

# Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

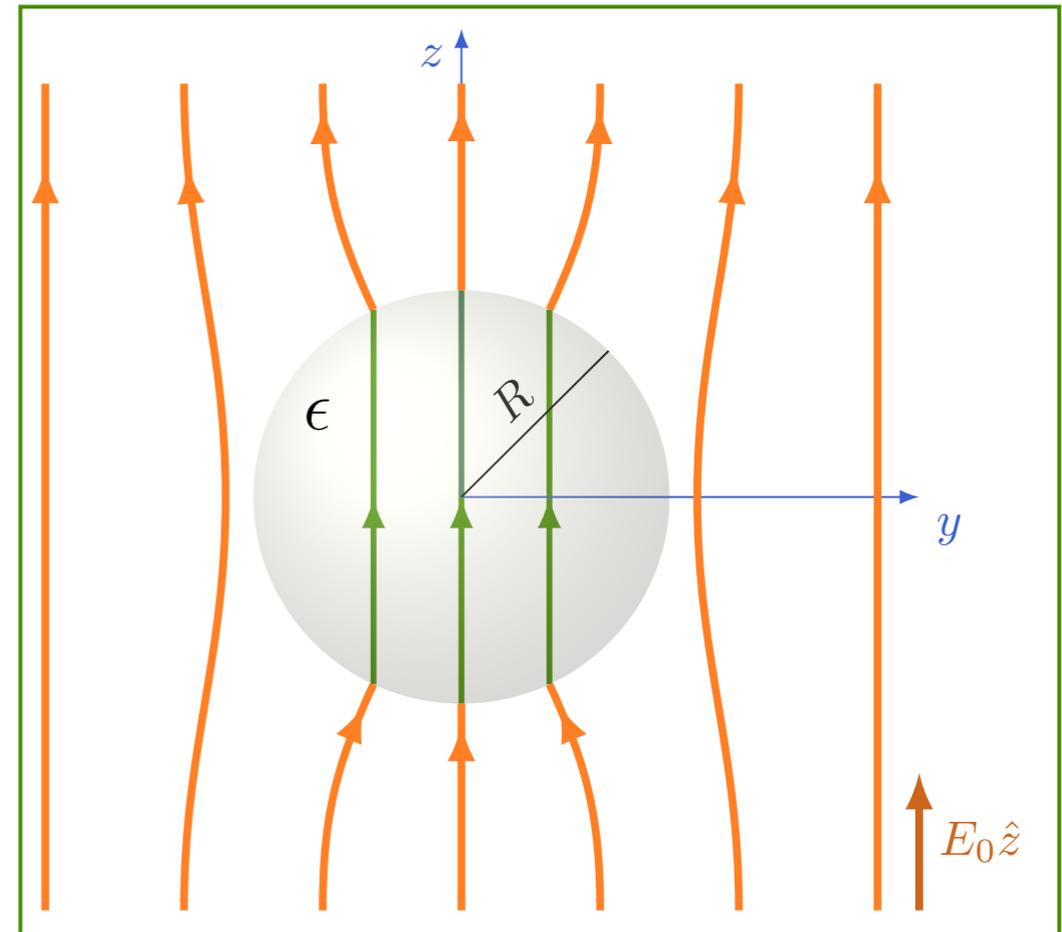
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

