

Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

30 de junho de 2021
Magnetostática

Potencial vector

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad \vec{\nabla} \cdot \vec{A} = 0$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\left[\nabla^2 V = -\frac{\rho}{\epsilon_0} \right]$$

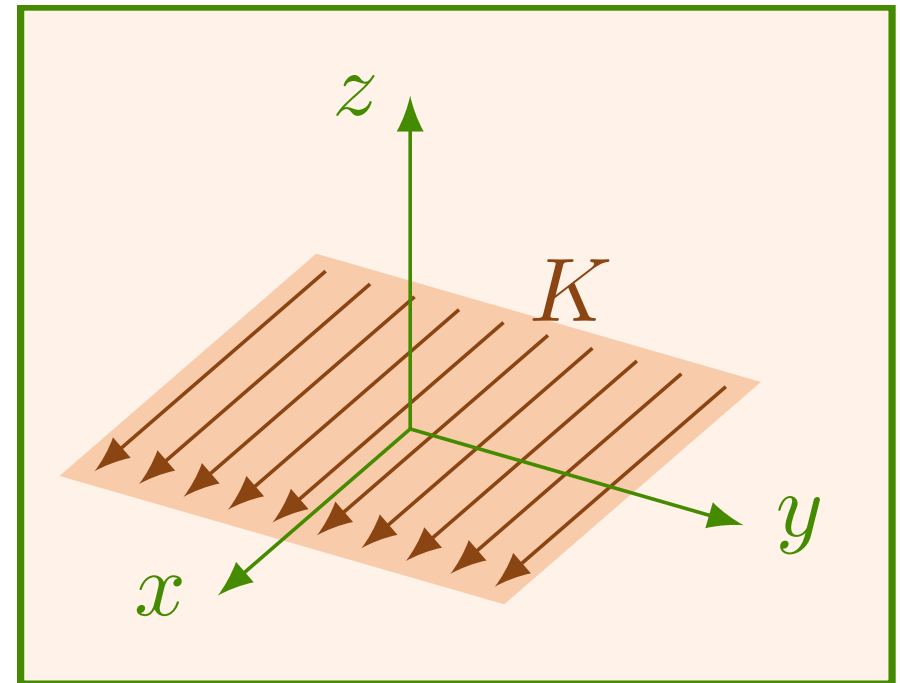
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau'$$

$$\left[V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{r} d\tau' \right]$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

Pratique o que aprendeu



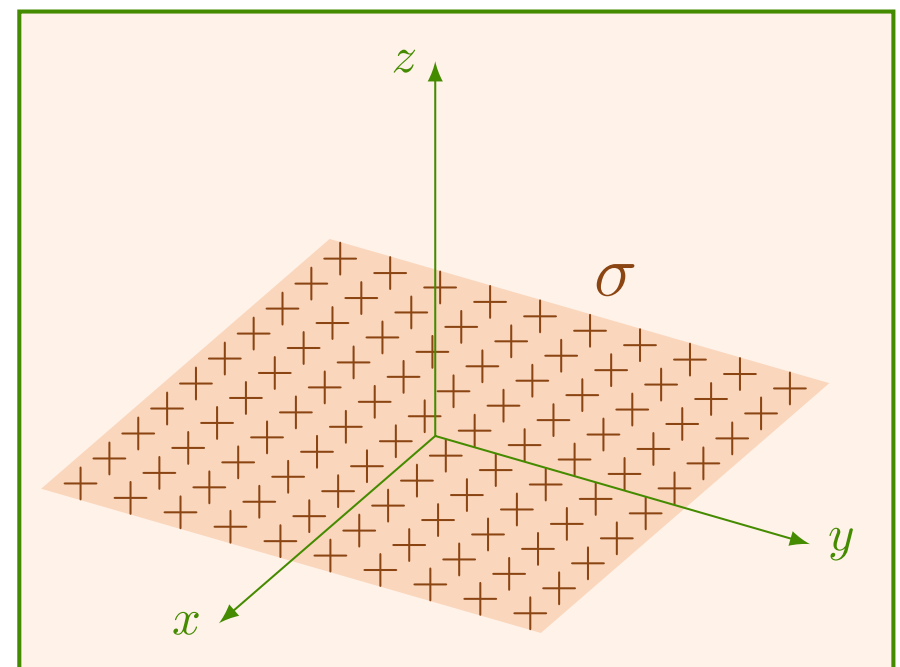
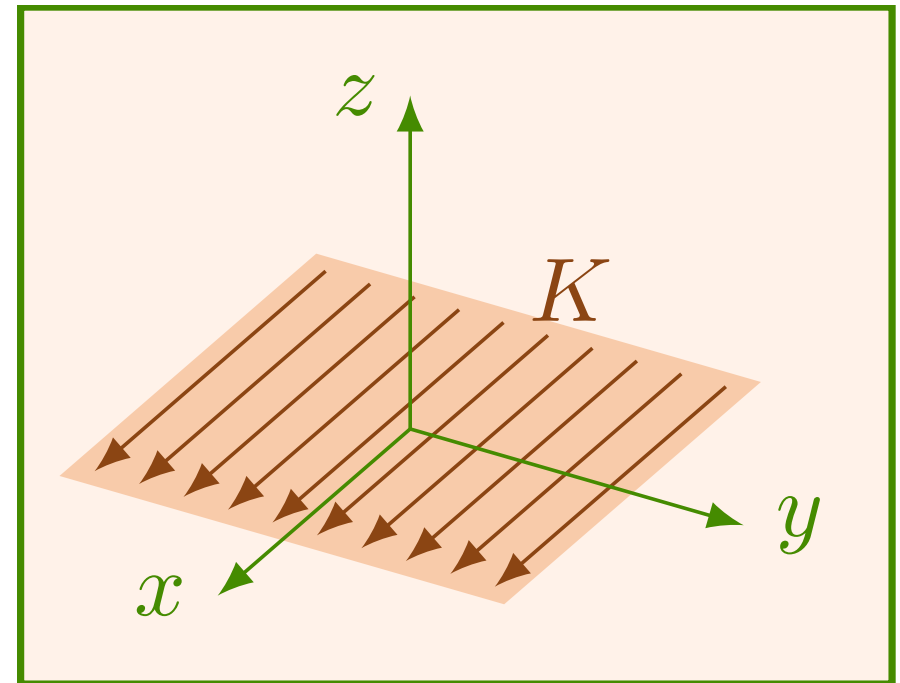
$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

