Lista 3 - Ex. 01 ; 2. b - c - d ; 4. b ; 5. d ; 6

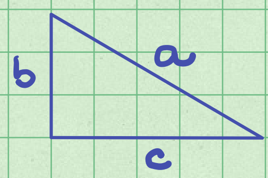
1

elipse $\begin{cases} e = \dfrac{2}{3} \\ C = O(0,0) \\ P(\underset{x}{2}, \underset{y}{1}) \in \text{elipse} \end{cases}$

Quantas elipses satisfazem esta condição?

$e = \dfrac{c}{a} = \dfrac{2}{3} \quad \therefore \quad \dfrac{c}{a} = \dfrac{2}{3} \quad \therefore \quad a = \dfrac{3}{2}c$

Como:



$a^2 = b^2 + c^2$

$\dfrac{9}{4}c^2 = b^2 + c^2 \longrightarrow b = \dfrac{\sqrt{5}}{2}c$

Duas possibilidades $\begin{cases} \text{EM} \rightarrow Ox: \dfrac{x^2}{\frac{9}{4}c^2} + \dfrac{y^2}{\frac{5}{4}c^2} = 1 & (1) \\ \text{EM} \rightarrow Oy: \dfrac{x^2}{\frac{5}{4}c^2} + \dfrac{y^2}{\frac{9}{4}c^2} = 1 & (2) \end{cases}$

$P \longrightarrow (1): \dfrac{4}{\frac{9}{4}c^2} + \dfrac{1}{\frac{5}{4}c^2} = 1$

$\dfrac{16}{9c^2} + \dfrac{4}{5c^2} = 1$

$80 + 36 = 45c^2 \quad \therefore \quad c^2 = \dfrac{116}{45}$ $\exists$ solução

$(1): \dfrac{x^2}{\frac{9}{4} \cdot \frac{116}{45}} + \dfrac{y^2}{\frac{5}{4} \cdot \frac{116}{45}} = 1 \quad \therefore \quad \dfrac{x^2}{29/5} + \dfrac{y^2}{29/9} = 1$

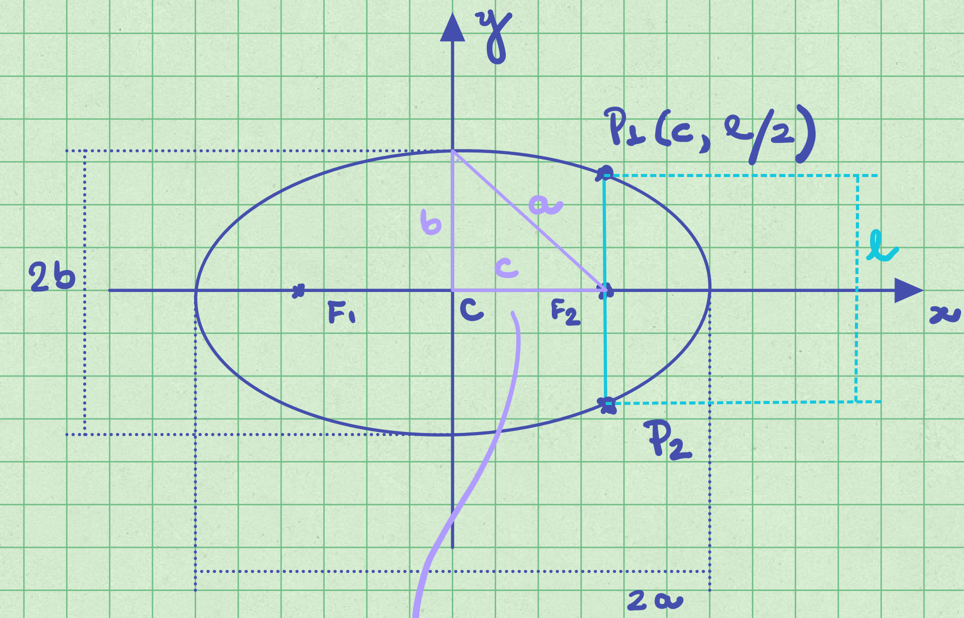
⋮

(2) c^2 (diferente do 1º)

Logo: 2 elipses satisfazem a condição.

