Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

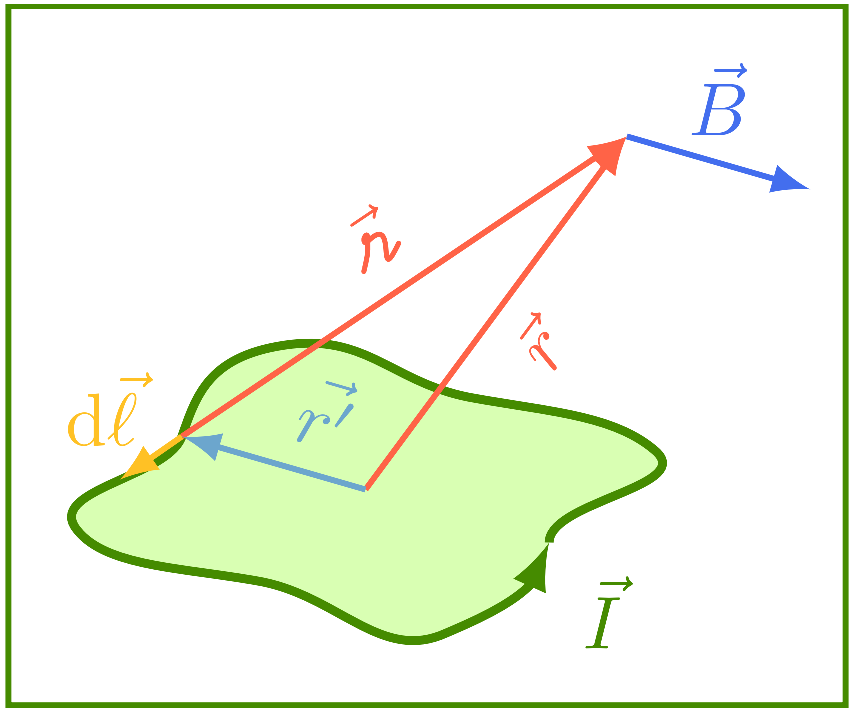
$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

5 de julho de 2021

Magnetostática

Expansão multipolar

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{\nabla} \cdot \vec{A} = 0$$



$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,')}{r} \, \mathrm{d}\tau'$$

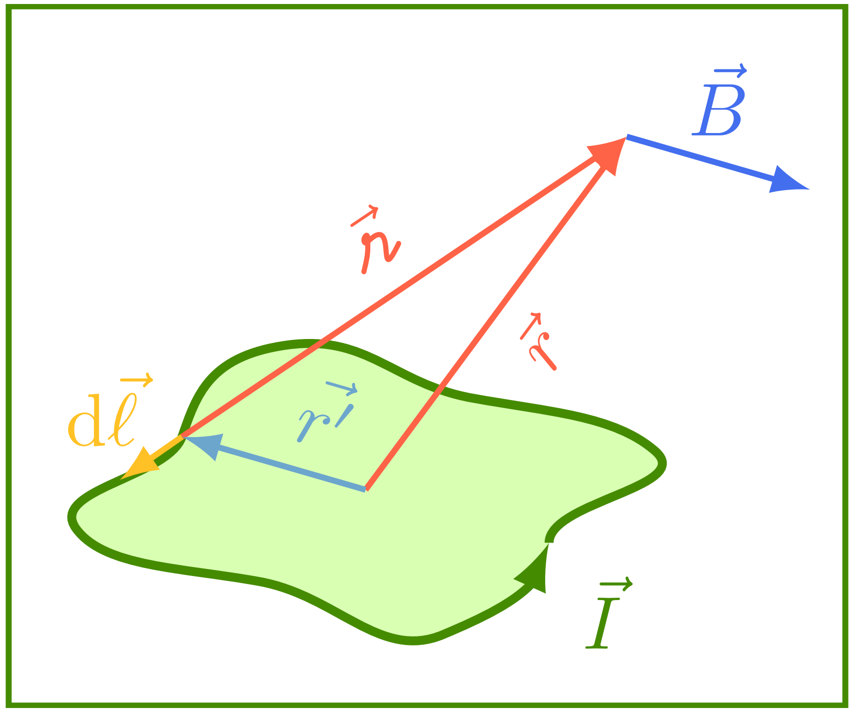
- QUEREMOS ACHAR $\vec{A}(\vec{r})$, PARA DEPOIS CALCULAR $\vec{B}$

Expansão multipolar

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{\nabla} \cdot \vec{A} = 0$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,')}{\mathscr{r}}\, \mathrm{d}\tau'$$

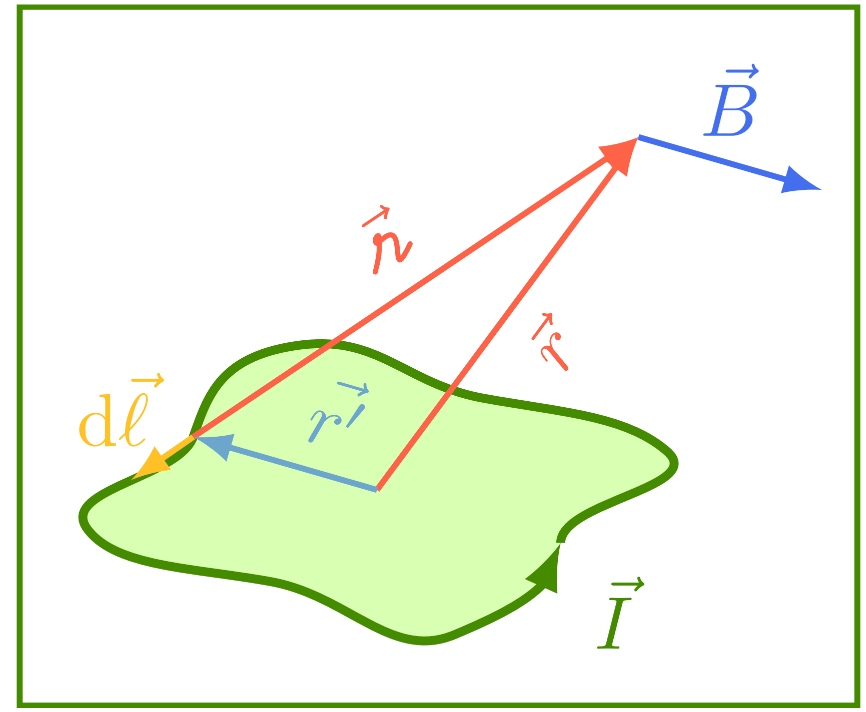


$$\frac{1}{\mathscr{r}} = \frac{1}{r} \sum_{\ell=0}^{\infty} \left(\frac{r'}{r}\right)^{\ell} P_\ell(\cos\theta')$$

← DERIVADA NO CAPÍTULO DE MÉTODOS MATEMÁTICOS

Expansão multipolar

$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{\nabla} \cdot \vec{A} = 0$



$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,')}{\mathscr{r}}\, d\tau' = \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r}\,')}{\mathscr{r}}\, d\ell'$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r} \sum_{\ell=0}^{\infty} \left(\frac{r'}{r}\right)^{\ell} P_\ell(\cos\theta')$$

→ APROXIMAÇÃO PARA r>>r'?

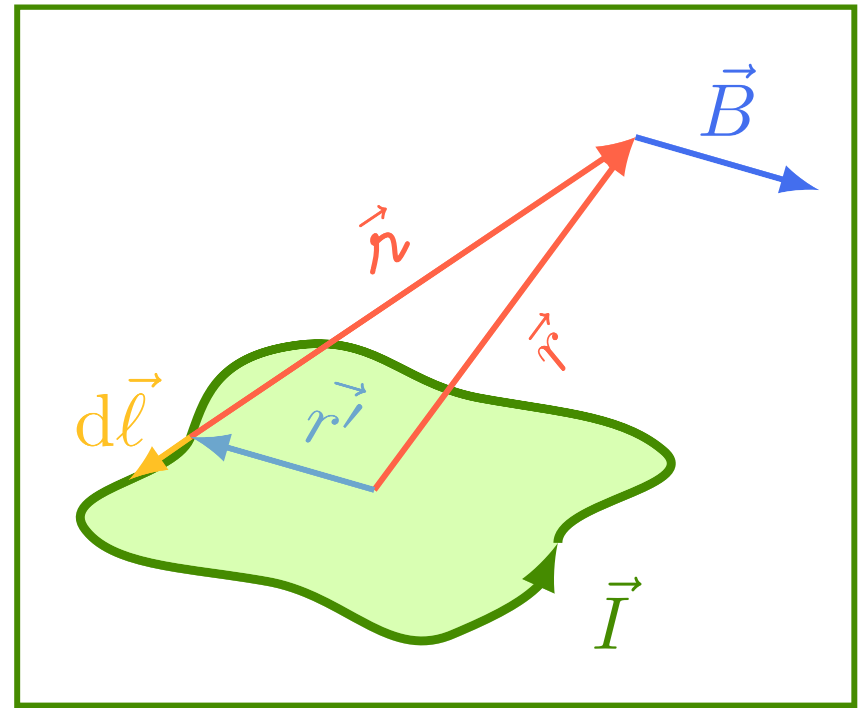
$$\ell = 0 \quad \Rightarrow \quad \frac{1}{\mathscr{r}} \approx \frac{1}{r} \quad \Rightarrow \quad \vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi r} \oint d\vec{\ell}'$$

