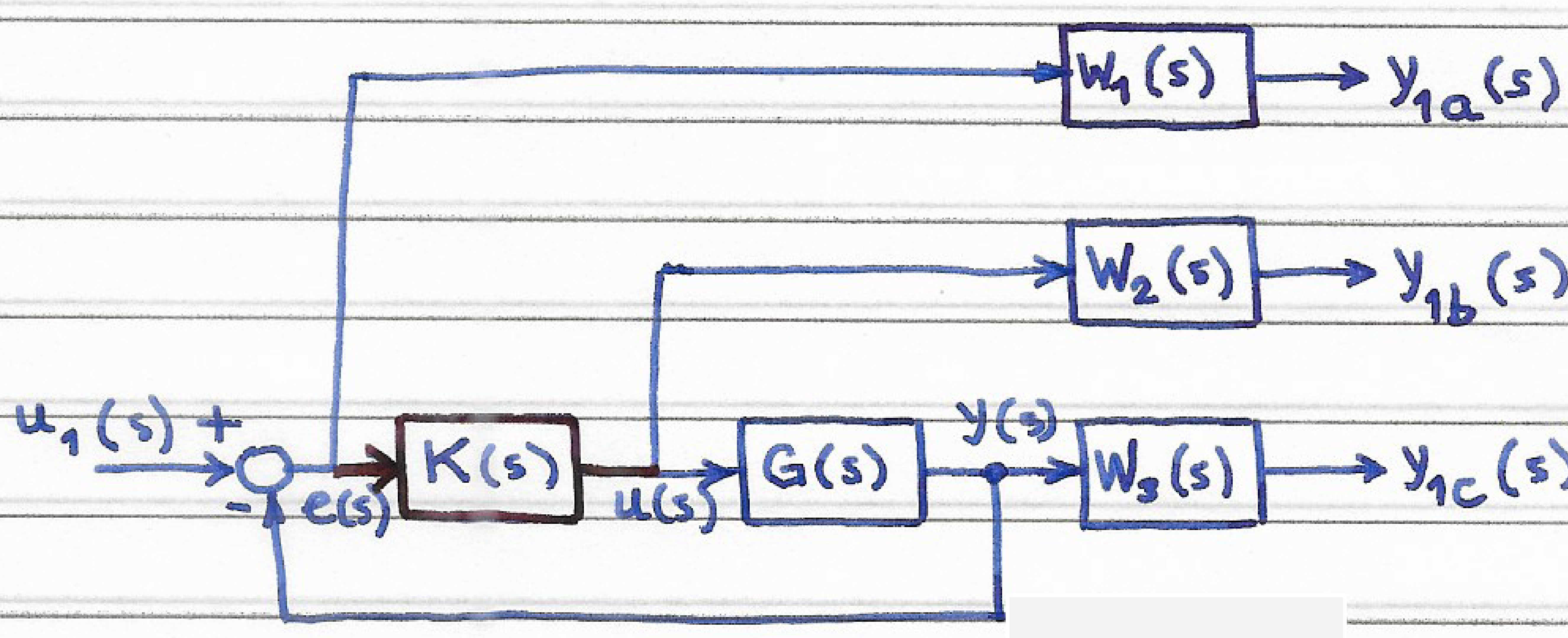
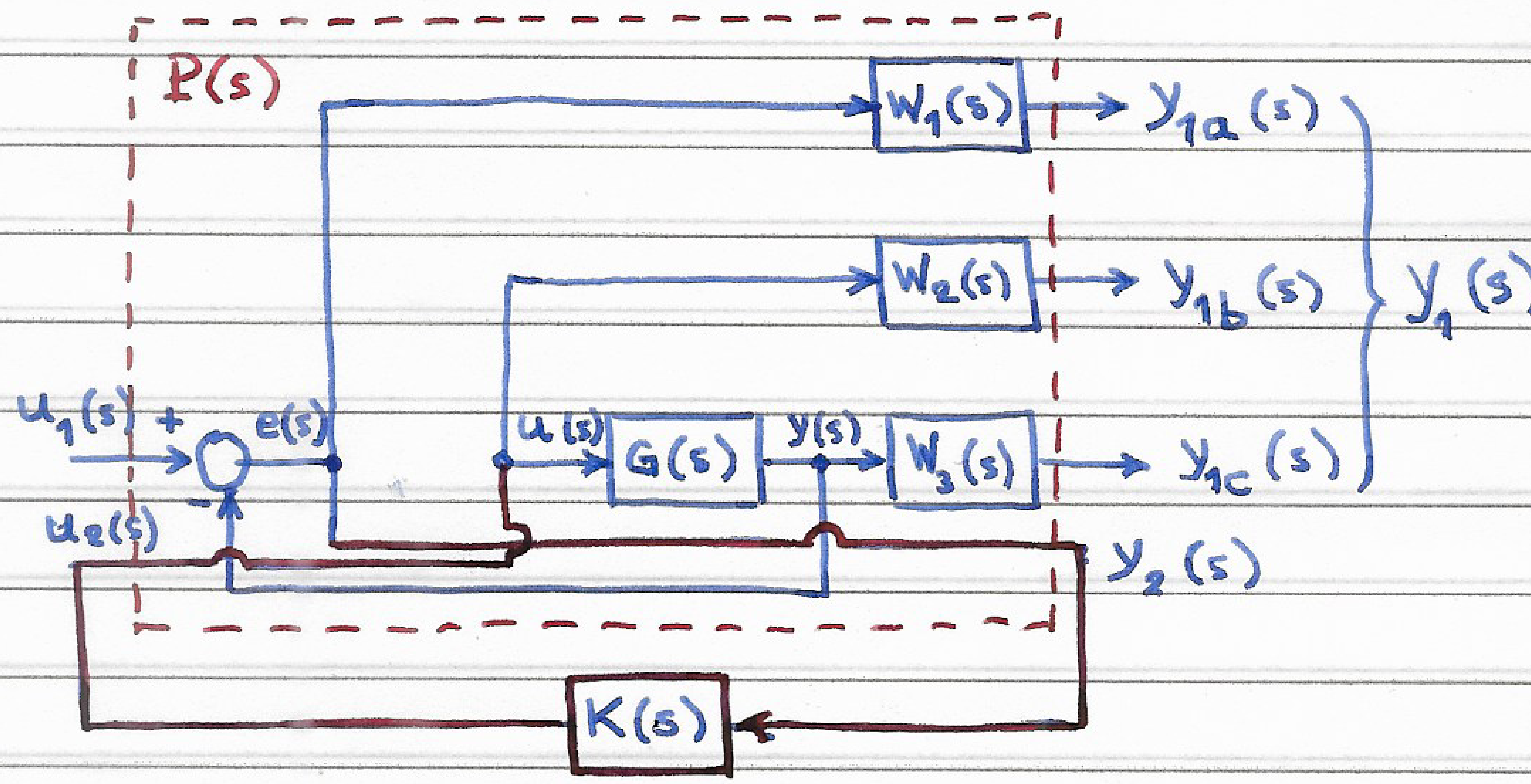
7.8 - UM NOVO PARADIGMA - A PLANTA GENERALIZADA



