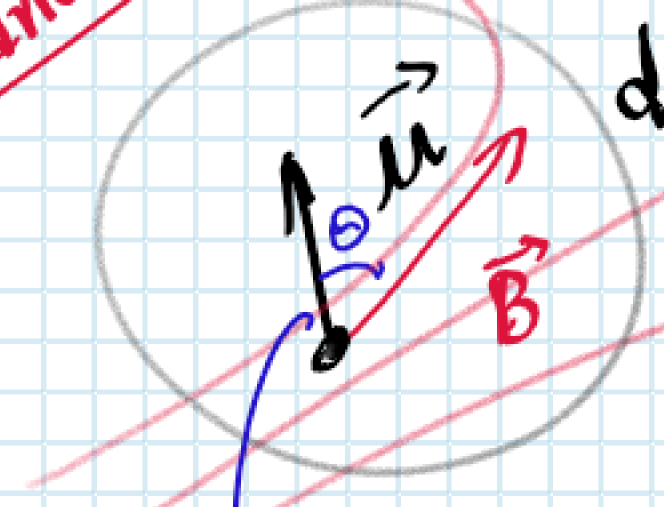
Relembrando



dipolo magnético

Física 3

$U_B \equiv$ energia potencial magnética

$U_E = -\vec{p}\cdot\vec{E}$; $p = qd$

$U_B = -\vec{\mu}\cdot\vec{B}$

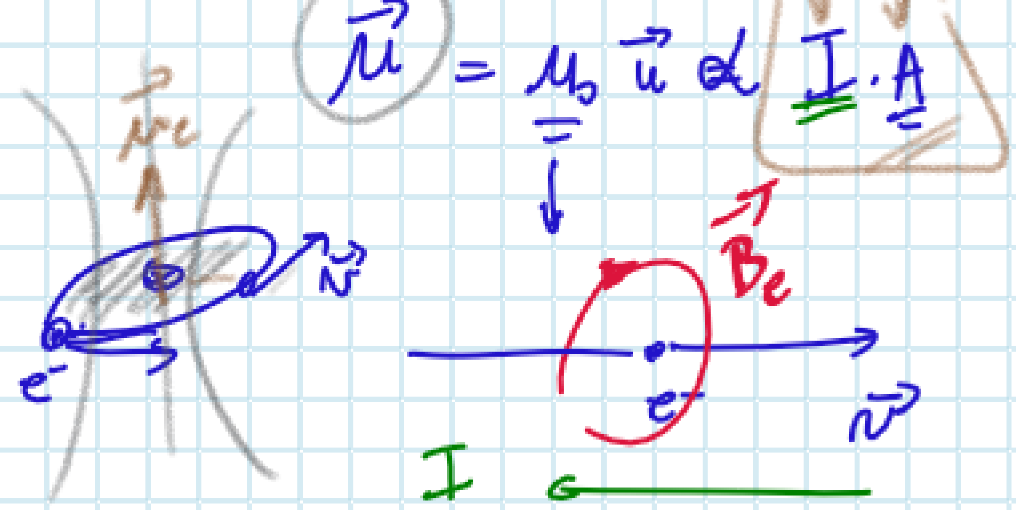
$\Downarrow$

$\vec{\nabla} = (\hat{i}\,\partial_x, \hat{j}\,\partial_y, \hat{k}\,\partial_z)$

Força $\Rightarrow$ $\vec{F} = -\vec{\nabla}\, U(\vec{r})$; F_x, F_y, F_z

$F_x = -\dfrac{d}{dx} U(x,y,z) = -\dfrac{d}{dx} U(r)$

$\vec{\mu} = \mu_0\, \vec{u} \propto I \cdot A$



• **Spin**: $\hat{S}_i = \dfrac{\hbar}{2}\hat{\sigma}_i \longrightarrow S_i = \dfrac{\hbar}{2}\sigma_i \rightarrow$

σ_i: Matriz de Pauli

$S_x = \dfrac{\hbar}{2}\sigma_x = \dfrac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$S_y = \dfrac{\hbar}{2}\sigma_y = \dfrac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$S_z = \dfrac{\hbar}{2}\sigma_z = \dfrac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$|+\rangle_z = \alpha|+\rangle_x + \beta|-\rangle_x$

$$\begin{cases} \hat{S}_i\,|+\rangle_i = \hat{S}_i\,|\uparrow\rangle_i = \dfrac{\hbar}{2}\,|+\rangle_i \\ \hat{S}_i\,|-\rangle_i = \hat{S}_i\,|\downarrow\rangle_i = -\dfrac{\hbar}{2}\,|-\rangle_i \end{cases}$$

base $i = \{x, y, z\}$

$\hat{S}_z\,|+\rangle_z = +\hbar/2\,|+\rangle$

$\hat{S}_z\,|-\rangle_z = -\dfrac{\hbar}{2}\,|-\rangle$

base $\hat{z}$

$|s, m\rangle$; $|1/2, 1/2\rangle$

Toda a algebra dos op. de spins está expressa nas relações

$[\hat{S}_i, \hat{S}_j] = i\hbar\, \epsilon_{ijk}\, S_k$

Levi-Civita: $\epsilon_{ijk} = \begin{cases} \epsilon_{123} = 1 & \text{(cíclico)} \\ \epsilon_{213} = -1 & \text{(ñ cíclico)} \\ \epsilon_{113} = 0 & \text{(repetição)} \end{cases}$