PRIMEIRA APROXIMAÇÃO

INTEGRAL DÁ 0 (CIRCUITO FECHADO)

Expansão multipolar

$$\vec{B} = \vec{\nabla} \times \vec{A} \qquad \vec{\nabla} \cdot \vec{A} = 0$$



$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,')}{\mathscr{r}}\, \mathrm{d}\tau' = \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r}\,')}{\mathscr{r}}\, \mathrm{d}\ell'$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r} \sum_{\ell=0}^{\infty} \left(\frac{r'}{r}\right)^{\ell} P_\ell(\cos\theta')$$

$$\ell \leq 1 \quad \Rightarrow \quad \frac{1}{\mathscr{r}} \approx \frac{1}{r} + \frac{r'}{r^2}\cos\theta \Rightarrow \vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi r} \oint \mathrm{d}\vec{\ell'} + \frac{\mu_0}{4\pi r^2} \oint r' \cos\theta' \mathrm{d}\vec{\ell}$$

0

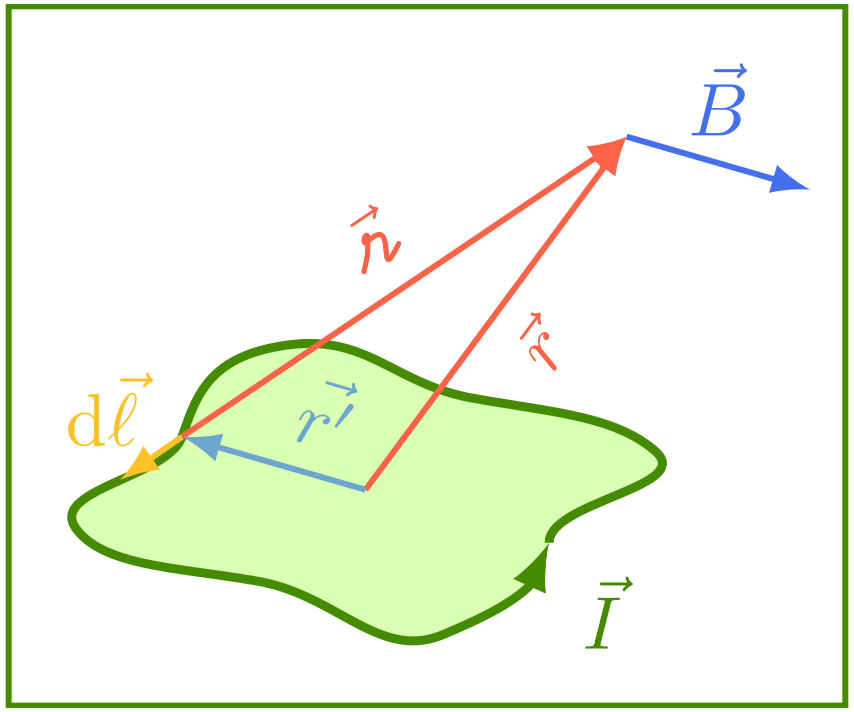
PRÓXIMA APROXIMAÇÃO

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

NOTAS DE AULA

Expansão multipolar

$\vec{B} = \vec{\nabla} \times \vec{A}$ $\qquad$ $\vec{\nabla} \cdot \vec{A} = 0$