21 de junho de 2021  
Dielétricos

# Pratique o que aprendeu

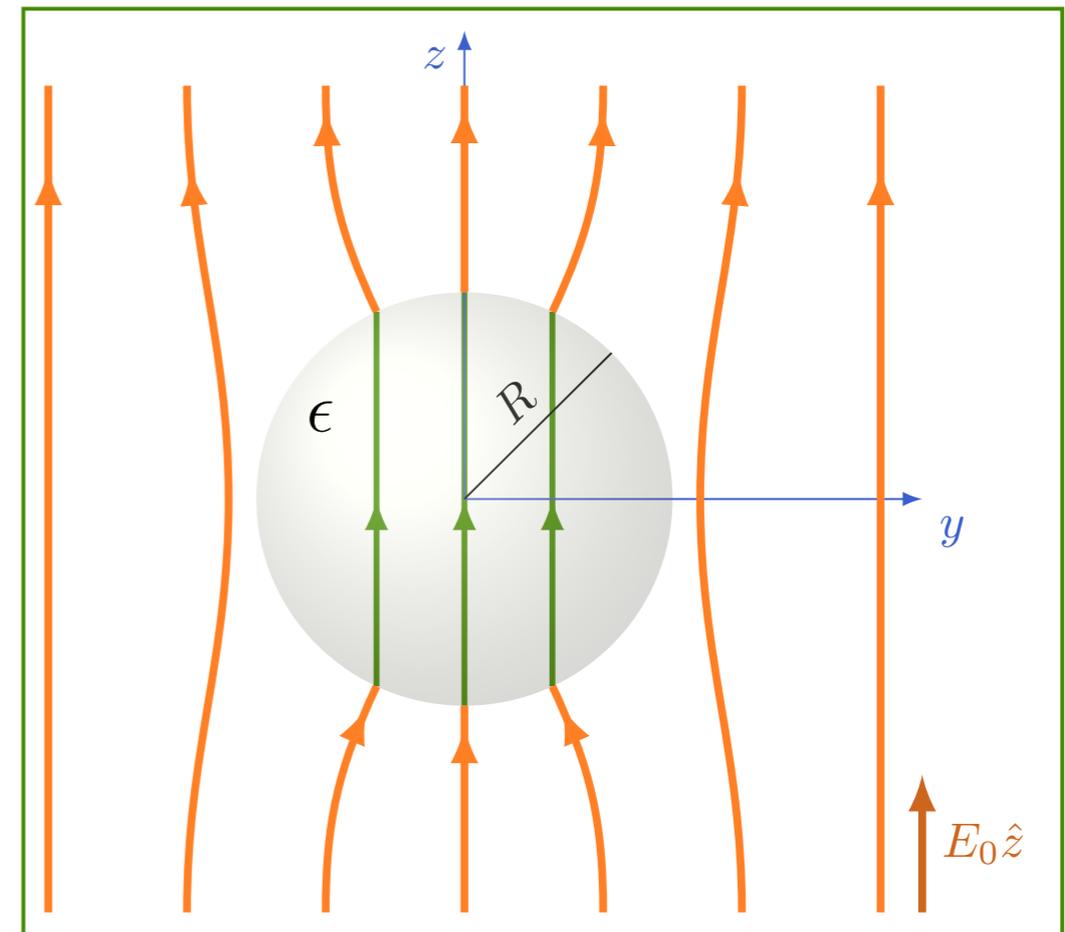
$$\epsilon_b \frac{\partial V_b}{\partial n} = \epsilon_a \frac{\partial V_a}{\partial n} \quad V_b = V_a$$



