yield strength greater than 50,000 psi and a % elongation of at least 10%. What range of final thicknesses must be obtained? (See Figure 8-23.)

Solution: YS > 50,000 psi requires CW > 20%

% E > 10% requires CW < 35%

$$\frac{1.2 - t_f}{1.2} = 0.20 \qquad \frac{1.2 - t_f}{1.2} = 0.35$$

$$t_f = 0.96 \text{ in.} \qquad t_f = 0.78 \text{ in.}$$

$$t_f = 0.78 \text{ to } 0.96 \text{ in.}$$

- 8-26** We want a copper sheet to have at least 50,000 psi yield strength and at least 10% elongation, with a final thickness of 0.12 in. What range of original thicknesses must be used? (See Figure 8-7.)

Solution: YS > 50 ksi requires CW \geq 25%

% E > 10% requires CW \leq 30%

$$\frac{t_o - 0.12}{t_o} = 0.25 \qquad \frac{t_o - 0.12}{t_o} = 0.30$$

$$t_o = 0.16 \text{ in.} \qquad t_o = 0.17 \text{ in.}$$

$$t_o = 0.16 \text{ to } 0.17 \text{ in.}$$

- 8-27** A 3105 aluminum plate previously cold worked 20% is 2-in. thick. It is then cold worked further to 1.3 in. Calculate the total percent cold work and determine the final properties of the plate? (See Figure 8-22.)

Solution: The original thickness (before the 20% cold work) must have been:

$$\frac{t_o - 2}{t_o} = 0.20 \qquad t_o = 2.5 \text{ in.}$$

The total cold work is then based on the prior 2.5 in. thickness:

$$\text{CW} = \frac{2.5 - 1.3}{2.5} \times 100\% = 48\% \qquad \text{TS} = 28 \text{ ksi}$$

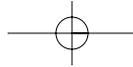
$$\qquad \qquad \qquad \qquad \qquad \qquad \text{YS} = 25 \text{ ksi}$$

$$\qquad \qquad \qquad \qquad \qquad \qquad \text{\%E} = 4\%$$

- 8-28** An aluminum-lithium (Al-Li) strap 0.25-in. thick and 2-in. wide is to be cut from a rolled sheet, as described in Figure 8-10. The strap must be able to support a 35,000-lb load without plastic deformation. Determine the range of orientations from which the strap can be cut from the rolled sheet.

Solution: $\sigma = \frac{35,000}{(0.25)(2)} \geq 70,000 \text{ psi}$

The properties can be obtained at angles of 0 to 20° from the rolling direction of the sheet.


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- 8-42** We want to draw a 0.3-in.-diameter copper wire having a yield strength of 20,000 psi into 0.25-in.-diameter wire. (a) Find the draw force, assuming no friction. (b) Will the drawn wire break during the drawing process? Show why. (See Figure 8-7.)

Solution: (a) Before drawing (0% CW), the yield strength is 20 ksi = 20,000 psi.

$$CW = \frac{(0.3)^2 - (0.25)^2}{(0.3)^2} = 30.6\% \quad \text{which gives YS} = 53,000 \text{ psi in the drawn wire}$$

(b) The force needed to draw the original wire is :

$$20,000 \text{ psi} = F/(\pi/4)(0.3)^2 \quad \text{or} \quad F = 1414 \text{ lb}$$

(c) The stress acting on the drawn wire is:

$$\sigma = 1414/(\pi/4)(0.25)^2 = 28,806 \text{ psi} < 53,000 \text{ psi}$$

Since the actual stress (28,806 psi) acting on the drawn wire is less than the yield strength (53,000 psi) of the drawn wire, the wire will not break during manufacturing.

- 8-43** A 3105 aluminum wire is to be drawn to give a 1-mm-diameter wire having a yield strength of 20,000 psi. (a) Find the original diameter of the wire, (b) calculate the draw force required, and (c) determine whether the as-drawn wire will break during the process. (See Figure 8-22.)

Solution: (a) We need to cold work 25% to obtain the required yield strength:

$$\frac{d_o^2 - 1^2}{d_o^2} = 0.25 \quad d_o = \sqrt{1 / 0.75} = 1.1547 \text{ mm} = 0.04546 \text{ in.}$$

(b) The initial yield strength of the wire (with 0% cold work) is 8000 psi, so the force required to deform the initial wire is:

$$F = 8000[(\pi/4)(0.04546)^2] = 12.98 \text{ lb}$$

(c) The stress acting on the drawn wire (which has a smaller diameter but is subjected to the same drawing force) is:

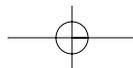
$$\sigma = \frac{12.98 \text{ lb}}{(\pi/4)(1 \text{ mm}/25.4 \text{ mm/in})^2} = 10,662 \text{ psi} < 20,000 \text{ psi}$$

Since the actual stress is less than the 20,000 psi yield strength of the drawn wire, the process will be successful and the wire will not break.

- 8-52** A titanium alloy contains a very fine dispersion of tiny Er_2O_3 particles. What will be the effect of these particles on the grain growth temperature and the size of the grains at any particular annealing temperature? Explain.

Solution: These particles, by helping pin the grain boundaries, will increase the grain growth temperature and decrease the grain size.

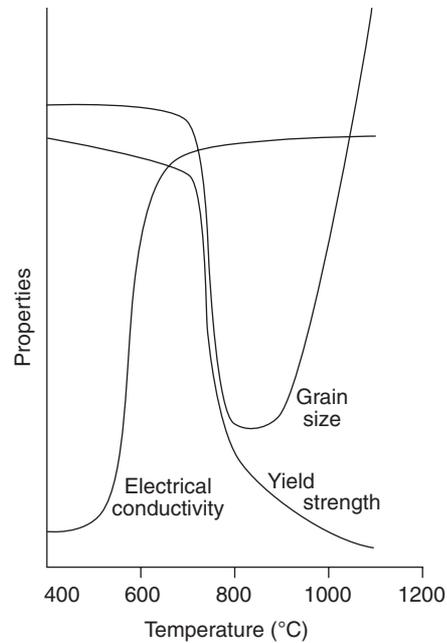
- 8-54** The following data were obtained when a cold-worked metal was annealed.
- Estimate the recovery, recrystallization, and grain growth temperatures.
 - Recommend a suitable temperature for a stress-relief heat treatment.
 - Recommend a suitable temperature for a hot-working process.
 - Estimate the melting temperature of the alloy.



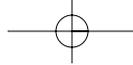
CHAPTER 8 Strain Hardening and Annealing 79

Annealing Temperature (°C)	Electrical Conductivity ($\text{ohm}^{-1} \cdot \text{cm}^{-1}$)	Yield Strength (MPa)	Grain Size (mm)
400	3.04×10^5	86	0.10
500	3.05×10^5	85	0.10
600	3.36×10^5	84	0.10
700	3.45×10^5	83	0.098
800	3.46×10^5	52	0.030
900	3.46×10^5	47	0.031
1000	3.47×10^5	44	0.070
1100	3.47×10^5	42	0.120

- Solution:**
- (a) recovery temperature $\approx 550^\circ\text{C}$
 recrystallization temperature $\approx 750^\circ\text{C}$
 grain growth temperature $\approx 950^\circ\text{C}$
- (b) Stress relief temperature = 700°C
- (c) Hot working temperature = 900°C
- (d) $0.4 T_{\text{mp}} \approx 750^\circ\text{C} = 1023 \text{ K}$
 $T_{\text{mp}} \approx 1023 / 0.4 = 2558 \text{ K} = 2285^\circ\text{C}$



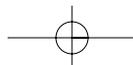
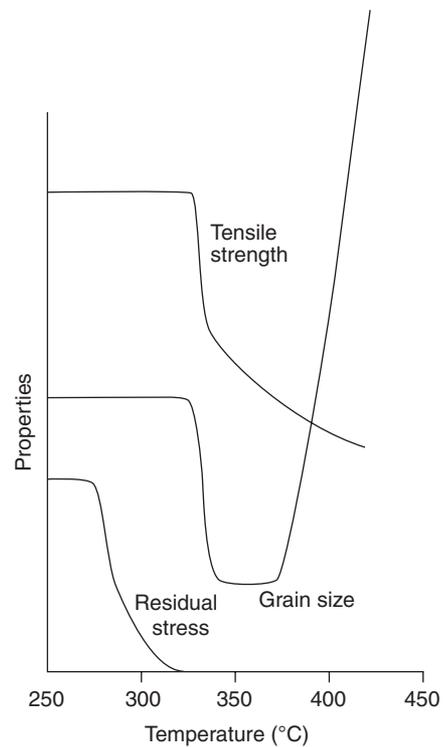
- 8-55** The following data were obtained when a cold-worked metal was annealed.
- (a) Estimate the recovery, recrystallization, and grain growth temperatures.
- (b) Recommend a suitable temperature for obtaining a high-strength, high-electrical conductivity wire. (c) Recommend a suitable temperature for a hot-working process.
- (d) Estimate the melting temperature of the alloy.


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Annealing Temperature (°C)	Residual Stresses (psi)	Tensile Strength (psi)	Grain Size (in.)
250	21,000	52,000	0.0030
275	21,000	52,000	0.0030
300	5,000	52,000	0.0030
325	0	52,000	0.0030
350	0	34,000	0.0010
375	0	30,000	0.0010
400	0	27,000	0.0035
425	0	25,000	0.0072

Solution:

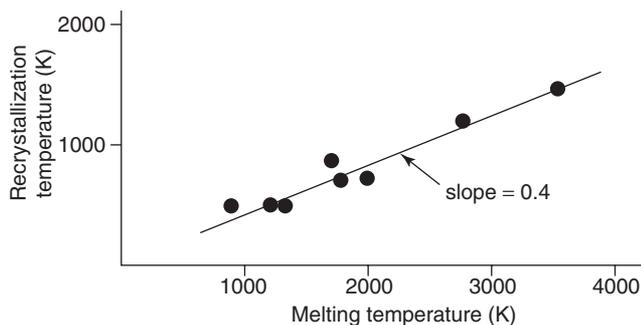
- (a) recovery temperature $\approx 280^\circ\text{C}$
 recrystallization temperature $\approx 330^\circ\text{C}$
 grain growth temperature $\approx 380^\circ\text{C}$
- (b) For a high strength, high conductivity wire, we want to heat into the recovery range. A suitable temperature might be 320°C .
- (c) Hot working temperature = 375°C
- (d) $0.4 T_{\text{mp}} \approx 330^\circ\text{C} = 603 \text{ K}$
 $T_{\text{mp}} \approx 603 / 0.4 = 1508 \text{ K} = 1235^\circ\text{C}$



- 8-63** Using the data in Table 8-4, plot the recrystallization temperature versus the melting temperature of each metal, using absolute temperatures (Kelvin). Measure the slope and compare with the expected relationship between these two temperatures. Is our approximation a good one?

Solution: Converting the recrystallization and melting temperatures to Kelvin, we can obtain the graph shown. The relationship of $T_r = 0.4T_m$ (K) is very closely followed.

	T_m	T_r
Al	933 K	423 K
Mg	923 K	473 K
Ag	1235 K	473 K
Cu	1358 K	473 K
Fe	1811 K	723 K
Ni	1726 K	873 K
Mo	2883 K	1173 K
W	3683 K	1473 K



- 8-64** We wish to produce a 0.3-in.-thick plate of 3105 aluminum having a tensile strength of at least 25,000 psi and a % elongation of at least 5%. The original thickness of the plate is 3 in. The maximum cold work in each step is 80%. Describe the cold working and annealing steps required to make this product. Compare this process with that you would recommend if you could do the initial deformation by hot working. (See Figure 8-22.)

Solution: For $TS \geq 25000$ $CW \geq 30\%$; For % elongation $\geq 5\%$ $CW \leq 30\%$
 \therefore required $CW = 30\%$

$$\frac{t_i - 0.3}{t_i} = 0.30 \quad \text{or} \quad t_i = 0.429 \text{ in.}$$

Cold work/anneal treatment

CW 75% from 3.0 to 0.75 in.
anneal

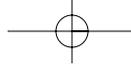
CW 42.8% from 0.75 to 0.429 in.
anneal

CW 30% from 0.429 to 0.3 in.

Hot work treatment

HW 85.7% from 3.0 to 0.429 in.

CW 30% from 0.429 to 0.3 in.


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8-65 We wish to produce a 0.2-in. diameter wire of copper having a minimum yield strength of 60,000 psi and a minimum % elongation of 5%. The original diameter of the rod is 2 in. and the maximum cold work in each step is 80%. Describe the cold working and annealing steps required to make this product. Compare this process with that you would recommend if you could do the initial deformation by hot working. (See Figure 8-7.)

Solution: For YS > 60 ksi, CW ≥ 40%; For % elongation > 5 CW ≥ 45%
 ∴ pick CW = 42%, the middle of the allowable range

$$\frac{d_i^2 - (0.2)^2}{d_i^2} = 0.42 \quad \text{or} \quad d_i = \sqrt{0.04 / 0.58} = 0.263 \text{ in.}$$

Cold work/anneal treatment

CW 75% from 2 to 1 in-diameter
 anneal

CW 75% from 1 to 0.5 in.
 anneal

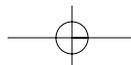
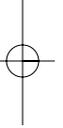
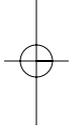
CW 72.3% from 0.5 to 0.263 in.
 anneal

CW 42% from 0.263 to 0.2 in.

Hot work treatment

HW 98.3% from 2 to 0.263 in.

CW 42% from 0.263 to 0.2 in.



9

Principles of Solidification

- 9-10** Suppose that liquid nickel is undercooled until homogeneous nucleation occurs. Calculate (a) the critical radius of the nucleus required, and (b) the number of nickel atoms in the nucleus. Assume that the lattice parameter of the solid FCC nickel is 0.356 nm.

Solution: From Table 9-1, $\Delta T_{\max} = 480^\circ\text{C}$

$$r^* = \frac{(2)(255 \times 10^{-7} \text{ J/cm}^2)(1453 + 273)}{(2756 \text{ J/cm}^3)(480)} = 6.65 \times 10^{-8} \text{ cm}$$

$$a_0 = 3.56 \text{ \AA} \quad V = 45.118 \times 10^{-24} \text{ cm}^3$$

$$V_{\text{nucleus}} = (4\pi/3)(6.65 \times 10^{-8} \text{ cm})^3 = 1232 \times 10^{-24} \text{ cm}^3$$

$$\text{number of unit cells} = 1232/45.118 = 27.3$$

$$\text{atoms per nucleus} = (4 \text{ atoms/cell})(27.3 \text{ cells}) = 109 \text{ atoms}$$

- 9-11** Suppose that liquid iron is undercooled until homogeneous nucleation occurs. Calculate (a) the critical radius of the nucleus required, and (b) the number of iron atoms in the nucleus. Assume that the lattice parameter of the solid BCC iron is 2.92 Å.

Solution: $r^* = \frac{(2)(204 \times 10^{-7} \text{ J/cm}^2)(1538 + 273)}{(1737 \text{ J/cm}^3)(420)} = 10.128 \times 10^{-8} \text{ cm}$

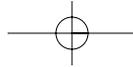
$$V = (4\pi/3)(10.128)^3 = 4352 \text{ \AA}^3 = 4352 \times 10^{-24} \text{ cm}^3$$

$$V_{\text{uc}} = (2.92 \text{ \AA})^3 = 24.897 \text{ \AA}^3 = 24.897 \times 10^{-24} \text{ cm}^3$$

$$\text{number of unit cells} = 4352/24.897 = 175$$

$$\text{atoms per nucleus} = (175 \text{ cells})(2 \text{ atoms/cell}) = 350 \text{ atoms}$$

- 9-12** Suppose that solid nickel was able to nucleate homogeneously with an undercooling of only 22°C . How many atoms would have to group together spontaneously for this to occur? Assume that the lattice parameter of the solid FCC nickel is 0.356 nm.