Pratique o que aprendeu

$$\vec{K} = K \hat{x}$$

$$A_x(\vec{r}) \leftrightarrow V(\vec{r})$$



$$\vec{B} = \vec{\nabla} \times \vec{A}$$

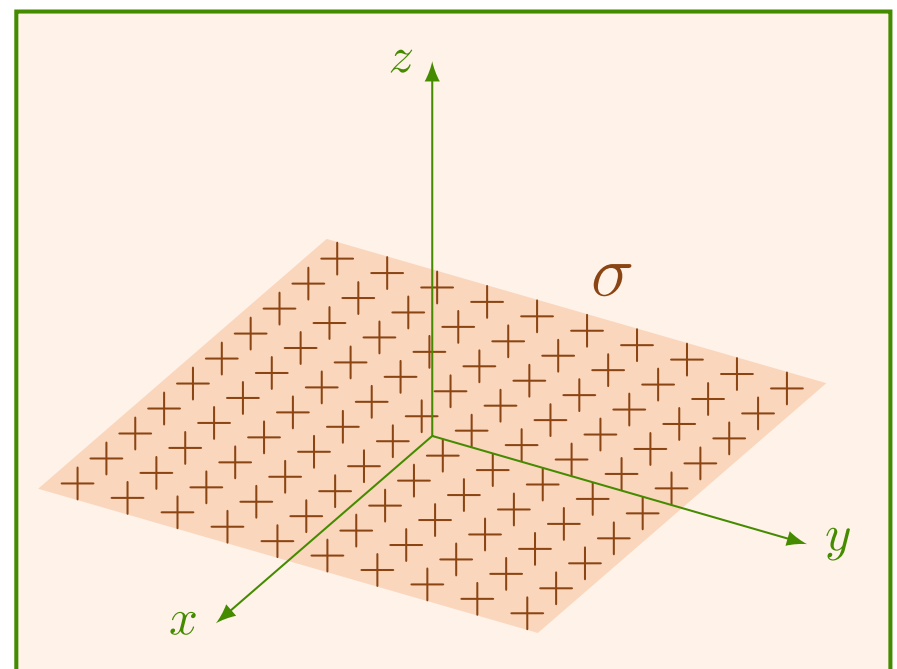
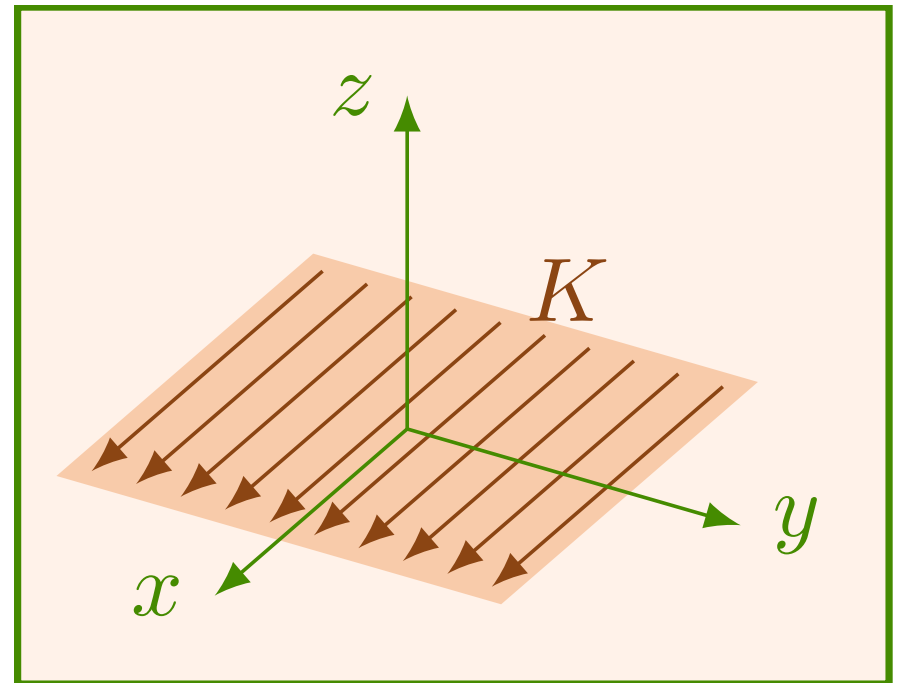
$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

Pratique o que aprendeu

$$\vec{K} = K \hat{x}$$

$$A_x(\vec{r}) \leftrightarrow V(\vec{r})$$

$$V(\vec{r}) = -\frac{\sigma}{2\epsilon_0} z$$



$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

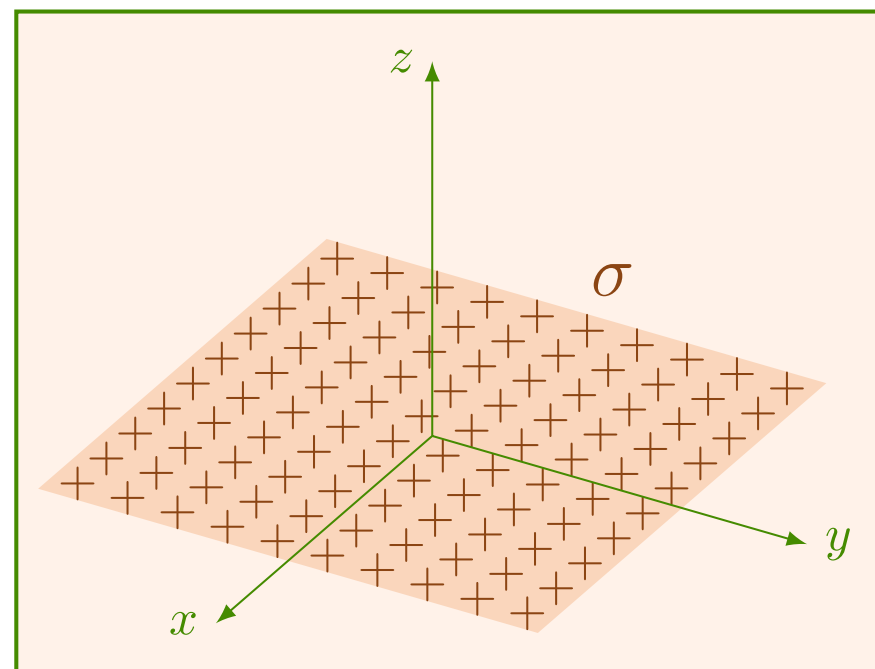
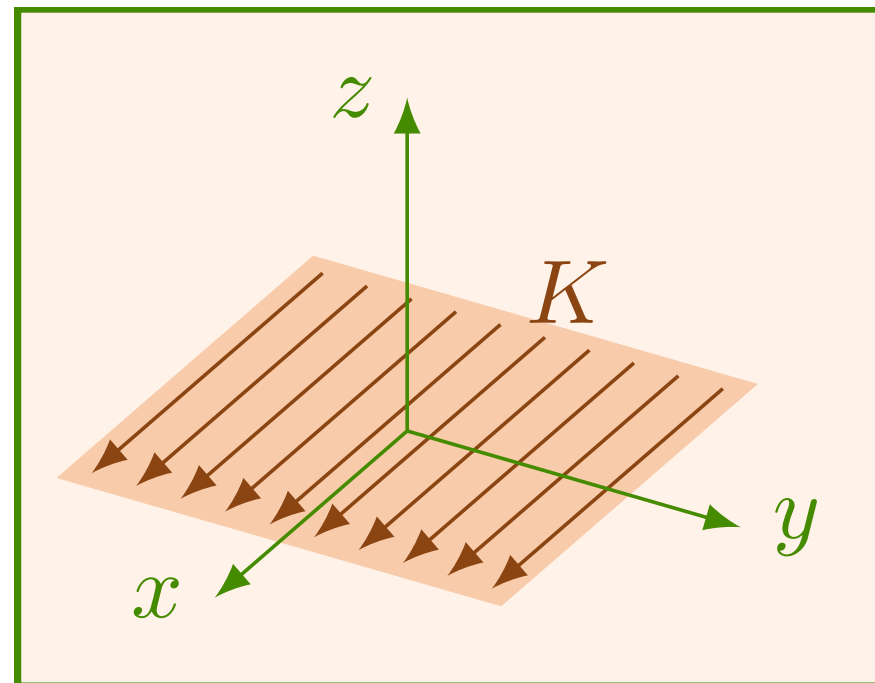
Pratique o que aprendeu