$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$\vec{m} = I\vec{a}$$

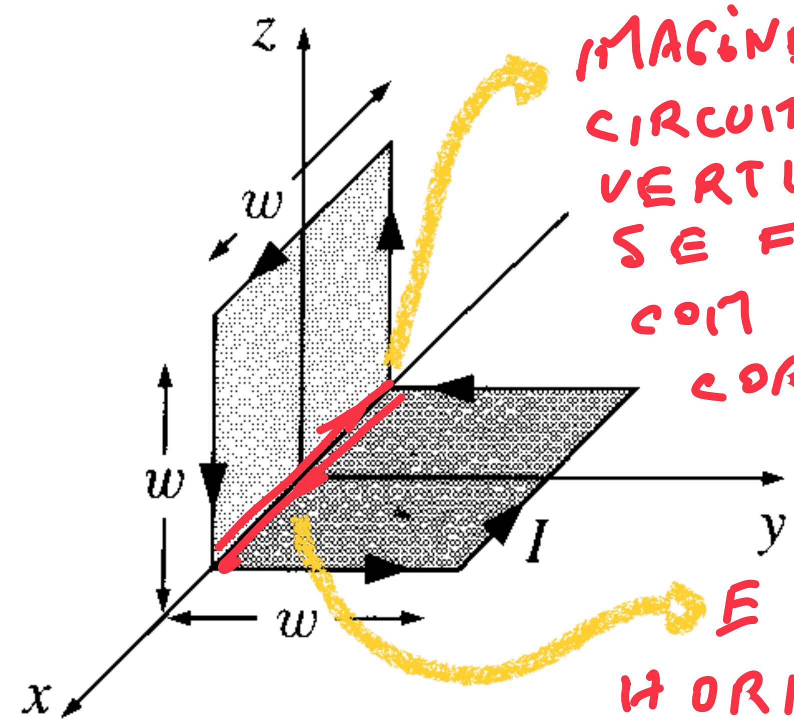
MOMENTO MAGNÉTICO DO CIRCUITO

É COMO SE FOSSE UM PEQUENO ÍMÃ

Pratique o que aprendeu

$$\vec{m} = I\vec{a}$$

$$\vec{m} = ?$$



IMAGINE QUE CIRCUITO VERTICAL SE FECHA COM ESTA CORRENTE

E O HORIZONTAL, COM ESTA

Figure 5.52

DOIS CIRCUITOS FECHADOS

Pratique o que aprendeu

$$\vec{m} = I\vec{a}$$

$$\vec{m} = Iw^2(\hat{y} + \hat{z})$$

ÁREA
DE
CADA CIRCUITO

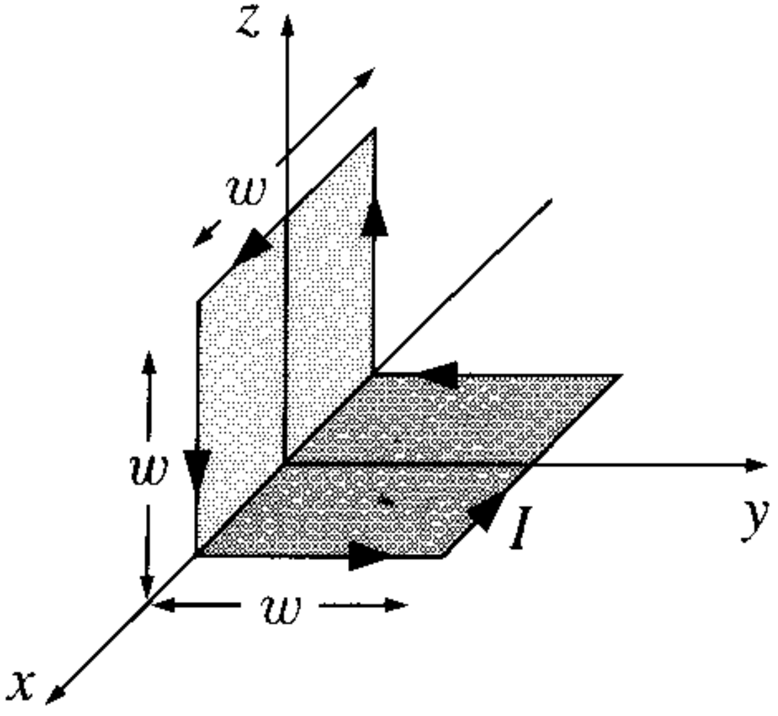


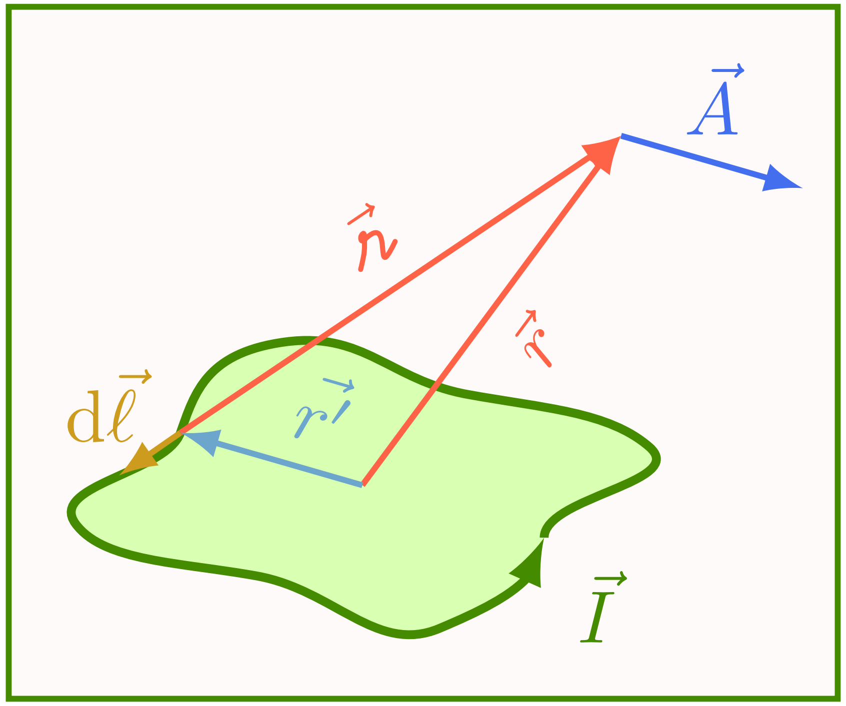
Figure 5.52

Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

$$\vec{m} = m\hat{z}$$

CALCULAR EM COORDENADAS CILÍNDRICAS

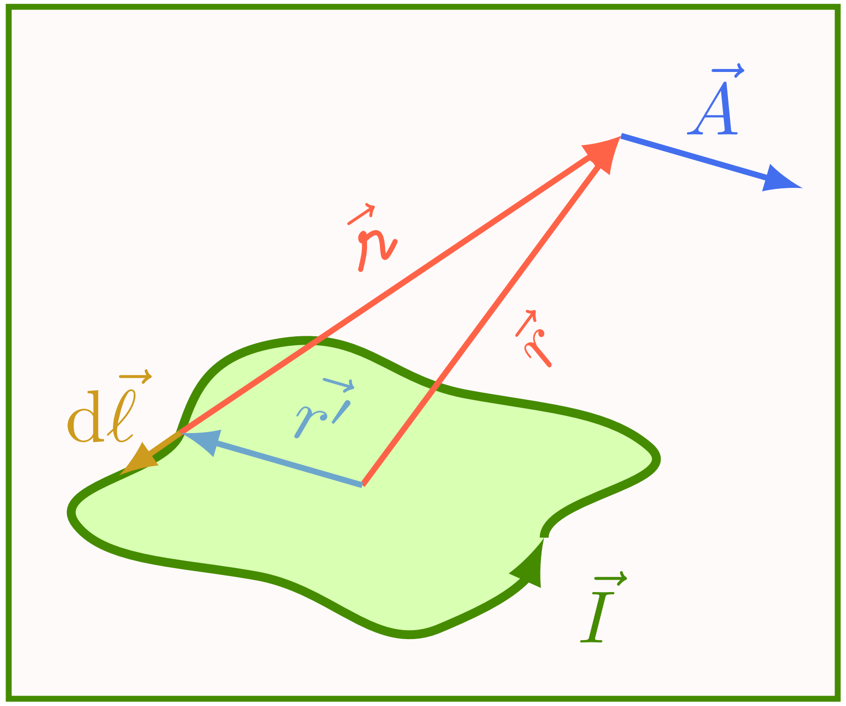


Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

$$\vec{m} = m\hat{z}$$

$$\vec{r} = s\hat{s} + z\hat{z}$$



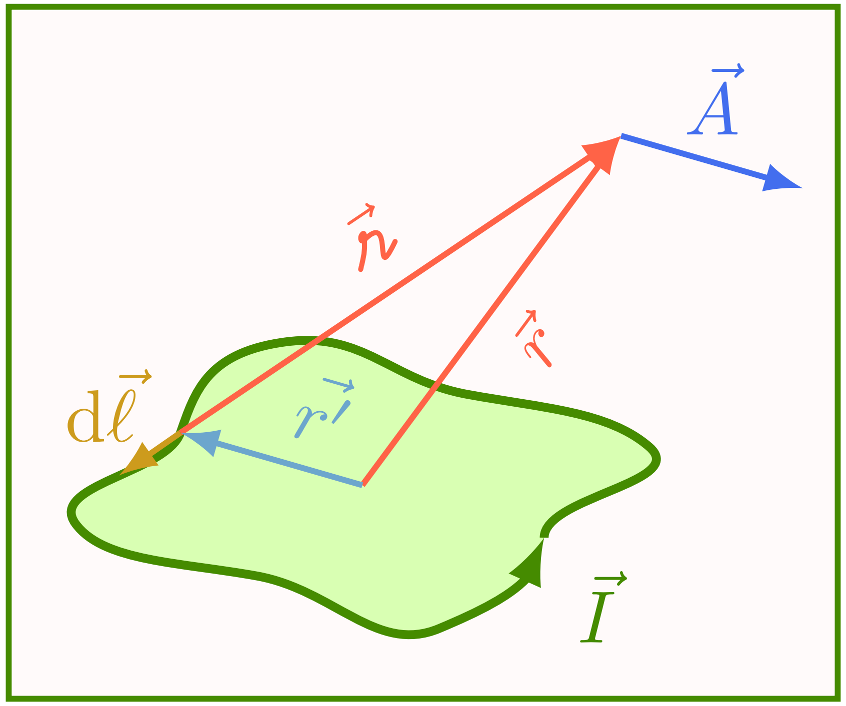
Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

$$\vec{m} = m\hat{z}$$

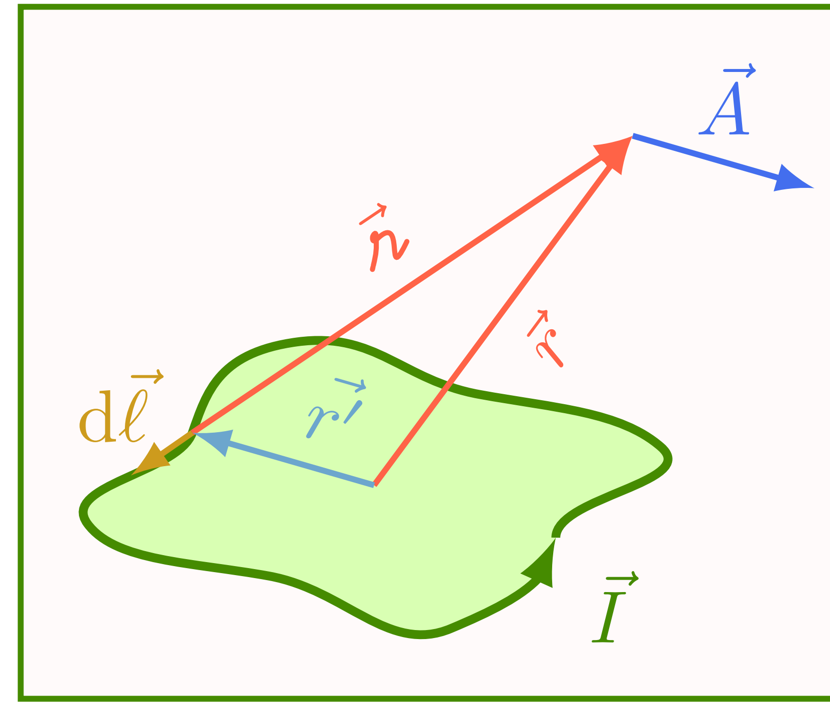
$$\vec{r} = s\hat{s} + z\hat{z}$$

$$\vec{m}\times\vec{r} = s\vec{m}\times\hat{s}$$



Expansão multipolar

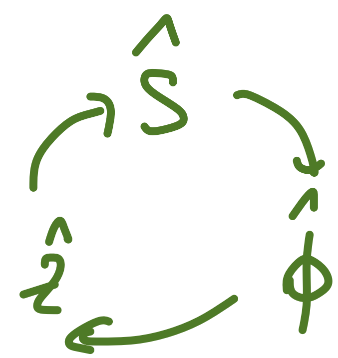
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$



$$\vec{m} = m\hat{z}$$

$$\vec{r} = s\hat{s} + z\hat{z}$$

$$\vec{m}\times\vec{r} = s\vec{m}\times\hat{s}$$



$$\vec{m}\times\vec{r} = sm\hat{z}\times\hat{s}$$

Expansão multipolar

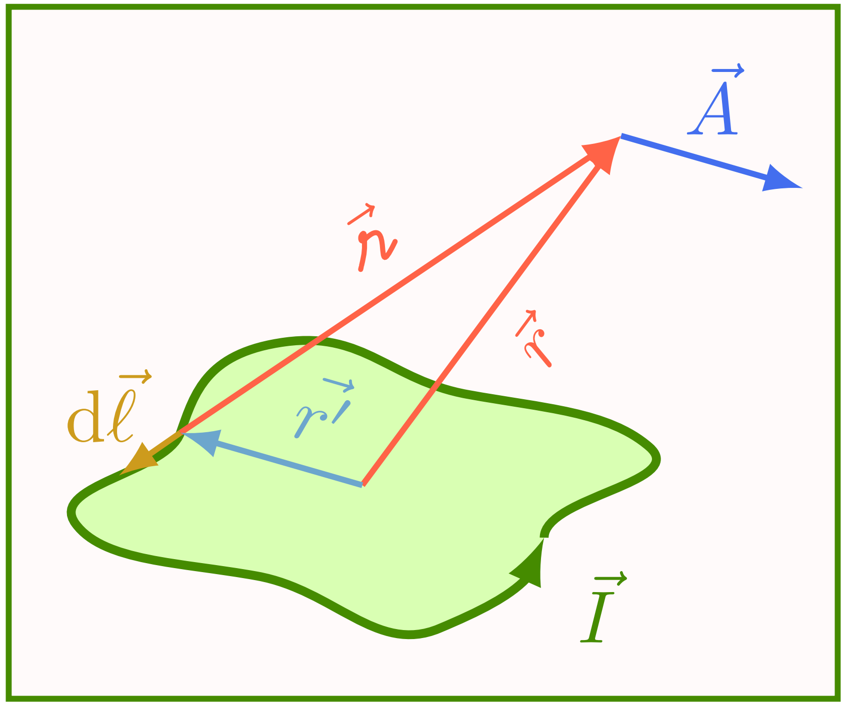
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