# Pratique o que aprendeu

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Condições de contorno



# Pratique o que aprendeu

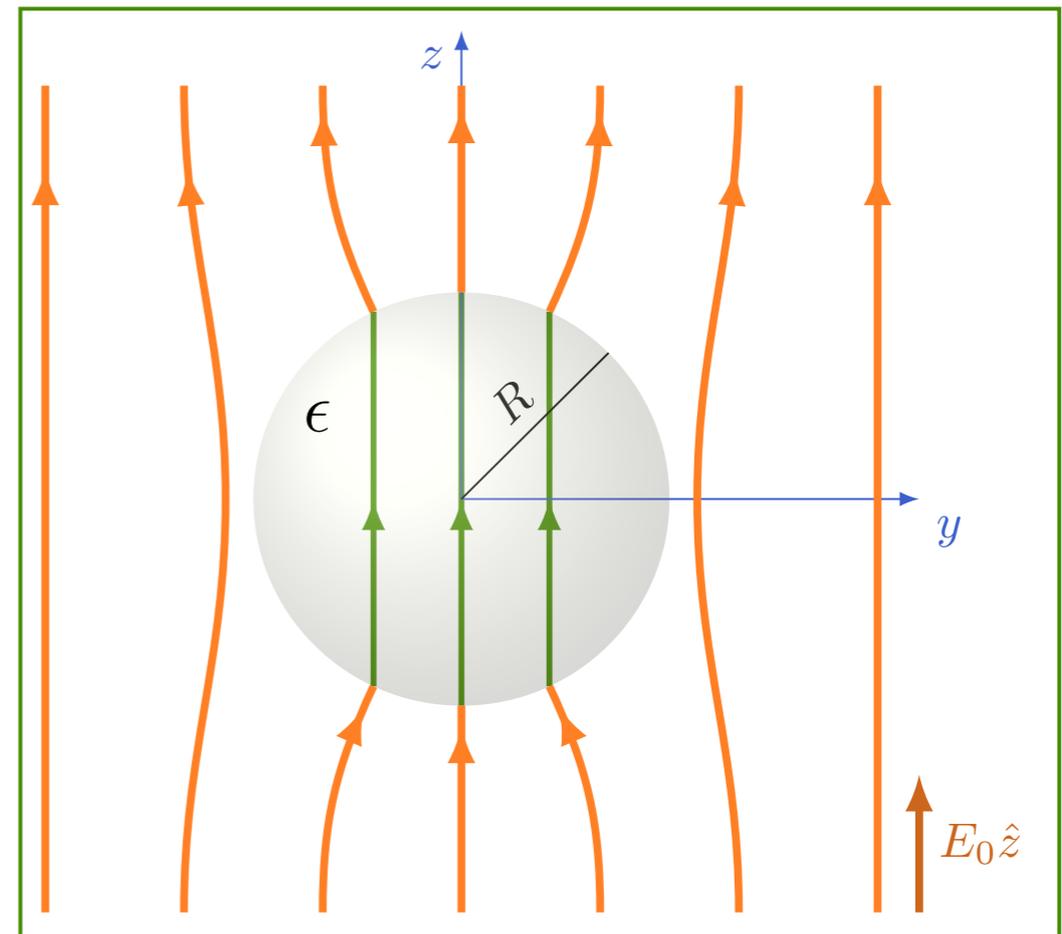
$$\epsilon_b \frac{\partial V_b}{\partial n} = \epsilon_a \frac{\partial V_a}{\partial n} \quad V_b = V_a$$

## Condições de contorno

$$V(r \rightarrow \infty) = -E_0 r \cos \theta$$

$$V_d(R) = V_f(R)$$

$$\epsilon \frac{\partial V_d}{\partial n} \Big|_R = \epsilon_0 \frac{\partial V_f}{\partial n} \Big|_R$$



# Pratique o que aprendeu

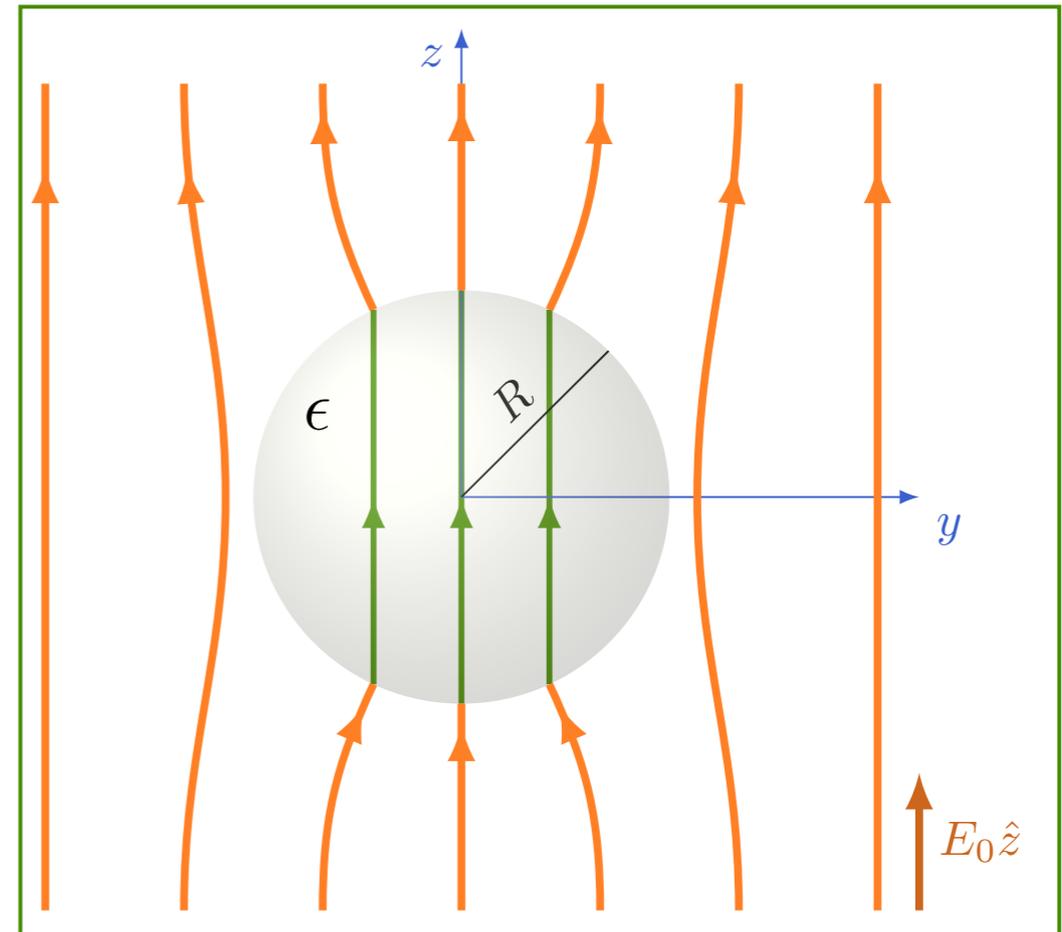
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## Condições de contorno