$$\vec{K} = K \hat{x}$$

$$A_x(\vec{r}) \leftrightarrow V(\vec{r})$$

$$V(\vec{r}) = -\frac{\sigma}{2\epsilon_0} z$$

$$\Rightarrow A_x(\vec{r}) = -\mu_0 \frac{K}{2} z$$



$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

Pratique o que aprendeu

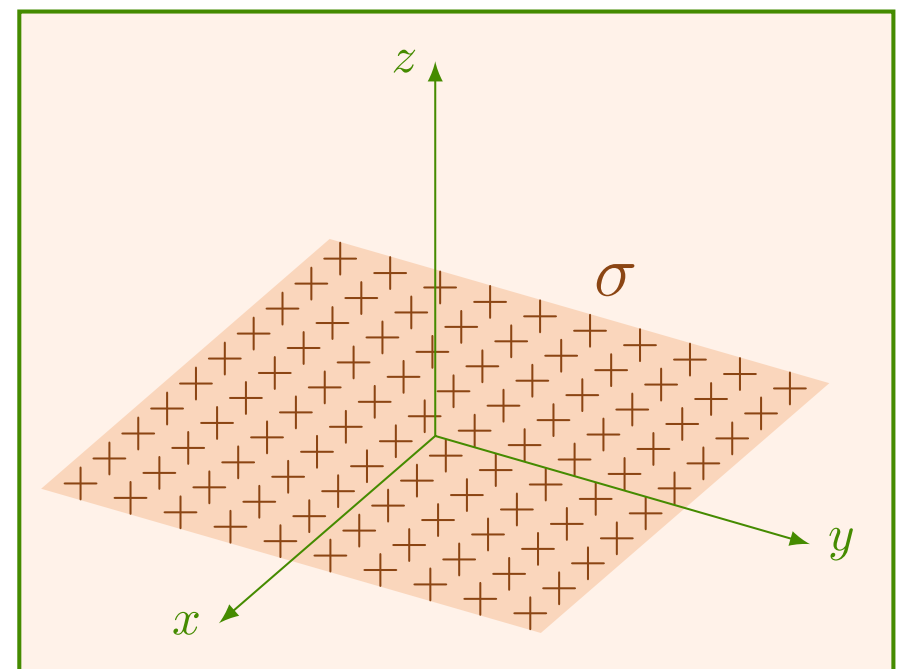
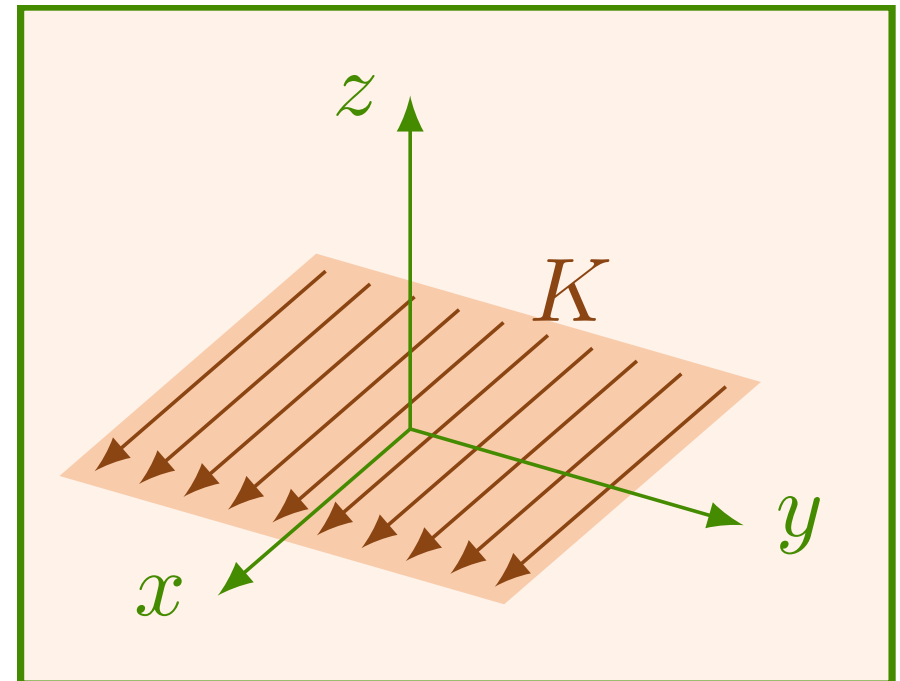
$$\vec{K} = K \hat{x}$$

$$A_x(\vec{r}) \leftrightarrow V(\vec{r})$$

$$V(\vec{r}) = -\frac{\sigma}{2\epsilon_0} z$$

$$\Rightarrow A_x(\vec{r}) = -\mu_0 \frac{K}{2} z$$

$$\vec{B} = -\mu_0 \frac{K}{2} \hat{y}$$



$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

Pratique o que aprendeu

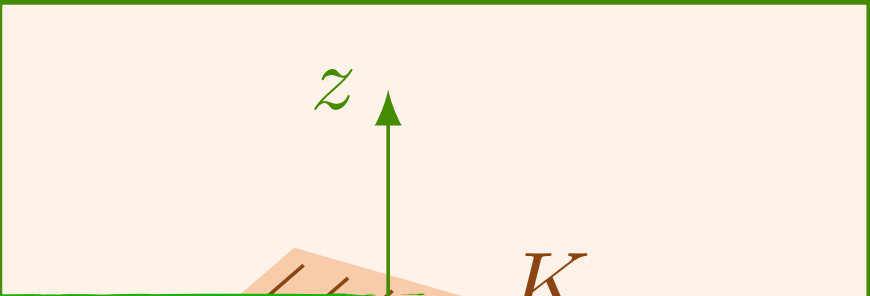
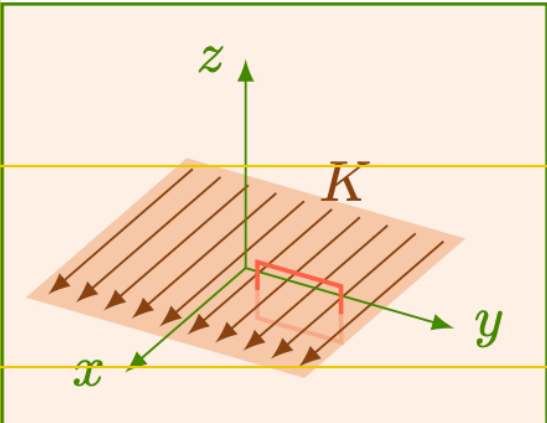
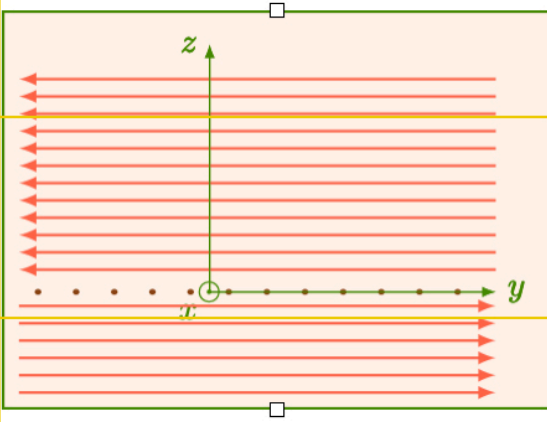
$$\vec{K} = K \hat{x}$$

$$A_x(\vec{r}) \leftrightarrow V(\vec{r})$$

$$V(\vec{r}) = -\frac{\sigma}{2\epsilon_0} z$$

$$\Rightarrow A_x(\vec{r}) = -\mu_0 \frac{K}{2} z$$

$$\vec{B} = -\mu_0 \frac{K}{2} \hat{y}$$

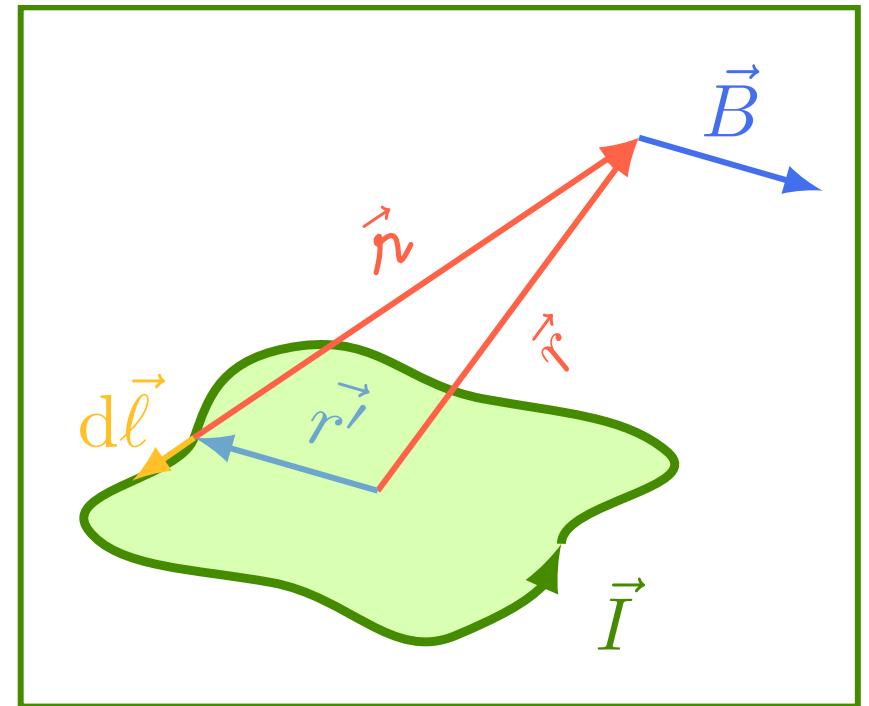
| Pratique o que aprendeu | |
|---|---|
| $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{int}}$ |  |
| $B_{2l} = \mu_0 K l$ |  |
| $B = \mu_0 \frac{K}{2}$ |  |

Expansão multipolar

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad \vec{\nabla} \cdot \vec{A} = 0$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau'$$