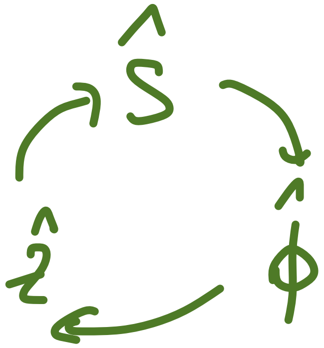
$$\vec{m} = m\hat{z}$$

$$\vec{r} = s\hat{s} + z\hat{z}$$

$$\vec{m}\times\vec{r} = s\vec{m}\times\hat{s}$$

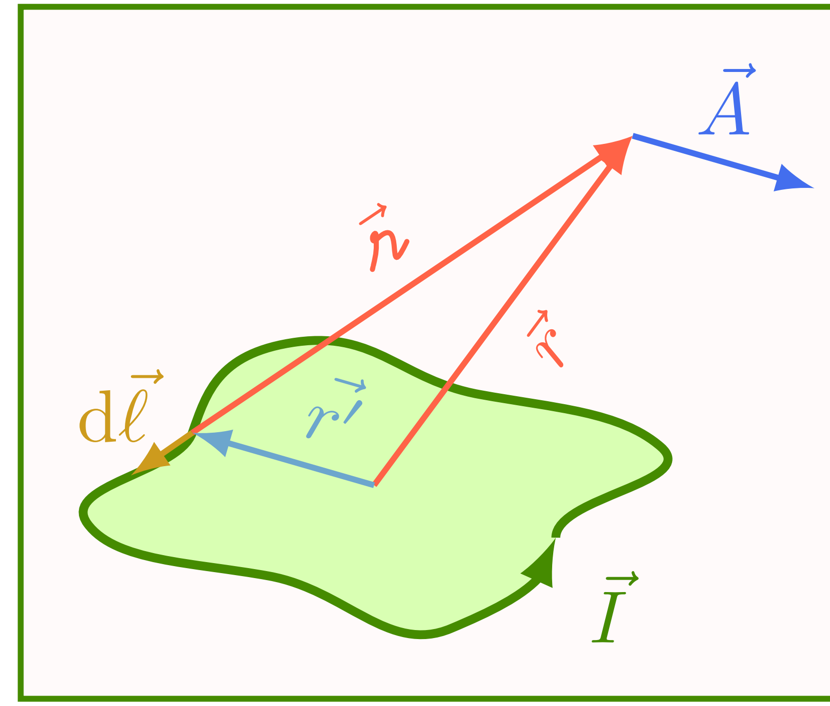
$$\vec{m}\times\vec{r} = sm\hat{z}\times\hat{s} = sm\hat{\phi}$$





Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$



$$\vec{m} = m\hat{z}$$

$$\vec{r} = s\hat{s} + z\hat{z}$$

$$\vec{m}\times\vec{r} = s\vec{m}\times\hat{s}$$

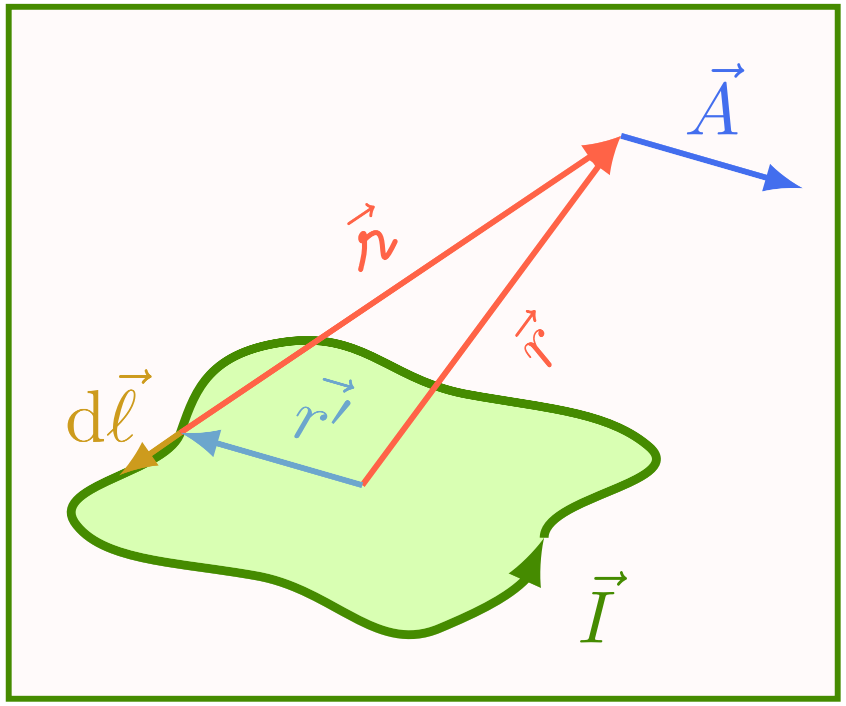
$$\vec{m}\times\vec{r} = sm\hat{z}\times\hat{s} = sm\hat{\phi}$$

$$\hat{r} = \frac{\vec{r}}{r}$$

$$\vec{m}\times\hat{r} = \frac{sm}{r}\hat{\phi}$$

Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$



$$\vec{m} = m\hat{z}$$

$$\vec{r} = s\hat{s} + z\hat{z}$$

$$\vec{m}\times\vec{r} = s\vec{m}\times\hat{s}$$

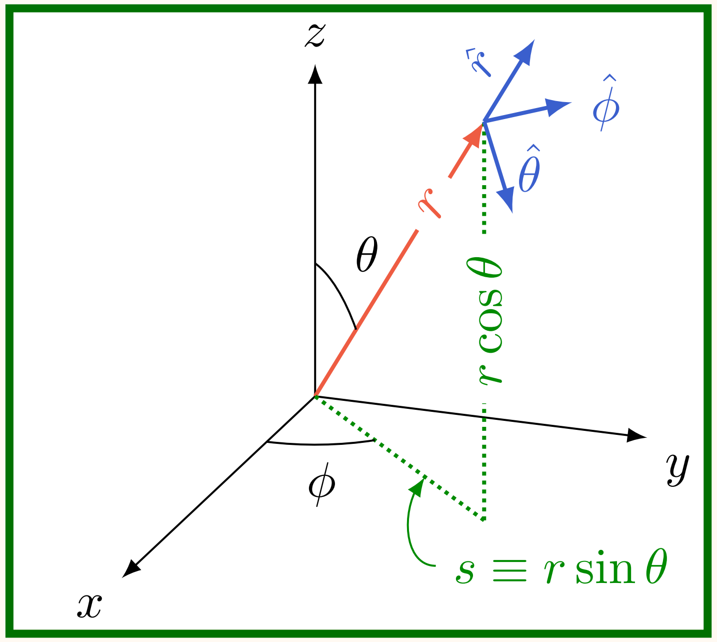
$$\vec{m}\times\vec{r} = sm\hat{z}\times\hat{s} = sm\hat{\phi}$$

POTENCIAL VETOR DO DIPOLO C/ $\vec{m} = m\hat{z}$

$$\vec{m}\times\hat{r} = \frac{sm}{r}\hat{\phi} \quad \Rightarrow \quad \vec{A} = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi}$$

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

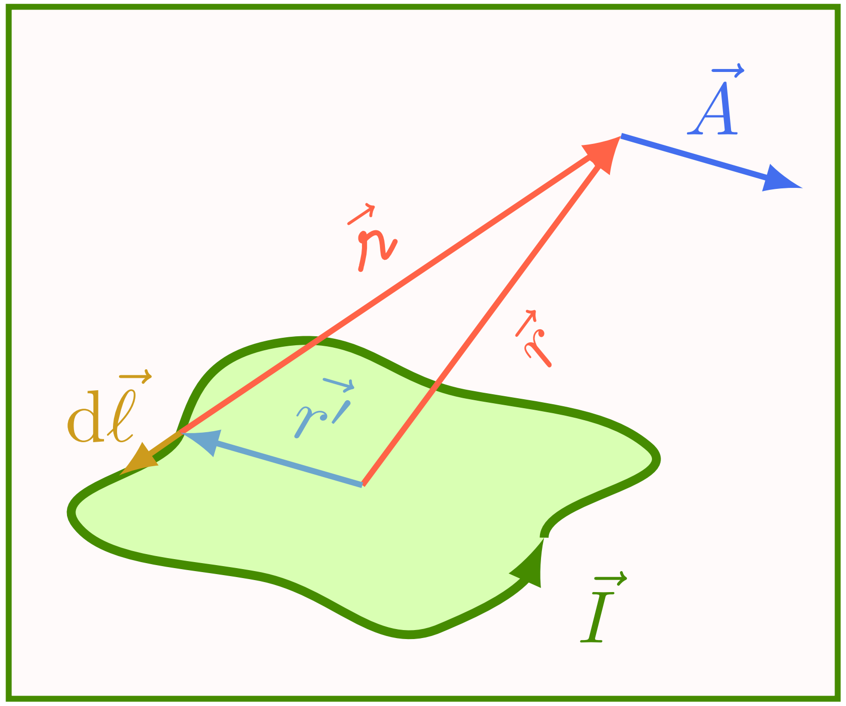
$$\vec{m} = m\hat{z}$$

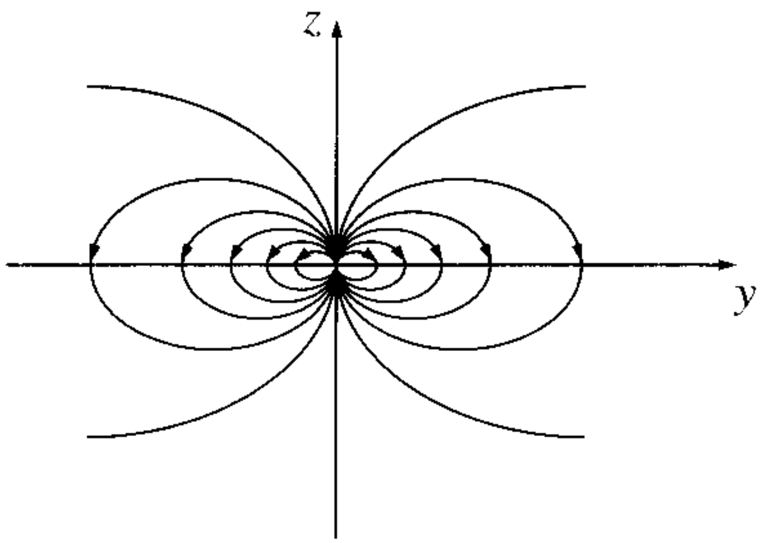
$$\vec{r} = s\hat{s} + z\hat{z}$$

$$\vec{m}\times\hat{r} = s\vec{m}\times\hat{s} = sm\hat{\phi}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi} \rightarrow \vec{B} = \vec{\nabla}\times\vec{A}$$