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Solution: $r^* = \frac{(2)(255 \times 10^{-7} \text{ J/cm}^2)(1453 + 273)}{(2756 \text{ J/cm}^3)(22)} = 145.18 \times 10^{-8} \text{ cm}$

$$V_{\text{uc}} = 45.118 \times 10^{-24} \text{ cm}^3 \quad (\text{see Problem 9-10})$$

$$V_{\text{nuc}} = (4\pi/3)(145.18 \times 10^{-8} \text{ cm})^3 = 1.282 \times 10^{-17} \text{ cm}^3$$

$$\text{number of unit cells} = 1.282 \times 10^{-17} / 45.118 \times 10^{-24} = 2.84 \times 10^5$$

$$\text{atoms per nucleus} = (4 \text{ atoms/cells})(2.84 \times 10^5 \text{ cell}) = 1.136 \times 10^6$$

- 9-13** Suppose that solid iron was able to nucleate homogeneously with an undercooling of only 15°C. How many atoms would have to group together spontaneously for this to occur? Assume that the lattice parameter of the solid BCC iron is 2.92 Å.

Solution: $r^* = \frac{(2)(204 \times 10^{-7} \text{ J/cm}^2)(1538 + 273)}{(1737 \text{ J/cm}^3)(15)} = 283.6 \times 10^{-8} \text{ cm}$

$$V_{\text{uc}} = 24.897 \times 10^{-24} \text{ cm}^3 \quad (\text{see Problem 9-10})$$

$$V_{\text{nuc}} = (4\pi/3)(283.6 \times 10^{-8} \text{ cm})^3 = 95,544,850 \times 10^{-24} \text{ cm}^3$$

$$\text{number of unit cells} = 95,544,850 / 24.897 = 3.838 \times 10^6$$

$$\text{atoms per nucleus} = (2 \text{ atoms/cells})(3.838 \times 10^6 \text{ cell}) = 7.676 \times 10^6$$

- 9-14** Calculate the fraction of solidification that occurs dendritically when iron nucleates (a) at 10°C undercooling, (b) at 100°C undercooling, and (c) homogeneously. The specific heat of iron is 5.78 J/cm³ · °C.

Solution: $f = \frac{c\Delta T}{\Delta H_f} = \frac{(5.78 \text{ J/cm}^3 \cdot \text{°C})(10\text{°C})}{1737 \text{ J/cm}^3} = 0.0333$

$$\frac{c\Delta T}{\Delta H_f} = \frac{(5.78 \text{ J/cm}^3 \cdot \text{°C})(100\text{°C})}{1737 \text{ J/cm}^3} = 0.333$$

$$\frac{c\Delta T}{\Delta H_f} = \frac{(5.78 \text{ J/cm}^3 \cdot \text{°C})(420\text{°C})}{1737 \text{ J/cm}^3}, \text{ therefore, all dendritically}$$

- 9-28** Calculate the fraction of solidification that occurs dendritically when silver nucleates (a) at 10°C undercooling, (b) at 100°C undercooling, and (c) homogeneously. The specific heat of silver is 3.25 J/cm³ · °C.

Solution: $f = \frac{c\Delta T}{\Delta H_f} = \frac{(3.25 \text{ J/cm}^3 \cdot \text{°C})(10\text{°C})}{965 \text{ J/cm}^3} = 0.0337$

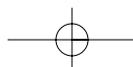
$$\frac{c\Delta T}{\Delta H_f} = \frac{(3.25 \text{ J/cm}^3 \cdot \text{°C})(100\text{°C})}{965 \text{ J/cm}^3} = 0.337$$

$$\frac{c\Delta T}{\Delta H_f} = \frac{(3.25 \text{ J/cm}^3 \cdot \text{°C})(250\text{°C})}{965 \text{ J/cm}^3} = 0.842$$

- 9-29** Analysis of a nickel casting suggests that 28% of the solidification process occurred in a dendritic manner. Calculate the temperature at which nucleation occurred. The specific heat of nickel is 4.1 J/cm³ · °C.

Solution: $f = \frac{c\Delta T}{\Delta H_f} = \frac{(4.1 \text{ J/cm}^3 \cdot \text{°C})(\Delta T)}{2756 \text{ J/cm}^3} = 0.28$

$$\Delta T = 188\text{°C} \quad \text{or} \quad T_n = 1453 - 188 = 1265\text{°C}$$



- 9-31** A 2-in. cube solidifies in 4.6 min. Calculate (a) the mold constant in Chvorinov's rule and (b) the solidification time for a 0.5 in. \times 0.5 in. \times 6 in. bar cast under the same conditions. Assume that $n = 2$.

Solution: (a) We can find the volume and surface area of the cube:

$$V = (2)^3 = 8 \text{ in.}^3 \quad A = 6(2)^2 = 24 \text{ in.}^2 \quad t = 4.6 = B(8/24)^2$$

$$B = 4.6/(0.333)^2 = 41.48 \text{ min/in.}^2$$

(b) For the bar, assuming that $B = 41.48 \text{ min/in.}^2$:

$$V = (0.5)(0.5)(6) = 1.5 \text{ in.}^2$$

$$A = 2(0.5)(0.5) + 4(0.5)(6) = 12.5 \text{ in.}^2$$

$$t = (41.48)(1.5/12.5)^2 = 0.60 \text{ min}$$

- 9-32** A 5-cm diameter sphere solidifies in 1050 s. Calculate the solidification time for a 0.3 cm \times 10 cm \times 20 cm plate cast under the same conditions. Assume that $n = 2$.

Solution:

$$t = 1050 \text{ s} = B \left[\frac{(4\pi/3)(2.5)^3}{4\pi(2.5)^2} \right]^2 = B[2.5/3]^2 \quad \text{or} \quad B = 1512 \text{ s/cm}^2$$

$$t = \frac{(1512)(0.3 \times 10 \times 20)^2}{[2(0.3)(10) + 2(0.3)(20) + 2(10)(20)]^2} = 1512[60/418]^2 = 31.15 \text{ s}$$

- 9-33** Find the constants B and n in Chvorinov's rule by plotting the following data on a log-log plot:

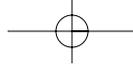
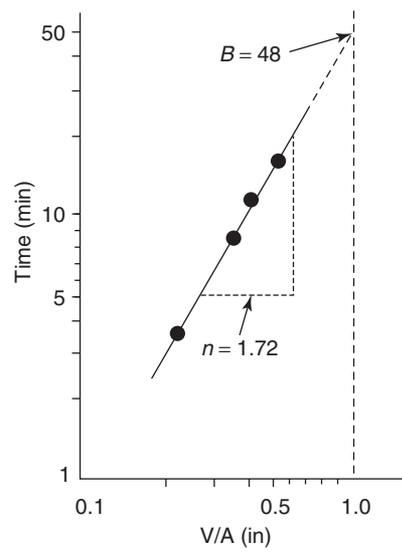
Casting dimensions (in.)	Solidification time (min)
0.5 \times 8 \times 12	3.48
2 \times 3 \times 10	15.78
2.5 cube	10.17
1 \times 4 \times 9	8.13

Solution:

$V(\text{in.}^3)$	$A(\text{in.}^2)$	V/A (in.)
48	212	0.226
60	112	0.536
15.6	37.5	0.416
36	98	0.367

From the graph on the following page, we find that

$$B = 48 \text{ min/in.}^2 \quad \text{and} \quad n = 1.72$$


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- 9-34** Find the constants B and n in Chvorinov's rule by plotting the following data on a log-log plot:

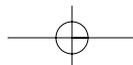
Casting dimensions (cm)	Solidification time (s)
$1 \times 1 \times 6$	28.58
$2 \times 4 \times 4$	98.30
$4 \times 4 \times 4$	155.89
$8 \times 6 \times 5$	306.15

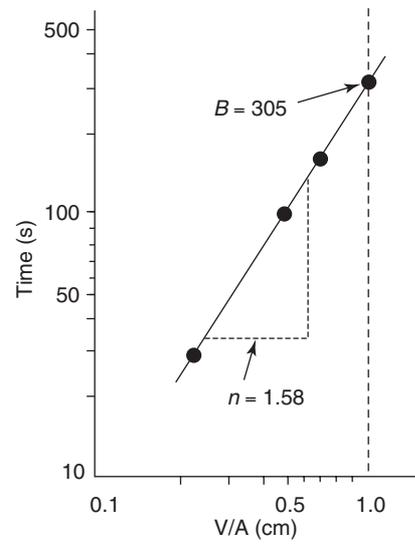
Solution:

$V(\text{cm}^3)$	$A(\text{cm}^2)$	V/A (cm)
6	26	0.23
32	64	0.5
64	96	0.67
240	236	1.02

From the graph on the next page, we find that

$$B = 305 \text{ s/cm}^2 \text{ and } n = 1.58$$





- 9-35** A 3-in.-diameter casting was produced. The times required for the solid-liquid interface to reach different distances beneath the casting surface were measured and are shown in the following table.

Distance from surface (in.)	Time (s)	\sqrt{t}
0.1	32.6	5.71
0.3	73.5	8.57
0.5	130.6	11.43
0.75	225.0	15.00
1.0	334.9	18.22

Determine (a) the time at which solidification begins at the surface and (b) the time at which the entire casting is expected to be solid. (c) Suppose the center of the casting actually solidified in 720 s. Explain why this time might differ from the time calculated in part (b).

Solution: We could plot d versus \sqrt{t} , as shown, finding $\sqrt{t_{\text{surface}}}$ from where the plot intersects the x -axis and $\sqrt{t_{\text{center}}}$ where the plot intersects $d = 1.5$ in. Or we could take two of the data points and solve for c and k .

$$d = k\sqrt{t} - c$$

$$0.1 = k\sqrt{32.6} - c$$

$$0.5 = k\sqrt{130.6} - c$$

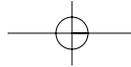
$$-0.4 = k[\sqrt{32.6} - \sqrt{130.6}] = -5.718k$$

$$k = 0.070$$

$$c = 0.070\sqrt{32.6} - 0.1 = 0.30$$

$$(a) \quad d = 0 = 0.070\sqrt{t} - 0.30$$

$$t_{\text{surface}} = (0.3/0.07)^2 = 18.4 \text{ s}$$

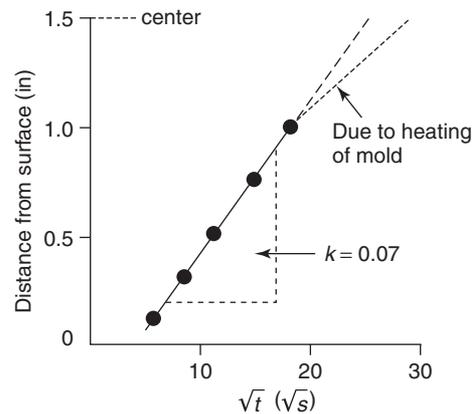


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$$(b) 1.5 = 0.070\sqrt{t} - 0.3$$

$$t_{\text{center}} = (1.8/0.07)^2 = 661 \text{ s}$$

- (c) The mold gets hot during the solidification process, and consequently heat is extracted from the casting more slowly. This in turn changes the constants in the equation and increases the time required for complete solidification.



- 9-36** Figure 9-5(b) shows a photograph of an aluminum alloy. Estimate (a) the secondary dendrite arm spacing and (b) the local solidification time for that area of the casting.

Solution: (a) The distance between adjacent dendrite arms can be measured. Although most people doing these measurements will arrive at slightly different numbers, the author's calculations obtained from four different primary arms are:

$$16 \text{ mm} / 6 \text{ arms} = 2.67 \text{ mm}$$

$$9 \text{ mm} / 5 \text{ arms} = 1.80 \text{ mm}$$

$$13 \text{ mm} / 7 \text{ arms} = 1.85 \text{ mm}$$

$$18 \text{ mm} / 9 \text{ arms} = 2.00 \text{ mm}$$

$$\text{average} = 2.08 \text{ mm} = 0.208 \text{ cm}$$

Dividing by the magnification of $\times 50$:

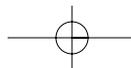
$$\text{SDAS} = 0.208 \text{ cm} / 50 = 4.16 \times 10^{-3} \text{ cm}$$

- (b) From Figure 9-6, we find that local solidification time (LST) = 90 s

- 9-37** Figure 9-25 shows a photograph of FeO dendrites that have precipitated from a glass (an undercooled liquid). Estimate the secondary dendrite arm spacing.

Solution: We can find 13 SDAS along a 3.5 cm distance on the photomicrograph. The magnification of the photomicrograph is $\times 450$, while we want the actual length (at magnification $\times 1$). Thus:

$$\text{SDAS} = (13 \text{ SDAS} / 3.5 \text{ cm}) (1/450) = 8.25 \times 10^{-3} \text{ cm}$$



- 9-38** Find the constants k and m relating the secondary dendrite arm spacing to the local solidification time by plotting the following data on a log-log plot:

Solidification Time (s)	SDAS (cm)
156	0.0176
282	0.0216
606	0.0282
1356	0.0374

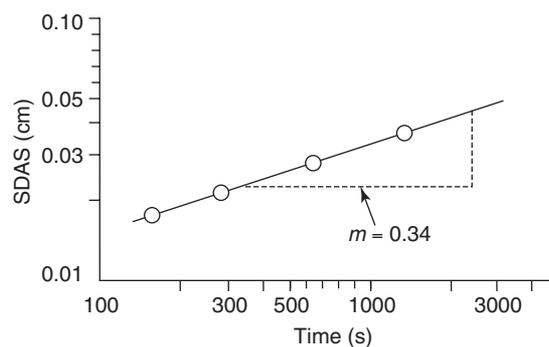
Solution: From the slope of the graph:

$$m = 34/100 = 0.34$$

We can then pick a point off the graph (say SDAS = 0.0225 cm when LST = 300 s) and calculate “ k ”:

$$0.0225 = k(300)^{0.34} = 6.954k$$

$$k = 0.0032$$



- 9-39** Figure 9-26 shows dendrites in a titanium powder particle that has been rapidly solidified. Assuming that the size of the titanium dendrites is related to solidification time by the same relationship as in aluminum, estimate the solidification time of the powder particle.

Solution: The secondary dendrite arm spacing can be estimated from the photomicrograph at several locations. The author’s calculations, derived from measurements at three locations, are

$$11 \text{ mm} / 8 \text{ arms} = 1.375 \text{ mm}$$

$$13 \text{ mm} / 8 \text{ arms} = 1.625 \text{ mm}$$

$$13 \text{ mm} / 8 \text{ arms} = 1.625 \text{ mm}$$

$$\text{average} = 1.540 \text{ mm}$$

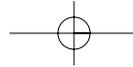
Dividing by the magnification of 2200:

$$\text{SDAS} = (1.540 \text{ mm})(0.1 \text{ cm/mm}) / 2200 = 7 \times 10^{-5} \text{ cm}$$

The relationship between SDAS and solidification time for aluminum is:

$$\text{SDAS} = 8 \times 10^{-4} t^{0.42} = 7 \times 10^{-5}$$

$$t = (0.0875)^{1/0.42} = 0.003 \text{ s}$$



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- 9-40** The secondary dendrite arm spacing in an electron-beam weld of copper is 9.5×10^{-4} cm. Estimate the solidification time of the weld.