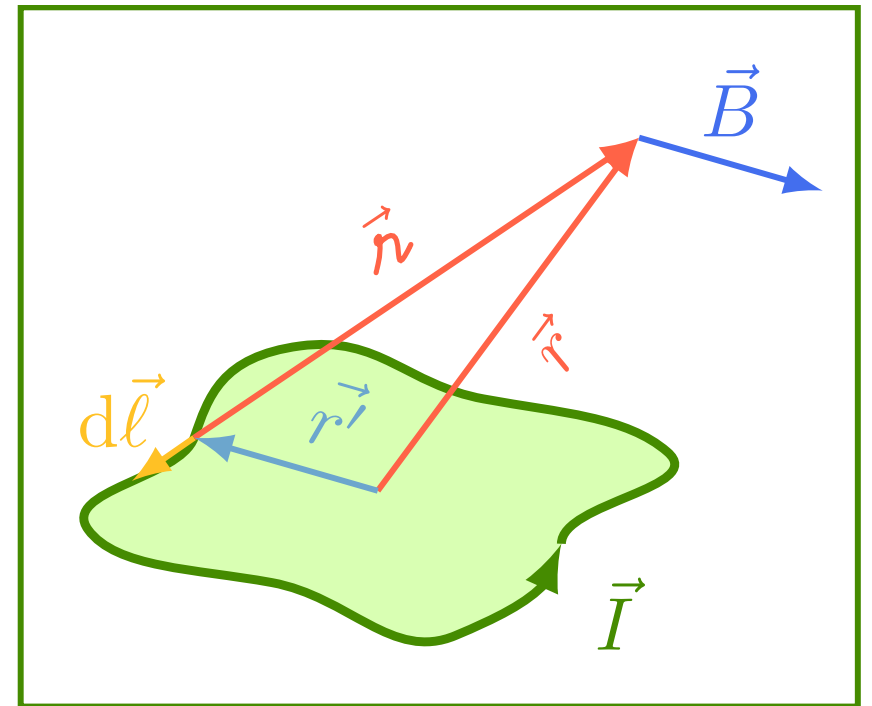


Expansão multipolar

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad \vec{\nabla} \cdot \vec{A} = 0$$

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$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau'$$



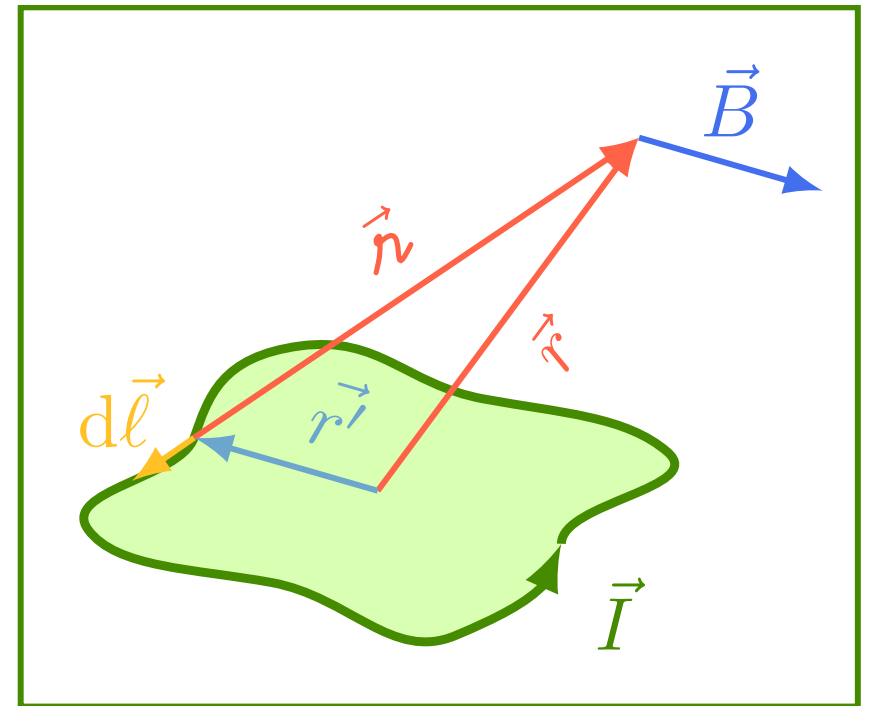
$$\frac{1}{r} = \frac{1}{r} \sum_{\ell=0}^{\infty} \left(\frac{r'}{r}\right)^{\ell} P_{\ell}(\cos \theta')$$

Expansão multipolar

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad \vec{\nabla} \cdot \vec{A} = 0$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau' = \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r}')}{r} d\ell'$$



$$\frac{1}{r} = \frac{1}{r} \sum_{\ell=0}^{\infty} \left(\frac{r'}{r}\right)^{\ell} P_{\ell}(\cos \theta')$$

$$\ell = 0 \Rightarrow \frac{1}{r} \approx \frac{1}{r} \Rightarrow$$

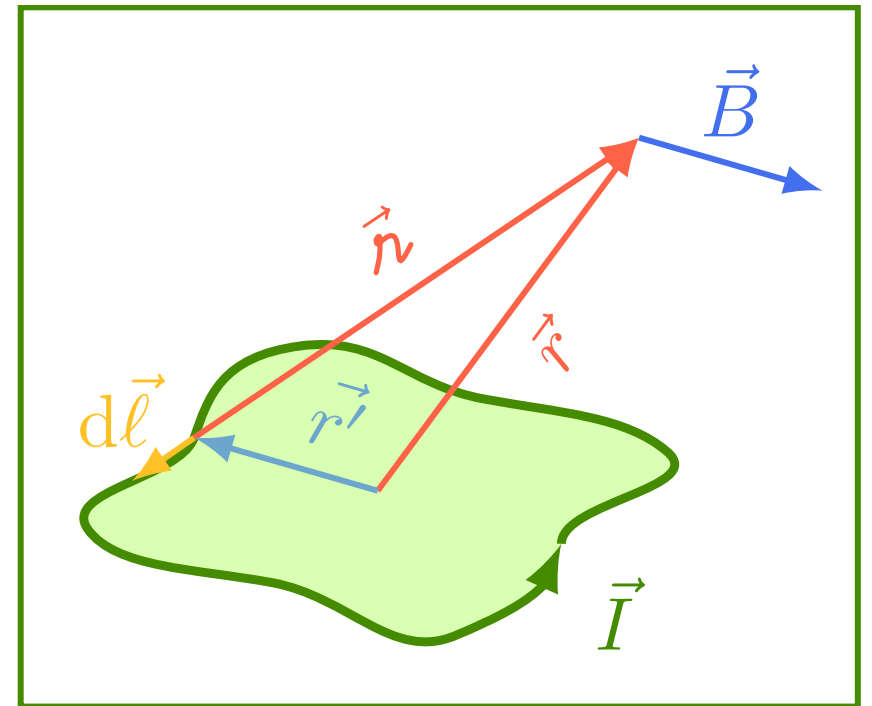
$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi r} \oint d\vec{\ell}'$$

Expansão multipolar

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad \vec{\nabla} \cdot \vec{A} = 0$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}')}{r} d\tau' = \frac{\mu_0}{4\pi} \int \frac{\vec{I}(\vec{r}')}{r} d\ell'$$



$$\frac{1}{r} = \frac{1}{r} \sum_{\ell=0}^{\infty} \left(\frac{r'}{r}\right)^{\ell} P_{\ell}(\cos \theta')$$

$$\ell \leq 1 \quad \Rightarrow \quad \frac{1}{r} \approx \frac{1}{r} + \frac{r'}{r^2} \cos \theta \quad \Rightarrow \quad \vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi r} \oint d\vec{\ell}' + \frac{\mu_0}{4\pi r^2} \oint r' \cos \theta' d\vec{\ell}$$

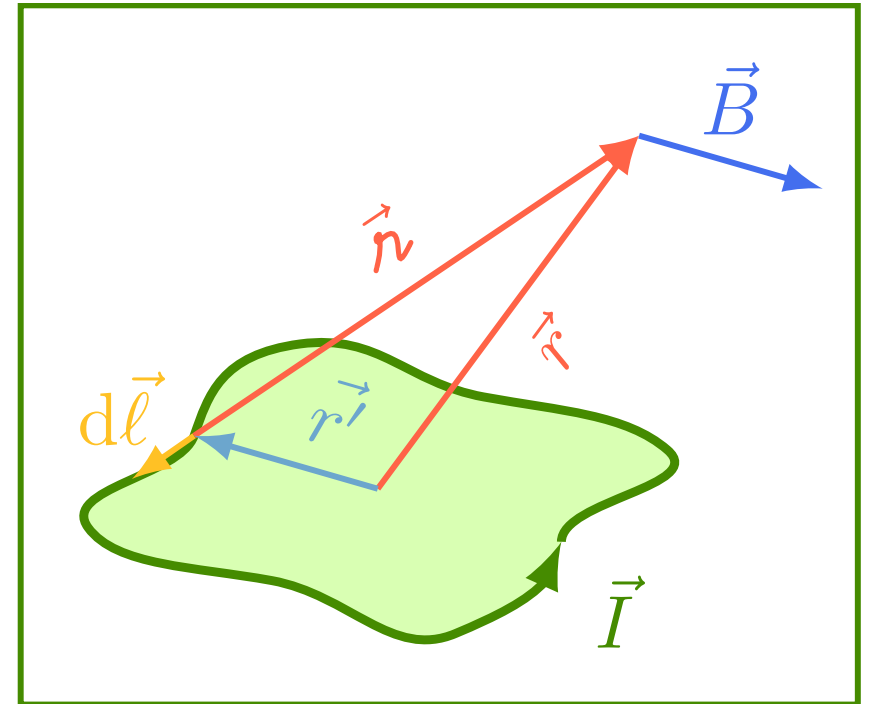
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