$i \to j \to k$: $\{1,2,3\} = \{x,y,z\}$ (permutações cíclicas)

Ex.: p/ $s = 1/2 = \begin{cases} m_s = -1/2 \\ m_s = 1/2 \end{cases}$ $(2s+1)$ elementos

Define-se:

$$\begin{cases} \hat{S}_+ = \dfrac{1}{2}(\hat{S}_x + \hat{S}_y) \\ \hat{S}_- = \dfrac{1}{2}(\hat{S}_x - \hat{S}_y) \end{cases}$$

Spin ; $m = \{-s, (-s+1), \dots (s-1), s\}$

$\Rightarrow$ • $S_\pm\,|s, m\rangle = \hbar\sqrt{s(s+1) - m(m\pm 1)}\;|s, m\pm 1\rangle$

$|s, m\rangle$: estado inicial na base Zeeman ; $|s, m\pm1\rangle$: estado final

• $S^2\,|s,m\rangle = \hbar^2\, s(s+1)\,|s,m\rangle$

• $S_z\,|s,m\rangle = \hbar m\,|s,m\rangle$

Ações dos operadores

$\vec{S} = (S_x, S_y, S_z)$

$$\begin{cases} s = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots \\ m = -s, (-s+1), \dots (s-1), s. \end{cases}$$

Energia: $\hat{H} = -\vec{\mu}\cdot\vec{B}$; U_B ; $\vec{\mu} = \gamma\vec{S} \Rightarrow \hat{H} = -\gamma\,\vec{S}\cdot\vec{B}$

Magneton de Bohr $(\mu_B = e\hbar/2mc)$

fator giromagnético $\gamma = -2\mu_B/\hbar$

$\hat{H} \Rightarrow$ Superposição $\qquad |0\rangle \rightarrow \hat{H}|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$\Downarrow$ $\qquad P_0 = \frac{1}{2} \; ; \; P_1 = 1/2$

$\hat{X} \Rightarrow$ "NOT"

$\hat{X}|0\rangle = |1\rangle$

$\hat{X}|1\rangle = |0\rangle \Rightarrow X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

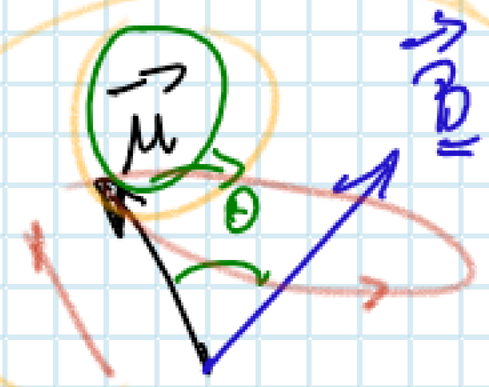
$\begin{pmatrix} 0 \\ 1 \end{pmatrix} ; \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Introdução às plataformas físicas ("hardware")

- **Spin**: sistema spin $1/2$ $\Rightarrow$ qbit : $\begin{pmatrix} 0 \\ 1 \end{pmatrix} ; \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

  $\hookrightarrow$ momento de dipolo magnético

* Clássico:



$U_B = -\vec{\mu} \cdot \vec{B} = -|\vec{\mu}||\vec{B}|.\cos\theta = -\mu B \cos\theta$

$\hookrightarrow \vec{F}_B = -\vec{\nabla} U_B$

* Quântico $\qquad \hat{H} = -\vec{\mu} \cdot \vec{B} \rightsquigarrow \vec{\mu} \propto \vec{S} \Rightarrow \boxed{\vec{\mu} = \gamma \vec{S}}$

$\Downarrow$

Spin $\Rightarrow$ Momento angular

$\vec{L}$: orbital

S : spin

$\boxed{\vec{J} = \vec{L} + \vec{S}}$

$\gamma = -2\mu_B/\hbar$

$\mu_B = \frac{e}{m.c}$

p/ o elétron

Exemplo 1: $\vec{B} = B_z \hat{z}$ $\qquad \vec{S} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$

op.