Solution: From Figure 9-6, we can determine the equation relating SDAS and solidification time for copper:

$$n = 19/50 = 0.38 \quad k = 4 \times 10^{-3} \text{ cm}$$

Then for the copper weld:

$$9.5 \times 10^{-4} = 4 \times 10^{-3}(\text{LST})^{0.38}$$

(Note: LST is local solidification time)

$$0.2375 = (\text{LST})^{0.38} \quad \text{or} \quad -1.438 = 0.38 \ln \text{LST}$$

$$\ln \text{LST} = -3.783 \quad \text{or} \quad \text{LST} = 0.023 \text{ s}$$

- 9-45** A cooling curve is shown in Figure 9-27. Determine (a) the pouring temperature, (b) the solidification temperature, (c) the superheat, (d) the cooling rate just before solidification begins, (e) the total solidification time, (f) the local solidification time, and (g) the probable identity of the metal. (h) If the cooling curve was obtained at the center of the casting sketched in the figure, determine the mold constant, assuming that $n = 2$.

Solution:

(a) $T_{\text{pour}} = 475^\circ\text{C}$	(e) $t_s = 470 \text{ s}$
(b) $T_{\text{sol}} = 320^\circ\text{C}$	(f) $\text{LST} = 470 - 170 = 300 \text{ s}$
(c) $\Delta T_s = 475 - 320 = 155^\circ\text{C}$	(g) Cadmium (Cd)
(d) $\Delta T/\Delta t = \frac{475 - 320}{150 - 0} = 1.0^\circ\text{C/s}$	(h) $t_s = 470 = B[38.4/121.6]^2$ $B = 4713 \text{ s/cm}^2$

- 9-46** A cooling curve is shown in Figure 9-28. Determine (a) the pouring temperature, (b) the solidification temperature, (c) the superheat, (d) the cooling rate just before solidification begins, (e) the total solidification time, (f) the local solidification time, (g) the undercooling, and (h) the probable identity of the metal. (i) If the cooling curve was obtained at the center of the casting sketched in the figure, determine the mold constant, assuming $n = 2$.

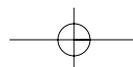
Solution:

(a) $T_{\text{pour}} = 900^\circ\text{C}$	(e) $t_s = 9.7 \text{ min}$
(b) $T_{\text{sol}} = 420^\circ\text{C}$	(f) $\text{LST} = 9.7 - 1.6 = 8.1 \text{ min}$
(c) $\Delta T_s = 900 - 420 = 480^\circ\text{C}$	(g) $420 - 360 = 60^\circ\text{C}$
(d) $\Delta T/\Delta t = \frac{900 - 400}{1.6 - 0} = 312 \text{ }^\circ\text{C/min}$	(h) Zn
(i) $t_s = 9.7 = B[8/24]^2$	or $B = 87.5 \text{ min/in.}^2$

- 9-47** Figure 9-29 shows the cooling curves obtained from several locations within a cylindrical aluminum casting. Determine the local solidification times and the SDAS at each location, then plot the tensile strength versus distance from the casting surface. Would you recommend that the casting be designed so that a large or small amount of material must be machined from the surface during finishing? Explain.

Solution: The local solidification times can be found from the cooling curves and can be used to find the expected SDAS values from Figure 9-6. The SDAS values can then be used to find the tensile strength, using Figure 9-7.

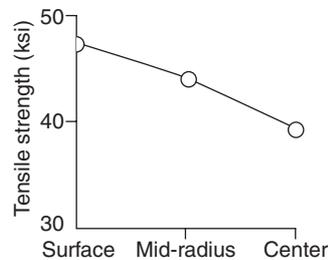
$$\text{Surface: LST} = 10 \text{ s} \Rightarrow \text{SDAS} = 1.5 \times 10^{-3} \text{ cm} \Rightarrow \text{TS} = 47 \text{ ksi}$$



Midradius: $LST = 100 \text{ s} \Rightarrow SDAS = 5 \times 10^{-3} \text{ cm} \Rightarrow TS = 44 \text{ ksi}$

Center: $LST = 500 \text{ s} \Rightarrow SDAS = 10 \times 10^{-3} \text{ cm} \Rightarrow TS = 39.5 \text{ ksi}$

You prefer to machine as little material off the surface of the casting as possible; the surface material has the finest structure and highest strength; any excessive machining simply removes the "best" material.



- 9-48** Calculate the volume, diameter, and height of the cylindrical riser required to prevent shrinkage in a 4 in. \times 10 in. \times 20 in. casting if the H/D of the riser is 1.5.

Solution:

$$(V/A)_c = \frac{(4)(10)(20)}{2(4)(10) + 2(4)(20) + 2(10)(20)} = \frac{800}{640} = 1.25$$

$$(V/A)_r = \frac{(\pi/4)D^2H}{2(\pi/4)D^2 + \pi DH} = \frac{(\pi/4)(3/2)D^3}{(\pi/2)D^2 + (3\pi/2)D^2} = \frac{3D/8}{2} = \frac{3D}{16} \geq 1.25$$

$$D \geq 6.67 \text{ in.} \quad H \geq 10 \text{ in.} \quad V \geq 349 \text{ in.}^3$$

- 9-55** Calculate the volume, diameter, and height of the cylindrical riser required to prevent shrinkage in a 1 in. \times 6 in. \times 6 in. casting if the H/D of the riser is 1.0.

Solution:

$$V = (1)(6)(6) = 36 \text{ in.}^3 \quad A = 2(1)(6) + 2(1)(6) + 2(6)(6) = 96 \text{ in.}^2$$

$$(V/A)_c = 36/96 = 0.375$$

$$(V/A)_r = \frac{(\pi/4)D^2H}{2(\pi/4)D^2 + \pi DH} = \frac{(\pi/4)D^3}{(3\pi/2)D^2} = D/6 \geq 0.375$$

$$D \geq 2.25 \text{ in.} \quad H \geq 2.25 \text{ in.} \quad V \geq 8.95 \text{ in.}^3$$

- 9-56** Figure 9-30 shows a cylindrical riser attached to a casting. Compare the solidification times for each casting section and the riser and determine whether the riser will be effective.

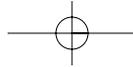
Solution:

$$(V/A)_{\text{thin}} = \frac{(8)(6)(3)}{(3)(6) + 2(3)(8) + 2(6)(8)} = 0.889$$

$$(V/A)_{\text{thick}} = \frac{(6)(6)(6)}{(6)(3) + 5(6)(6) - (\pi/4)(3)^2} = 1.13$$

$$(V/A)_{\text{riser}} = \frac{(\pi/4)(3)^2(7)}{\pi(3)(7) + (\pi/4)(3)^2} = 0.68$$

Note that the riser area in contact with the casting is not included in either the riser or casting surface area; no heat is lost across this interface. In a like manner, the area of contact between the thick and thin



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portions of the casting are not included in the calculation of the casting area.

The riser will not be effective; the thick section of the casting has the largest V/A ratio and therefore requires the longest solidification time. Consequently the riser will be completely solid before the thick section is solidified; no liquid metal will be available to compensate for the solidification shrinkage.

- 9-57** Figure 9-31 shows a cylindrical riser attached to a casting. Compare the solidification times for each casting section and the riser and determine whether the riser will be effective.

Solution:

$$(V/A)_{\text{thick}} = \frac{(4)(4)(4)}{5(4)(4) + 1(2)(4)} = 0.73$$

$$(V/A)_{\text{thin}} = \frac{(2)(2)(4)}{3(2)(4) + 2(2)(2)} = 0.50$$

$$(V/A)_R = \frac{(\pi/4)(4^2)(8)}{\pi(4)(8) + 2(\pi/4)4^2} = 0.8$$

The area between the thick and thin sections of the casting are not included in calculating casting area; no heat is lost across this interface.

The riser will not be effective; the thin section has the smallest V/A ratio and therefore freezes first. Even though the riser has the longest solidification time, the thin section isolates the thick section from the riser, preventing liquid metal from feeding from the riser to the thick section. Shrinkage will occur in the thick section.

- 9-58** A 4-in.-diameter sphere of liquid copper is allowed to solidify, producing a spherical shrinkage cavity in the center of the casting. Compare the volume and diameter of the shrinkage cavity in the copper casting to that obtained when a 4-in. sphere of liquid iron is allowed to solidify.

Solution: Cu: 5.1% Fe: 3.4% $r_{\text{sphere}} = 4/2 = 2$ in.

$$\text{Cu: } V_{\text{shrinkage}} = (4\pi/3)(2)^3(0.051) = 1.709 \text{ in.}^3$$

$$(4\pi/3)r^3 = 1.709 \text{ in.}^3 \quad \text{or} \quad r = 0.742 \text{ in.} \quad d_{\text{pore}} = 1.48 \text{ in.}$$

$$\text{Fe: } V_{\text{shrinkage}} = (4\pi/3)(2)^3(0.034) = 1.139 \text{ in.}^3$$

$$(4\pi/3)r^3 = 1.139 \text{ in.}^3 \quad \text{or} \quad r = 0.648 \text{ in.}$$

$$d_{\text{cavity}} = 1.30 \text{ in.}$$

- 9-59** A 4-in. cube of a liquid metal is allowed to solidify. A spherical shrinkage cavity with a diameter of 1.49 in. is observed in the solid casting. Determine the percent volume change that occurs during solidification.

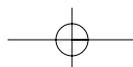
Solution:

$$V_{\text{liquid}} = (4 \text{ in.})^3 = 64 \text{ in.}^3$$

$$V_{\text{shrinkage}} = (4\pi/3)(1.49/2)^3 = 1.732 \text{ in.}^3$$

$$V_{\text{solid}} = 64 - 1.732 = 62.268 \text{ in.}^3$$

$$\% \text{ volume change} = \frac{64 - 62.268}{64} \times 100 = 2.7\%$$



- 9-60** A 2 cm × 4 cm × 6 cm magnesium casting is produced. After cooling to room temperature, the casting is found to weigh 80 g. Determine (a) the volume of the shrinkage cavity at the center of the casting and (b) the percent shrinkage that must have occurred during solidification.

Solution: The density of the magnesium is 1.738 g/cm³

$$(a) V_{\text{initial}} = (2)(4)(6) = 48 \text{ cm}^3$$

$$V_{\text{final}} = 80 \text{ g} / 1.738 \text{ g/cm}^3 = 46.03 \text{ cm}^3$$

$$(b) \% \text{ shrinkage} = \frac{48 - 46.03}{48} \times 100\% = 4.1\%$$

- 9-61** A 2 in. × 8 in. × 10 in. iron casting is produced and, after cooling to room temperature, is found to weigh 43.9 lb. Determine (a) the percent shrinkage that must have occurred during solidification and (b) the number of shrinkage pores in the casting if all of the shrinkage occurs as pores with a diameter of 0.05 in.

Solution: The density of the iron is 7.87 g/cm³

$$(a) V_{\text{actual}} = \frac{(43.9 \text{ lb})(454 \text{ g})}{7.87 \text{ g/cm}^3} = 2532.5 \text{ cm}^3$$

$$V_{\text{intended}} = (2)(8)(10) = 160 \text{ in.}^3 \times (2.54 \text{ cm/in.})^3 = 2621.9 \text{ cm}^3$$

$$\text{shrinkage} = \frac{2621.9 - 2532.5}{2621.9} \times 100\% = 3.4\%$$

$$(b) V_{\text{pores}} = 2621.9 - 2532.5 = 89.4 \text{ cm}^3$$

$$r_{\text{pores}} = (0.05 \text{ in.}/2)(2.54 \text{ cm/in.}) = 0.0635 \text{ cm}$$

$$\# \text{ pores} = \frac{89.4 \text{ cm}^3}{(\frac{4\pi}{3})(0.0635 \text{ cm})^3} = 83,354 \text{ pores}$$

- 9-65** Liquid magnesium is poured into a 2 cm × 2 cm × 24 cm mold and, as a result of directional solidification, all of the solidification shrinkage occurs along the length of the casting. Determine the length of the casting immediately after solidification is completed.

Solution: $V_{\text{initial}} = (2)(2)(24) = 96 \text{ cm}^3$

$$\% \text{ contraction} = 4 \quad \text{or} \quad 0.04 \times 96 = 3.84 \text{ cm}^3$$

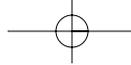
$$V_{\text{final}} = 96 - 3.84 = 92.16 \text{ cm}^3 = (2)(2)(L)$$

$$\text{Length (L)} = 23.04 \text{ cm}$$

- 9-66** A liquid cast iron has a density of 7.65 g/cm³. Immediately after solidification, the density of the solid cast iron is found to be 7.71 g/cm³. Determine the percent volume change that occurs during solidification. Does the cast iron expand or contract during solidification?

Solution: $\frac{1/7.65 - 1/7.71}{1/7.65} \times 100\% = \frac{0.1307 \text{ cm}^3 - 0.1297 \text{ cm}^3}{0.1307 \text{ cm}^3} \times 100\% = 0.77\%$

The casting contracts.

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- 9-67** From Figure 9-14, find the solubility of hydrogen in liquid aluminum just before solidification begins when the partial pressure of hydrogen is 1 atm. Determine the solubility of hydrogen (in $\text{cm}^3/100 \text{ g Al}$) at the same temperature if the partial pressure were reduced to 0.01 atm.

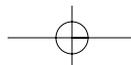
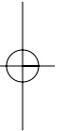
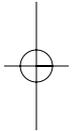
Solution: $0.46 \text{ cm}^3 \text{ H}_2/100 \text{ g Aluminum}$

$$0.46/x = \frac{\sqrt{1}}{\sqrt{0.01}}$$

$$x = 0.46\sqrt{0.01} = 0.046 \text{ cm}^3/100 \text{ g AL}$$

- 9-68** The solubility of hydrogen in liquid aluminum at 715°C is found to be $1 \text{ cm}^3/100 \text{ g Al}$. If all of this hydrogen precipitated as gas bubbles during solidification and remained trapped in the casting, calculate the volume percent gas in the solid aluminum.

Solution: $(1 \text{ cm}^3 \text{ H}_2/100 \text{ g Al})(2.699 \text{ g/cm}^3) = 0.02699 \text{ cm}^3 \text{ H}_2/\text{cm}^3 \text{ Al} = 2.699\%$



10

Solid Solutions and Phase Equilibrium

10–10 The unary phase diagram for SiO_2 is shown in Figure 10–19. Locate the triple point where solid, liquid, and vapor coexist and give the temperature and the type of solid present. What do the other “triple” points indicate?

Solution: (a) The solid-liquid-vapor triple point occurs at 1713°C ; the solid phase present at this point is β -cristobalite.

(b) The other triple points describe the equilibrium between two solids and a vapor phase.

10–22 Based on Hume-Rothery’s conditions, which of the following systems would be expected to display unlimited solid solubility? Explain.

- (a) Au–Ag (b) Al–Cu (c) Al–Au (d) U–W
 (e) Mo–Ta (f) Nb–W (g) Mg–Zn (h) Mg–Cd

Solution:

(a)	$r_{\text{Au}} = 1.442$	$v = +1$	FCC
	$r_{\text{Ag}} = 1.445$	$v = +1$	FCC
	$\phi r = 0.2\%$		Yes
(b)	$r_{\text{Al}} = 1.432$	$v = +3$	FCC
	$r_{\text{Cu}} = 1.278$	$v = +1$	FCC
	$\phi r = 10.7\%$		No
(c)	$r_{\text{Al}} = 1.432$	$v = +3$	FCC
	$r_{\text{Au}} = 1.442$	$v = +1$	FCC
	$\phi r = 0.7\%$		No
(d)	$r_{\text{U}} = 1.38$	$v = +4$	Ortho
	$r_{\text{W}} = 1.371$	$v = +4$	FCC
	$\phi r = 0.7\%$		No

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- (e) $r_{\text{Mo}} = 1.363$ $v = +4$ BCC
 $r_{\text{Ta}} = 1.43$ $v = +5$ BCC
 $\phi r = 4.7\%$ No
- (f) $r_{\text{Nb}} = 1.426$ $v = +4$ BCC
 $r_{\text{W}} = 1.371$ $v = +4$ BCC
 $\phi r = 3.9\%$ Yes
- (g) $r_{\text{Mg}} = 1.604$ $v = +2$ HCP
 $r_{\text{Zn}} = 1.332$ $v = +2$ HCP
 $\phi r = 17\%$ No
- (h) $r_{\text{Mg}} = 1.604$ $v = +2$ HCP
 $r_{\text{Cd}} = 1.490$ $v = +2$ HCP
 $\phi r = 7.1\%$ Yes

The Au–Ag, Mo–Ta, and Mg–Cd systems have the required radius ratio, the same crystal structures, and the same valences. Each of these might be expected to display complete solid solubility. [The Au–Ag and Mo–Ta do have isomorphous phase diagrams. In addition, the Mg–Cd alloys all solidify like isomorphous alloys; however a number of solid-state phase transformations complicate the diagram.]

