

Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

5 de julho de 2021
Magnetostática

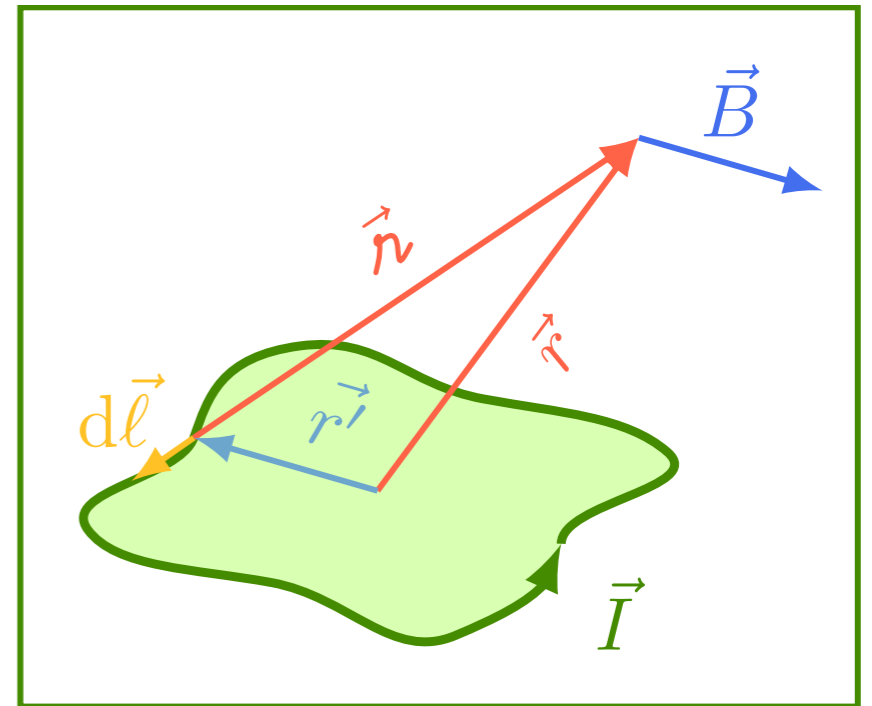
Expansão multipolar

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad \vec{\nabla} \cdot \vec{A} = 0$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau'$$

- QUEREMOS ACHAR $\vec{A}(\vec{r})$,
PARA DEPOIS CALCULAR \vec{B}

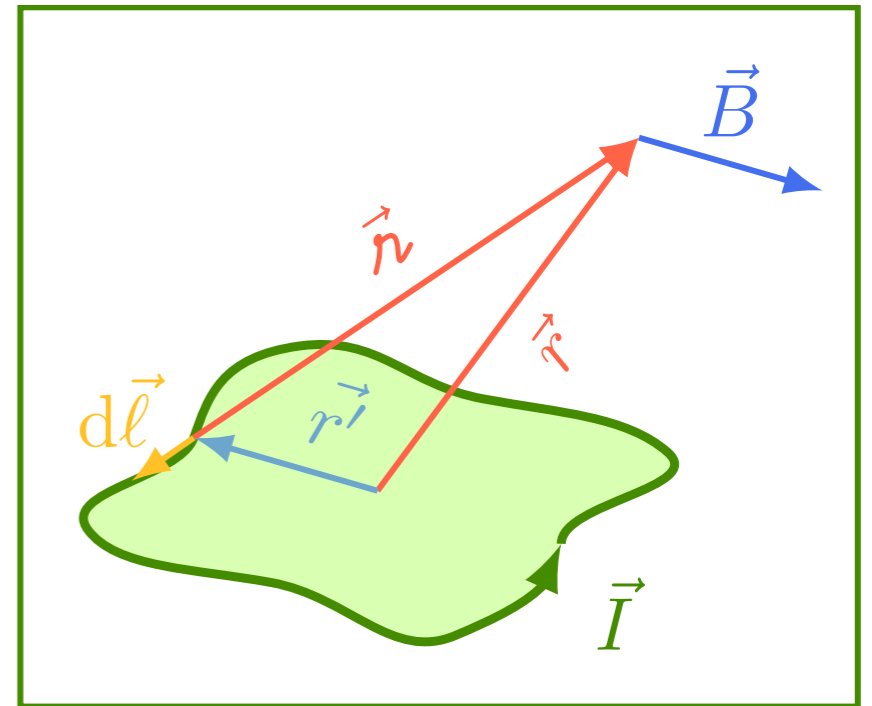


Expansão multipolar

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$$\frac{1}{r} = \frac{1}{r} \sum_{\ell=0}^{\infty} \left(\frac{r'}{r}\right)^{\ell} P_{\ell}(\cos \theta')$$

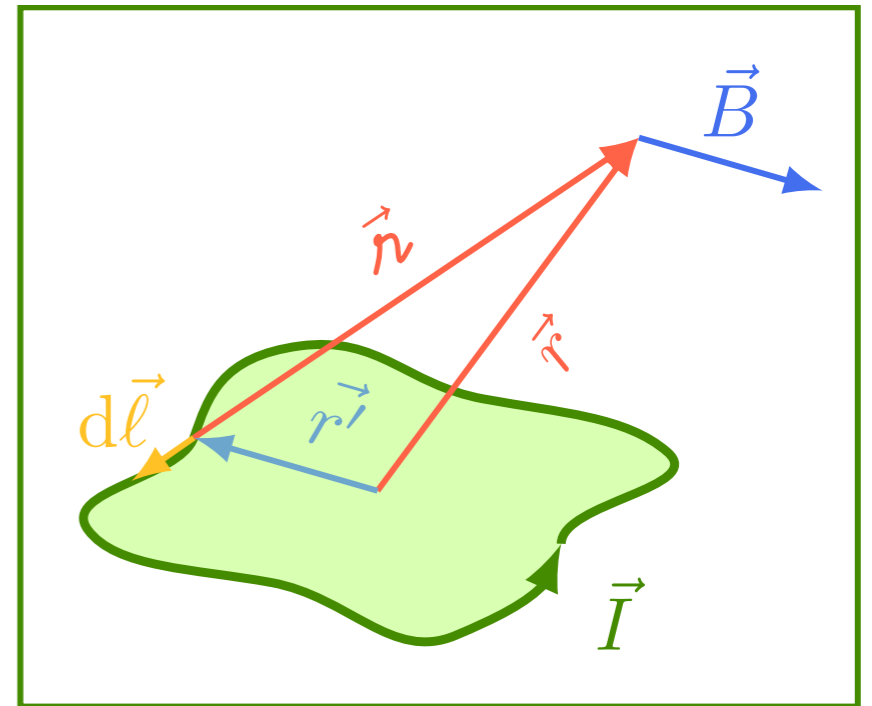
← DERIVADA NO CAPÍTULO DE
MÉTODOS MATEMÁTICOS

Expansão multipolar

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad \vec{\nabla} \cdot \vec{A} = 0$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau' = \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r}')}{r} dl'$$



$$\frac{1}{r} = \frac{1}{r} \sum_{\ell=0}^{\infty} \left(\frac{r'}{r}\right)^{\ell} P_{\ell}(\cos \theta')$$

→ APROXIMAÇÃO PARA $r \gg r'$?

$$\ell = 0 \Rightarrow \frac{1}{r} \approx \frac{1}{r}$$

PRIMEIRA APROXIMAÇÃO

$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi r} \oint dl'$$

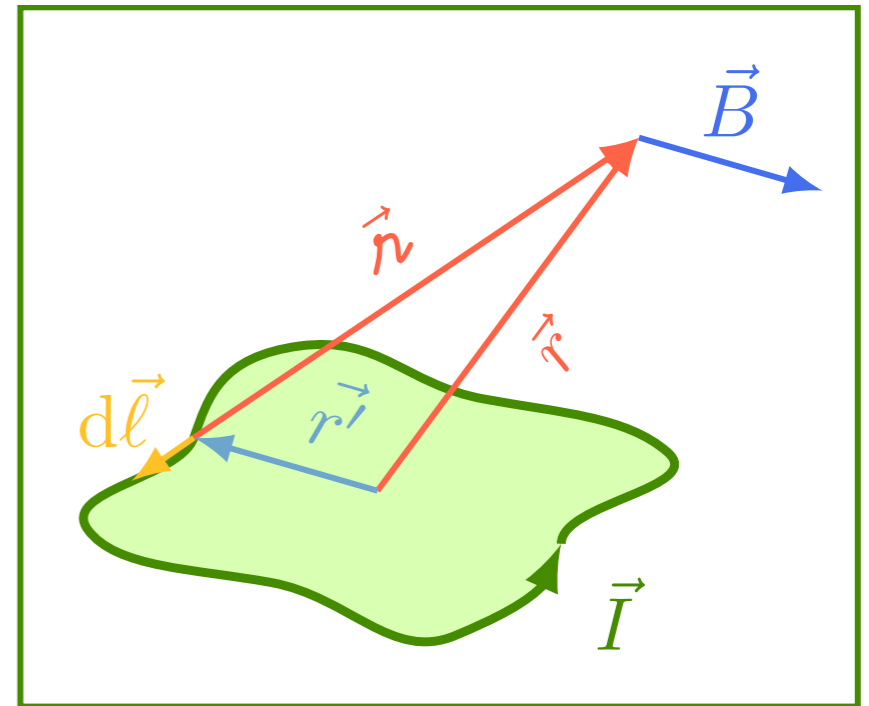
INTEGRAL DÁ O (CIRCUITO FECHADO)