2 b) $2a, 2b$

$\ell = 2\dfrac{b^2}{a^2}$



$c = \sqrt{a^2 - b^2}$

$F_1(-c, 0)$

$F_2(c, 0)$

Da def. elipse:

$$\delta(P, F_1) + \delta(P, F_2) = 2a$$

$$\delta(P, F_1) = |\overrightarrow{PF_1}| \quad , \quad \overrightarrow{PF_1} = F_1 - P = (-2c, -\ell/2)$$

$$\delta(P, F_1) = \sqrt{4c^2 + \frac{\ell^2}{4}} = \frac{\sqrt{16c^2 + \ell^2}}{2}$$

$$\therefore \frac{\sqrt{16c^2 + \ell^2}}{2} + \frac{\ell}{2} = 2a$$

$$\frac{\sqrt{16c^2 + \ell^2}}{2} = 2a - \frac{\ell}{2}$$

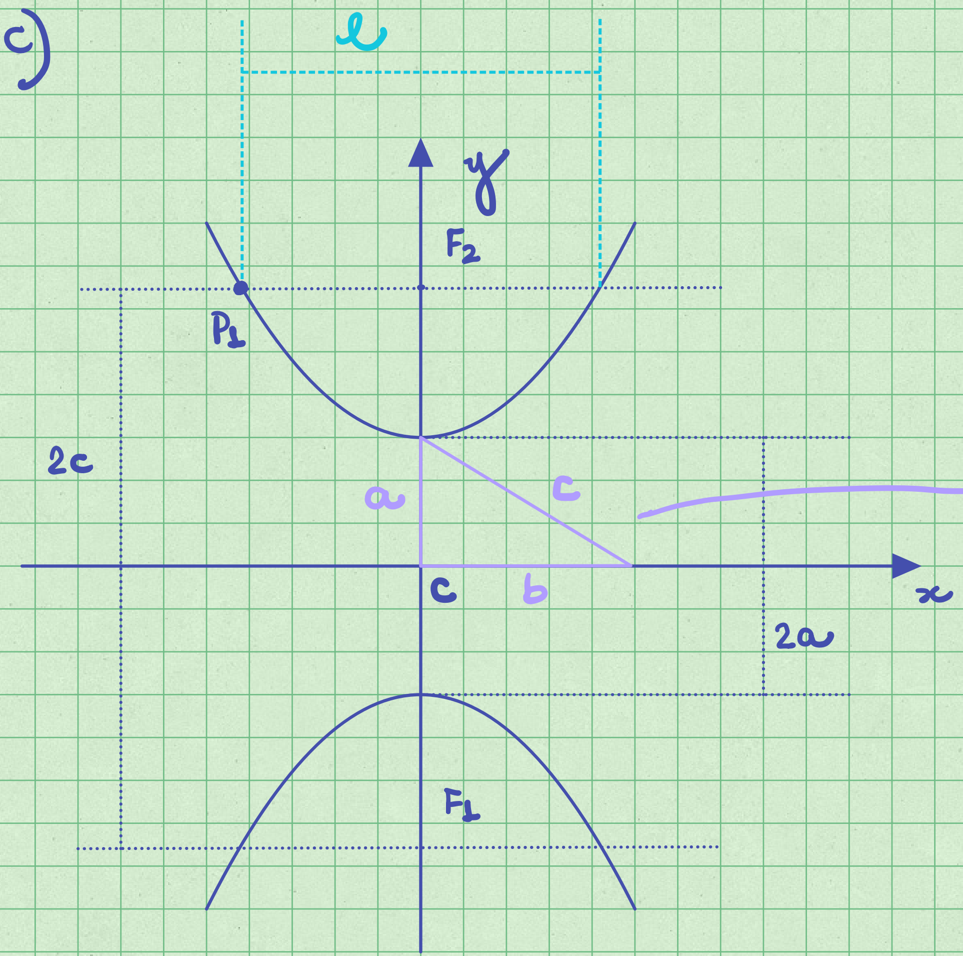
$$\sqrt{16c^2 + \ell^2} = \cancel{2}\left(\frac{4a - \ell}{\cancel{2}}\right) \qquad (\ )^2$$

$$16c^2 + \cancel{\ell^2} = 16a^2 - 8a\ell + \cancel{\ell^2}$$

$$16(\cancel{a^2} - b^2) = \cancel{16a^2} - 8a\ell$$

$$-16b^2 = -8a\ell \qquad \therefore \quad \ell = \frac{2b^2}{a}$$

c)