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left[2\cos\theta\,\hat{r} + \operatorname{sen}\theta\,\hat{\theta}\right]$$





(a) Field of a "pure" dipole

Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

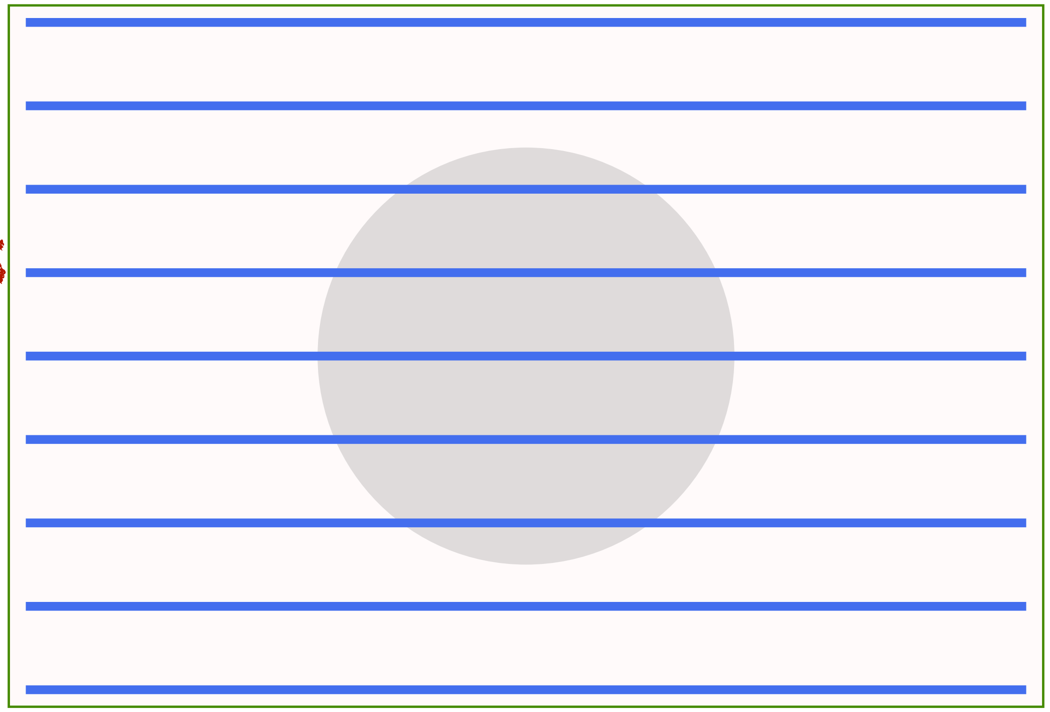
5 de julho de 2021

Magnetismo em materiais

Magnetismo em materiais

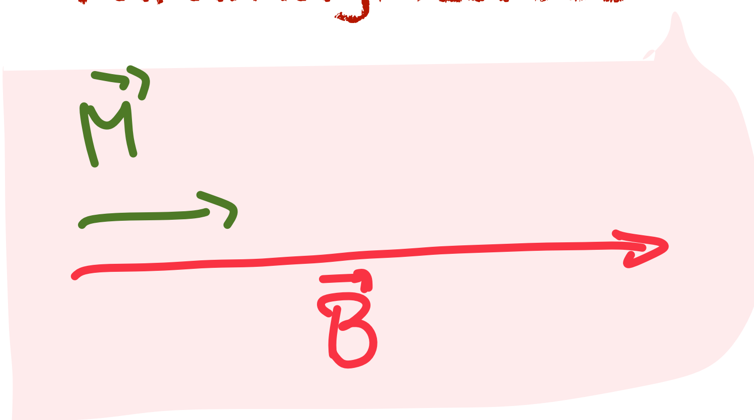
- Paramagnéticos
- Diamagnéticos
- Ferromagnéticos
- Antiferromagnéticos

