Monitoria Mat 4 17/06

k-forma é $\omega = \sum_{i_1,\dots,i_k} f_{i_1,\dots,i_k}\, dx_{i_1} \wedge \dots \wedge dx_{i_k}$

$f_{i_1,\dots,i_k} : U \to \mathbb{R}$ de classe C^r

$dx_j \wedge dx_i \wedge dx_k = -dx_i \wedge dx_j \wedge dx_k$

$i_1, \dots, i_k \in \{1, \dots, n\}$ t.q. $i_1 < i_2 < \dots < i_k$

$\Omega^k(U) = \{k\text{-formas em } U\}$

$d : \Omega^k(U) \to \Omega^{k+1}(U)$ (derivada exterior)

$d(f\, dx_{i_1} \wedge \dots \wedge dx_{i_k}) = \underbrace{\sum_{j=1}^{n} \left(\frac{\partial f}{\partial x_j}\right) dx_j}_{df_j} \wedge dx_{i_1} \wedge \dots \wedge dx_{i_k}$

$$d\left(\sum f_{i_1,\dots,i_k}\, dx_{i_1}\wedge\dots\wedge dx_{i_k}\right) =$$

$$\sum d\left(f_{i_1,\dots,i_k}\, dx_{i_1}\wedge\dots\wedge dx_{i_k}\right)$$

Ja definimes

$k=0$:

$$df = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}\, dx_i \cong \nabla f$$

$k=n-1$:

$$\widehat{dx_i} = dx_1\wedge dx_2\wedge\dots\wedge dx_{i-1}\wedge dx_{i+1}\wedge\dots\wedge dx_n$$

ω $n-1$ forma $\cong F:\mathbb{R}^n\to\mathbb{R}^n$

$$\omega = F_1\,\widehat{dx_1}+\dots+F_n\,\widehat{dx_n}$$

$$d\omega_F = \nabla \cdot F \; dx_1 \wedge \cdots \wedge dx_n = \nabla \cdot F \, dV$$

<u>$k=1, n=3$:</u>

ω 1-forma $= f_1 dx_1 + \cdots + f_n dx_n \cong (f_1, \ldots, f_n)$

$n=3: \omega = F_x dx_1 + F_y dx_2 + F_z dx_3$

$$d\omega = (\nabla \times F)_1 \, dx_2 \wedge dx_3 + (\nabla \times F)_2 \, dx_3 \wedge dx_1$$
$$+ (\nabla \times F)_3 \, dx_1 \wedge dx_2$$

Teoremas de Green, Stokes e Gauss (e outros) em:

$$\int_{\partial M} \omega = \int_M d\omega$$

LISTA 2

13) $f: \mathbb{R}^2 \to \mathbb{R}$

$$f(x,y) = \begin{cases} \dfrac{x^3 y}{x^4+y^2} & , x,y \neq 0 \\ 0 & , x,y = 0 \end{cases}$$

$$0 \leq \| f(x,y) - f(0,0) \| \leq \text{TRAMBOLHO} \xrightarrow[x,y \to 0]{} 0$$

TEOR. DO CONFRONTO

$x,y \neq 0$

$$\left| \frac{x^3 y}{x^4+y^2} \right| = \frac{|x^3 y|}{x^4+y^2} = \frac{|x|^3 |y|}{\underbrace{|x|^4 + |y|^2}_{\geq |y|^2}} \leq \frac{|x|^3 |y|}{|y|^2} =$$

$$\frac{|x|^3}{|y|}$$

$g(t) = (t, t^4)$

$$\frac{|g_1(t)|^3}{|g_2(t)|} = \frac{|t|^3}{|t|^4} = \frac{1}{|t|} \to \infty$$

$$\left|\frac{g_1(t)^3\, g_2(t)}{g_1(t)^4+g_2(t)^2}\right| = \left|\frac{t^7}{t^4+t^8}\right| = \left|\frac{t^3}{1+t^4}\right| \to 0$$

$\nexists$ DIF:

$$\frac{\left|\frac{x^3y}{x^4+y^2}-0-0\right|}{\|(x,y)\|_1} = \frac{|x^3y|}{(|x|+|y|)(x^4+y^2)} =$$

$$= \frac{|x|^3|y|}{(|x|+|y|)(|x|^4+|y|^2)} \qquad g(t)=(t,t^2)$$

$$\hookrightarrow \frac{|t|^3|t^2|}{(|t|+|t|^2)(|t|^4+|t^2|^2)} = \frac{|t|^5}{2|t|^5+|t|^6} = \frac{1}{2+|t|}$$

$$\downarrow \frac{1}{2}$$