Expansão multipolar

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad \vec{\nabla} \cdot \vec{A} = 0$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau' = \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r}')}{r} dl'$$



$$\frac{1}{r} = \frac{1}{r} \sum_{\ell=0}^{\infty} \left(\frac{r'}{r}\right)^{\ell} P_{\ell}(\cos \theta')$$

$$\ell \leq 1 \Rightarrow \frac{1}{r} \approx \frac{1}{r} + \frac{r'}{r^2} \cos \theta \Rightarrow \vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi r} \oint dl' + \frac{\mu_0}{4\pi r^2} \oint r' \cos \theta' dl'$$

PRÓXIMA APROXIMAÇÃO

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

NOTAS DE AULA

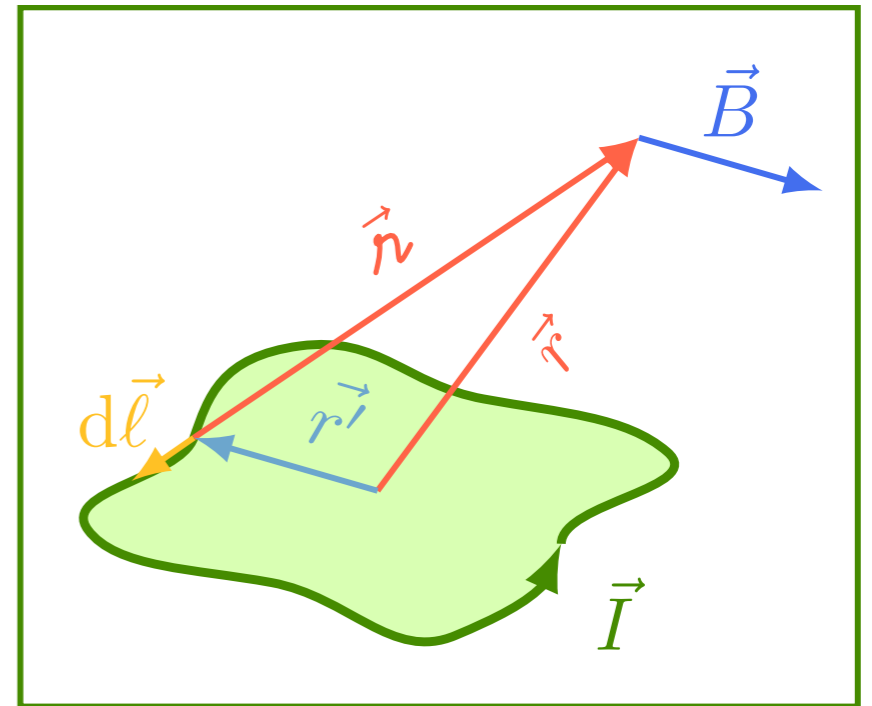
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$$\vec{m} = I \vec{a}$$