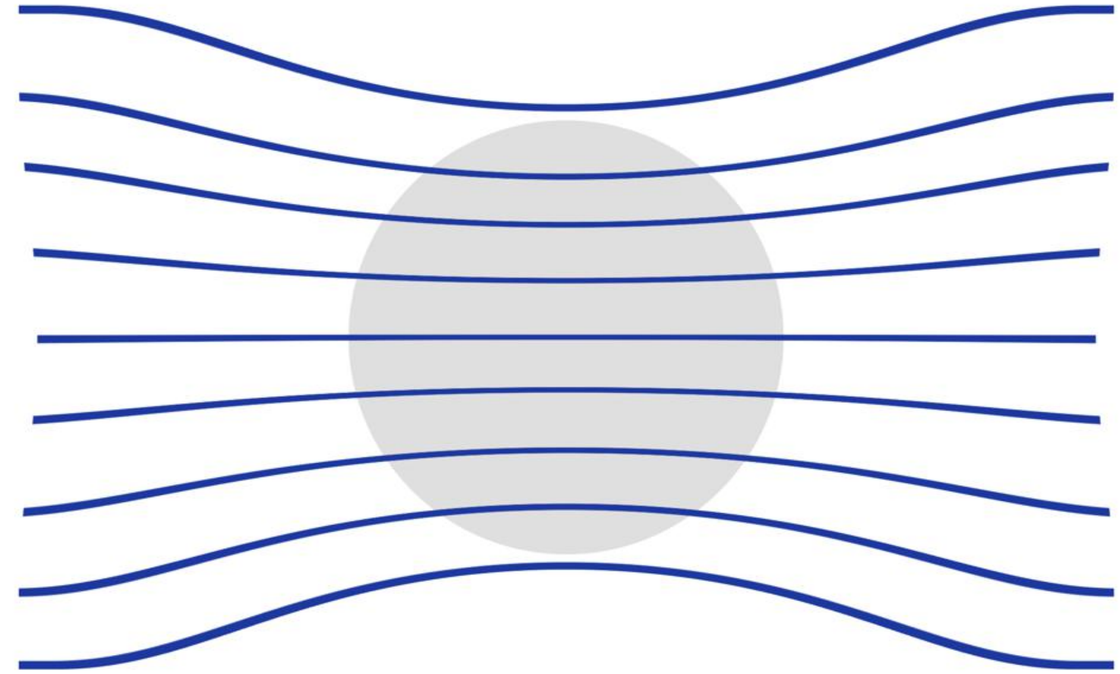
} 4 CLASSES



Magnetismo em materiais

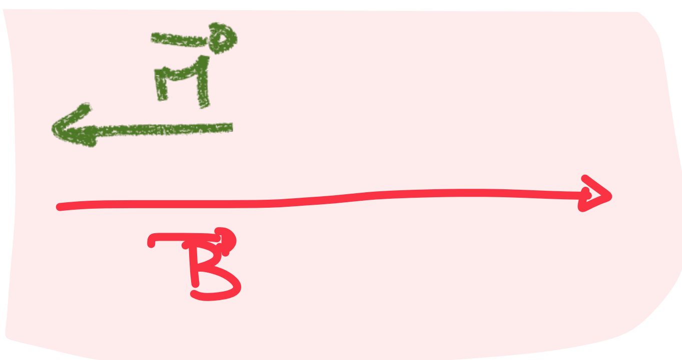
• Paramagnéticos

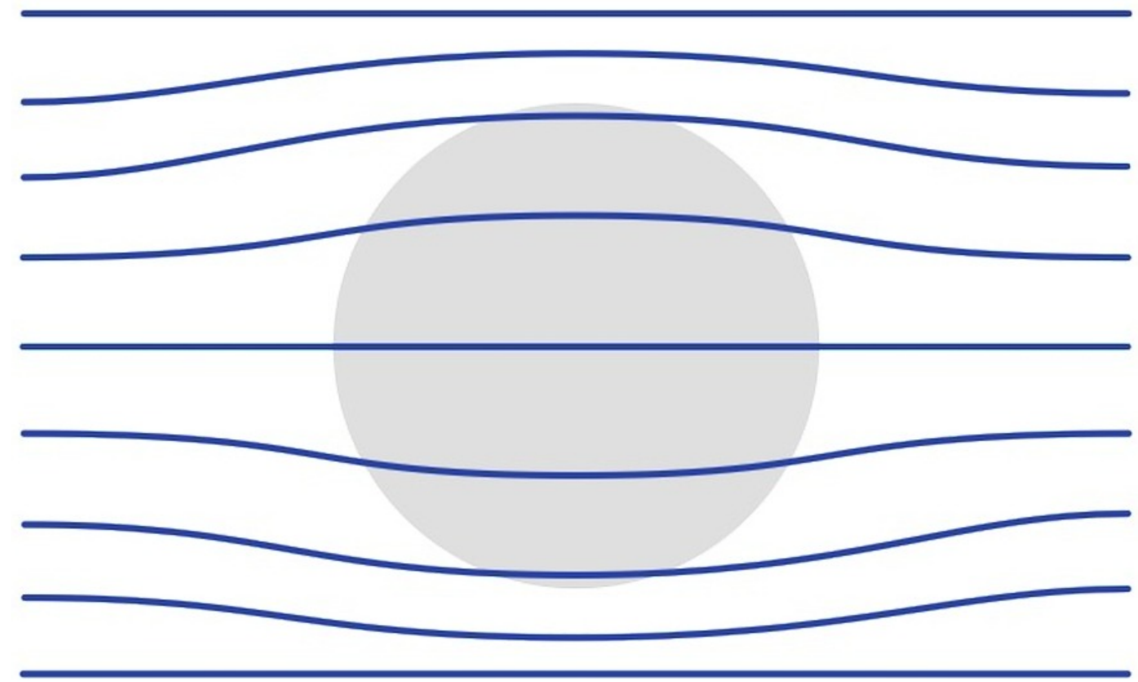




Magnetismo em materiais

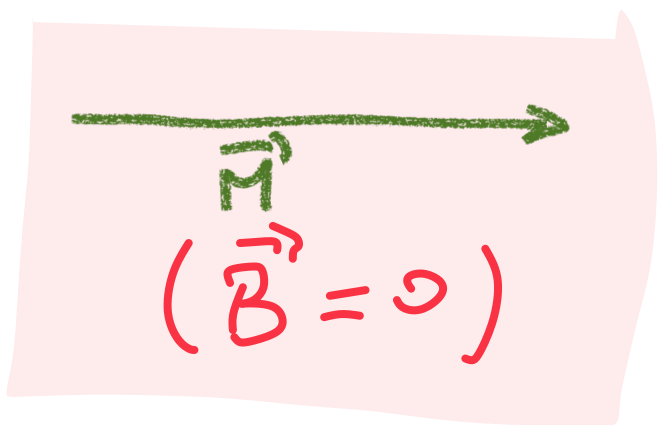
- Diamagnéticos

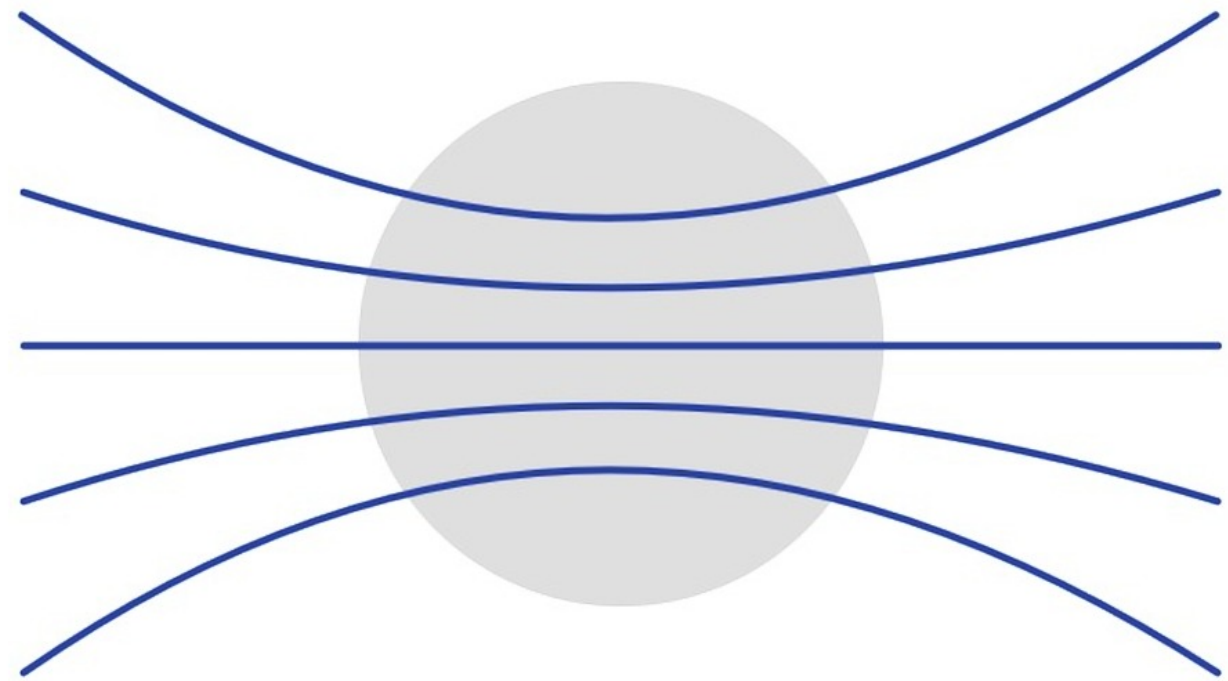




Magnetismo em materiais

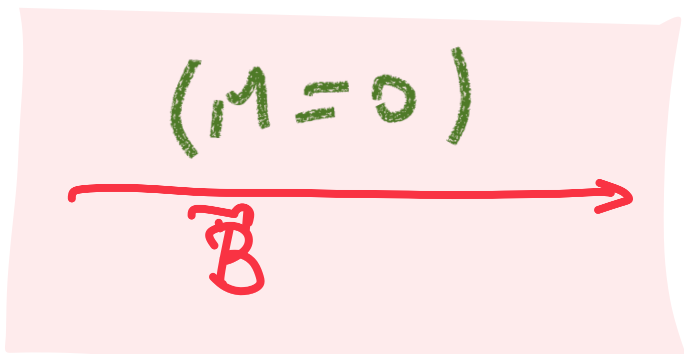
• Ferromagnéticos

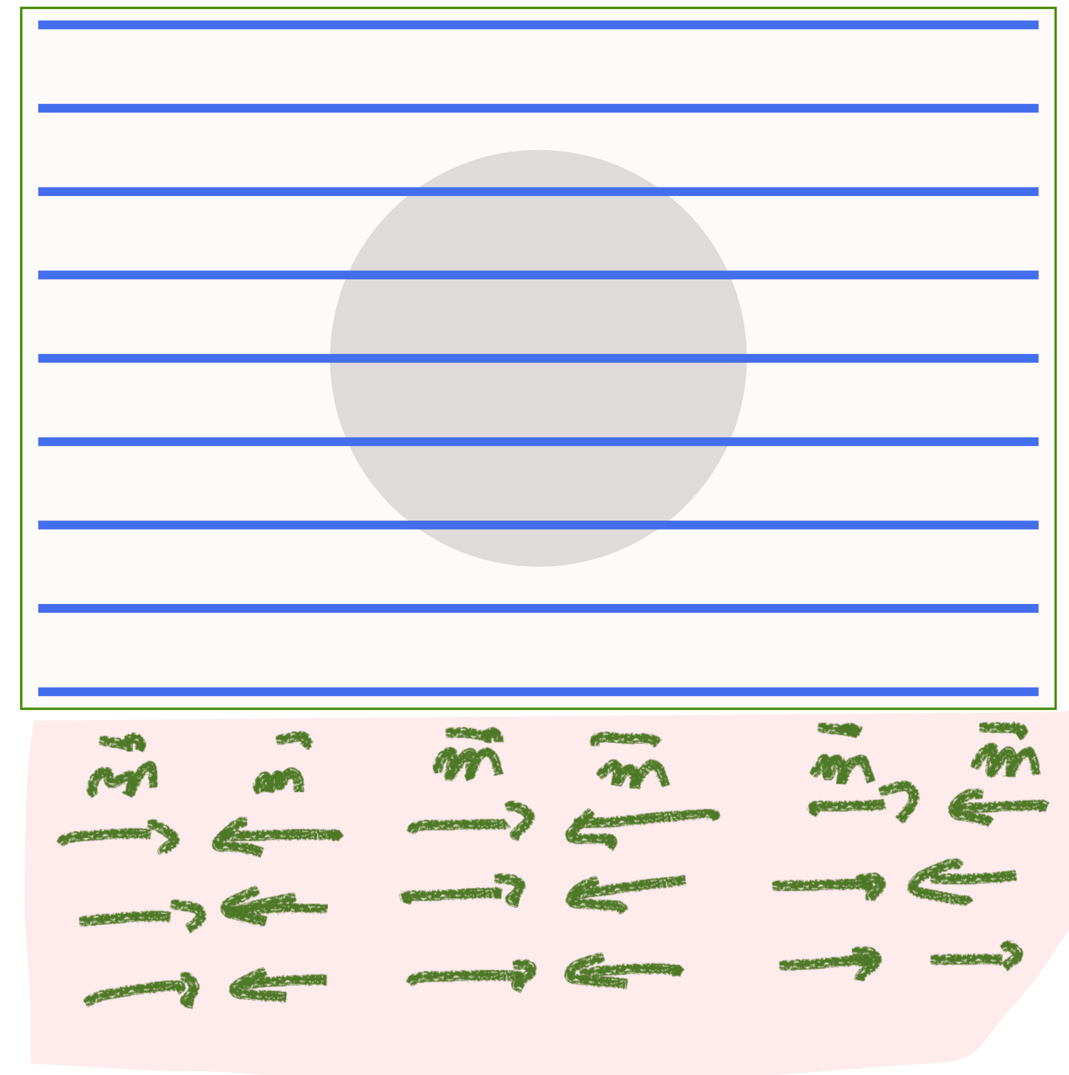




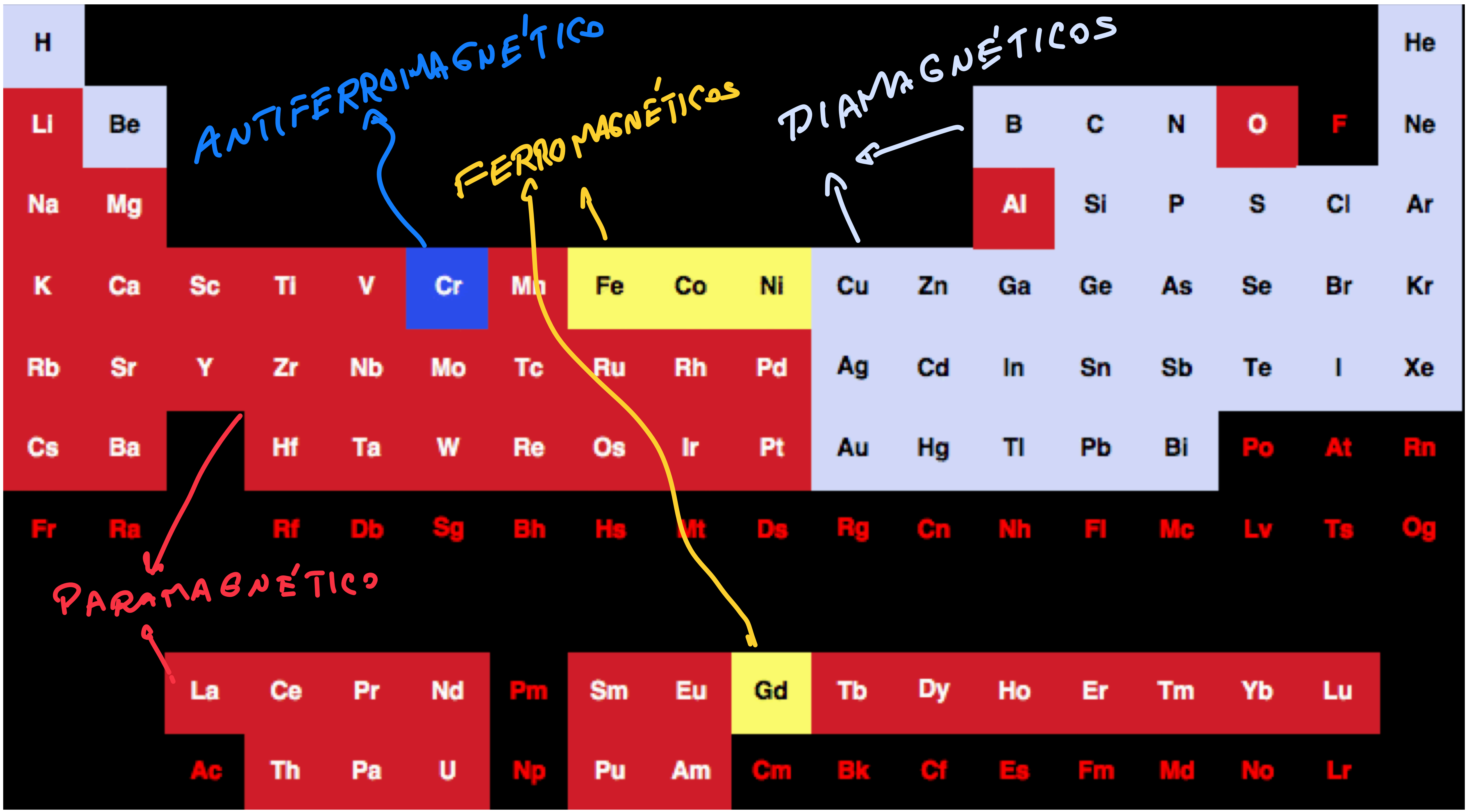
Magnetismo em materiais

- Antiferromagnéticos





Magnetismo em materiais

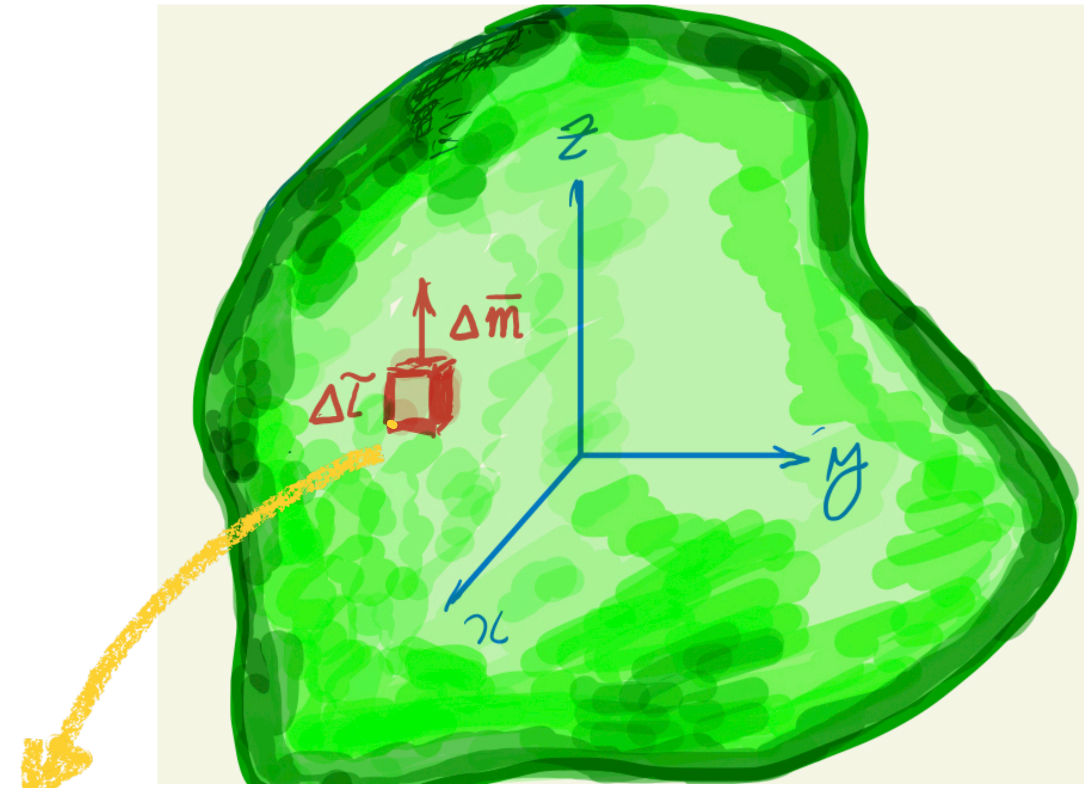


M

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