$\hat{H} = -\gamma B_z \hat{S}_z \Rightarrow \hat{H}|0\rangle = -\gamma B_z \hat{S}_z |0\rangle = \begin{cases} -\gamma B_z \frac{\hbar}{2} \Rightarrow E_1 \\ +\gamma B \hbar/2 \Rightarrow E_2 \end{cases}$

Número

$|+\rangle = |0\rangle$
$|-\rangle = |1\rangle$ $\Rightarrow \lambda = \pm \frac{\hbar}{2}$

Resumindo

P/ Campo estático ($\vec{n}$ varia no tempo)

$$\boxed{\vec{B} = B_z\hat{z}} \Rightarrow \vec{S} \Rightarrow \hat{S}_z$$

$(\hat{S}_x, \hat{S}_y, \hat{S}_z)$

$$\Rightarrow 0$$

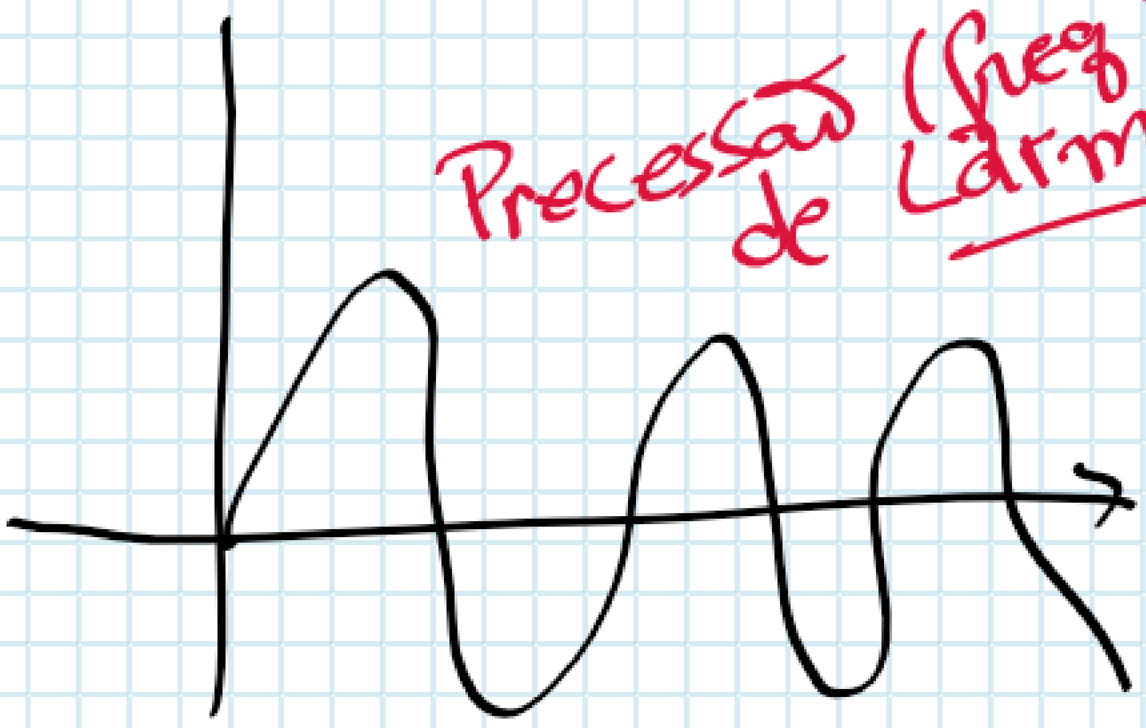
$$E_1 = \frac{\hbar}{2}\gamma B_z = \epsilon_0 \rightarrow \omega_0 = \omega_{Larmor} = \epsilon_0/\hbar$$

$$E_0 = -\frac{\hbar}{2}\gamma B_z = -\epsilon_0$$

$$\Rightarrow \hat{S}_z \Rightarrow |\vec{S}|$$

$$|\psi(t)\rangle = e^{-E_i/\hbar\, t} \rightsquigarrow \boxed{e^{+\frac{\epsilon_0}{\hbar}t}|0\rangle + e^{-\frac{\epsilon_0}{\hbar}t}|1\rangle}$$

$$e^{\alpha t}\left(|0\rangle + e^{\beta t}|1\rangle\right)$$



Precessão (freq.) de Larmor

Exemplo 2

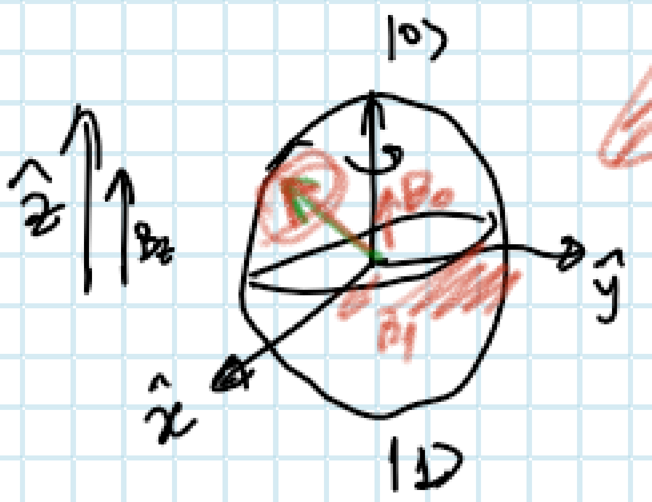
$$\boxed{\vec{B} = B_x\hat{x} + B_z\hat{z}}$$