MOMENTO MAGNÉTICO
DO CIRCUITO

\vec{I} COMO SE FOSSE UM
PEQUENO ÍMÃ

Pratique o que aprendeu

$$\vec{m} = I\vec{a}$$

$$\vec{m} = ?$$

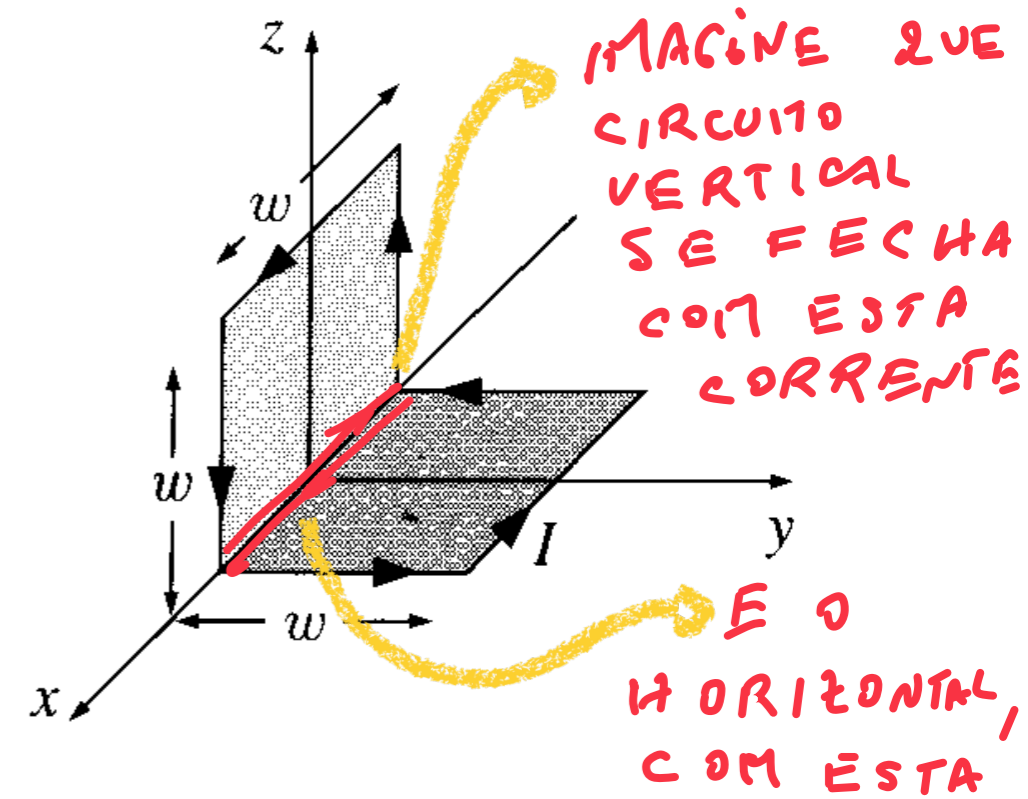


Figure 5.52

DOIS
CIRCUITOS
FECHADOS

Pratique o que aprendeu

$$\vec{m} = I\vec{a}$$

$$\vec{m} = Iw^2(\hat{y} + \hat{z})$$

ÁREA
DE
CADA CIRCUITO

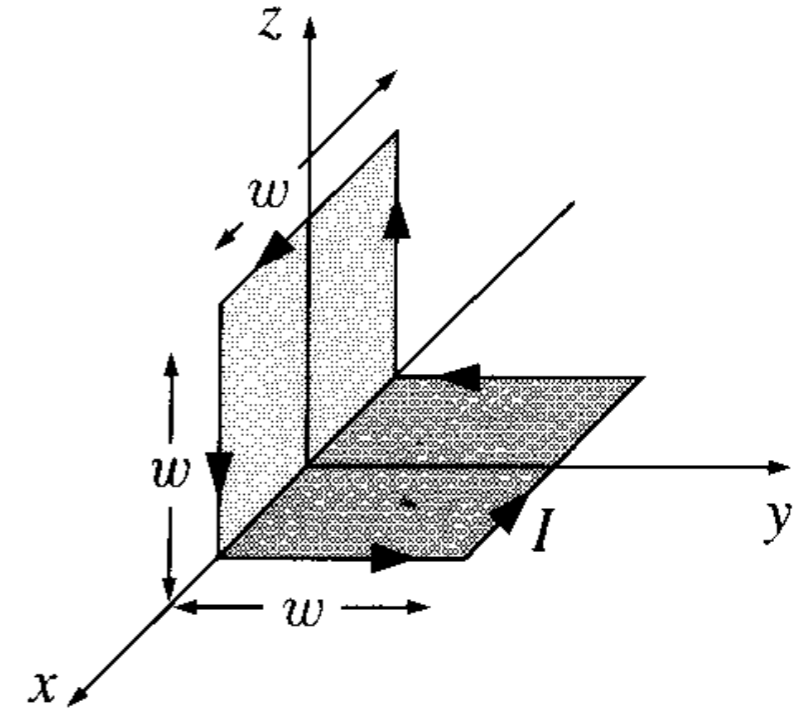


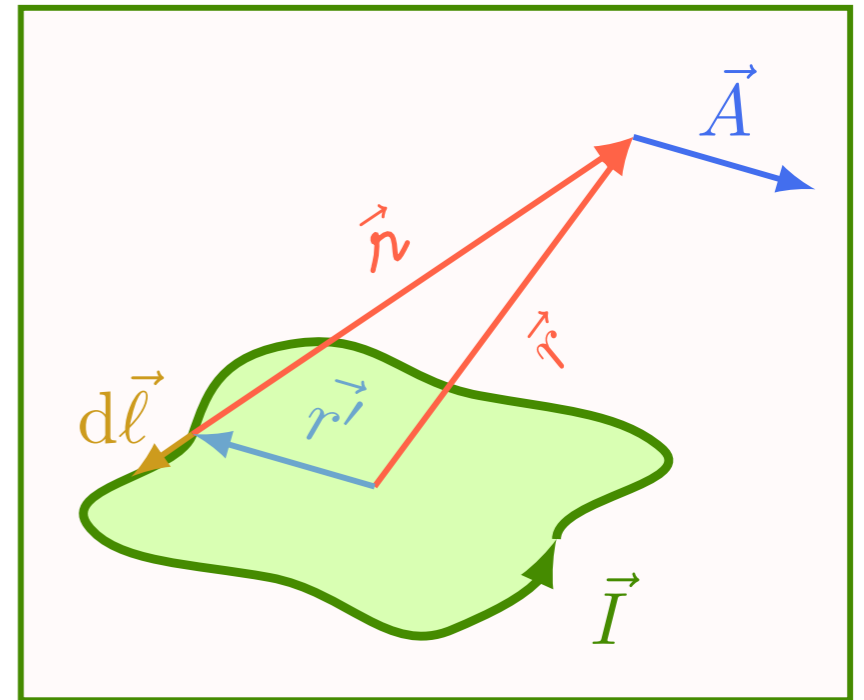
Figure 5.52

Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$\vec{m} = m\hat{z}$$

CALCULAR
EM
COORDENADAS
CILÍNDRICAS

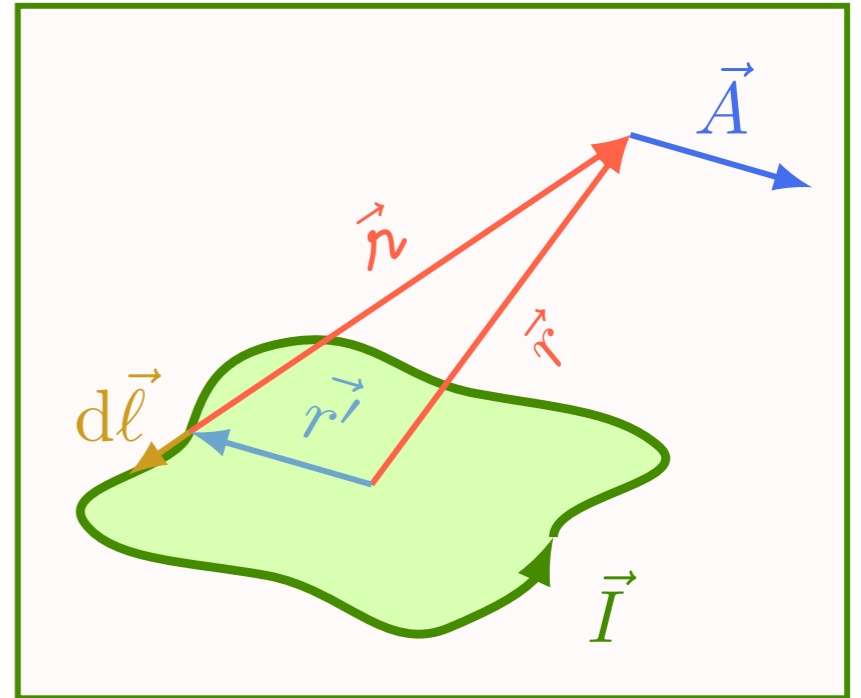


Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$\vec{m} = m\hat{z}$$

$$\vec{r} = s\hat{s} + z\hat{z}$$



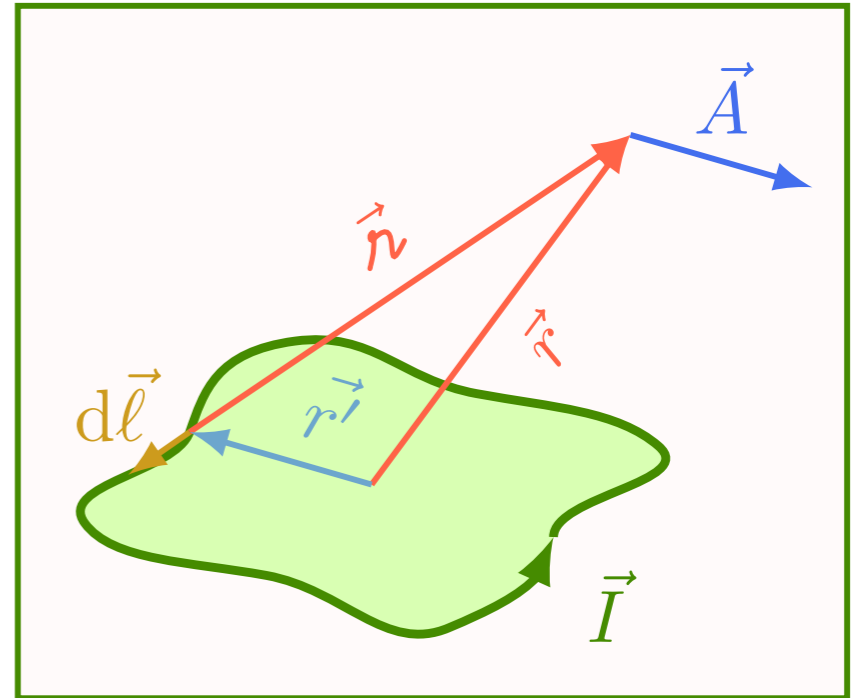
Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

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$$\vec{m} \times \vec{r} = sm\hat{s} \times \hat{s}$$



Expansão multipolar

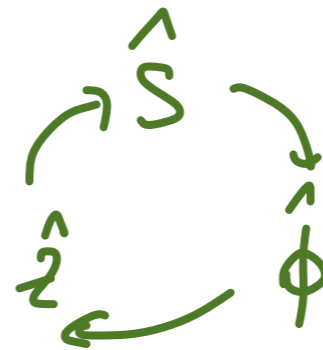
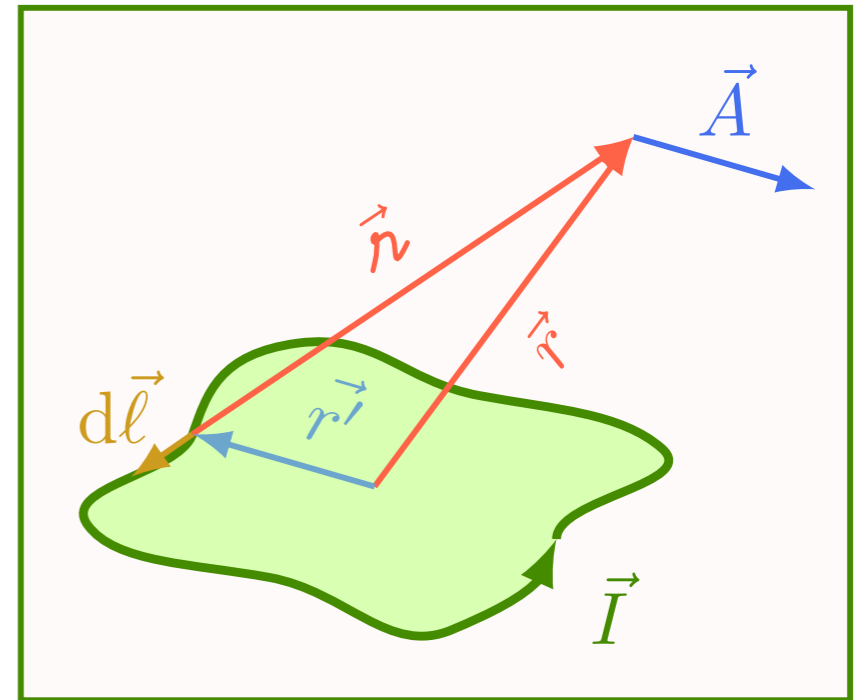
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$\vec{m} = m\hat{z}$$