10–23 Suppose 1 at% of the following elements is added to copper (forming a separate alloy with each element) without exceeding the solubility limit. Which one would be expected to give the higher strength alloy? Is any of the alloying elements expected to have unlimited solid solubility in copper?

- (a) Au (b) Mn (c) Sr (d) Si (e) Co

Solution: For copper: $r_{\text{Cu}} = 1.278 \text{ \AA}$

- | | | |
|---------------------|--|-----------------------------|
| (a) Au: $r = 1.442$ | $\phi r = \frac{r_{\text{Au}} - r_{\text{Cu}}}{r_{\text{Cu}}} = +12.8\%$ | May be Unlimited Solubility |
| (b) Mn: $r = 1.12$ | $\phi r = -12.4\%$ | Different structure |
| (c) Sr: $r = 2.151$ | $\phi r = +68.3\%$ | Highest Strength |
| (d) Si: $r = 1.176$ | $\phi r = -8.0\%$ | Different structure |
| (e) Co: $r = 1.253$ | $\phi r = -2.0\%$ | Different structure |

The Cu–Sr alloy would be expected to be strongest (largest size difference). The Cu–Au alloy satisfies Hume-Rothery's conditions and might be expected to display complete solid solubility—in fact it freezes like an isomorphous series of alloys, but a number of solid-state transformations occur at lower temperatures.

10–24 Suppose 1 at% of the following elements is added to aluminum (forming a separate alloy with each element) without exceeding the solubility limit. Which one would be expected to give the least reduction in electrical conductivity? Is any of the alloy elements expected to have unlimited solid solubility in aluminum?

- (a) Li (b) Ba (c) Be (d) Cd (e) Ga

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Solution: For aluminum: $r = 1.432 \text{ \AA}$ (FCC structure with valence of 3)

- | | | | |
|---------------------|--------------------|--------------|-------------|
| (a) Li: $r = 1.519$ | $\phi r = 6.1\%$ | BCC | valence = 1 |
| (b) Ba: $r = 2.176$ | $\phi r = -52.0\%$ | BCC | valence = 2 |
| (c) Be: $r = 1.143$ | $\phi r = -20.2\%$ | HCP | valence = 2 |
| (d) Cd: $r = 1.49$ | $\phi r = 4.1\%$ | HCP | valence = 2 |
| (e) Ga: $r = 1.218$ | $\phi r = 14.9\%$ | Orthorhombic | valence = 3 |

The cadmium would be expected to give the smallest reduction in electrical conductivity, since the Cd atoms are most similar in size to the aluminum atoms.

None are expected to have unlimited solid solubility, due either to difference in valence, atomic radius, or crystal structure.

10–25 Which of the following oxides is expected to have the largest solid solubility in Al_2O_3 ?

- (a) Y_2O_3 (b) Cr_2O_3 (c) Fe_2O_3

Solution: The ionic radius of $\text{Al}^{3+} = 0.51 \text{ \AA}$

- | | |
|---------------------------------|---|
| (a) $r_{\text{Y}^{3+}} = 0.89$ | $\phi r = \frac{0.63 - 0.51}{0.51} \times 100 = 74.5\%$ |
| (b) $r_{\text{Cr}^{3+}} = 0.63$ | $\phi r = 23.5\%$ |
| (c) $r_{\text{Fe}^{3+}} = 0.64$ | $\phi r = 25.5\%$ |

We would expect Cr_2O_3 to have a high solubility in Al_2O_3 ; in fact, they are completely soluble in one another.

10–29 Determine the liquidus temperature, solidus temperature, and freezing range for the following NiO–MgO ceramic compositions. [See Figure 10–9(b).]

- (a) NiO–30 mol% MgO (b) NiO–45 mol% MgO
(c) NiO–60 mol% MgO (d) NiO–85 mol% MgO

- Solution:**
- | | | |
|--------------------------------|----------------------------|--------------------------|
| (a) $T_L = 2330^\circ\text{C}$ | $T_S = 2150^\circ\text{C}$ | FR = 180°C |
| (b) $T_L = 2460^\circ\text{C}$ | $T_S = 2250^\circ\text{C}$ | FR = 210°C |
| (c) $T_L = 2570^\circ\text{C}$ | $T_S = 2380^\circ\text{C}$ | FR = 190°C |
| (d) $T_L = 2720^\circ\text{C}$ | $T_S = 2610^\circ\text{C}$ | FR = 110°C |

10–30 Determine the liquidus temperature, solidus temperature, and freezing range for the following MgO–FeO ceramic compositions. (See Figure 10–21.)

- (a) MgO–25 wt% FeO (b) MgO–45 wt% FeO
(c) MgO–65 wt% FeO (d) MgO–80 wt% FeO

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Solution: (a) $T_L = 2600^\circ\text{C}$ $T_S = 2230^\circ\text{C}$ FR = 370°C
 (b) $T_L = 2340^\circ\text{C}$ $T_S = 1900^\circ\text{C}$ FR = 440°C
 (c) $T_L = 2000^\circ\text{C}$ $T_S = 1610^\circ\text{C}$ FR = 390°C
 (d) $T_L = 1750^\circ\text{C}$ $T_S = 1480^\circ\text{C}$ FR = 270°C

10-31 Determine the phases present, the compositions of each phase, and the amount of each phase in mol% for the following NiO–MgO ceramics at 2400°C . [See Figure 10-9(b).]

(a) NiO–30 mol% MgO (b) NiO–45 mol% MgO
 (c) NiO–60 mol% MgO (d) NiO–85 mol% MgO

Solution: (a) L: NiO–30 mol% MgO 100% L

$$(b) L: 38\% \text{ MgO} \quad \% L = \frac{62 - 45}{62 - 38} \times 100\% = 70.8\%$$

$$S: 62\% \text{ MgO} \quad \% L = \frac{45 - 38}{62 - 38} \times 100\% = 29.2\%$$

$$(c) L: 38\% \text{ MgO} \quad \% L = \frac{62 - 60}{62 - 38} \times 100\% = 8.3\%$$

$$S: 62\% \text{ MgO} \quad \% L = \frac{60 - 38}{62 - 38} \times 100\% = 91.7\%$$

(d) S: 85% MgO 100% S

10-32 (a) Determine the phases present, the compositions of each phase, and the amount of each phase in wt% for the following MgO–FeO ceramics at 2000°C . (See Figure 10-21.)

(i) MgO–25 wt% FeO (ii) MgO–45 wt% FeO
 (iii) MgO–60 wt% FeO (iv) MgO–80 wt% FeO

Solution: (i) S: 25% FeO 100% S

$$(ii) S: 39\% \text{ FeO} \quad \% S = \frac{65 - 45}{65 - 39} \times 100\% = 76.9\%$$

$$L: 65\% \text{ FeO} \quad \% L = \frac{45 - 39}{65 - 39} \times 100\% = 23.1\%$$

$$(iii) S: 39\% \text{ FeO} \quad \% S = \frac{65 - 60}{65 - 39} \times 100\% = 19.2\%$$

$$L: 65\% \text{ MgO} \quad \% L = \frac{60 - 39}{65 - 39} \times 100\% = 80.8\%$$

(iv) S: 80% MgO 100% L

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(b) Consider an alloy of 65 wt% Cu and 35 wt% Al. Calculate the composition of the alloy in at%.

$$\text{Solution: (b) at \% Cu} = \frac{65/63.54}{65/63.54 + 35/26.981} \times 100\% = 44.1\%$$

$$\text{at \% Al} = \frac{35/26.981}{65/63.54 + 35/26.981} \times 100\% = 55.9\%$$

10-33 Consider a ceramic composed of 30 mol% MgO and 70 mol% FeO. Calculate the composition of the ceramic in wt%.

$$\text{Solution: } MW_{\text{MgO}} = 24.312 + 16 = 40.312 \text{ g/mol}$$

$$MW_{\text{FeO}} = 55.847 + 16 = 71.847 \text{ g/mol}$$

$$\text{wt\% MgO} = \frac{30 \times 40.312}{30 \times 40.312 + 70 \times 71.847} \times 100\% = 19.4\%$$

$$\text{wt\% FeO} = \frac{70 \times 71.847}{30 \times 40.312 + 70 \times 71.847} \times 100\% = 80.6\%$$

10-34 A NiO–20 mol% MgO ceramic is heated to 2200°C. Determine (a) the composition of the solid and liquid phases in both mol% and wt% and (b) the amount of each phase in both mol% and wt%. (c) Assuming that the density of the solid is 6.32 g/cm³ and that of the liquid is 7.14 g/cm³, determine the amount of each phase in vol% (see Figure 10-9(b)).

$$\text{Solution: } MW_{\text{MgO}} = 24.312 + 16 = 40.312 \text{ g/mol}$$

$$MW_{\text{NiO}} = 58.71 + 16 = 74.71 \text{ g/mol}$$

(a) L: 15 mol% MgO

$$\text{wt\% MgO} = \frac{15 \times 40.312}{15 \times 40.312 + 85 \times 74.71} \times 100\% = 8.69\%$$

S: 38 mol% MgO

$$\text{wt\% MgO} = \frac{38 \times 40.312}{38 \times 40.312 + 62 \times 74.71} \times 100\% = 24.85\%$$

(b) mol% L = $\frac{38 - 20}{38 - 15} \times 100\% = 78.26\%$ mol% S = 21.74%

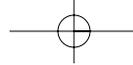
The original composition, in wt% MgO, is:

$$\frac{20 \times 40.312}{20 \times 40.312 + 80 \times 74.71} \times 100\% = 11.9\%$$

$$\text{wt\% L} = \frac{24.85 - 11.9}{24.85 - 8.69} \times 100\% = 80.1\% \quad \text{wt\% S} = 19.9\%$$

(c) vol% L = $\frac{80.1/7.14}{80.1/7.14 + 19.9/6.32} \times 100\% = 78.1\%$

$$\text{vol\% S} = 21.9\%$$



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10–35 A Nb–60 wt% W alloy is heated to 2800°C. Determine (a) the composition of the solid and liquid phases in both wt% and at% and (b) the amount of each phase in both wt% and at%. (c) Assuming that the density of the solid is 16.05 g/cm³ and that of the liquid is 13.91 g/cm³, determine the amount of each phase in vol%. (See Figure 10–22.)

Solution: (a) *L*: 49 wt% W

$$\text{at\% W} = \frac{49/183.85}{49/183.852 + 51/92.912} \times 100\% = 32.7\%$$

α: 70 wt% W

$$\text{at\% W} = \frac{70/183.852}{70/183.852 + 30/92.912} \times 100\% = 54.1\%$$

$$(b) \text{ wt\% } L = \frac{70 - 60}{70 - 49} \times 100\% = 47.6\% \quad \text{wt\% } \alpha = 52.4\%$$

The original composition, in wt% MgO, is:

$$\frac{60/183.85}{60/183.852 + 40/92.912} \times 100\% = 43.1\%$$

$$\text{at\% } L = \frac{54.1 - 43.1}{54.1 - 32.7} \times 100\% = 51.4\% \quad \text{wt\% } \alpha = 48.6\%$$

$$(c) \text{ vol\% } L = \frac{47.6/13.91}{47.6/13.912 + 52.4/16.052} \times 100\% = 51.2\%$$

$$\text{vol\% } \alpha = 48.8\%$$

10–36 How many grams of nickel must be added to 500 grams of copper to produce an alloy that has a liquidus temperature of 1350°C? What is the ratio of the number of nickel atoms to copper atoms in this alloy?

Solution: We need 60 wt% Ni to obtain the correct liquidus temperature.

$$\%Ni = 60 = \frac{x}{x + 500 \text{ g}} \times 100\% \quad \text{or} \quad x = 750 \text{ g Ni}$$

$$\frac{\text{Ni atoms}}{\text{Cu atoms}} = \frac{750 \text{ g Ni} / 58.71 \text{ g/mol}}{500 \text{ g Cu} / 63.54 \text{ g/mol}} = 1.62$$

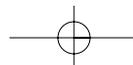
10–37 How many grams of nickel must be added to 500 grams of copper to produce an alloy that contains 50 wt% *α* at 1300°C?

Solution: At 1300°C, the composition of the two phases in equilibrium are

L: 46 wt% Ni and *α*: 58 wt% Ni

The alloy required to give 50% *α* is then

$$\frac{x - 46}{58 - 46} \times 100 = 50\% \alpha \quad \text{or} \quad x = 52 \text{ wt\% Ni}$$



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The number of grams of Ni must be:

$$\frac{x}{x + 500} \times 100\% = 52 \quad \text{or} \quad x = 541.7 \text{ g Ni}$$

10–38 How many grams of MgO must be added to 1 kg of NiO to produce a ceramic that has a solidus temperature of 2200°C?

Solution: $MW_{\text{MgO}} = 40.312 \text{ g/mol}$ $MW_{\text{NiO}} = 74.71 \text{ g/mol}$

38 mol% MgO is needed to obtain the correct solidus temperature.

$$\text{wt\% MgO} = \frac{38 \times 40.312}{38 \times 40.312 + 62 \times 74.71} \times 100\% = 24.9\%$$

The number of grams required is:

$$\frac{x}{x + 1000} \times 100\% = 24.9\% \quad \text{or} \quad x = 332 \text{ g of MgO}$$

10–39 How many grams of MgO must be added to 1 kg of NiO to produce a ceramic that contains 25 mol% solid at 2400°C?

Solution: L: 38 mol% MgO $MW_{\text{MgO}} = 40.312 \text{ g/mol}$

S: 62 mol% MgO $MW_{\text{NiO}} = 74.71 \text{ g/mol}$

$$\frac{x - 38}{62 - 38} \times 100\% = 25\% \text{S} \quad \text{or} \quad x = 44 \text{ mol\% MgO}$$

$$\text{wt\% MgO} = \frac{44 \times 40.312}{44 \times 40.312 + 56 \times 74.71} \times 100\% = 29.77\%$$

The number of grams of MgO is then:

$$\frac{x}{x + 1000} \times 100\% = 29.77\% \quad \text{or} \quad x = 424 \text{ g MgO}$$

10–40 We would like to produce a solid MgO–FeO ceramic that contains equal mol percentages of MgO and FeO at 1200°C. Determine the wt% FeO in the ceramic. (See Figure 10–21.)

Solution: Only solid is present at 1200°C. $MW_{\text{MgO}} = 40.312 \text{ g/mol}$

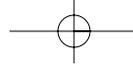
$MW_{\text{FeO}} = 71.847 \text{ g/mol}$

$$50 \text{ mol\% FeO: } \frac{50 \times 71.847}{50 \times 40.312 + 50 \times 71.847} = 64.1 \text{ wt\% FeO}$$

10–41 We would like to produce a MgO–FeO ceramic that is 30 wt% solid at 2000°C. Determine the original composition of the ceramic in wt%. (See Figure 10–21.)