$$\hat{H} = -\gamma\left(B_x\hat{S}_x + B_z\hat{S}_z\right)$$

$$\hat{S}_x = \frac{\hbar}{2}\hat{\sigma}_x$$

$(\hat{\sigma}_x)$ $(\hat{\sigma}_z)$

$$\boxed{\hat{S}_i = \frac{\hbar}{2}\hat{\sigma}_i}$$



Exemplo 3

ω_0: Larmor

"rádio" frequência

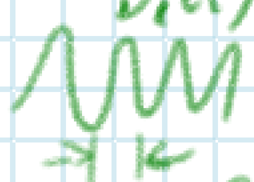
$$\vec{B}(t) = B_0\hat{z} + \boxed{B_1\cos(\omega t)\,\hat{x}}$$

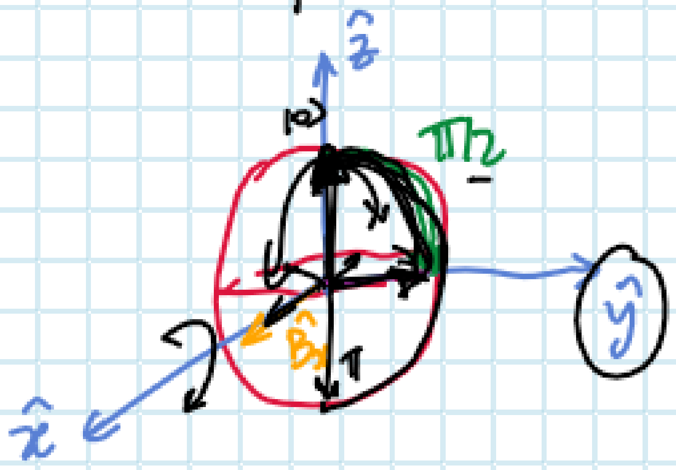
$B_1(t)$

ω: freq. externa

$$\cos(\omega t + \phi)$$

fase





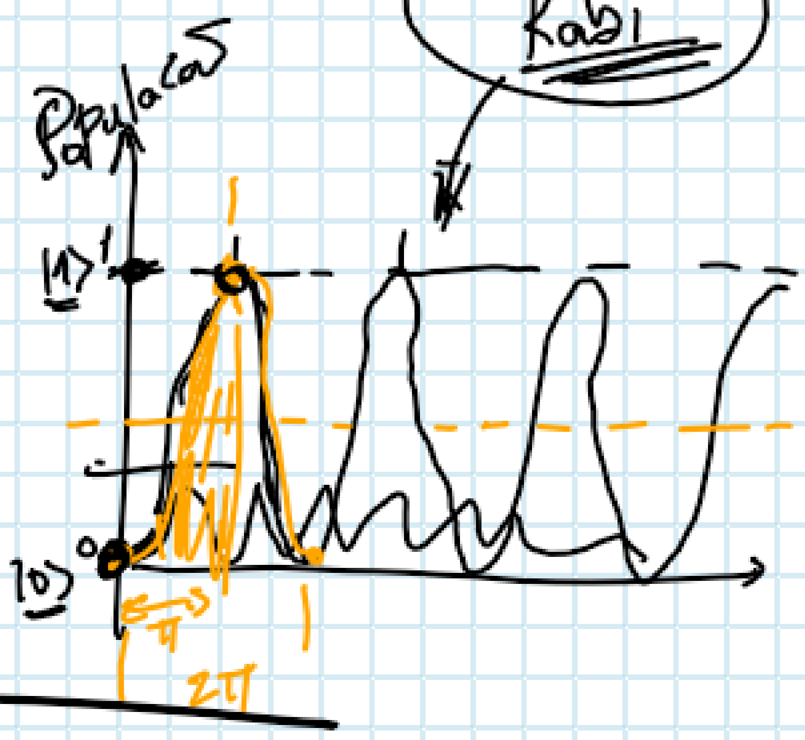
$$\hat{x} \times \hat{y} = \hat{z}$$

$$\tau_\pi$$

$$[\tau_{\pi/2}]$$

$$\rightsquigarrow$$

Rabi



$$\omega_R = |\vec{B}_1 \cdot \vec{\mu}|$$