$$\vec{r} = s\hat{s} + z\hat{z}$$

$$\vec{m} \times \vec{r} = sm\hat{z} \times \hat{s}$$

$$\vec{m} \times \vec{r} = sm\hat{z} \times \hat{s}$$



Expansão multipolar

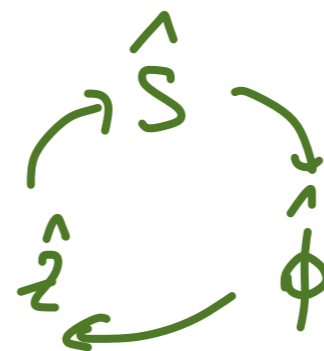
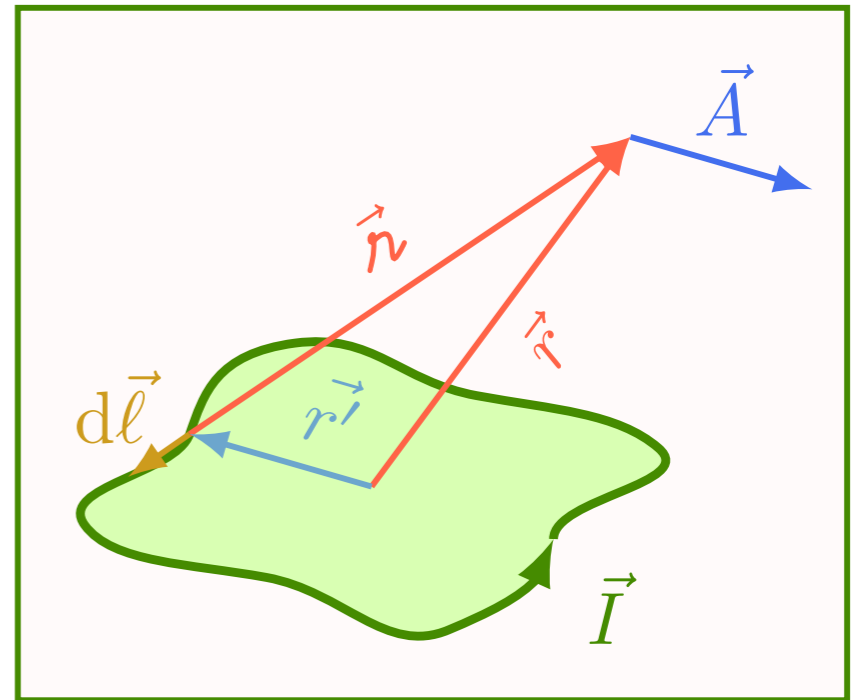
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$\vec{m} = m\hat{z}$$

$$\vec{r} = s\hat{s} + z\hat{z}$$

$$\vec{m} \times \vec{r} = sm\hat{z} \times \hat{s}$$

$$\vec{m} \times \vec{r} = sm\hat{z} \times \hat{s} = sm\hat{\phi}$$



Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

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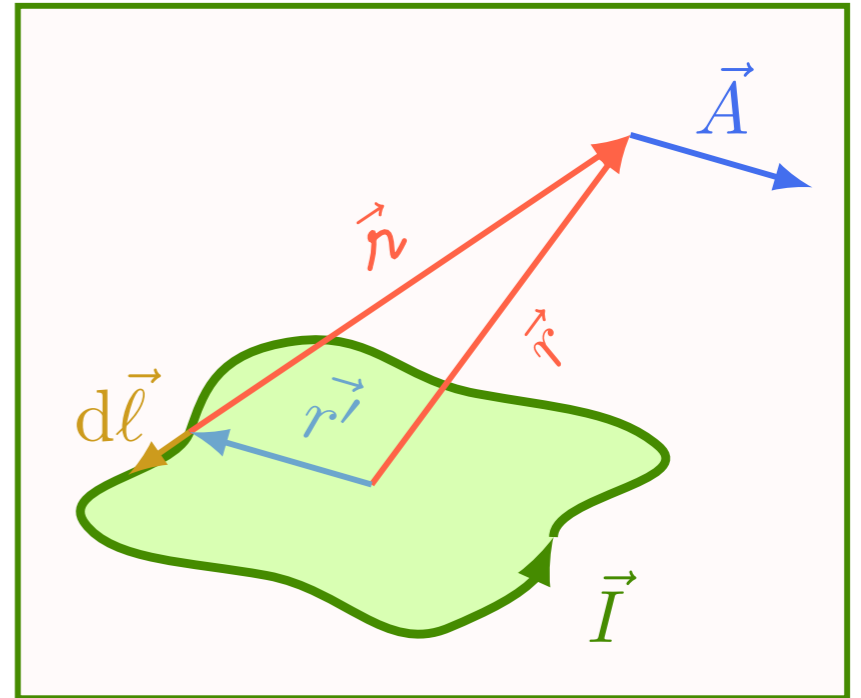
$$\vec{r} = s\hat{s} + z\hat{z}$$

$$\vec{m} \times \vec{r} = sm\hat{z} \times \hat{s}$$

$$\vec{m} \times \vec{r} = sm\hat{z} \times \hat{s} = sm\hat{\phi}$$

$$\vec{m} \times \hat{r} = \frac{sm}{r} \hat{\phi}$$

$\hat{r} = \frac{\vec{r}}{r}$



Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

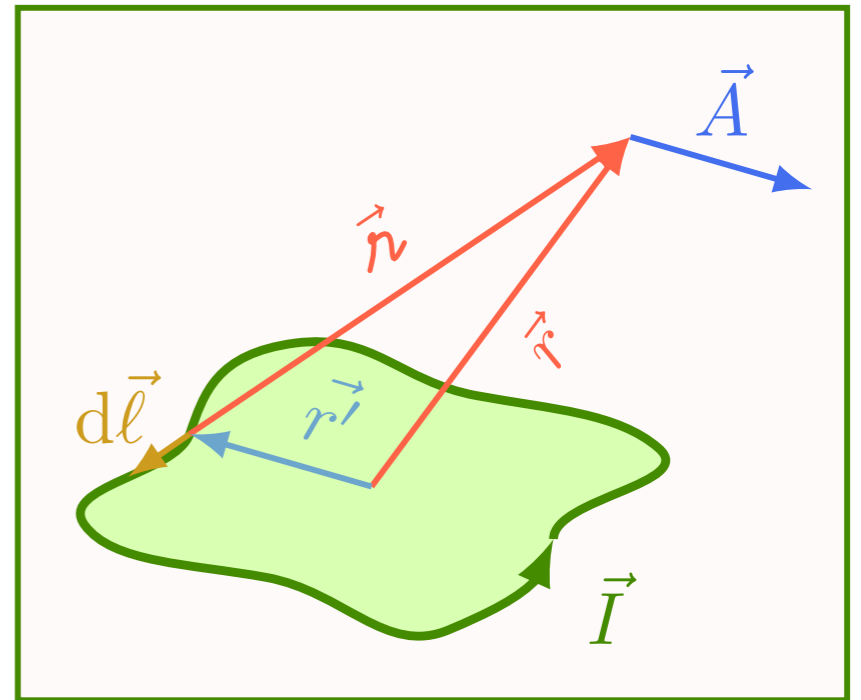
$$\vec{m} = m\hat{z}$$

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$$\vec{m} \times \vec{r} = sm\hat{z} \times \hat{s}$$

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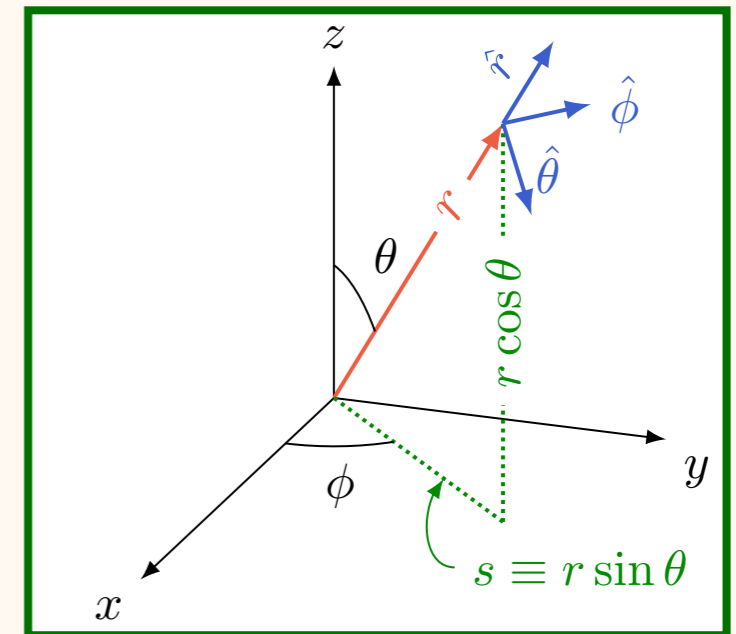
POTENCIAL VETOR
DO DIPLOLO C/ $\vec{m} = m\hat{z}$

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{m \sin \theta}{r^2} \hat{\phi}$$

Coordenadas esféricas

$$d\vec{\ell} = dr \hat{r} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}$$

$$\vec{\nabla} t = \frac{\partial t}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial t}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial t}{\partial \phi} \hat{\phi}$$