Solution: L: 65 wt% FeO S: 38 wt% FeO

$$30 \text{ wt\%} = \frac{65 - x}{65 - 38} \times 100\% \quad \text{or} \quad x = 56.9 \text{ wt\% FeO}$$



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10-42 A Nb–W alloy held at 2800°C is partly liquid and partly solid. (a) If possible, determine the composition of each phase in the alloy; and (b) if possible, determine the amount of each phase in the alloy. (See Figure 10–22.)

Solution: (a) L : 49 wt% W α : 70 wt% W

(b) Not possible unless we know the original composition of the alloy.

10-43 A Nb–W alloy contains 55% α at 2600°C. Determine (a) the composition of each phase; and (b) the original composition of the alloy. (See Figure 10–22.)

Solution: (a) L : 22 wt% W α : 42 wt% W

$$(b) 0.55 = \frac{x - 22}{42 - 22} \quad \text{or} \quad x = 33 \text{ wt\% W}$$

10-44 Suppose a 1200-lb bath of a Nb–40 wt% W alloy is held at 2800°C. How many pounds of tungsten can be added to the bath before any solid forms? How many pounds of tungsten must be added to cause the entire bath to be solid? (See Figure 10–22.)

Solution: Solid starts to form at 2800°C when 49 wt% W is in the alloy. In 1200 lb of the original Nb–40% W alloy, there are $(0.4)(1200) = 480$ lb W and 720 lb Nb. The total amount of tungsten that must be in the final alloy is:

$$0.49 = \frac{x}{x + 720} \quad \text{or} \quad x = 692 \text{ lb W total}$$

or $692 - 480 = 212$ additional pounds of W must be added

To be completely solid at 2800°C, the alloy must contain 70 wt% W. The total amount of tungsten required in the final alloy is:

$$0.70 = \frac{x}{x + 720} \quad \text{or} \quad x = 1680 \text{ lb W total}$$

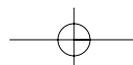
or $1680 - 480 = 1200$ additional pounds of W must be added

10-45 A fiber-reinforced composite material is produced, in which tungsten fibers are embedded in a Nb matrix. The composite is composed of 70 vol% tungsten. (a) Calculate the wt% of tungsten fibers in the composite. (b) Suppose the composite is heated to 2600°C and held for several years. What happens to the fibers? Explain. (See Figure 10–22.)

$$\text{Solution: (a) wt\%} = \frac{70 \text{ cm}^3 \cdot 219.254 \text{ g/cm}^3}{70 \cdot 219.254 + 30 \cdot 28.572} = 83.98 \text{ wt\% W}$$

(b) The fibers will dissolve. Since the W and Nb are completely soluble in one another, and the temperature is high enough for rapid diffusion, a single solid solution will eventually be produced.

10-46 Suppose a crucible made of pure nickel is used to contain 500 g of liquid copper at 1150°C. Describe what happens to the system as it is held at this temperature for several hours. Explain.



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Solution: Cu dissolves Ni until the Cu contains enough Ni that it solidifies completely. When 10% Ni is dissolved, freezing begins:

$$0.10 = \frac{x}{x + 500} \quad \text{or} \quad x = 55.5 \text{ g Ni}$$

When 18% Ni dissolved, the bath is completely solid:

$$0.18 = \frac{x}{x + 500} \quad \text{or} \quad x = 109.8 \text{ g Ni}$$

10-49 Equal moles of MgO and FeO are combined and melted. Determine (a) the liquidus temperature, the solidus temperature, and the freezing range of the ceramic and (b) determine the phase(s) present, their composition(s), and their amount(s) at 1800°C. (See Figure 10-21.)

Solution: $MW_{\text{MgO}} = 40.312 \text{ g/mol}$ $MW_{\text{FeO}} = 71.847 \text{ g/mol}$

$$\text{wt\% FeO} = \frac{1 \text{ mol FeO} \cdot 71.847 \text{ g/mol}}{1 \text{ mol FeO} \cdot 71.847 \text{ g/mol} + 1 \text{ mol MgO} \cdot 40.312 \text{ g/mol}} = 64.1\%$$

(a) $T_{\text{Liq}} = 2000^\circ\text{C}$ $T_s = 1620^\circ\text{C}$ $\text{FR} = 380^\circ\text{C}$

(b) $L: 75\% \text{ FeO}$ $S: 50\% \text{ FeO}$

$$\% L = \frac{64.1 - 50}{75 - 50} \times 100\% = 56.4\% \quad \% S = 43.6\%$$

10-50 Suppose 75 cm³ of Nb and 45 cm³ of W are combined and melted. Determine (a) the liquidus temperature, the solidus temperature, and the freezing range of the alloy and (b) determine the phase(s) present, their composition(s), and their amount(s) at 2800°C. (See Figure 10-22.)

Solution: $\text{wt\% W} = \frac{45 \text{ cm}^3 \cdot 19.254 \text{ g/cm}^3}{45 \cdot 19.254 + 75 \cdot 28.572} \times 100 = 57.4 \text{ wt\% W}$

(a) $T_{\text{Liq}} = 2900^\circ\text{C}$ $T_{\text{Sol}} = 2690^\circ\text{C}$ $\text{FR} = 210^\circ\text{C}$

(b) $L: 49\% \text{ W}$ $\% L = \frac{70 - 57.4}{70 - 49} = 60\%$

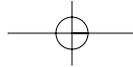
$\alpha: 70\% \text{ W}$ $\% \alpha = 40\%$

10-51 A NiO-60 mol% MgO ceramic is allowed to solidify. Determine (a) the composition of the first solid to form and (b) the composition of the last liquid to solidify under equilibrium conditions.

Solution: (a) 1st $\alpha: 80\% \text{ MgO}$ (b) Last $L: 35\% \text{ MgO}$

10-52 A Nb-35% W alloy is allowed to solidify. Determine (a) the composition of the first solid to form and (b) the composition of the last liquid to solidify under equilibrium conditions. (See Figure 10-22.)

Solution: (a) 1st $\alpha: 55\% \text{ W}$ (b) Last $L: 18\% \text{ W}$



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10-53 For equilibrium conditions and a MgO–65 wt% FeO ceramic, determine (a) the liquidus temperature, (b) the solidus temperature, (c) the freezing range, (d) the composition of the first solid to form during solidification, (e) the composition of the last liquid to solidify, (f) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 1800°C, and (g) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 1600°C. (See Figure 10–21.)

Solution: (a) Liquidus = 2000°C

(b) Solidus = 1605°C

(c) Freezing range = 2000 – 1605 = 395°C

(d) First solid: 40% FeO

(e) Last liquid: 88% FeO

(f) L: 75% FeO % L = $\frac{65 - 51}{75 - 51} \times 100\% = 58\%$

α: 51% FeO % α = 42%

(g) α: 65% FeO 100% α

10-54 Figure 10–23 shows the cooling curve for a NiO–MgO ceramic. Determine (a) the liquidus temperature, (b) the solidus temperature, (c) the freezing range, (d) the pouring temperature, (e) the superheat, (f) the local solidification time, (g) the total solidification time, and (h) the composition of the ceramic.

Solution: (a) Liquidus = 2690°C

(b) Solidus = 2570°C

(c) Freezing range = 2690 – 2570 = 120°C

(d) Pouring temperature = 2775°C

(e) Superheat = 2775 – 2690 = 85°C

(f) Local solidification time = 27 – 5 = 22 min

(g) Total solidification time = 27 min

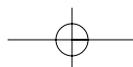
(h) 80% MgO

10-55 For equilibrium conditions and a Nb–80 wt% W alloy, determine (a) the liquidus temperature, (b) the solidus temperature, (c) the freezing range, (d) the composition of the first solid to form during solidification, (e) the composition of the last liquid to solidify, (f) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 3000°C, and (g) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 2800°C. (see Figure 10–22.)

Solution: (a) Liquidus = 3100°C

(b) Solidus = 2920°C

(c) Freezing range = 3100 – 2920 = 180°C



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- (d) First solid: 90% W
 (e) Last liquid: 64% W
 (f) $L: 70\% \text{ W} \quad \% L = \frac{85 - 80}{85 - 70} \times 100\% = 33.3\%$
 $\alpha: 85\% \text{ W} \quad \% \alpha = 66.7\%$
 (g) $\alpha: 80\% \text{ W} \quad 100\% \alpha$

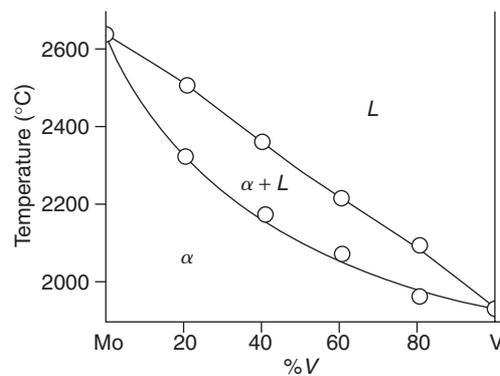
10-56 Figure 10-24 shows the cooling curve for a Nb-W alloy. Determine (a) the liquidus temperature, (b) the solidus temperature, (c) the freezing range, (d) the pouring temperature, (e) the superheat, (f) the local solidification time, (g) the total solidification time, and (h) the composition of the alloy.

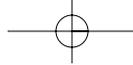
- Solution:** (a) Liquidus = 2900°C
 (b) Solidus = 2710°C
 (c) Freezing range = 2900 - 2710 = 190°C
 (d) Pouring temperature = 2990°C
 (e) Superheat = 2990 - 2900 = 90°C
 (f) Local solidification time = 340 - 40 = 300 s
 (g) Total solidification time = 340 sec
 (h) Nb-60 wt% W

10-57 Cooling curves are shown in Figure 10-25 for several Mo-V alloys. Based on these curves, construct the Mo-V phase diagram.

Solution:

	T_{Liquidus}	T_{Solidus}
0% V	2630°C	
20% V	2500°C	2320°C
40% V	2360°C	2160°C
60% V	2220°C	2070°C
80% V	2100°C	1970°C
100% V	1930°C	





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10–59 For the nonequilibrium conditions shown for the MgO–65 wt% FeO ceramic, determine (a) the liquidus temperature, (b) the nonequilibrium solidus temperature, (c) the freezing range, (d) the composition of the first solid to form during solidification, (e) the composition of the last liquid to solidify, (f) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 1800°C, and (g) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 1600°C. (See Figure 10–20.)

Solution: (a) Liquidus = 2000°C

(b) Solidus = 1450°C

(c) Freezing range = 2000 – 1450 = 550°C

(d) First solid: 40% FeO

(e) Last liquid: 92% FeO

$$(f) L: 75\% \text{ FeO} \quad \% L = \frac{65 - 46}{75 - 46} \times 100\% = 65.5\%$$

$$S: 46\% \text{ FeO} \quad \% S = 34.5\%$$

$$(g) L: 88\% \text{ FeO} \quad \% L = \frac{65 - 55}{88 - 55} \times 100\% = 30.3\%$$

$$S: 55\% \text{ FeO} \quad \% S = 69.7\%$$

10–60 For the nonequilibrium conditions shown for the Nb–80 wt% W alloy, determine (a) the liquidus temperature, (b) the nonequilibrium solidus temperature, (c) the freezing range, (d) the composition of the first solid to form during solidification, (e) the composition of the last liquid to solidify, (f) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 3000°C, and (g) the phase(s) present, the composition of the phase(s), and the amount of the phase(s) at 2800°C. (See Figure 10–21.)

Solution: (a) Liquidus = 3100°C

(b) Solidus = 2720°C

(c) Freezing range = 3100 – 2720 = 380°C

(d) First solid: 90% W

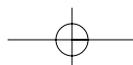
(e) Last liquid: 40% W

$$(f) L: 70\% \text{ W} \quad \% L = \frac{88 - 80}{88 - 70} \times 100\% = 44.4\%$$

$$\alpha: 88\% \text{ W} \quad \% \alpha = 55.6\%$$

$$(g) L: 50\% \text{ W} \quad \% L = \frac{83 - 80}{83 - 50} \times 100\% = 9.1\%$$

$$\alpha: 83\% \text{ W} \quad \% \alpha = 90.9\%$$



11

Dispersion Strengthening and Eutectic Phase Diagrams

- 11-7** A hypothetical phase diagram is shown in Figure 11-25. (a) Are any intermetallic compounds present? If so, identify them and determine whether they are stoichiometric or nonstoichiometric. (b) Identify the solid solutions present in the system. Is either material *A* or *B* allotropic? Explain. (c) Identify the three-phase reactions by writing down the temperature, the reaction in equation form, the composition of each phase in the reaction, and the name of the reaction.

Solution: (a) θ = non-stoichiometric intermetallic compound.

(b) α , η , γ , and β ; material *B* is allotropic, existing in three different forms at different temperatures

(c) 1100°C: $\gamma + L \rightleftharpoons \beta$; peritectic; L : 82% *B*
 γ : 97% *B* β : 90% *B*

900°C: $L_1 \rightleftharpoons L_2 + \alpha$; monotectic; L_1 : 28% *B*
 L_2 : 50% *B* α : 5% *B*

680°C: $L \rightleftharpoons \alpha + \beta$; eutectic; L : 60% *B*
 α : 5% *B* β : 90% *B*

600°C: $\alpha + \beta \rightleftharpoons \theta$; peritectoid; α : 5% *B*
 β : 80% *B* θ : 37% *B*

300°C: $\beta \rightleftharpoons \theta + \eta$; eutectoid; β : 90% *B*
 θ : 40% *B* η : 95% *B*

- 11-8** The Cu–Zn phase diagram is shown in Figure 11-26. (a) Are any intermetallic compounds present? If so, identify them and determine whether they are stoichiometric or nonstoichiometric. (b) Identify the solid solutions present in the system. (c) Identify the three-phase reactions by writing down the temperature, the reaction in equation form, and the name of the reaction.

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Solution: (a) $\beta, \beta', \gamma, \delta, \varepsilon$: all nonstoichiometric.

(b) α, θ

(c) 900°C: $\alpha + L \rightleftharpoons \beta$; peritectic

830°C: $\beta + L \rightleftharpoons \gamma$; peritectic

700°C: $\gamma + L \rightleftharpoons \delta$; peritectic

600°C: $\delta + L \rightleftharpoons \varepsilon$; peritectic

550°C: $\delta \rightleftharpoons \gamma + \varepsilon$; eutectoid

420°C: $\varepsilon + L \rightleftharpoons \theta$; peritectic

250°C: $\beta \rightleftharpoons \alpha + \gamma$; eutectoid

11-9 The Al–Li phase diagram is shown in Figure 11-27. (a) Are any intermetallic compounds present? If so, identify them and determine whether they are stoichiometric or nonstoichiometric. Determine the formula for each compound. (b) Identify the three-phase reactions by writing down the temperature, the reaction in equation form, the composition of each phase in the reaction, and the name of the reaction.

Solution: (a) β is non-stoichiometric @ 21 wt% Li:

$$\text{at\% Li} = \frac{21 \text{ g}/6.94 \text{ g/mol}}{21/6.94 + 79/26.981} \times 100\% = 50 \text{ at\% Li} \quad \therefore \text{AlLi}$$

γ , is stoichiometric @ 34 wt% Li:

$$\text{at\% Li} = \frac{34 \text{ g}/6.94 \text{ g/mol}}{34/6.94 + 66/26.981} \times 100\% = 66.7\% \text{ Li} \quad \therefore \text{AlLi}_2$$

(b) 600°C: $L \rightleftharpoons \alpha + \beta$ eutectic L : 9.9% Li α : 4% Li β : 20.4% Li

510°C: $\beta + L \rightleftharpoons \gamma$ peritectic β : 25% Li L : 47% Li γ : 34% Li

170°C: $L \rightleftharpoons \gamma + \alpha \text{Li}_2$ eutectic L : 98% Li γ : 34% Li αLi_2 : 99% Li

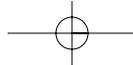
11-11 An intermetallic compound is found for 10 wt% Si in the Cu–Si phase diagram. Determine the formula for the compound.