Expansão multipolar

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad \vec{\nabla} \cdot \vec{A} = 0$$

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2} \quad \vec{m} = I \vec{a}$$



Pratique o que aprendeu

$$\vec{m} = I\vec{a}$$

$$\vec{m} = ?$$

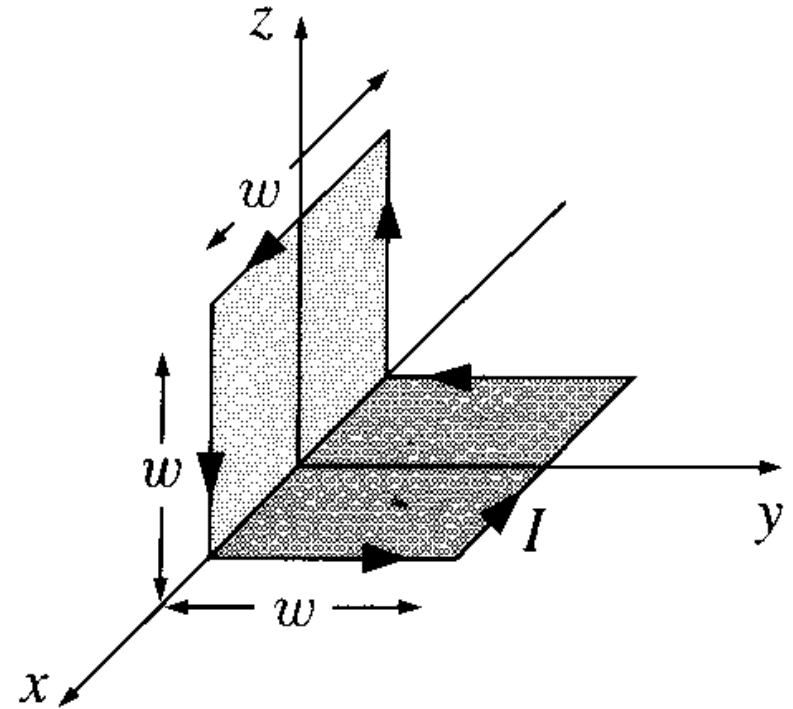


Figure 5.52

Pratique o que aprendeu

$$\vec{m} = I\vec{a}$$

$$\vec{m} = Iw^2(\hat{y} + \hat{z})$$

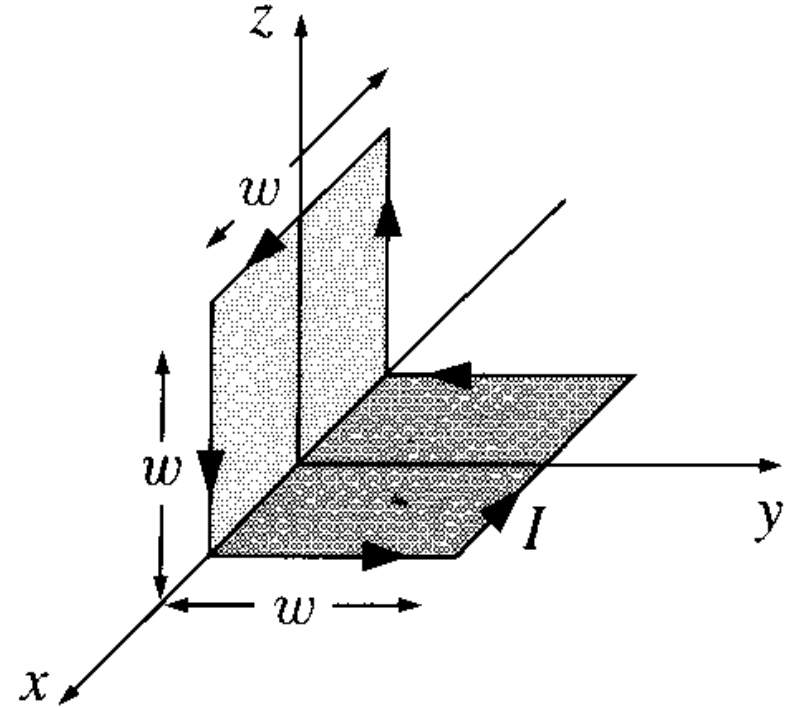
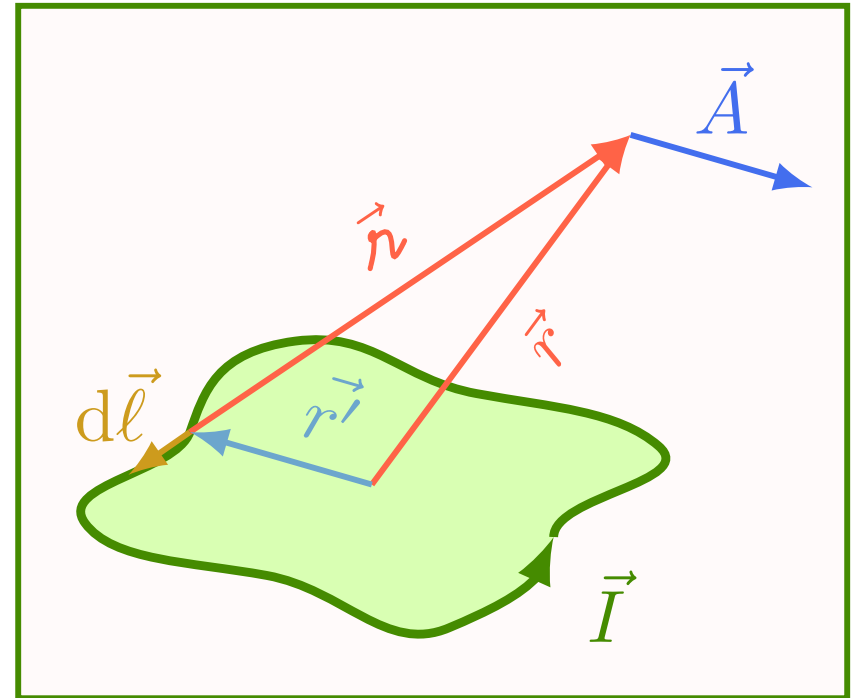


Figure 5.52

Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$\vec{m} = m\hat{z}$$



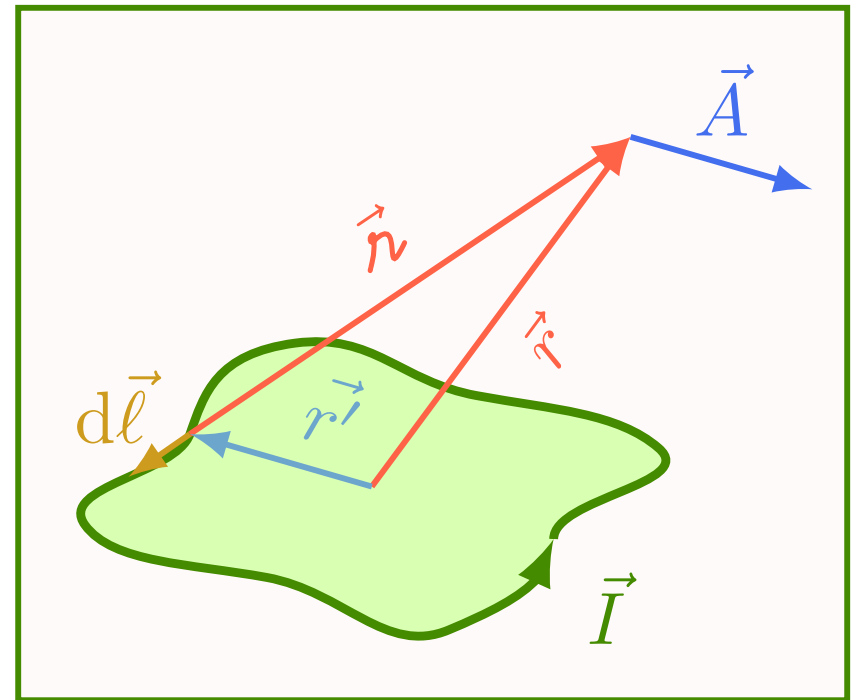
Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$\vec{m} = m\hat{z}$$