$$\vec{M} = \frac{\Delta\vec{m}}{\Delta\tau}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{M}(\vec{r'})\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,\mathrm{d}\tau'$$



CAMPO

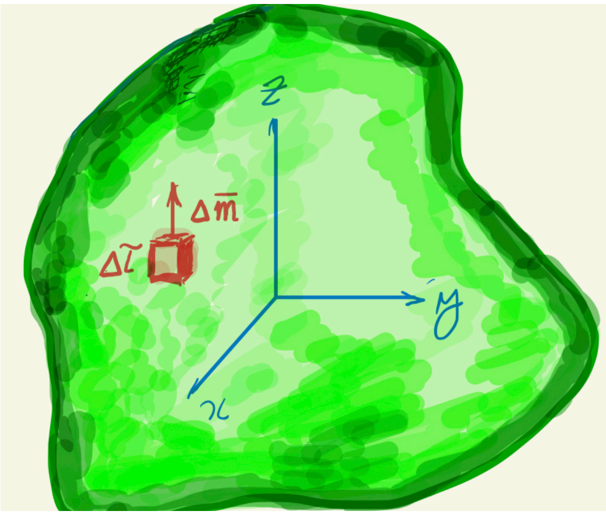
MAGNÉTICO

INDUZ

MOMENTO

MAGNÉTICO

Magnetização



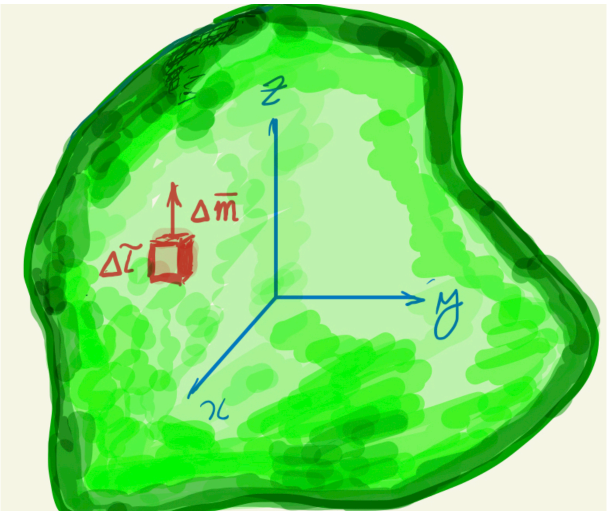
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$\vec{M} = \frac{\Delta \vec{m}}{\Delta \tau}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r'}) \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, \mathrm{d}\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \vec{M}(\vec{r'}) \times \vec{\boldsymbol{\nabla}}' \left(\frac{1}{\mathscr{r}} \right) \mathrm{d}\tau'$$