Redesenhando:



- $P(s)$: planta generalizada

- Entradas: $u_1(s)$, $u_2(s)$

- Saídas: $y_1(s) = \begin{bmatrix} y_{1a}(s) \\ y_{1b}(s) \\ y_{1c}(s) \end{bmatrix}$, $y_2(s)$

- $P(s)$ é conhecida e contém o modelo nominal, as especificações de desempenho robusto e as restrições referentes à robustez da estabilidade

- $\underset{(4\times1)}{y(s)} = \underset{(4\times2)}{P(s)}\,\underset{(2\times1)}{u(s)} \rightarrow P(s) = ?$

$$y_{1a}(s) = W_1(s)\,e(s) =$$

$$= W_1(s)\left[u_1(s) - y(s)\right] =$$

$$= W_1(s)\left[u_1(s) - G(s)\,u(s)\right] =$$

$$= W_1(s)\left[u_1(s) - G(s)\,u_2(s)\right] =$$

$$= \underline{W_1(s)\,u_1(s) - W_1(s)\,G(s)\,u_2(s)}$$

$$y_{1b}(s) = \underline{W_2(s)\,u_2(s)}$$

$$y_{1c}(s) = W_3(s)\,y(s) =$$

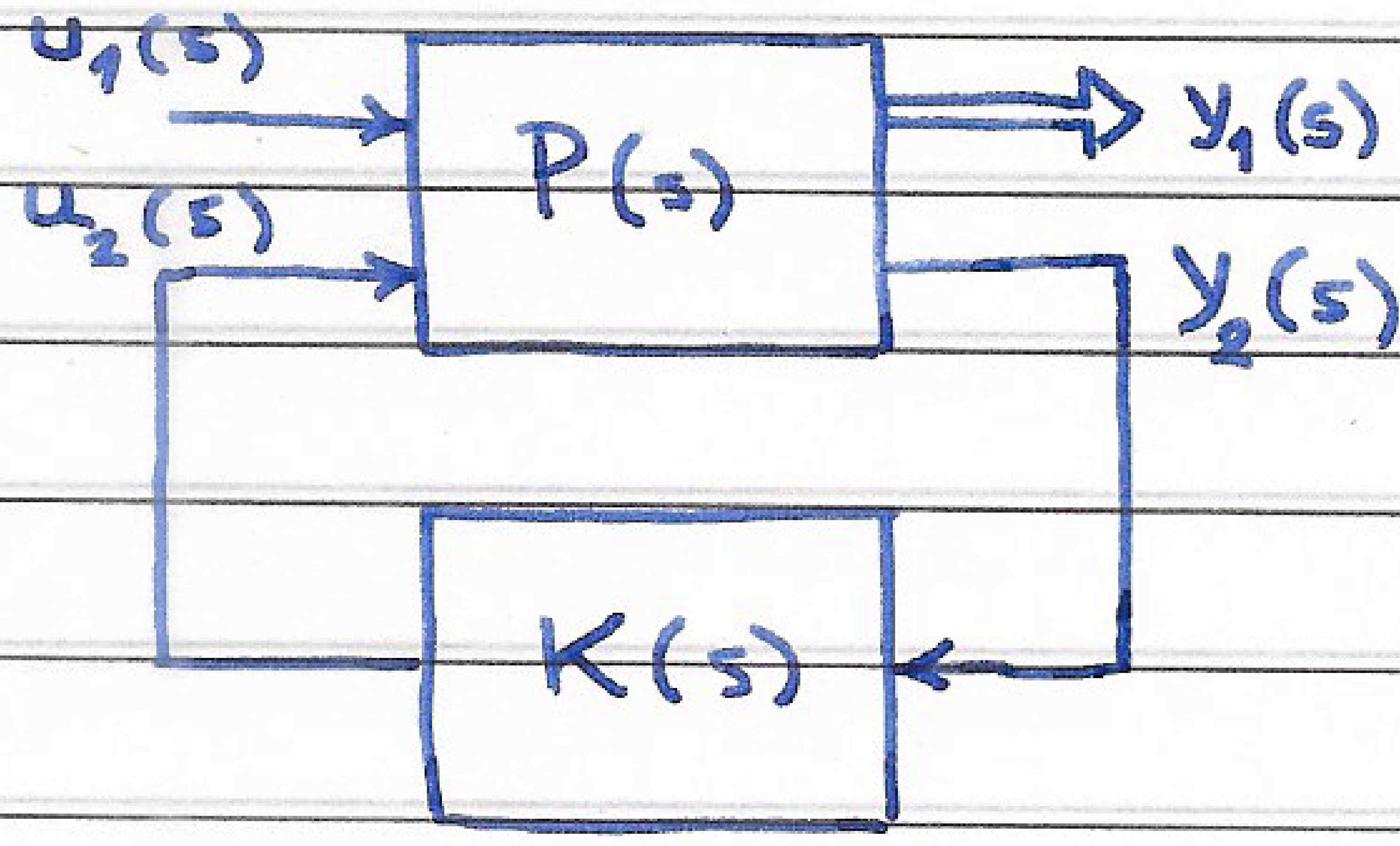
$$= \underline{W_3(s)\,G(s)\,u_2(s)}$$

$$y_2(s) = e(s) =$$

$$= \underline{u_1(s) - G(s)\,u_2(s)}$$

$$\boxed{\underbrace{\begin{bmatrix} \begin{bmatrix} y_{1a}(s) \\ y_{1b}(s) \\ y_{1c}(s) \end{bmatrix} \\ y_2(s) \end{bmatrix}}_{y(s)} = \underbrace{\begin{bmatrix} W_1(s) & -W_1(s)G(s) \\ 0 & W_2(s) \\ 0 & W_3(s)G(s) \\ 1 & -G(s) \end{bmatrix}}_{P(s)} \underbrace{\begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}}_{u(s)}}$$