Solution: $\text{at\% Si} = \frac{10 \text{ g}/28.08 \text{ g/mol}}{10/28.08 + 90/63.54} = 0.20 \quad \text{or} \quad \text{SiCu}_4$

11-12 Consider a Pb–15% Sn alloy. During solidification, determine (a) the composition of the first solid to form, (b) the liquidus temperature, solidus temperature, solvus temperature, and freezing range of the alloy, (c) the amounts and compositions of each phase at 260°C, (d) the amounts and compositions of each phase at 183°C, and (e) the amounts and compositions of each phase at 25°C.

Solution: (a) 8% Sn

(b) liquidus = 290°C, solidus = 240°C,
solvus = 170°C, freezing range = 50°C



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(c) L : 30% Sn α : 12% Sn;

$$\% L = \frac{15 - 12}{30 - 12} \times 100\% = 17\% \quad \% \alpha = 83\%$$

(d) α : 15% Sn 100% α (e) α : 2% Pb β : 100% Sn

$$\% \alpha = \frac{100 - 15}{100 - 2} \times 100 = 87\% \quad \% \beta = 13\%$$

- 11-13** Consider an Al-12% Mg alloy (Figure 11-28). During solidification, determine (a) the composition of the first solid to form, (b) the liquidus temperature, solidus temperature, solvus temperature, and freezing range of the alloy, (c) the amounts and compositions of each phase at 525°C, (d) the amounts and compositions of each phase at 450°C, and (e) the amounts and compositions of each phase at 25°C.

Solution: (a) 2.5% Mg(b) liquidus = 600°C, solidus = 470°C,
solvus = 400°C, freezing range = 130°C(c) L : 26% Mg α : 7% Mg;

$$\% \alpha = \frac{26 - 12}{26 - 7} \times 100\% = 74\% \quad \% L = 26\%$$

(d) α : 12% Mg 100% α (e) α : 1% Mg β : 34% Mg

$$\% \alpha = \frac{34 - 12}{34 - 1} \times 100\% = 67\% \quad \% \beta = 33\%$$

- 11-14** Consider a Pb-35% Sn alloy. Determine (a) if the alloy is hypoeutectic or hypereutectic, (b) the composition of the first solid to form during solidification, (c) the amounts and compositions of each phase at 184°C, (d) the amounts and compositions of each phase at 182°C, (e) the amounts and compositions of each microconstituent at 182°C, and (f) the amounts and compositions of each phase at 25°C.

Solution: (a) hypoeutectic (b) 14% Sn(c) α : 19% Sn L : 61.9% Sn

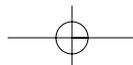
$$\% \alpha = \frac{61.9 - 35}{61.9 - 19} \times 100\% = 63\% \quad \% L = 37\%$$

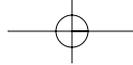
(d) α : 19% Sn β : 97.5% Sn

$$\% \alpha = \frac{97.5 - 35}{97.5 - 19} \times 100\% = 80\% \quad \% \beta = 20\%$$

(e) primary α : 19% Sn $\%$ primary α = 63%
eutectic: 61.9% Sn $\%$ eutectic = 37%(f) α : 2% Sn β : 100% Sn

$$\% \alpha = \frac{100 - 35}{100 - 2} \times 100\% = 66\% \quad \% \beta = 34\%$$




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- 11-15** Consider a Pb-70% Sn alloy. Determine (a) if the alloy is hypoeutectic or hypereutectic, (b) the composition of the first solid to form during solidification, (c) the amounts and compositions of each phase at 184°C, (d) the amounts and compositions of each phase at 182°C, (e) the amounts and compositions of each microconstituent at 182°C, and (f) the amounts and compositions of each phase at 25°C.

Solution: (a) hypereutectic (b) 98% Sn

(c) β : 97.5% Sn L: 61.9% Sn

$$\% \beta = \frac{70 - 61.9}{97.5 - 61.9} \times 100\% = 22.8\% \quad \% L = 77.2\%$$

(d) α : 19% Sn β : 97.5% Sn

$$\% \alpha = \frac{97.5 - 70}{97.5 - 19} \times 100\% = 35\% \quad \% \beta = 65\%$$

(e) primary β : 97.5% Sn $\% \text{ primary } \beta = 22.8\%$

eutectic: 61.9% Sn $\% \text{ eutectic} = 77.2\%$

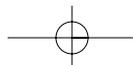
(f) α : 2% Sn β : 100% Sn

$$\% \alpha = \frac{100 - 70}{100 - 2} \times 100\% = 30\% \quad \% \beta = 70\%$$

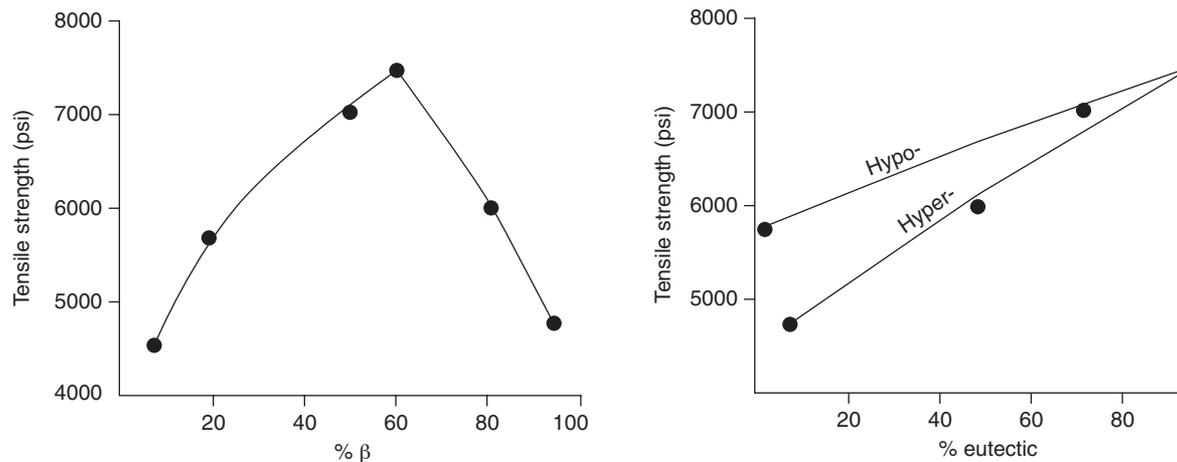
- 11-16** Calculate the total % β and the % eutectic microconstituent at room temperature for the following lead-tin alloys: 10% Sn, 20% Sn, 50% Sn, 60% Sn, 80% Sn, and 95% Sn. Using Figure 11-18, plot the strength of the alloys versus the % β and the % eutectic and explain your graphs.

Solution:

	$\% \beta$	$\% \text{ eutectic}$
10% Sn	$\frac{10 - 2}{99 - 2} = 8.2\%$	0%
20% Sn	$\frac{20 - 2}{99 - 2} = 18.6\%$	$\frac{20 - 19}{61.9 - 19} = 2.3\%$
50% Sn	$\frac{50 - 2}{99 - 2} = 49.5\%$	$\frac{50 - 19}{61.9 - 19} = 72.3\%$
60% Sn	$\frac{60 - 2}{99 - 2} = 59.8\%$	$\frac{60 - 19}{61.9 - 19} = 95.6\%$
80% Sn	$\frac{80 - 2}{99 - 2} = 80.4\%$	$\frac{97.5 - 80}{97.5 - 61.9} = 49.2\%$
95% Sn	$\frac{95 - 2}{99 - 2} = 95.9\%$	$\frac{97.5 - 95}{97.5 - 61.9} = 7.0\%$



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- 11–17** Consider an Al–4% Si alloy. (See Figure 11–19.) Determine (a) if the alloy is hypoeutectic or hypereutectic, (b) the composition of the first solid to form during solidification, (c) the amounts and compositions of each phase at 578°C, (d) the amounts and compositions of each phase at 576°C, the amounts and compositions of each microconstituent at 576°C, and (e) the amounts and compositions of each phase at 25°C.

Solution: (a) hypoeutectic

(b) 1% Si

(c) α : 1.65% Si L : 12.6% Si

$$\% \alpha = \frac{12.6 - 4}{12.6 - 1.65} = 78.5\% \quad \% L = 21.5\%$$

(d) α : 1.65% Si β : 99.83% Si

$$\% \alpha = \frac{99.83 - 4}{99.83 - 1.65} = 97.6\% \quad \% \beta = 2.4\%$$

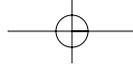
primary α : 1.65% Si % primary α = 78.5%
eutectic: 12.6% Si % eutectic = 21.5%

(e) α : 0% Si β : 100% Si % α = $\frac{100 - 4}{100 - 0} = 96\%$ % β = 4%

- 11–18** Consider a Al–25% Si alloy. (See Figure 11–19.) Determine (a) if the alloy is hypoeutectic or hypereutectic, (b) the composition of the first solid to form during solidification, (c) the amounts and compositions of each phase at 578°C, (d) the amounts and compositions of each phase at 576°C, (e) the amounts and compositions of each microconstituent at 576°C, and (f) the amounts and compositions of each phase at 25°C.

Solution: (a) hypereutectic

(b) 100% Si



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(c) β : 99.83% Si L: 12.6% Si

$$\% L = \frac{99.83 - 25}{99.83 - 12.6} = 85.8\% \quad \% \beta = 14.2\%$$

(d) α : 1.65% Si β : 99.83% Si

$$\% \alpha = \frac{99.83 - 25}{99.83 - 1.65} = 76.2\% \quad \% \beta = 23.8\%$$

(e) primary β : 99.83% Si $\% \text{ primary } \beta = 14.2\%$
eutectic: 12.6% Si $\% \text{ eutectic} = 85.8\%$

$$(f) \alpha: 0\% \text{ Si} \quad \beta: 100\% \text{ Si} \quad \% \alpha = \frac{100 - 25}{100 - 0} = 75\% \quad \% \beta = 25\%$$

11-19 A Pb–Sn alloy contains 45% α and 55% β at 100°C. Determine the composition of the alloy. Is the alloy hypoeutectic or hypereutectic?

$$\text{Solution: } \% \alpha = 45 = \frac{98.0 - x}{98.0 - 5} \times 100 \quad \text{or } x = 56.15\% \text{ Sn Hypoeutectic}$$

11-20 An Al–Si alloy contains 85% α and 15% β at 500°C. Determine the composition of the alloy. Is the alloy hypoeutectic or hypereutectic?

$$\text{Solution: } \% \alpha = 85 = \frac{100 - x}{100 - 1} \times 100 \quad \text{or } x = 15.85\% \text{ Si Hypereutectic}$$

11-21 A Pb–Sn alloy contains 23% primary α and 77% eutectic microconstituent. Determine the composition of the alloy.

$$\text{Solution: } \% \text{ primary } \alpha = 23 = \frac{61.9 - x}{61.9 - 19} \times 100 \quad \text{or } x = 52\% \text{ Sn}$$

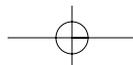
11-22 An Al–Si alloy contains 15% primary β and 85% eutectic microconstituent. Determine the composition of the alloy.

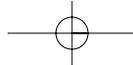
$$\text{Solution: } \% \text{ eutectic} = 85 = \frac{100 - x}{100 - 12.6} \times 100 \quad \text{or } x = 25.71\% \text{ Si}$$

11-23 Observation of a microstructure shows that there is 28% eutectic and 72% primary β in an Al–Li alloy (Figure 11-27). (a) Determine the composition of the alloy and whether it is hypoeutectic or hypereutectic. (b) How much α and β are in the eutectic microconstituent?

$$\text{Solution: (a) } 28 = \frac{20.4 - x}{20.4 - 9.9} \times 100 \quad \text{or } x = 17.46\% \text{ Li Hypereutectic}$$

$$(b) \% \alpha_{\text{Eut}} = \frac{20.4 - 9.9}{20.4 - 4} \times 100\% = 64\% \quad \text{and} \quad \% \beta_{\text{Eut}} = 36\%$$




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- 11-24** Write the eutectic reaction that occurs, including the compositions of the three phases in equilibrium, and calculate the amount of α and β in the eutectic microconstituent in the Mg–Al system, (Figure 11–28).

Solution: $L_{32.3} \rightleftharpoons \alpha_{12.7} + \gamma_{40.2}$

$$\therefore \% \alpha_{\text{Eut}} = \frac{40.2 - 32.3}{40.2 - 12.7} \times 100\% = 28.7\% \quad \text{and} \quad \% \gamma_{\text{Eut}} = 71.3\%$$

- 11-25** Calculate the total amount of α and β and the amount of each microconstituent in a Pb–50% Sn alloy at 182°C. What fraction of the total α in the alloy is contained in the eutectic microconstituent?

Solution: $\alpha_{\text{total}} = \frac{97.5 - 50}{97.5 - 19} \times 100\% = 60.5\% \quad \beta_{\text{Total}} = 39.5\%$

$$\alpha_{\text{Primary}} = \frac{61.9 - 50}{61.9 - 19} \times 100\% = 27.7\% \quad \text{Eutectic} = 72.3\%$$

$$\alpha_{\text{in eutectic}} = 60.5 - 27.7 = 32.8\%$$

$$f = 32.8/60.5 = 0.54$$

- 11-26** Figure 11–29 shows a cooling curve for a Pb–Sn alloy. Determine (a) the pouring temperature, (b) the superheat, (c) the liquidus temperature, (d) the eutectic temperature, (e) the freezing range, (f) the local solidification time, (g) the total solidification time, and (h) the composition of the alloy.

Solution: (a) pouring temperature = 360°C

$$(b) \text{ superheat} = 360 - 250 = 110^\circ\text{C}$$

$$(c) \text{ liquidus temperature} = 250^\circ\text{C}$$

$$(d) \text{ eutectic temperature} = 183^\circ\text{C}$$

$$(e) \text{ freezing range} = 250 - 183 = 67^\circ\text{C}$$

$$(f) \text{ local solidification time} = 600 - 110 = 490 \text{ s}$$

$$(g) \text{ total solidification time} = 600 \text{ s}$$

$$(h) \text{ approximately } 32\% \text{ Sn}$$

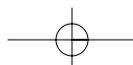
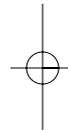
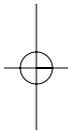
- 11-27** Figure 11–30 shows a cooling curve for an Al–Si alloy. Determine (a) the pouring temperature, (b) the superheat, (c) the liquidus temperature, (d) the eutectic temperature, (e) the freezing range, (f) the local solidification time, (g) the total solidification time, and (h) the composition of the alloy.