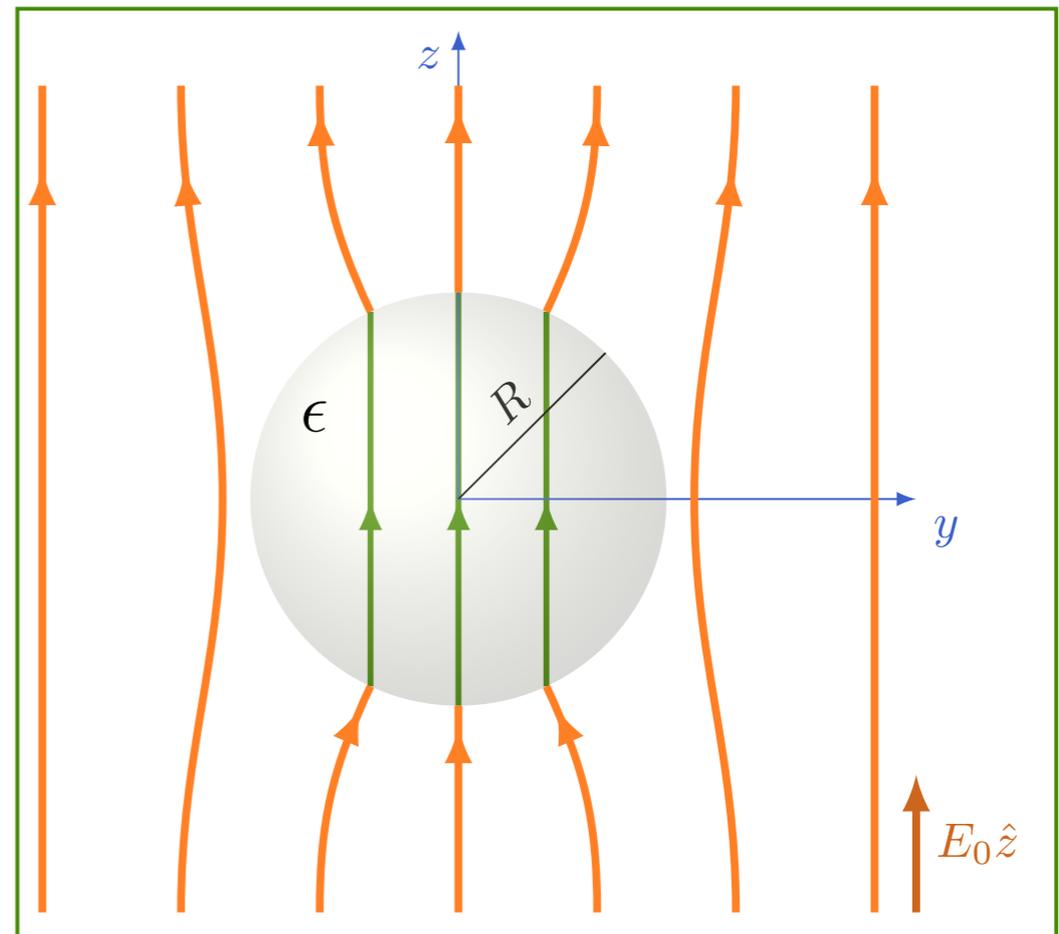
$$V(r \rightarrow \infty) = -E_0 r \cos \theta$$