$F_1(0,-c)$

$F_2(0,c)$

$P_1(-\ell/2, c)$

$c^2 = a^2 + b^2$

$\delta(P_1,F_2) = \ell/2$

Da definição hipérbole :

$$|\,\delta(P_1,F_1) - \delta(P_1,F_2)\,| = 2a$$

$$\delta(P_1,F_1) = |\overrightarrow{P_1F_1}| \quad , \quad \overrightarrow{P_1F_1} = F_1 - P_1 = (\ell/2, -2c)$$

$$\delta(P_1,F_1) = \sqrt{\frac{\ell^2}{4} + 4c^2} = \frac{\sqrt{\ell^2+16c^2}}{2}$$

$$\therefore \quad \left| \frac{\sqrt{\ell^2+16c^2}}{2} - \frac{\ell}{2} \right| = 2a$$

$$\frac{\sqrt{\ell^2+16c^2}}{2} - \frac{\ell}{2} = \pm 2a$$

$$\frac{\sqrt{\ell^2+16c^2}}{2} = \frac{\ell}{2} \pm 2a$$

$$\sqrt{\ell^2+16c^2} = \cancel{2}\left(\frac{\ell \pm 4a}{\cancel{2}}\right) \qquad (\quad)^2$$

$$\cancel{\ell^2} + 16c^2 = \cancel{\ell^2} \pm 8a\ell + 16a^2$$

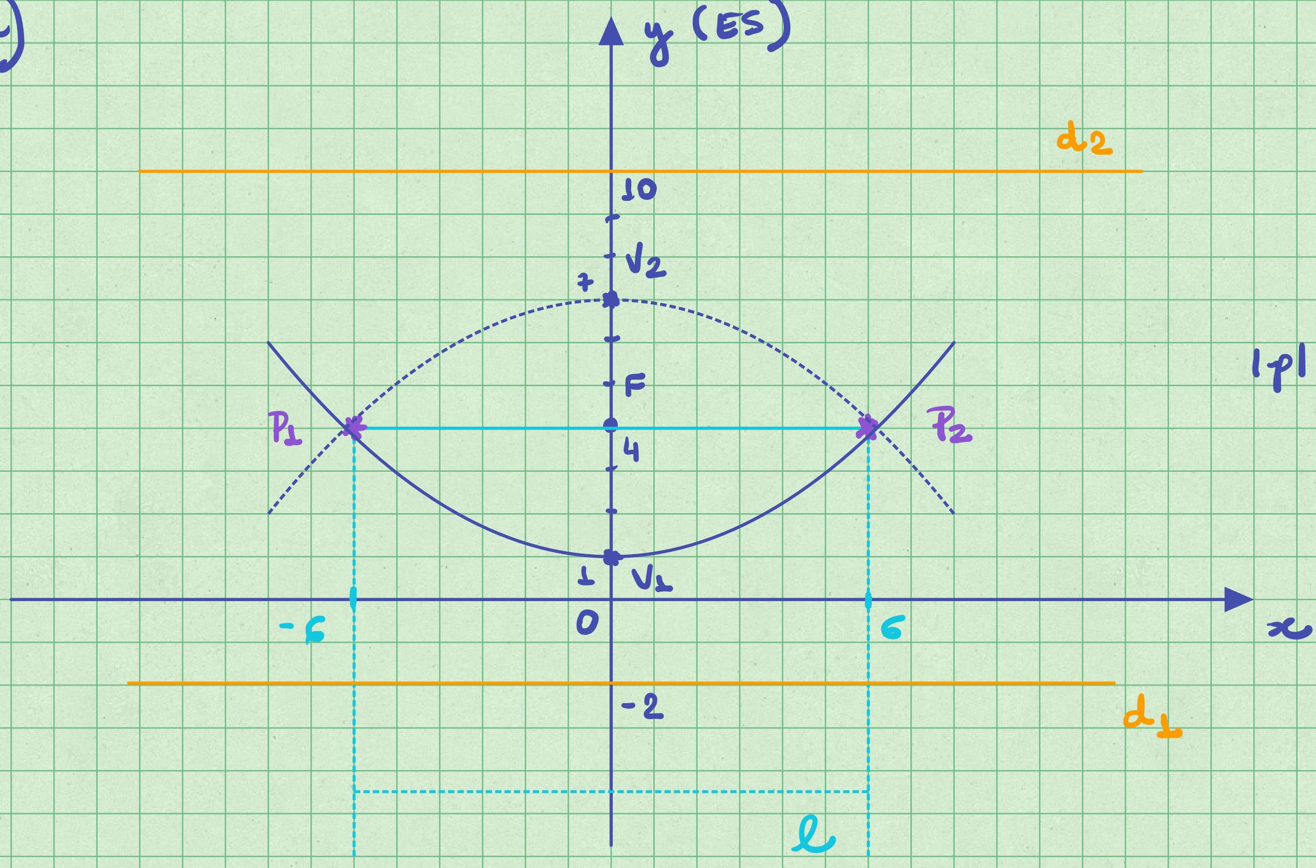
$$16(\cancel{a^2}+b^2) = \pm 8al + \cancel{16a^2}$$