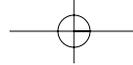
Solution: (a) pouring temperature = 1150°C

$$(b) \text{ superheat} = 1150 - 1000 = 150^\circ\text{C}$$

$$(c) \text{ liquidus temperature} = 1000^\circ\text{C}$$

$$(d) \text{ eutectic temperature} = 577^\circ\text{C}$$





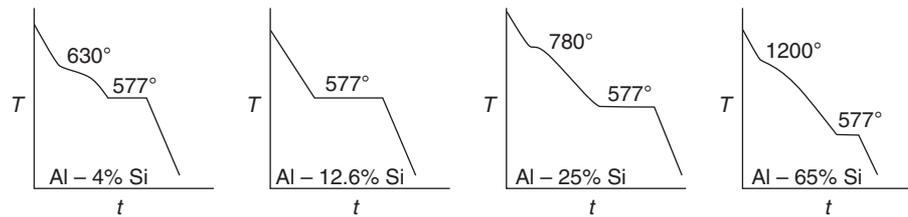
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- (e) freezing range = $1000 - 577 = 423^{\circ}\text{C}$
 (f) local solidification time = $11.5 - 1 = 10.5$ min
 (g) total solidification time = 11.5 min
 (h) approximately 45% Si

11-28 Draw the cooling curves, including appropriate temperatures, expected for the following Al-Si alloys.

- (a) Al-4% Si (b) Al-12.6% Si (c) Al-25% Si (d) Al-65% Si

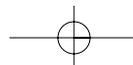
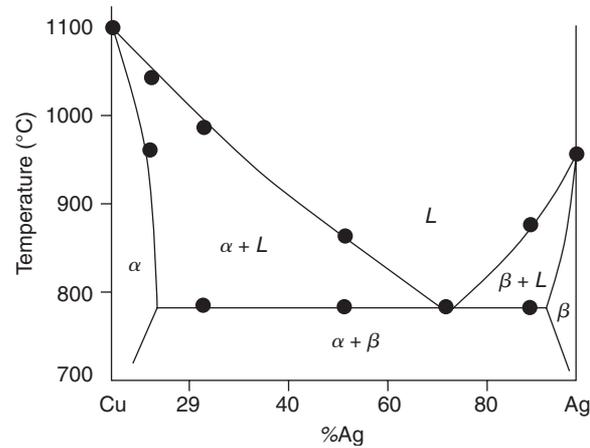
Solution:



11-29 Cooling curves are obtained for a series of Cu-Ag alloys, (Figure 11-31). Use this data to produce the Cu-Ag phase diagram. The maximum solubility of Ag in Cu is 7.9% and the maximum solubility of Cu in Ag is 8.8%. The solubilities at room temperature are near zero.

Solution:

	T_{liq}	T_{sol}
0% Ag	→ 1085°C	
8% Ag	→ 1030°C	950°C
20% Ag	→ 975°C	780°C
50% Ag	→ 860°C	780°C
71.9% Ag	→ 780°C	780°C
90% Ag	→ 870°C	780°C
100% Ag	→ 961°C	



12

Dispersion Strengthening by Phase Transformations and Heat Treatment

- 12-2 Determine the constants c and n in Equation 12-2 that describe the rate of crystallization of polypropylene at 140°C . (See Figure 12-30)

Solution: $f = 1 - \exp(-ct^n)$ $T = 140^\circ\text{C} = 413\text{ K}$

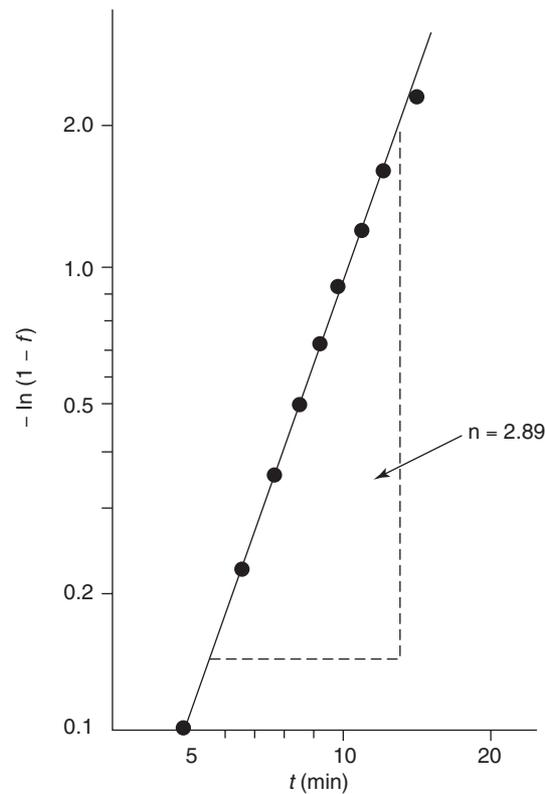
We can rearrange the equation and eliminate the exponential by taking natural logarithms of both sides of the rearranged equation. We can then note that $\ln(1 - f)$ versus t is a power equation; if these terms are plotted on a log-log plot, we should obtain a linear relationship, as the graph of the data below indicates. Note that in setting up the equation for plotting, we switch the minus sign from the right hand to the left hand side, since we don't have negative numbers on the log-log paper.

	f	$t(\text{min})$	$-\ln(1 - f)$
$1 - f = \exp(-ct^n)$	0.1	28	0.1
$\ln(1 - f) = -ct^n$	0.2	37	0.22
$\ln(3 - \ln(1 - f)) = \ln(3 - ct^n)$	0.3	44	0.36
$\ln(3 - \ln(1 - f)) = \ln(3 - ct^n)$	0.4	50	0.51
	0.5	55	0.69
	0.6	60	0.92
	0.7	67	1.20
	0.8	73	1.61
	0.9	86	2.302

A log-log plot of “ $-\ln(1 - f)$ ” versus “ t ” is shown. From the graph, we find that the slope $n = 2.89$ and the constant c can be found from one of the points from the curve:

if $f = 0.5$, $t = 55$. Then
 $1 - 0.5 = \exp(-c(55)^{2.89})$
 $c = 6.47 \times 10^{-6}$

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12-3 Determine the constants c and n in Equation 12-2 that describe the rate of recrystallization of copper at 135°C . (See Figure 12-2)

Solution: $f = 1 - \exp(-ct^n)$ $T = 135^{\circ}\text{C} = 408\text{ K}$

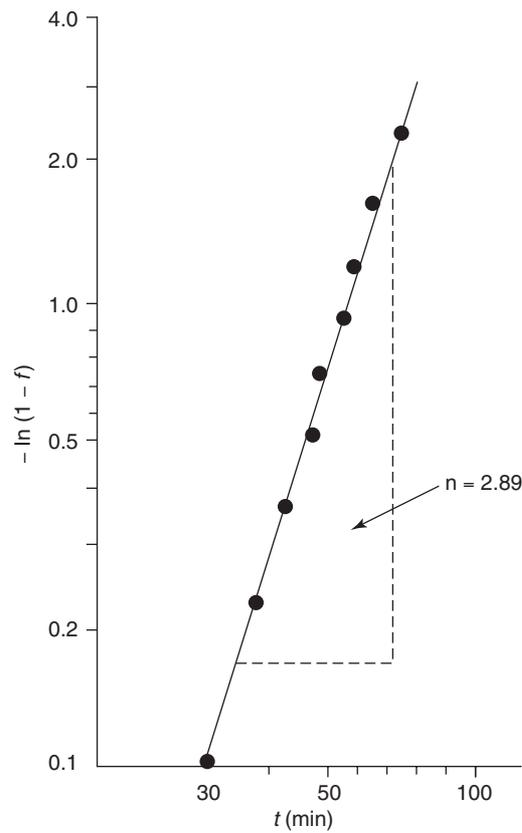
We can rearrange the equation and eliminate the exponential by taking natural logarithms of both sides of the rearranged equation. We can then note that $\ln(1-f)$ versus t is a power equation and should give a linear relationship in a log-log plot. Note that in setting up the equation for plotting, we switch the minus sign from the right hand to the left hand side, since we don't have negative numbers on the log-log paper.

$1-f = \exp(-ct^n)$	f	t (min)	$-\ln(1-f)$
$\ln(1-f) = -ct^n$	0.1	5.0	0.10
$\ln(1-f) = -ct^n$	0.2	6.6	0.22
$\ln(1-f) = -ct^n$	0.3	7.7	0.36
$\ln(1-f) = -ct^n$	0.4	8.5	0.51
$\ln(1-f) = -ct^n$	0.5	9.0	0.69
$\ln(1-f) = -ct^n$	0.6	10.0	0.92
$\ln(1-f) = -ct^n$	0.7	10.5	1.20
$\ln(1-f) = -ct^n$	0.8	11.5	1.61
$\ln(1-f) = -ct^n$	0.9	13.7	2.30

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A log-log plot of “ $-\ln(1 - f)$ ” versus “ t ” is shown. From the graph, we find that the slope $n = 3.1$ and the constant c can be found from one of the points from the curve:

if $f = 0.6$, then $t = 10$. Then
 $1 - 0.6 = \exp\{-c(10)^{3.1}\}$
 $c = 7.28 \times 10^{-4}$

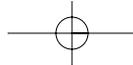


12-4 Determine the activation energy for crystallization of polypropylene, using the curves in Figure 12-30.

Solution: We can determine how the rate (equal to $1/\tau$) changes with temperature:

$$\text{rate} = 1/\tau = c \exp\{-Q/RT\}$$

$1/\tau \text{ s}^{-1}$	$1/T \text{ K}^{-1}$
$1/19 \text{ min} = 260 \text{ s/min} = 1.85 \times 10^{-3}$	$1/1130 + 2732 = 2.48 \times 10^{-3}$
$1/155 \text{ min} = 260 \text{ s/min} = 3.03 \times 10^{-4}$	$1/1140 + 2732 = 2.42 \times 10^{-3}$
$1/136 \text{ min} = 260 \text{ s/min} = 5.27 \times 10^{-5}$	$1/1150 + 2732 = 2.36 \times 10^{-3}$



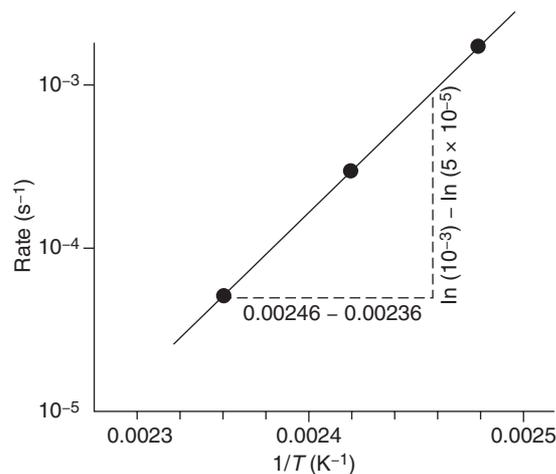
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From the semilog graph of rate versus reciprocal temperature, we find that the slope is:

$$Q/R = \frac{\ln 10^{-3} 2 - \ln 5 \times 10^{-5} 2}{0.00246 - 0.00236}$$

$$Q/R = 29,957$$

$$Q = 59,525 \text{ cal/mol}$$



- 12-16** (a) Recommend an artificial age-hardening heat treatment for a Cu-1.2% Be alloy (see Figure 12-34). Include appropriate temperatures. (b) Compare the amount of the γ_2 precipitate that forms by artificial aging at 400°C with the amount of the precipitate that forms by natural aging.

Solution: (a) For the Cu-1.2% Be alloy, the peritectic temperature is 870°C; above this temperature, liquid may form. The solvus temperature is about 530°C. Therefore:

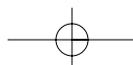
- 1) Solution treat between 530°C and 870°C (780°C is typical for beryllium copper alloys)
- 2) Quench
- 3) Age below 530°C (330°C is typical for these alloys)

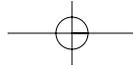
(b) We can perform lever law calculations at 400°C and at room temperature. The solubility of Be in Cu at 400°C is about 0.6% Be and that at room temperature is about 0.2% Be:

$$\gamma_2 \text{ at } 400^\circ\text{C} = \frac{1.2 - 0.6}{11.7 - 0.6} \times 100 = 5.4\%$$

$$\gamma_2 \text{ room } T = \frac{1.2 - 0.2}{12 - 0.2} \times 100 = 8.5\%$$

- 12-17** Suppose that age hardening is possible in the Al-Mg system (see Figure 12-10). (a) Recommend an artificial age-hardening heat treatment for each of the following alloys, and (b) compare the amount of the β precipitate that forms from your treatment of each alloy. (i) Al-4% Mg (ii) Al-6% Mg (iii) Al-12% Mg (c) Testing of the alloys after the heat treatment reveals that little strengthening occurs as a result of the heat treatment. Which of the requirements for age hardening is likely not satisfied?




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Solution: (a) The heat treatments for each alloy might be:

	Al-4% Mg	Al-6% Mg	Al-12% Mg
$T_{\text{Eutectic}} =$	451°C	451°C	451°C
$T_{\text{Solvus}} =$	210°C	280°C	390°C
Solution			
Treat at:	210–451°C	280–451°C	390–451°C
	Quench	Quench	Quench
Age at:	<210°C	<280°C	<390°C

(b) Answers will vary depending on aging temperature selected. If all three are aged at 200°C, as an example, the tie line goes from about 3.8 to 35% Mg:

$$\text{Al-4\% Mg: } \% \beta = \frac{4 - 3.8}{35 - 3.8} \times 100 = 0.6\%$$

$$\text{Al-6\% Mg: } \% \beta = \frac{6 - 3.8}{35 - 3.8} \times 100 = 7.1\%$$

$$\text{Al-12\% Mg: } \% \beta = \frac{12 - 3.8}{35 - 3.8} \times 100 = 26.8\%$$

(c) Most likely, a coherent precipitate is not formed; simple dispersion strengthening, rather than age hardening, occurs.

12-18 An Al-2.5% Cu alloy is solution-treated, quenched, and overaged at 230°C to produce a stable microstructure. If the spheroidal θ precipitates so that form has a diameter of 9000 Å and a density of 4.26 g/cm³, determine the number of precipitate particles per cm³.

Solution: $\text{wt\% } \alpha = \frac{53 - 2.5}{53 - 1} = 97.12\% \quad \text{wt\% } \theta = 2.88\%$

$$\text{vol fraction } \theta = \frac{2.88 \text{ g}/4.26 \text{ g/cm}^3}{2.88/4.26 + 97.12/2.669} = 0.0182 \text{ cm}^3 \theta/\text{cm}^3 \text{ alloy}$$

$$d_{\theta} = 9000 \times 10^{-10} \text{ m} = 9 \times 10^{-5} \text{ cm} \quad r_{\theta} = 4.5 \times 10^{-5} \text{ cm}$$

$$V_{\theta} = \frac{4\pi}{3} (4.5 \times 10^{-5} \text{ cm})^3 = 382 \times 10^{-15} \text{ cm}^3$$

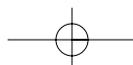
$$\# \text{ of particles} = \frac{0.0182 \text{ cm}^3}{382 \times 10^{-15} \text{ cm}^3} = 4.76 \times 10^{10} \text{ particles}$$

12-33 Figure 12-31 shows a hypothetical phase diagram. Determine whether each of the following alloys might be good candidates for age hardening and explain your answer. For those alloys that might be good candidates, describe the heat treatment required, including recommended temperatures.

- (a) A-10% B (b) A-20% B (c) A-55% B
 (d) A-87% B (e) A-95% B

Solution: (a) A-10% B is a good candidate: Solution Treatment @ $T = 290$ to 400°C
 Quench
 Age @ $T < 290^{\circ}\text{C}$

(b) A-20% B: Some age hardening effect may occur when alloy is solution treated below 400°C and quenched. However, eutectic is also present and the strengthening effect will not be as dramatic as in (a).



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- (c) A–55% B: almost all θ is formed. The alloy is expected to be very brittle.
- (d) A–87% B: the alloy cools from a two-phase ($\beta + \theta$) region to a one-phase (β) region, opposite of what we need for age hardening.
- (e) A–95% B: the alloy is single phase (β) at all temperatures and thus cannot be age hardened.