$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2} \frac{\partial (r^2 v_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (\sin \theta v_\theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$$

$$\begin{aligned} \vec{\nabla} \times \vec{v} = & \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta v_\phi) - \frac{\partial v_\theta}{\partial \phi} \right] \hat{r} \\ & + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi} \end{aligned}$$

$$\nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}$$

Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

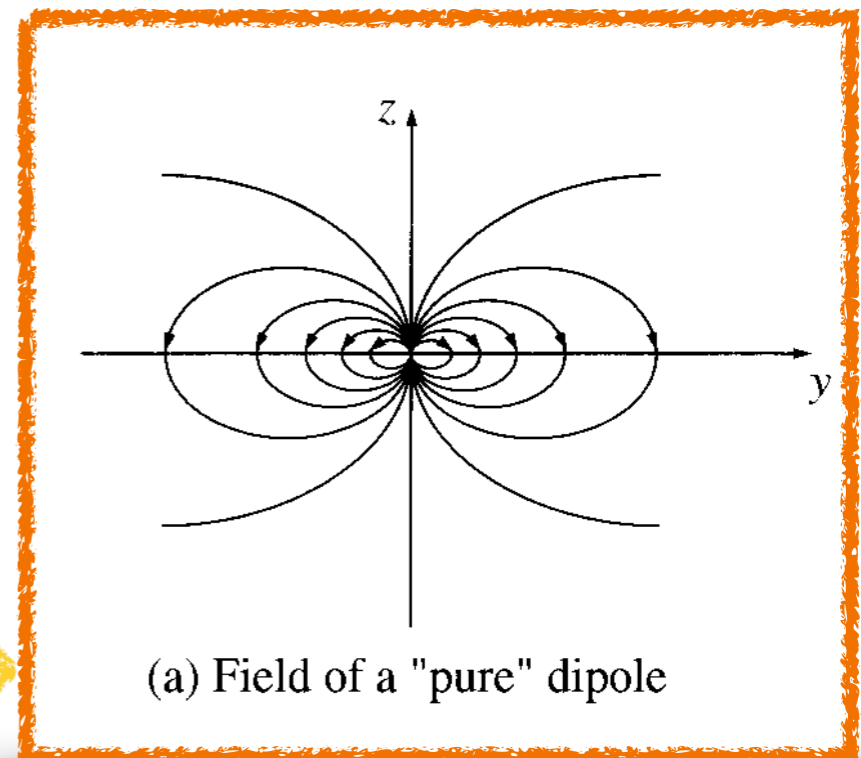
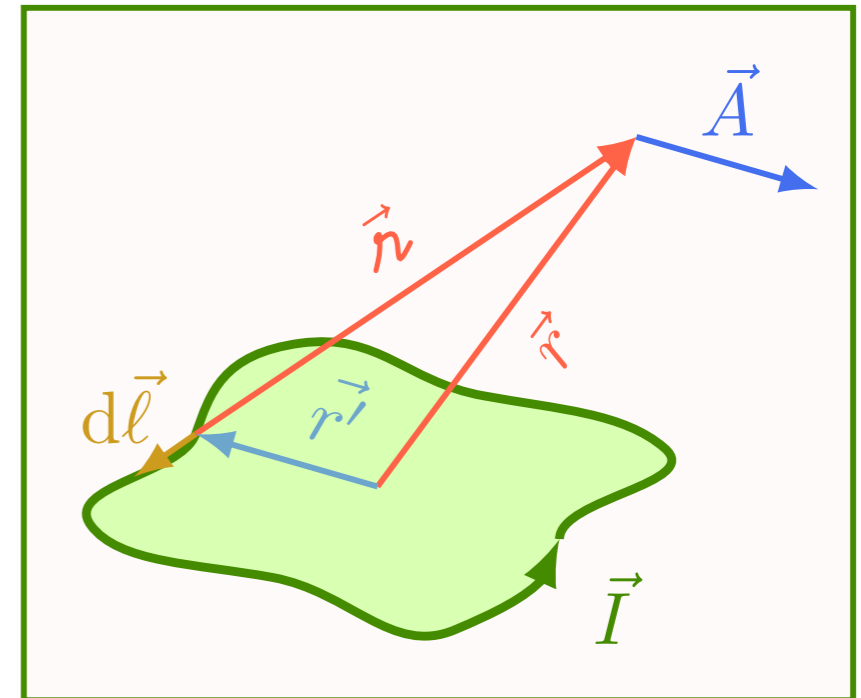
$$\vec{m} = m\hat{z}$$

$$\vec{r} = s\hat{s} + z\hat{z}$$

$$\vec{m} \times \hat{r} = s\vec{m} \times \hat{s} = sm\hat{\phi}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{m \sin \theta}{r^2} \hat{\phi} \quad \rightarrow \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3} \left[2\cos\theta \hat{r} + \sin\theta \hat{\theta} \right]$$



Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

5 de julho de 2021

Magnetismo em materiais

Magnetismo em materiais

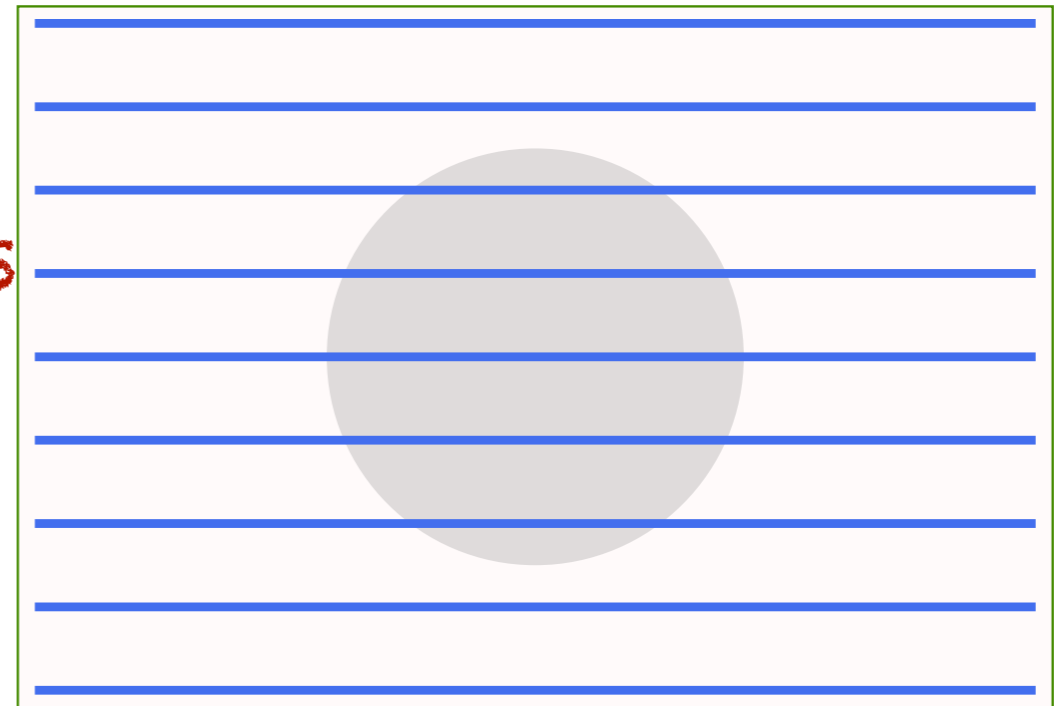
• Paramagnéticos

• Diamagnéticos

• Ferromagnéticos

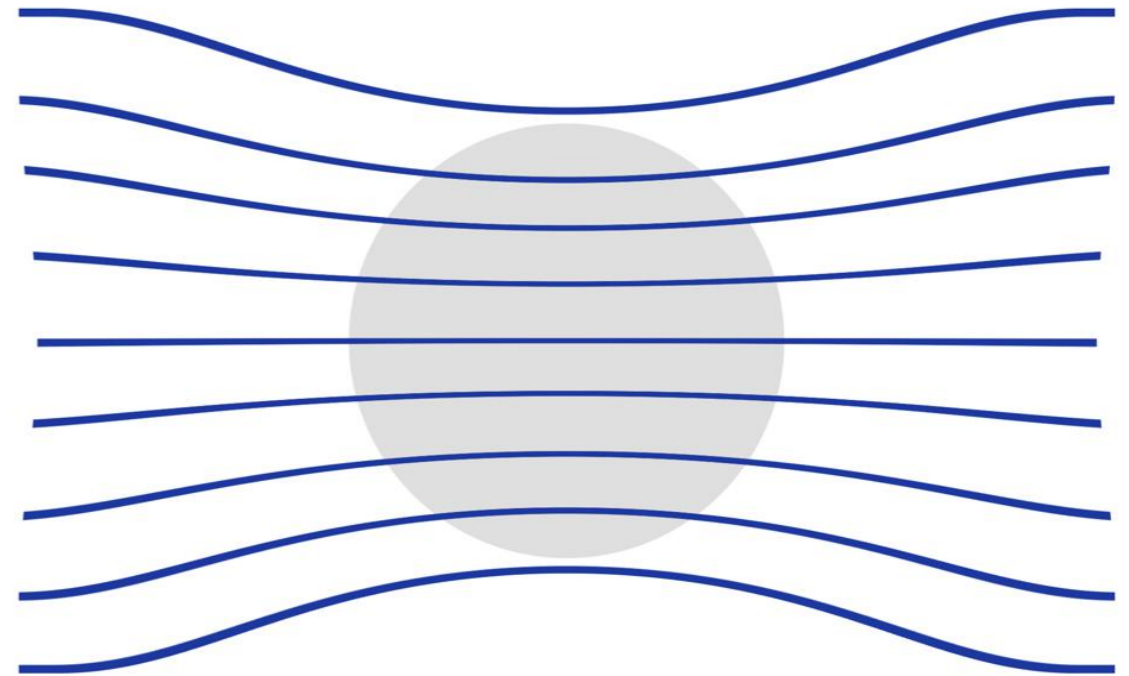
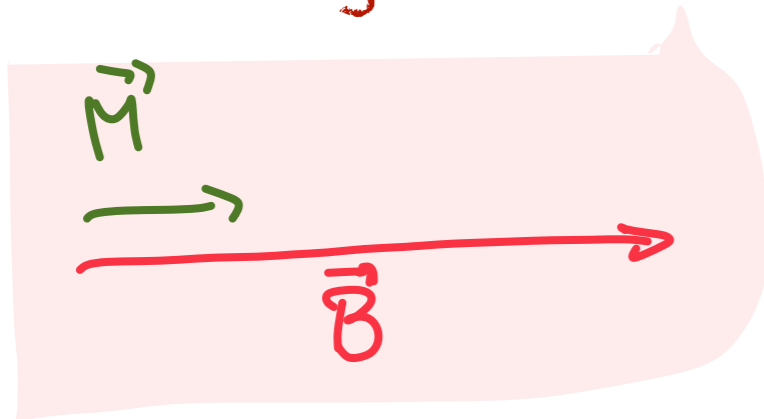
• Antiferromagnéticos