$$\vec{r} = s\hat{s} + z\hat{z}$$

$$\vec{m} \times \hat{r} = s\vec{m} \times \hat{s}$$



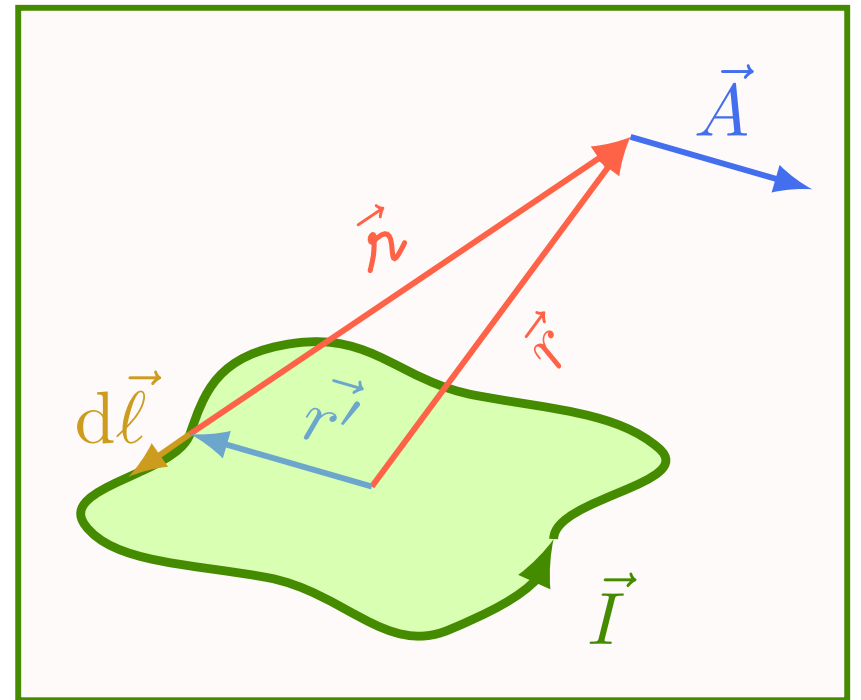
Expansão multipolar

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$\vec{m} = m\hat{z}$$

$$\vec{r} = s\hat{s} + z\hat{z}$$

$$\vec{m} \times \hat{r} = s\vec{m} \times \hat{s} = sm\hat{\phi}$$



Expansão multipolar

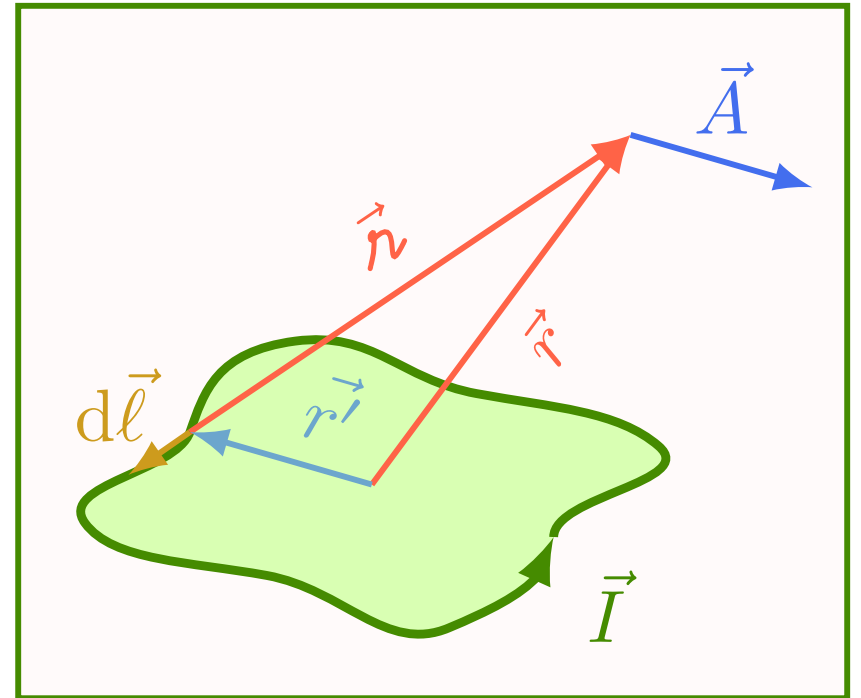
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

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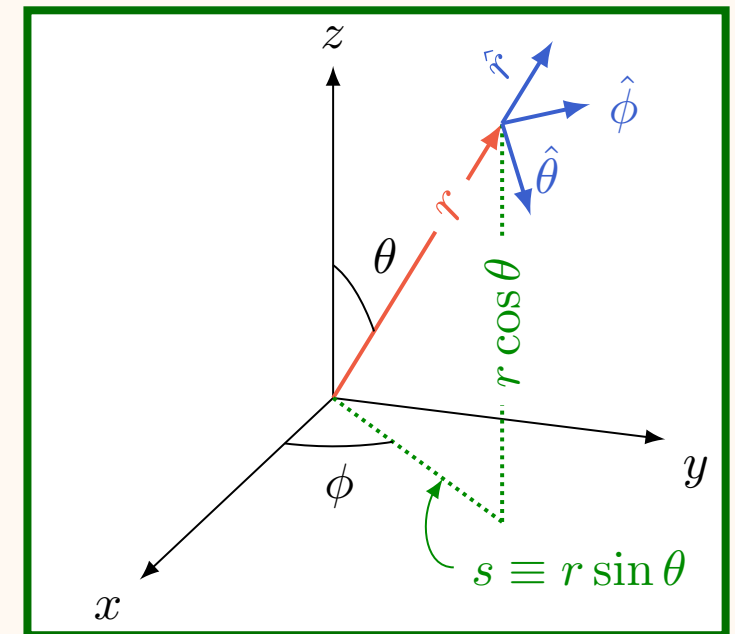
$$\vec{A} = \frac{\mu_0}{4\pi} \frac{m \sin \theta}{r^2} \hat{\phi}$$



Coordenadas esféricas

$$d\vec{\ell} = dr \hat{r} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}$$

$$\vec{\nabla} t = \frac{\partial t}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial t}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial t}{\partial \phi} \hat{\phi}$$



$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2} \frac{\partial (r^2 v_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (\sin \theta v_\theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$$

$$\begin{aligned} \vec{\nabla} \times \vec{v} = & \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta v_\phi) - \frac{\partial v_\theta}{\partial \phi} \right] \hat{r} \\ & + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi} \end{aligned}$$