12–46 For an Fe–0.35% C alloy, determine (a) the temperature at which austenite first begins to transform on cooling, (b) the primary microconstituent that forms, (c) the composition and amount of each phase present at 728°C, (d) the composition and amount of each phase present at 726°C, and (e) the composition and amount of each microconstituent present at 726°C.

Solution: (a) 795°C (b) primary α -ferrite

$$\begin{aligned} \text{(c) } \alpha: 0.0218\% \text{ C} \quad \% \alpha &= \frac{0.77 - 0.35}{0.77 - 0.0218} \times 100 = 56.1\% \\ \gamma: 0.77\% \text{ C} \quad \% \gamma &= 43.9\% \end{aligned}$$

$$\begin{aligned} \text{(d) } \alpha: 0.0218\% \text{ C} \quad \% \alpha &= \frac{6.67 - 0.35}{6.67 - 0.0218} \times 100 = 95.1\% \\ \text{Fe}_3\text{C}: 6.67\% \text{ C} \quad \% \text{Fe}_3\text{C} &= 4.9\% \end{aligned}$$

$$\begin{aligned} \text{(e) primary } \alpha: 0.0218\% \text{ C} \quad \% \text{ primary } \alpha &= 56.1\% \\ \text{pearlite: } 0.77\% \text{ C} \quad \% \text{ Pearlite} &= 43.9\% \end{aligned}$$

12–47 For an Fe–1.15% C alloy, determine (a) the temperature at which austenite first begins to transform on cooling, (b) the primary microconstituent that forms, (c) the composition and amount of each phase present at 728°C, (d) the composition and amount of each phase present at 726°C, and (e) the composition and amount of each microconstituent present at 726°C.

Solution: (a) 880°C (b) primary Fe_3C

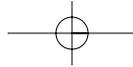
$$\begin{aligned} \text{(c) } \text{Fe}_3\text{C}: 6.67\% \text{ C} \quad \% \text{Fe}_3\text{C} &= \frac{1.15 - 0.77}{6.67 - 0.77} \times 100 = 6.4\% \\ \gamma: 0.77\% \text{ C} \quad \% \gamma &= 93.6\% \end{aligned}$$

$$\begin{aligned} \text{(d) } \alpha: 0.0218\% \text{ C} \quad \% \alpha &= \frac{6.67 - 1.15}{6.67 - 0.0218} \times 100 = 83\% \\ \text{Fe}_3\text{C}: 6.67\% \text{ C} \quad \% \text{Fe}_3\text{C} &= 17\% \end{aligned}$$

$$\begin{aligned} \text{(e) primary } \text{Fe}_3\text{C}: 6.67\% \text{ C} \quad \% \text{ primary } \text{Fe}_3\text{C} &= 6.4\% \\ \text{pearlite: } 0.77\% \text{ C} \quad \% \text{ Pearlite} &= 93.6\% \end{aligned}$$

12–48 A steel contains 8% cementite and 92% ferrite at room temperature. Estimate the carbon content of the steel. Is the steel hypoeutectoid or hypereutectoid?

Solution: $\alpha = 0.92 = \frac{6.67 - x}{6.67 - 0}$ $x = 0.53\% \text{ C}$, \therefore Hypoeutectoid


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- 12-49** A steel contains 18% cementite and 82% ferrite at room temperature. Estimate the carbon content of the steel. Is the steel hypoeutectoid or hypereutectoid?

Solution: $\alpha = 0.82 = \frac{6.67 - x}{6.67 - 0}$ $x = 1.20\% \text{ C}$, \therefore Hypereutectoid

- 12-50** A steel contains 18% pearlite and 82% primary ferrite at room temperature. Estimate the carbon content of the steel. Is the steel hypoeutectoid or hypereutectoid?

Solution: primary $\alpha = 0.82 = \frac{0.77 - x}{0.77 - 0.0218}$
 $x = 0.156\% \text{ C}$, \therefore Hypoeutectoid

- 12-51** A steel contains 94% pearlite and 6% primary cementite at room temperature. Estimate the carbon content of the steel. Is the steel hypoeutectoid or hypereutectoid?

Solution: Pearlite = $0.94 = \frac{6.67 - x}{6.67 - 0.77}$ $x = 1.124\% \text{ C}$, \therefore Hypereutectoid

- 12-52** A steel contains 55% α and 45% γ at 750°C. Estimate the carbon content of the steel.

Solution: $\alpha = 0.02\% \text{ C}$ and $\gamma = 0.6\% \text{ C}$ (from the tie line at 750°C)

$$\% \alpha = 55 = \frac{0.6 - x}{0.6 - 0.02} \times 100 \quad x = 0.281\% \text{ C}$$

- 12-53** A steel contains 96% γ and 4% Fe_3C at 800°C. Estimate the carbon content of the steel.

Solution: $\gamma = 0.92\% \text{ C}$ and $\text{Fe}_3\text{C} = 6.67\% \text{ C}$ (from the tie line at 800°C)

$$\gamma = 0.96 = \frac{6.67 - x}{6.67 - 0.92} \quad x = 1.15\% \text{ C}$$

- 12-54** A steel is heated until 40% austenite, with a carbon content of 0.5%, forms. Estimate the temperature and the overall carbon content of the steel.

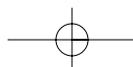
Solution: In order for γ to contain 0.5% C, the austenitizing temperature must be about 760°C (from the tie line). At this temperature:

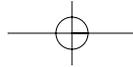
$$0.4 = \frac{x - 0.02}{0.5 - 0.02} \quad x = 0.212\% \text{ C}$$

- 12-55** A steel is heated until 85% austenite, with a carbon content of 1.05%, forms. Estimate the temperature and the overall carbon content of the steel.

Solution: In order for γ to contain 1.05% C, the austenitizing temperature must be about 845°C (from the tie line). At this temperature:

$$0.85 = \frac{6.67 - x}{6.67 - 1.05} \quad x = 1.893\% \text{ C}$$





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12-56 Determine the eutectoid temperature, the composition of each phase in the eutectoid reaction, and the amount of each phase present in the eutectoid microconstituent for the following systems. For the metallic systems, comment on whether you expect the eutectoid microconstituent to be ductile or brittle.

- (a) $\text{ZrO}_2\text{-CaO}$ (See Figure 12-32)
 (b) Cu-Al at 11.8%Al (See Figure 12-33(c))
 (c) Cu-Zn at 47%Zn (See Figure 12-33(a))
 (d) Cu-Be (See Figure 12-33(d))

Solution: (a) @900°C: Tetragonal_{12% CaO} \rightarrow Monoclinic_{3% CaO} + Cubic_{14% CaO}

$$\% \text{ Monoclinic} = \frac{14 - 12}{14 - 3} \times 100 = 18\% \quad \% \text{ Cubic} = 82\%$$

The eutectoid microconstituent (and the entire material, for that matter) will be brittle because the materials are ceramics

(b) @565°C: $\beta_{11.8\% \text{ Al}} \rightarrow \alpha_{9.4\% \text{ Al}} + \gamma_{15.6\% \text{ Al}}$

$$\% \alpha = \frac{15.6 - 11.8}{15.6 - 9.4} \times 100 = 61.3\% \quad \% \beta = 38.7\%$$

Most of the eutectoid microconstituent is α (solid solution strengthened copper) and is expected to be ductile.

(c) @250°C: $\beta'_{47\% \text{ Zn}} \rightarrow \alpha_{36\% \text{ Zn}} + \gamma_{59\% \text{ Zn}}$

$$\% \alpha = \frac{59 - 47}{59 - 36} \times 100 = 52.2\% \quad \% \gamma = 47.8\%$$

Slightly more than half of the eutectoid is the copper solid solution; there is a good chance that the eutectoid would be ductile.

(d) @605°C: $\gamma_{1.6\% \text{ Be}} \rightarrow \alpha_{1.5\% \text{ Be}} + \beta_{11\% \text{ Be}}$

$$\% \alpha = \frac{11 - 6}{11 - 1.5} \times 100 = 52.6\% \quad \% \beta = 47.4\%$$

Slightly more than half of the eutectoid is the copper solid solution; we might then expect the eutectoid to be ductile.

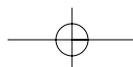
12-58 Compare the interlamellar spacing and the yield strength when an eutectoid steel is isothermally transformed to pearlite at (a) 700°C, and (b) 600°C.

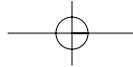
Solution: We can find the interlamellar spacing from Figure 12-19 and then use this spacing to find the strength from Figure 12-18.

$$(a) \lambda = 7.5 \times 10^{-5} \text{ cm} \quad 1/\lambda = 13,333 \quad \text{YS} = 200 \text{ MPa } 29,400 \text{ psi}$$

$$(b) \lambda = 1.5 \times 10^{-5} \text{ cm} \quad 1/\lambda = 66,667 \quad \text{YS} = 460 \text{ MPa } 67,600 \text{ psi}$$

12-66 An isothermally transformed eutectoid steel is found to have a yield strength of 410 MPa. Estimate (a) the transformation temperature and (b) the interlamellar spacing in the pearlite.




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Solution: We can first find the interlamellar spacing from Figure 12–18; then using this interlamellar spacing, we can find the transformation temperature from Figure 12–19.

(a) transformation temperature = 615°C

(b) $1/\lambda = 60,000$ or $\lambda = 1.67 \times 10^{-5}$ cm

12–67 Determine the required transformation temperature and microconstituent if an eutectoid steel is to have the following hardness values:

(a) HRC 38 (b) HRC 42 (c) HRC 48 (d) HRC 52

Solution: (a) 600°C (b) 400°C (c) 340°C (d) 300°C
 pearlite bainite bainite bainite

12–68 Describe the hardness and microstructure in an eutectoid steel that has been heated to 800°C for 1 h, quenched to 350°C and held for 750 s, and finally quenched to room temperature.

Solution: HRC = 47 and the microstructure is all bainite.

12–69 Describe the hardness and microstructure in an eutectoid steel that has been heated to 800°C, quenched to 650°C and held for 500 s, and finally quenched to room temperature.

Solution: HRC = 25 and the microstructure is all pearlite.

12–70 Describe the hardness and microstructure in an eutectoid steel that has been heated to 800°C, quenched to 300°C and held for 10 s, and finally quenched to room temperature.

Solution: HRC = 66 and the microstructure is all martensite.

12–71 Describe the hardness and microstructure in an eutectoid steel that has been heated to 800°C, quenched to 300°C and held for 10 s, quenched to room temperature, and then reheated to 400°C before finally cooling to room temperature again.

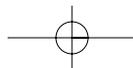
Solution: HRC = 42 and the microstructure is all tempered martensite.

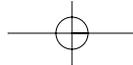
12–72 A steel containing 0.3% C is heated to various temperatures above the eutectoid temperature, held for 1 h, and then quenched to room temperature. Using Figure 12–34, determine the amount, composition, and hardness of any martensite that forms when the heating temperature is

(a) 728°C (b) 750°C (c) 790°C (d) 850°C

Solution: (a) γ : 0.77% C % M = $\frac{0.3 - 0.0218}{0.77 - 0.0218} \times 100\% = 37.2\%$ HRC 65

(b) γ : 0.60% C % M = $\frac{0.3 - 0.02}{0.6 - 0.02} \times 100\% = 48.3\%$ HRC 65





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$$(c) \gamma: 0.35\% \text{ C} \quad \% M = \frac{0.3 - 0.02}{0.35 - 0.02} \times 100\% = 84.8\% \quad \text{HRC 58}$$

$$(d) \gamma: 0.3\% \text{ C} \quad \% M = 100\% \quad \text{HRC 55}$$

12-80 A steel containing 0.95% C is heated to various temperatures above the eutectoid temperature, held for 1 h, and then quenched to room temperature. Using Figure 12-34, determine the amount and composition of any martensite that forms when the heating temperature is

(a) 728°C (b) 750°C (c) 780°C (d) 850°C

$$\text{Solution: (a) } \gamma = 0.77\% \text{ C} \quad \% M = \frac{6.67 - 0.95}{6.67 - 0.77} \times 100\% = 96.9\% \quad \text{HRC 65}$$

$$(b) \gamma = 0.82\% \text{ C} \quad \% M = \frac{6.67 - 0.95}{6.67 - 0.82} \times 100\% = 97.8\% \quad \text{HRC 65}$$

$$(c) \gamma = 0.88\% \text{ C} \quad \% M = \frac{6.67 - 0.95}{6.67 - 0.88} \times 100\% = 98.8\% \quad \text{HRC 65}$$

$$(d) \gamma = 0.95\% \text{ C} \quad \% M = 100\% \quad \text{HRC 65}$$

12-81 A steel microstructure contains 75% martensite and 25% ferrite; the composition of the martensite is 0.6% C. Using Figure 12-34, determine (a) the temperature from which the steel was quenched and (b) the carbon content of the steel.

Solution: In order for γ (and therefore martensite) to contain 0.6% C, the austenitizing $T = 750^\circ\text{C}$. Then:

$$M = \gamma = 0.25 = \frac{0.6 - x}{6.67 - 0.02} \quad x = 0.455\% \text{ C}$$

12-82 A steel microstructure contains 92% martensite and 8% Fe_3C ; the composition of the martensite is 1.10% C. Using Figure 12-34, determine (a) the temperature from which the steel was quenched and (b) the carbon content of the steel.

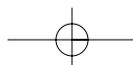
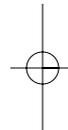
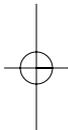
Solution: In order for γ (and therefore martensite) to contain 1.10% C, the austenitizing $T = 865^\circ\text{C}$. Then:

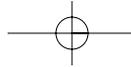
$$M = \gamma = 0.92 = \frac{6.67 - x}{6.67 - 1.10} \quad x = 1.55\% \text{ C}$$

12-83 A steel containing 0.8% C is quenched to produce all martensite. Estimate the volume change that occurs, assuming that the lattice parameter of the austenite is 3.6 Å. Does the steel expand or contract during quenching?

$$\text{Solution: } V_\gamma = 3.6 \text{ \AA}^3 = 46.656 \times 10^{-24} \text{ cm}^3$$

$$V_M = a^3c = 2.85 \times 10^{-8} \text{ cm}^2 \cdot 2.96 \times 10^{-8} \text{ cm} = 24.0426 \times 10^{-24} \text{ cm}^3$$




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But to assure that we have the same number of atoms, we need to consider two unit cells of martensite (2 atoms/cell) for each cell of FCC austenite (4 atoms/cell)

$$\% \Delta V = c \frac{2 \cdot 2 \cdot 4.04262 - 46.656}{46.656} d \times 100\% = 3.06\%, \therefore \text{expansion}$$

- 12-84** Describe the complete heat treatment required to produce a quenched and tempered eutectoid steel having a tensile strength of at least 125,000 psi. Include appropriate temperatures.

Solution: Austenitize at approximately 750°C,
Quench to below 130°C (the M_f temperature)
Temper at 620°C or less.

- 12-85** Describe the complete heat treatment required to produce a quenched and tempered eutectoid steel having a HRC hardness of less than 50. Include appropriate temperatures.

Solution: Austenitize at approximately 750°C,
Quench to below the M_f (less than 130°C)
Temper at a temperature higher than 330°C, but less than 727°C.

- 12-86** In eutectic alloys, the eutectic microconstituent is generally the continuous one, but in the eutectoid structures, the primary microconstituent is normally continuous. By describing the changes that occur with decreasing temperature in each reaction, explain why this difference is expected.

Solution: In a eutectoid reaction, the original grain boundaries serve as nucleation sites; consequently the primary microconstituent outlines the original grain boundaries and isolates the eutectoid product as a discontinuous constituent.

In a eutectic reaction, the primary phase nucleates from the liquid and grows. When the liquid composition approaches the eutectic composition, the eutectic constituent forms around the primary constituent, making the eutectic product the continuous constituent.