4 CLASSES



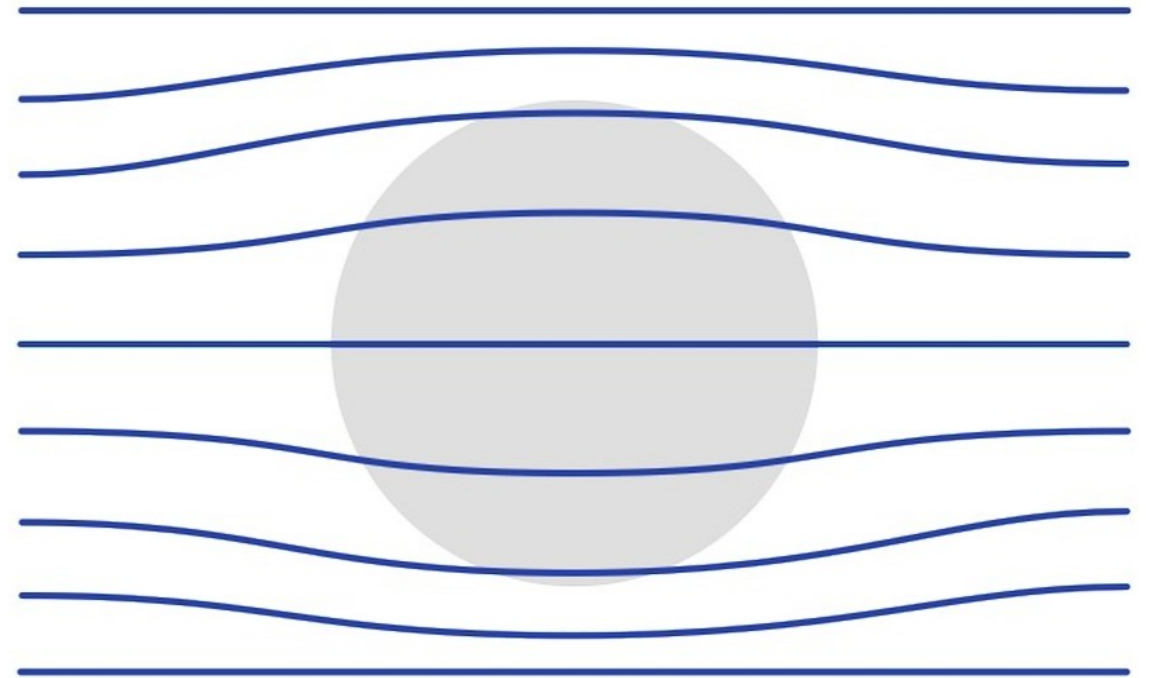
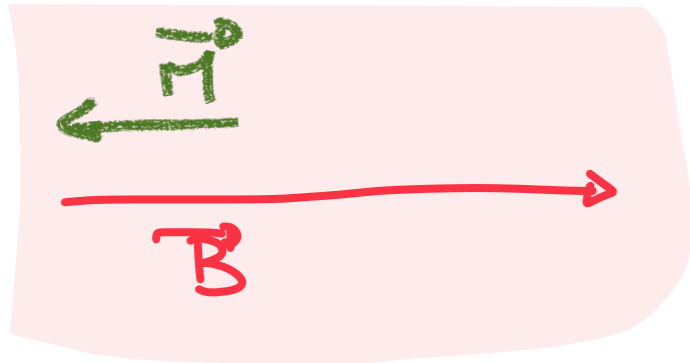
Magnetismo em materiais

• Paramagnéticos



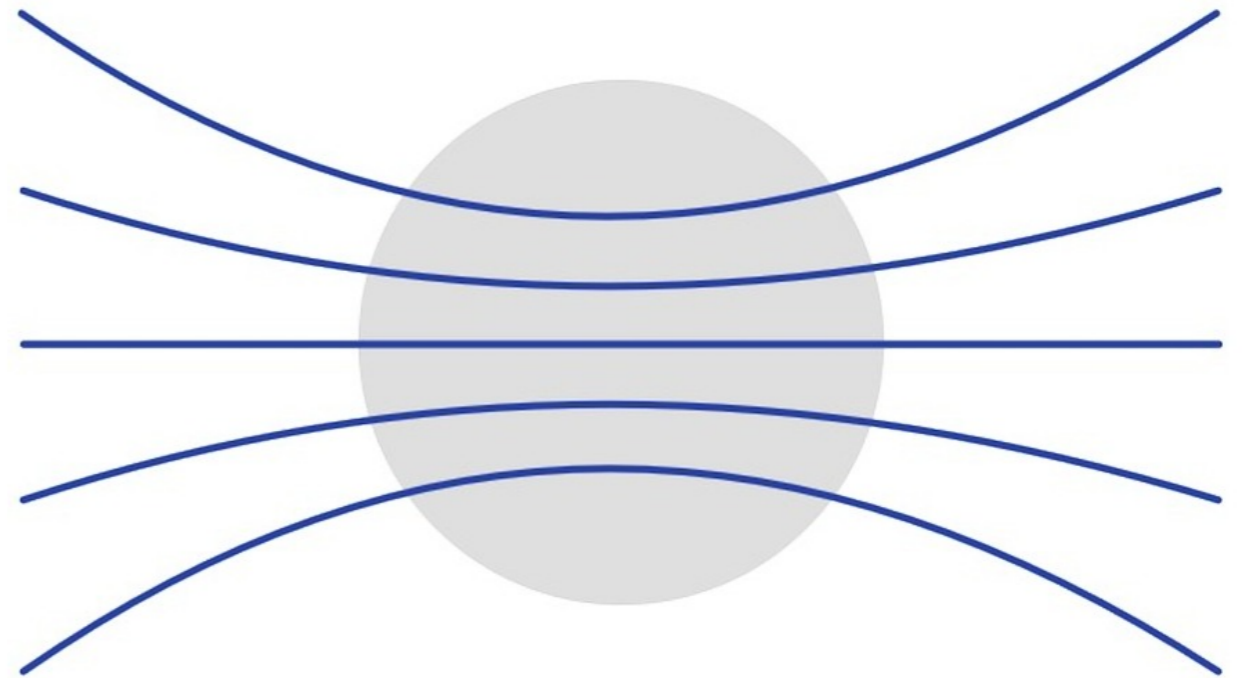
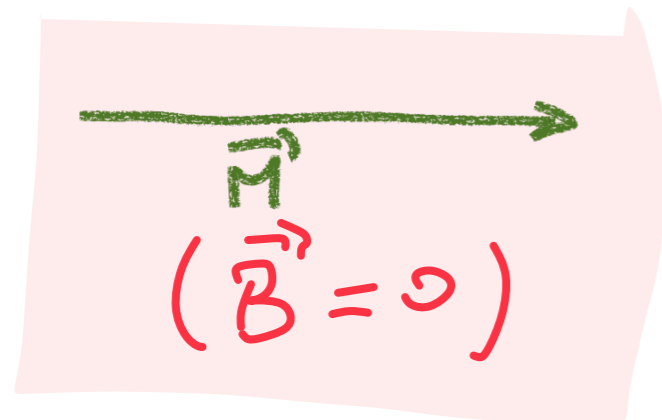
Magnetismo em materiais