Magnetização



$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

$$\vec{M} = \frac{\Delta\vec{m}}{\Delta\tau}$$

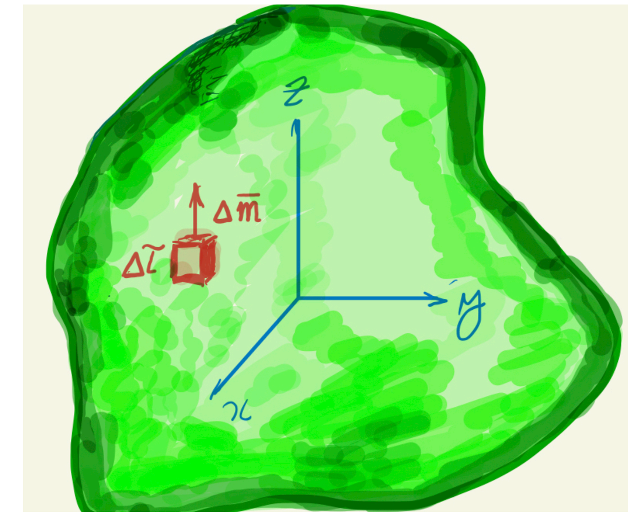
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{\vec{M}(\vec{r'})\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,\mathrm{d}\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int\vec{M}(\vec{r'})\times\vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right)\mathrm{d}\tau'$$

$$\vec{\nabla}'\times\left(\frac{\vec{M}(\vec{r'})}{\mathscr{r}}\right) = \vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right)\times\vec{M}(\vec{r'}) + \frac{1}{\mathscr{r}}\vec{\nabla}'\times\vec{M}(\vec{r'})$$

$$-\vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right)\times\vec{M}(\vec{r'}) = -\vec{\nabla}'\times\left(\frac{\vec{M}(\vec{r'})}{\mathscr{r}}\right) + \frac{1}{\mathscr{r}}\vec{\nabla}'\times\vec{M}(\vec{r'})$$

Magnetização



$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

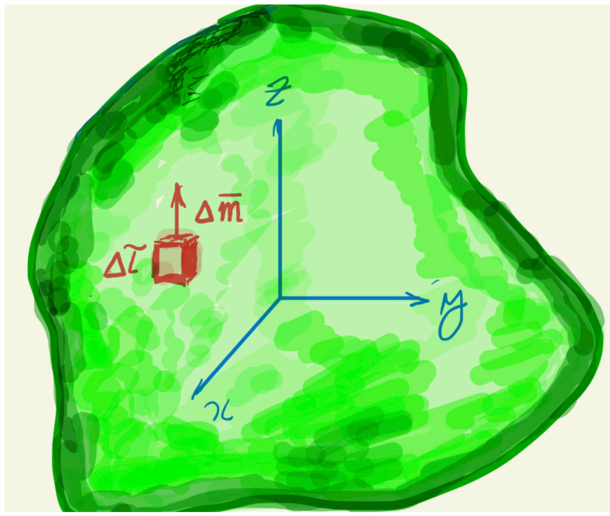
$$\vec{M} = \frac{\Delta \vec{m}}{\Delta \tau}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{r'}) \times \hat{\mathscr{r}}}{\mathscr{r}^2} \, \mathrm{d}\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \vec{M}(\vec{r'}) \times \vec{\nabla}' \left(\frac{1}{\mathscr{r}} \right) \mathrm{d}\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{1}{\mathscr{r}} \vec{\nabla} \times \vec{M}(\vec{r'}) \, \mathrm{d}\tau' - \frac{\mu_0}{4\pi} \int \vec{\nabla} \times \left(\frac{\vec{M}(\vec{r'})}{\mathscr{r}} \right) \mathrm{d}\tau'$$

Magnetização



$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$

$$\vec{M} = \frac{\Delta\vec{m}}{\Delta\tau}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{\vec{M}(\vec{r'})\times\hat{\mathscr{r}}}{\mathscr{r}^2}\,\mathrm{d}\tau'$$

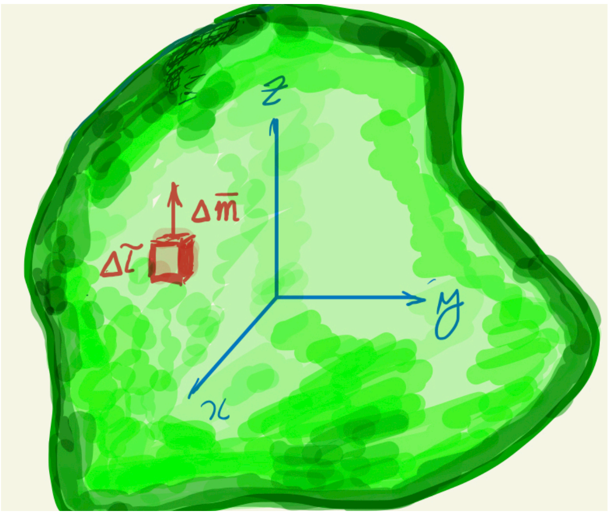
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int\vec{M}(\vec{r'})\times\vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right)\mathrm{d}\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{1}{\mathscr{r}}\vec{\nabla}\times\vec{M}(\vec{r'})\,\mathrm{d}\tau' - \frac{\mu_0}{4\pi}\int\vec{\nabla}\times\left(\frac{\vec{M}(\vec{r'})}{\mathscr{r}}\right)\mathrm{d}\tau'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int\frac{1}{\mathscr{r}}\vec{\nabla}\times\vec{M}(\vec{r'})\,\mathrm{d}\tau' - \frac{\mu_0}{4\pi}\int\frac{1}{\mathscr{r}}\vec{M}(\vec{r'})\times\mathrm{d}\vec{a'}$$

NOTAS DE AULA

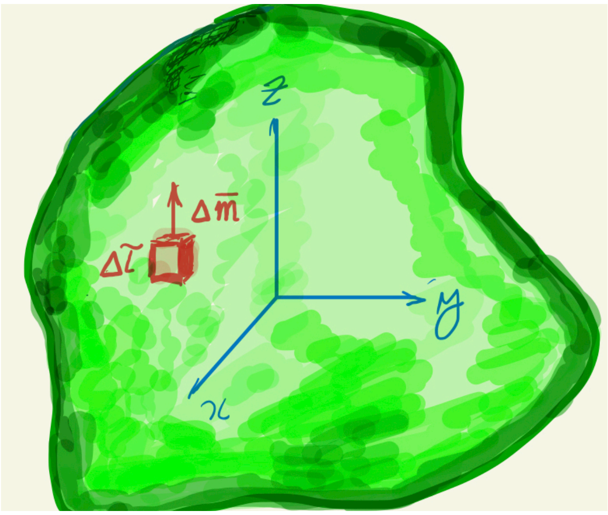
Magnetização



$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}\vec{\nabla}\times\vec{M}(\vec{r'})\,\mathrm{d}\tau' - \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}\vec{M}(\vec{r'})\times \mathrm{d}\vec{a'}$$

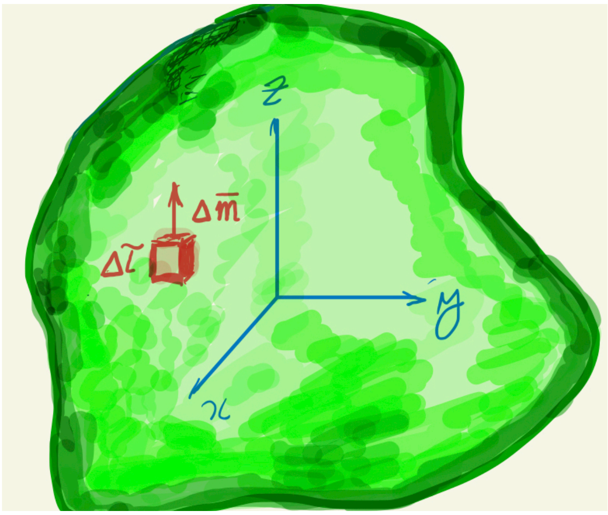
Magnetização

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}\vec{\nabla}\times\vec{M}(\vec{r'})\,\mathrm{d}\tau' - \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}\vec{M}(\vec{r'})\times \underbrace{\mathrm{d}\vec{a'}}_{\hat{n}\,da'}$$



Magnetização

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}\vec{\nabla}\times\vec{M}(\vec{r'})\,\mathrm{d}\tau' + \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}\vec{M}(\vec{r'})\times\hat{n}\,\mathrm{d}a'$$



CORRENTES DE MAGNETIZAÇÃO

$$\vec{\nabla}\times\vec{M}(\vec{r'}) \equiv \vec{J}_b$$

VOLUMÉTRICA

$$\vec{M}(\vec{r'})\times\hat{n} \equiv \vec{K}_b$$

SUPERFICIAL

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}_b(\vec{r'})}{\mathscr{r}}\,\mathrm{d}\tau' + \frac{\mu_0}{4\pi}\int \frac{\vec{K}_b(\vec{r'})}{\mathscr{r}}\,\mathrm{d}a'$$