$$\nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}$$

Expansão multipolar

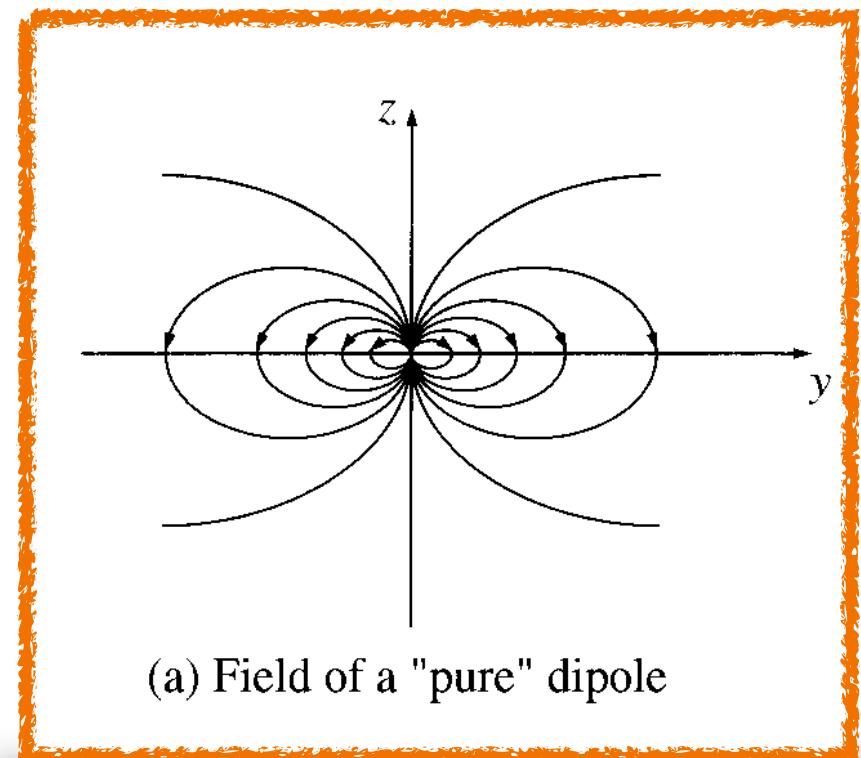
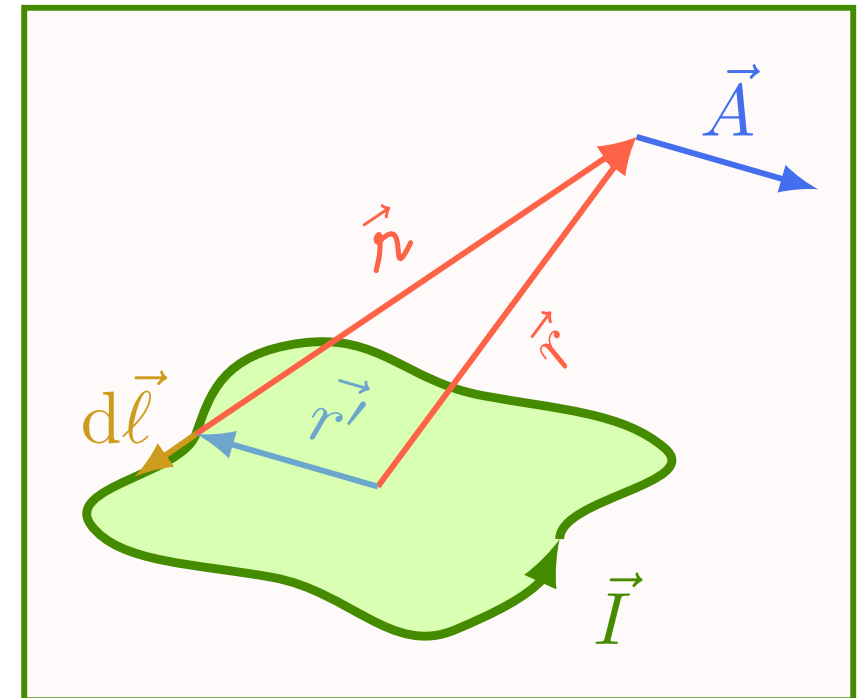
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$\vec{m} = m\hat{z}$$

$$\vec{r} = s\hat{s} + z\hat{z}$$

$$\vec{m} \times \hat{r} = s\vec{m} \times \hat{s} = sm\hat{\phi}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{m \sin \theta}{r^2} \hat{\phi}$$



Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

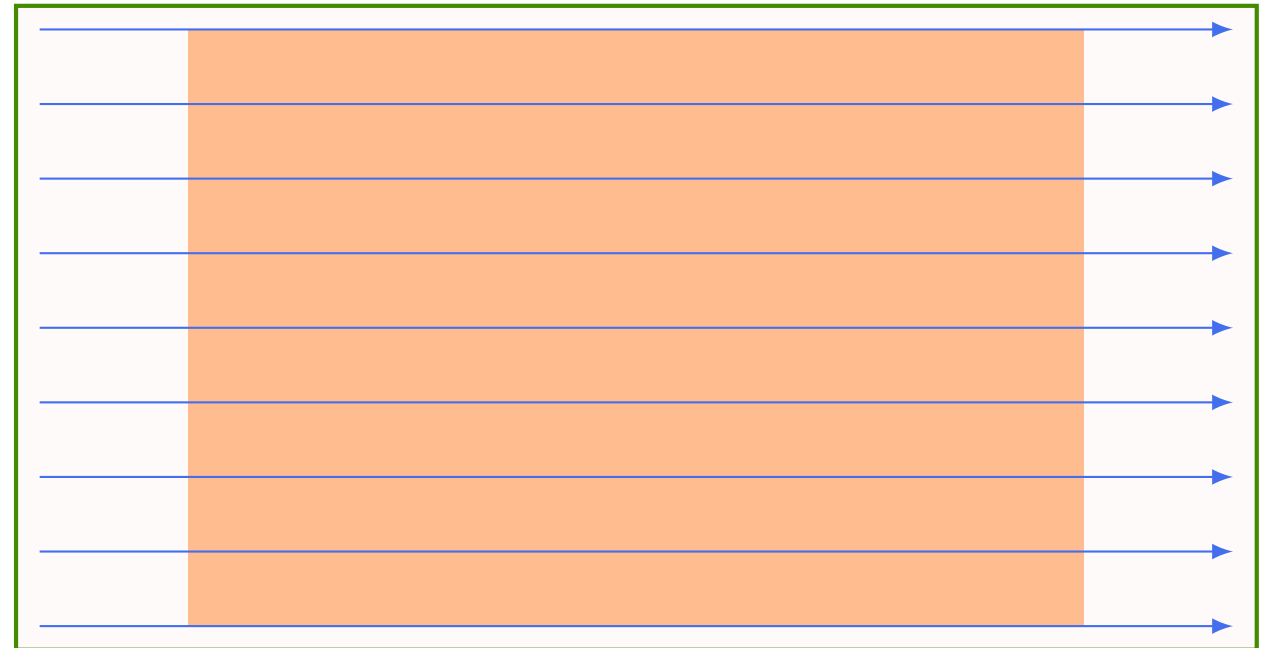
$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