• Diamagnéticos



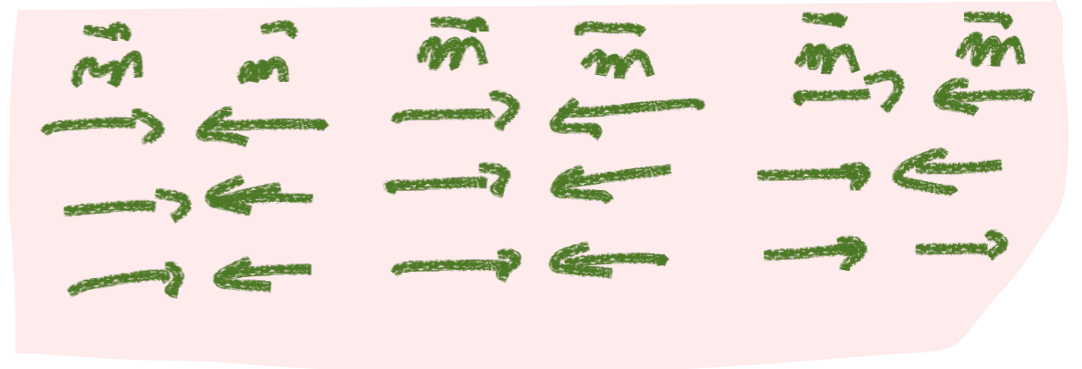
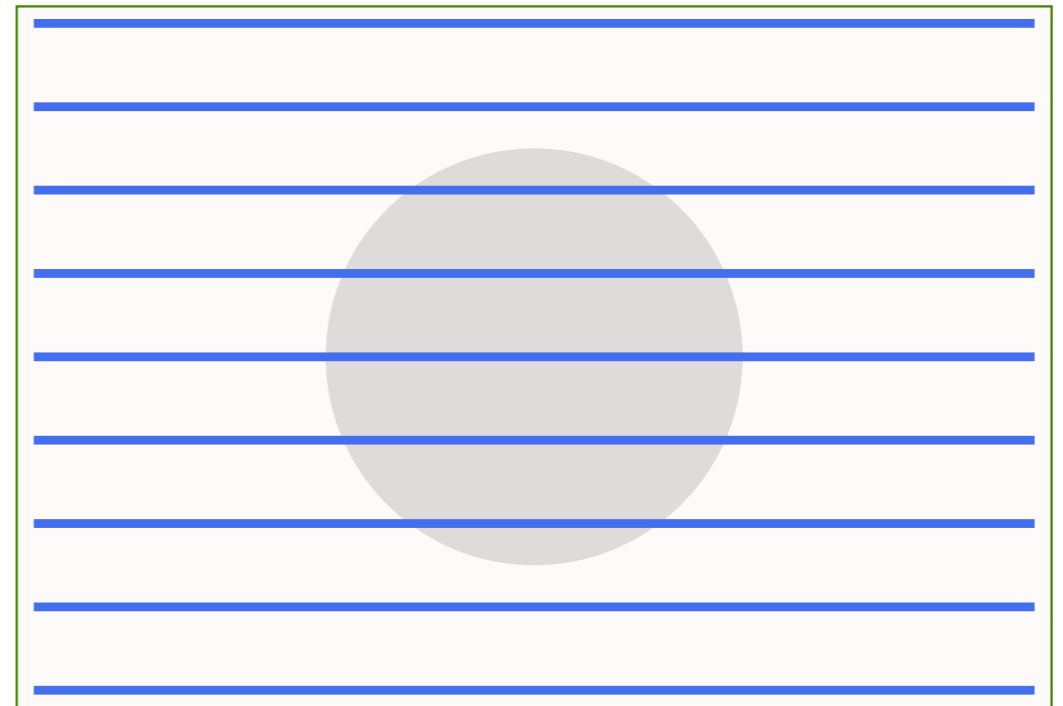
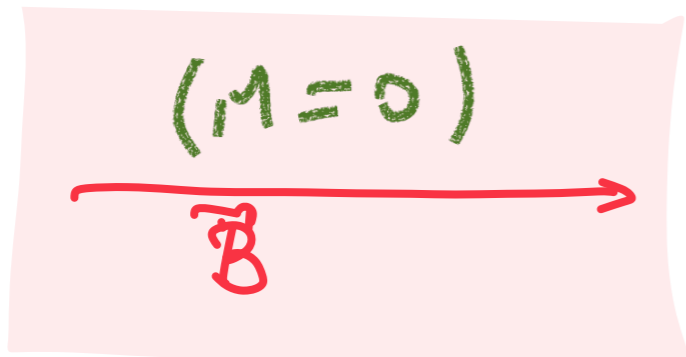
Magnetismo em materiais

• Ferromagnéticos



Magnetismo em materiais

• Antiferromagnéticos



Magnetismo em materiais

The periodic table is color-coded to show magnetic properties. Handwritten labels with arrows point to specific groups:

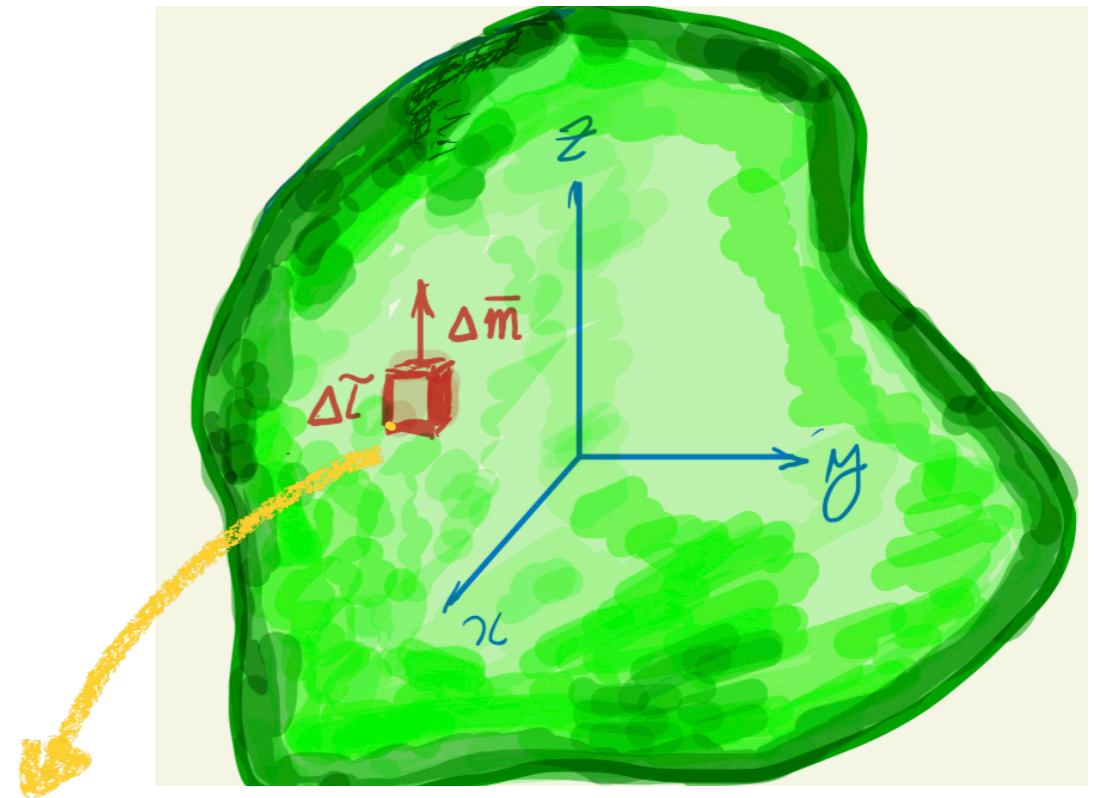
- ANTIFERROMAGNÉTICO** (blue): Cr
- FERROMAGNÉTICOS** (yellow): Fe, Co, Ni, Gd
- DIAMAGNÉTICOS** (white): B, C, N, O, F, Al, Si, P, S, Cl, Ar, Kr, Xe, Rn, Og
- PARAMAGNÉTICO** (red): All other elements, including the lanthanide and actinide series.

H																He	
Li	Be										B	C	N	O	F	Ne	
Na	Mg										Al	Si	P	S	Cl	Ar	
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba		Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra		Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Nh	Fl	Mc	Lv	Ts	Og
		La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu	
		Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr	

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$\vec{M} = \frac{\Delta \vec{m}}{\Delta \tau}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}') \times \hat{r}}{r^2} d\tau'$$



CAMPO

MAGNE'TICO

INDUZ

MOMENTO

MAGNE'TICO

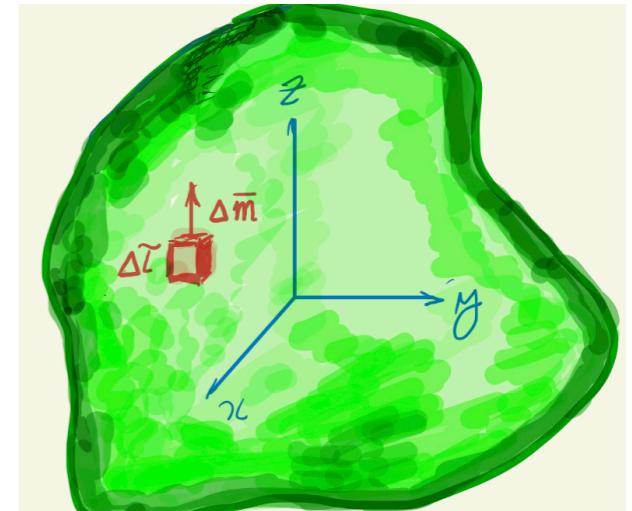
Magnetização

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

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$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}') \times \hat{r}}{r^2} d\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \vec{M}(\vec{r}') \times \vec{\nabla}' \left(\frac{1}{r} \right) d\tau'$$