- Novo diagrama:



- $P(s)$ contém:

$$G(s) \rightarrow \dim = n_G$$

$$W_1(s) \rightarrow \dim = n_{W_1}$$

$$W_2(s) \rightarrow \dim = n_{W_2}$$

$$W_3(s) \rightarrow \dim = n_{W_3}$$

$$P(s) \rightarrow \dim = n_G + n_{W_1} + n_{W_2} + n_{W_3} = n$$

- Modelo de estados de $P(s)$:

$$\underset{(n\times 1)}{\dot{x}(t)} = \underset{(n\times n)}{A}\,\underset{(n\times 1)}{x(t)} + \underset{(n\times 2)}{B}\,\underset{(2\times 1)}{u(t)}$$

$$\underset{(4\times 1)}{y(t)} = \underset{(4\times n)}{C}\,\underset{(n\times 1)}{x(t)} + \underset{(4\times 2)}{D}\,\underset{(2\times 1)}{u(t)}$$

- Partições

$$\dot{x}(t) = A\,x(t) + \underset{(n\times1)\;\;(n\times1)}{\left[\begin{array}{c:c} B_1 & B_2 \end{array}\right]} \left[\begin{array}{c} u_1(t) \\ \hdashline u_2(t) \end{array}\right]$$

$$\underset{(4\times1)}{y(t)} = \left[\begin{array}{c} y_1(t) \\ \hdashline y_2(t) \end{array}\right] \begin{array}{c} \updownarrow 3 \\ \updownarrow 1 \end{array} = \begin{array}{c} 3 \updownarrow \\ 1 \updownarrow \end{array} \underbrace{\left[\begin{array}{c} C_1 \\ \hdashline C_2 \end{array}\right]}_{n} \underset{(n\times1)}{x(t)} + \begin{array}{c} 3 \updownarrow \\ 1 \updownarrow \end{array} \underbrace{\left[\begin{array}{c:c} 0 & D_{12} \\ \hdashline D_{21} & 0 \end{array}\right]}_{1 \;\; 1} \left[\begin{array}{c} u_1 \\ \hdashline u_2 \end{array}\right]$$

- Portanto :

$$\underset{(n\times1)}{\dot{x}(t)} = \underset{(n\times n)}{A}\,\underset{(n\times1)}{x(t)} + \underset{(n\times1)}{B_1}\,\underset{(1\times1)}{u_1(t)} + \underset{(n\times1)}{B_2}\,\underset{(1\times1)}{u_2(t)}$$

$$\underset{(3\times1)}{y_1(t)} = \underset{(3\times n)}{C_1}\,\underset{(n\times1)}{x(t)} + \underset{(3\times1)}{D_{12}}\,\underset{(1\times1)}{u_2(t)}$$

$$\underset{(1\times1)}{y_2(t)} = \underset{(1\times n)}{C_2}\,\underset{(n\times1)}{x(t)} + \underset{(1\times1)}{D_{21}}\,\underset{(1\times1)}{u_1(t)}$$

- IMPORTANTE!

Sempre é possível obter uma realização de estados tal que

$$D = \left[\begin{array}{c:c} 0 & D_{12} \\ \hdashline D_{21} & 0 \end{array}\right]$$

Com isso, controlador (e demonstrações) mais simples.

- Notação para o modelo de estados de $P(s)$:

$$P(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

Serve para simplificar a escrita!

- Significado:

$$P(s) = C(sI - A)^{-1}B + D$$

- Para quê a forma de estados?

Resposta: O cálculo da solução do problema H_∞ Sub-Ótimo fica mais simples no domínio do tempo (variáveis de estado):

7.9 - UMA SOLUÇÃO DO PROBLEMA H_∞ SUB-ÓTIMO

Ver Notas de Aula, pgs. 106-107.