30 de junho de 2021

Magnetismo em materiais

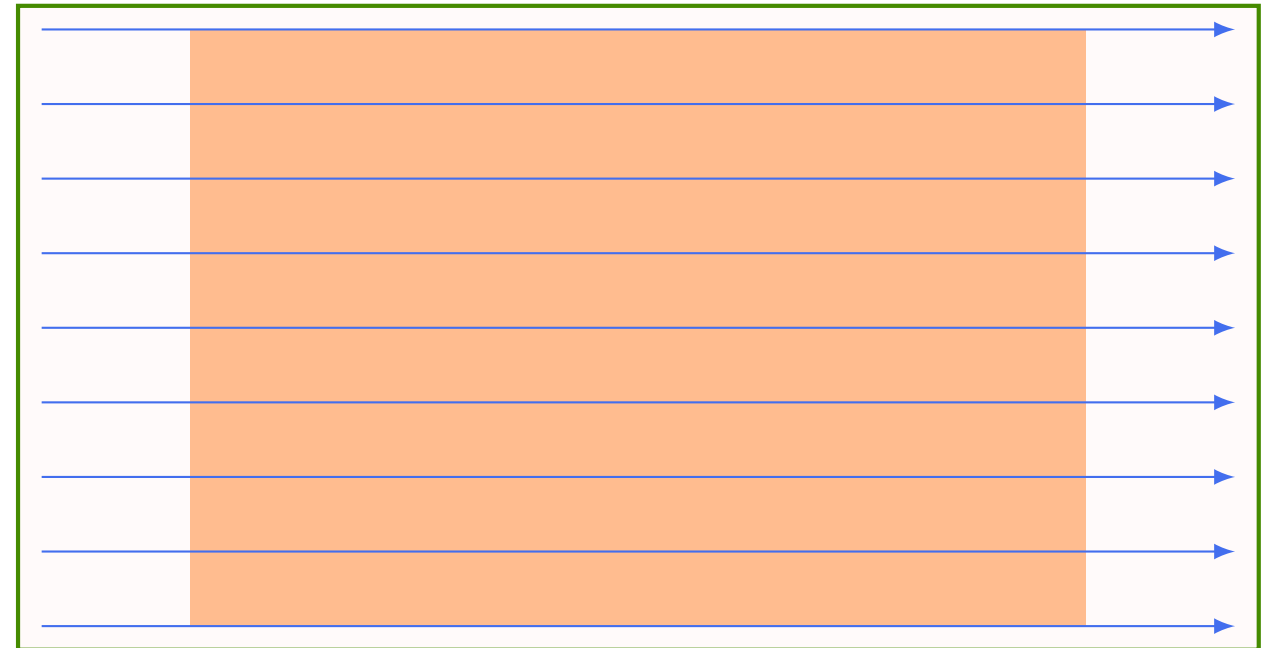
Magnetismo em materiais

- Paramagnéticos
- Diamagnéticos
- Ferromagnéticos
- Antiferromagnéticos



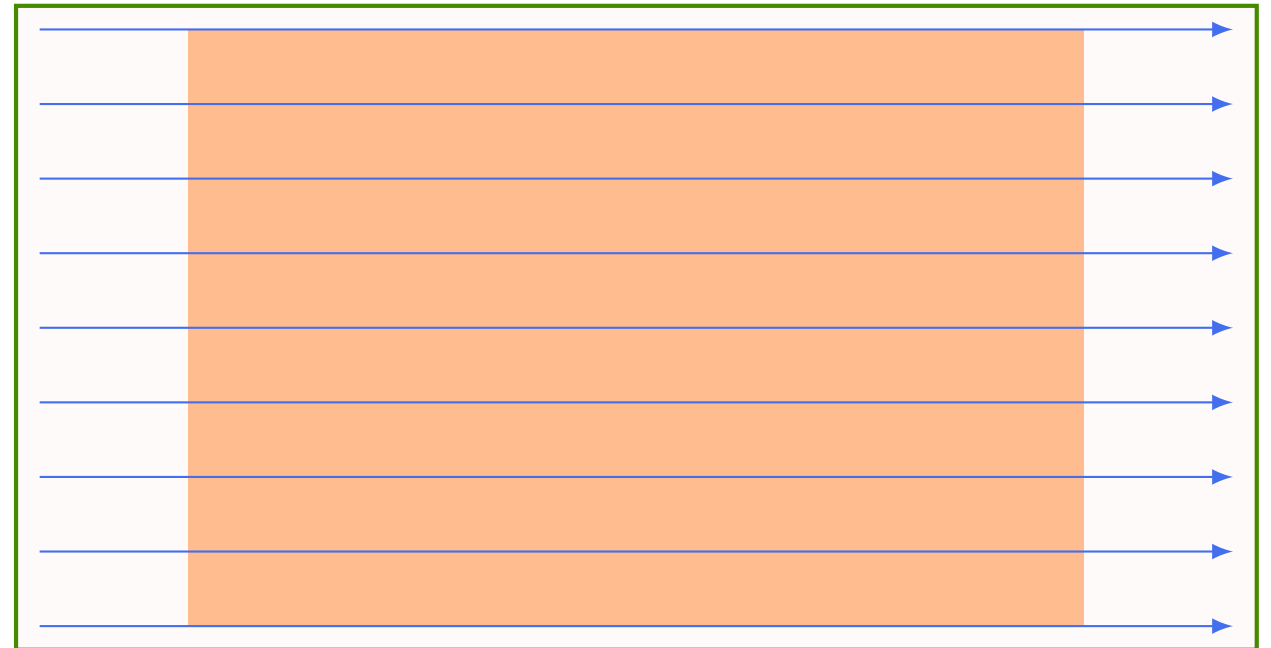
Magnetismo em materiais

• Paramagnéticos



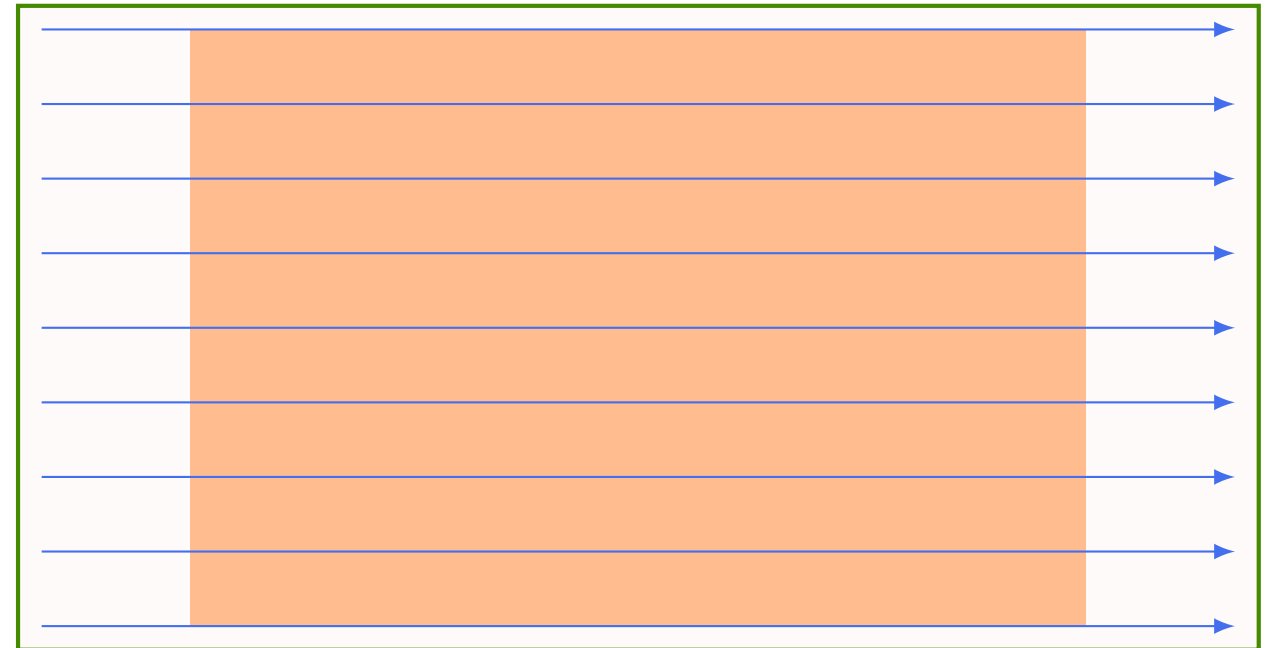
Magnetismo em materiais

• Diamagnéticos



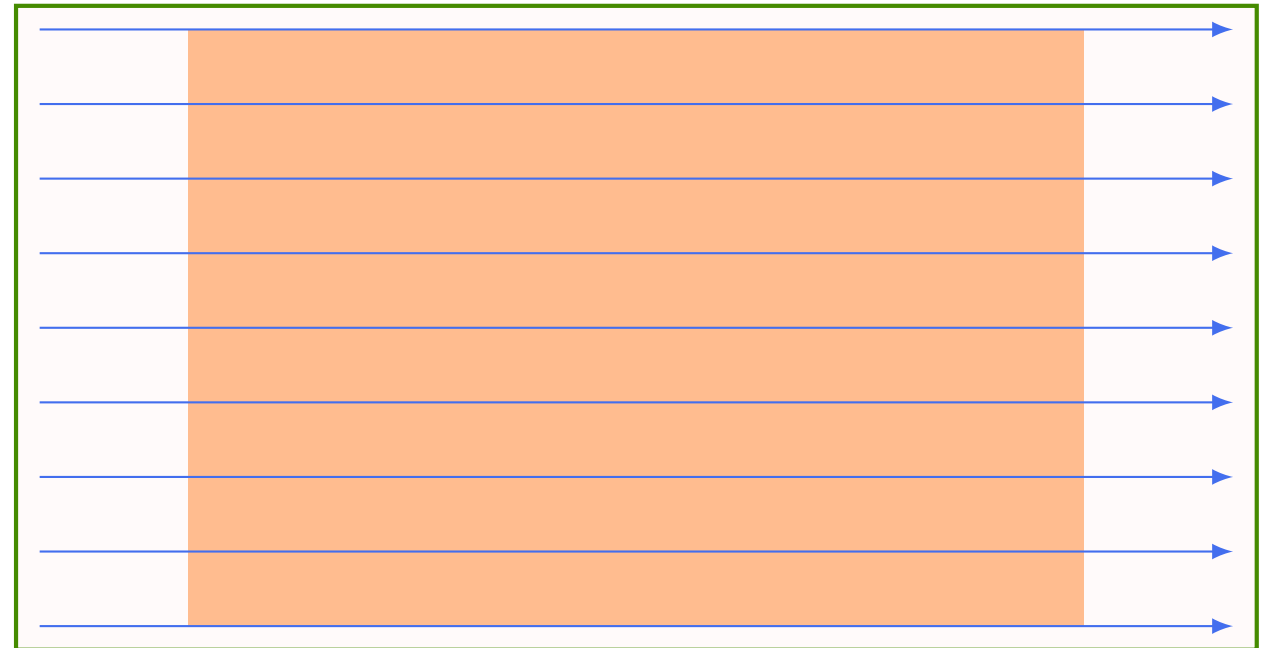
Magnetismo em materiais

• Ferromagnéticos



Magnetismo em materiais

• Antiferromagnéticos



Magnetização

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