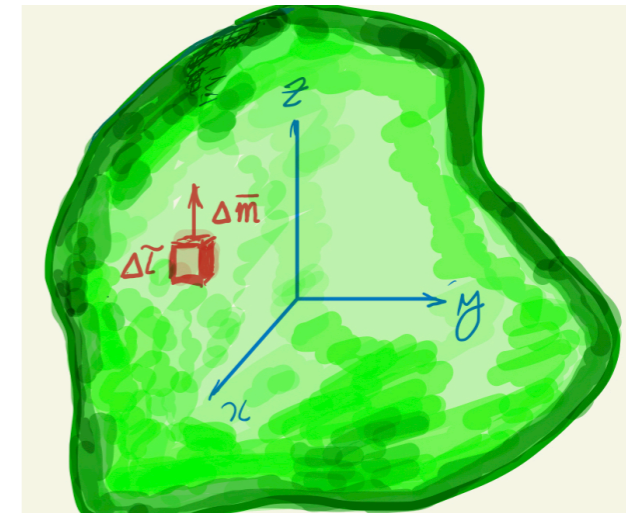


Magnetização

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$$\vec{M} = \frac{\Delta \vec{m}}{\Delta \tau}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}') \times \hat{r}}{r^2} d\tau'$$



$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \vec{M}(\vec{r}') \times \vec{\nabla}' \left(\frac{1}{r} \right) d\tau'$$

$$\vec{\nabla}' \times \left(\frac{\vec{M}(\vec{r}')}{r} \right) = \vec{\nabla}' \left(\frac{1}{r} \right) \times \vec{M}(\vec{r}') + \frac{1}{r} \vec{\nabla}' \times \vec{M}(\vec{r}')$$

$$-\vec{\nabla}' \left(\frac{1}{r} \right) \times \vec{M}(\vec{r}') = -\vec{\nabla}' \times \left(\frac{\vec{M}(\vec{r}')}{r} \right) + \frac{1}{r} \vec{\nabla}' \times \vec{M}(\vec{r}')$$

Magnetização

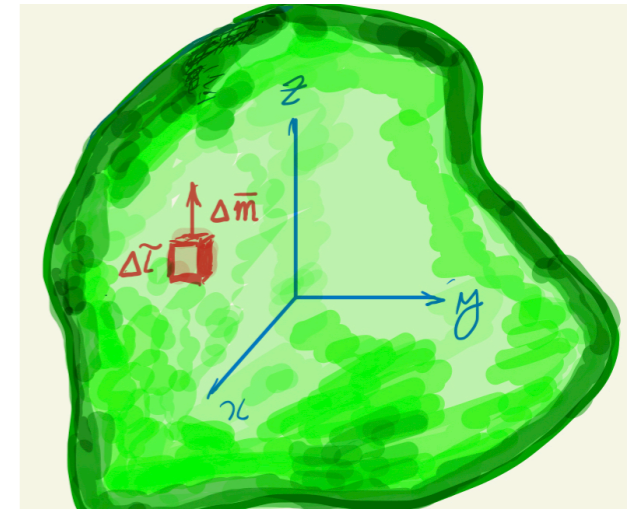
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$$\vec{M} = \frac{\Delta \vec{m}}{\Delta \tau}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}') \times \hat{r}}{r^2} d\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \vec{M}(\vec{r}') \times \vec{\nabla}' \left(\frac{1}{r} \right) d\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{1}{r} \vec{\nabla} \times \vec{M}(\vec{r}') d\tau' - \frac{\mu_0}{4\pi} \int \vec{\nabla} \times \left(\frac{\vec{M}(\vec{r}')}{r} \right) d\tau'$$



Magnetização

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

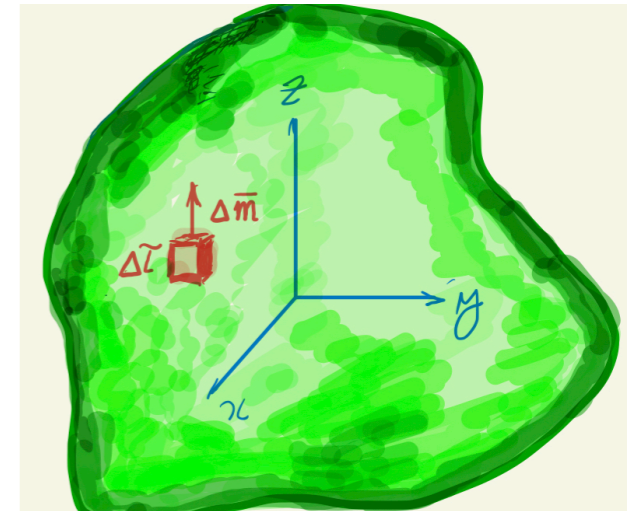
$$\vec{M} = \frac{\Delta \vec{m}}{\Delta \tau}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r}') \times \hat{r}}{r^2} d\tau'$$

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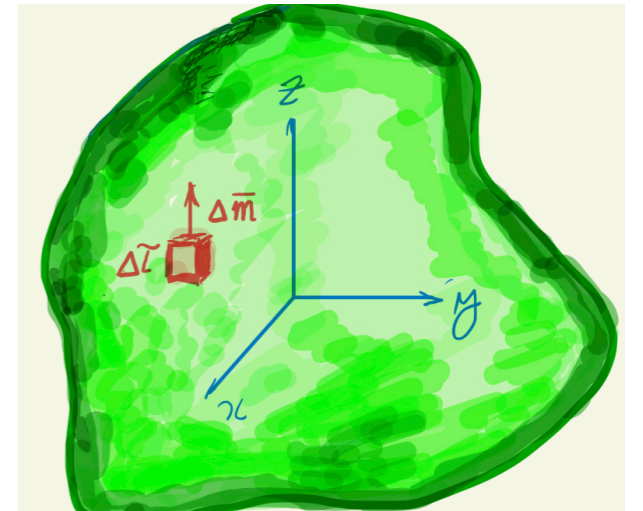
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{1}{r} \vec{\nabla} \times \vec{M}(\vec{r}') d\tau' - \frac{\mu_0}{4\pi} \int \vec{\nabla} \times \left(\frac{\vec{M}(\vec{r}')}{r} \right) d\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{1}{r} \vec{\nabla} \times \vec{M}(\vec{r}') d\tau' - \frac{\mu_0}{4\pi} \int \frac{1}{r} \vec{M}(\vec{r}') \times d\vec{a}'$$



NOTAS
DE AULA

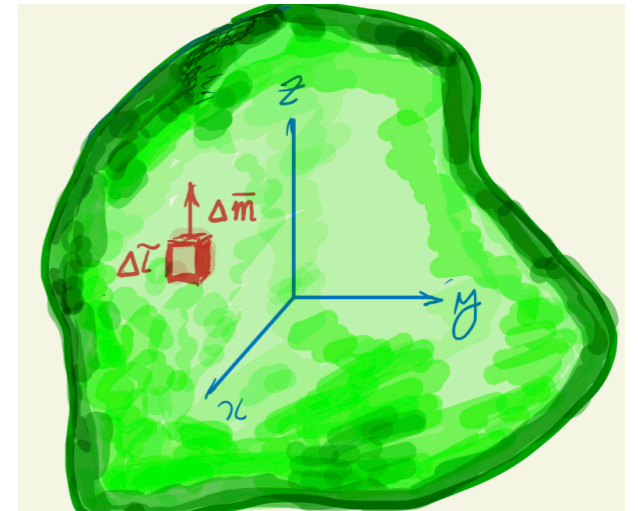
Magnetização



$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{1}{r} \vec{\nabla} \times \vec{M}(\vec{r}') d\tau' - \frac{\mu_0}{4\pi} \int \frac{1}{r} \vec{M}(\vec{r}') \times d\vec{a}'$$

Magnetização

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{1}{r} \vec{\nabla} \times \vec{M}(\vec{r}') d\tau' - \frac{\mu_0}{4\pi} \int \frac{1}{r} \vec{M}(\vec{r}') \times \underbrace{d\vec{a}'}_{\hat{n} da'}$$



Magnetização

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{1}{r} \vec{\nabla} \times \vec{M}(\vec{r}') d\tau' + \frac{\mu_0}{4\pi} \int \frac{1}{r} \vec{M}(\vec{r}') \times \hat{n} da'$$

CORRENTES DE MAGNETIZAÇÃO

$$\vec{\nabla} \times \vec{M}(\vec{r}') \equiv \vec{J}_b$$

VOLUME TRICA

$$\vec{M}(\vec{r}') \times \hat{n} \equiv \vec{K}_b$$

SUPERFICIAL

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}_b(\vec{r}')}{r} d\tau' + \frac{\mu_0}{4\pi} \int \frac{\vec{K}_b(\vec{r}')}{r} da'$$