$$16b^2 = \pm 8al$$

$$l = \pm \frac{2b^2}{a} \quad , \quad \text{mas } l > 0 \text{ , logo:}$$

$$l = \frac{2b^2}{a}$$

d)



$|p| = 3$

Parábola : P_1 , $P_2 \in$ parábola

$$\delta(P, F) = \delta(P, d) \quad , \quad P \in \text{parábola}$$

Parábola (1) : $V_1(0,1)$

Conc (+)

Eixo Oy

Parábola (2) : $V_2(0,7)$

Conc (−)

Eixo Oy

4.b $x^2 + ax + by + c = 0$ $\delta(F,V) = p$?

$-by = x^2 + ax + c$

$y = -\frac{1}{b}x^2 - \frac{a}{b}x - \frac{c}{b}$: F. Explícita

Eq. Geral (ES // Oy) : $(x-h)^2 = 4p(y-k)$

$x^2 - 2xh + h^2 = 4py - 4pk$

$4py = x^2 - 2xh + h^2 + 4pk$

$y = \frac{1}{4p}x^2 - \frac{h}{2p}x + \frac{1}{4p}(h^2 + 4pk)$

COMPARANDO

$y = -\frac{1}{b}x^2 - \frac{a}{b}x - \frac{c}{b}$

$\frac{1}{4p} = -\frac{1}{b}$

$p = -\frac{b}{4}$, $p > 0$ (distância) $\therefore$ $p = b/4$

concav (−)

5.d $\overset{A}{9x^2} - \overset{B}{16y^2} + \overset{D}{18x} - \overset{E}{64y} = -89$

FQ. $C = 0$ $\therefore$ $\nexists$ rotação

$D, E \neq 0$ $\therefore$ $\exists$ translação

1°) Id. cônica

$A * B = 0$: parábola ou degenerações

$A * B > 0$: elipse "

$A * B < 0$: hipérbole "

2º) Montar quad. perfeitos:

$$9\,(x^2 + 2x + 1) - 16\,(y^2 + 4y + 4) = -89 + 9 - 64$$

$\sqrt{\ }$ $a = x$ $\quad$ $+b^2$ $\quad$ $\sqrt{\ }$ $a = y$ $\quad$ $+b^2$

$2ab = 2x$ $\qquad$ $2ab = 4y$

$2ab = 2a$ $\qquad$ $2ab = 4a$

$b = 1$ $\qquad$ $b = 2$

$$9\,(x+1)^2 - 16\,(y+2)^2 = -144 \quad : -144$$

$$-\frac{(x+1)^2}{16} + \frac{(y+2)^2}{9} = 1$$

Hipérbole $\begin{cases} ER \parallel Oy \\ C(-1, -2) \\ a = 3;\ b = 4 \end{cases}$

⋮

Esboço