$$V_d(R) = V_f(R)$$

$$\epsilon \frac{\partial V_d}{\partial n} \Big|_R = \epsilon_0 \frac{\partial V_f}{\partial n} \Big|_R$$

$$V(r) = \sum_{\ell} \left( A_{\ell} r^{\ell} + \frac{B_{\ell}}{r^{\ell+1}} \right) P_{\ell}(\cos \theta)$$

$$r \rightarrow \infty \Rightarrow V_f(r) = -E_0 r \cos \theta + \sum_{\ell} \frac{B_{\ell}}{r^{\ell+1}} P_{\ell}(\cos \theta)$$

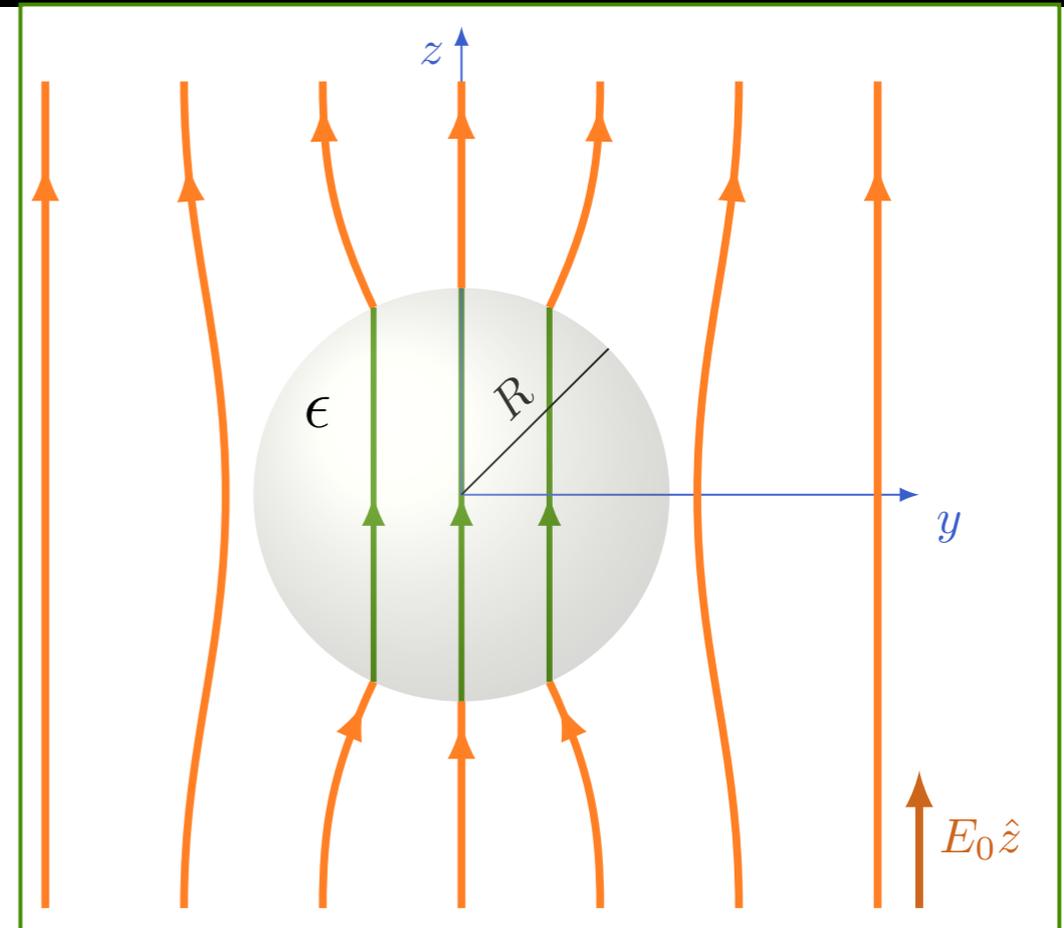


# Pratique o que aprendeu

$$V_f(r) = -E_0 r \cos \theta + \sum_{\ell} \frac{B_{\ell}}{r^{\ell+1}} P_{\ell}(\cos \theta)$$

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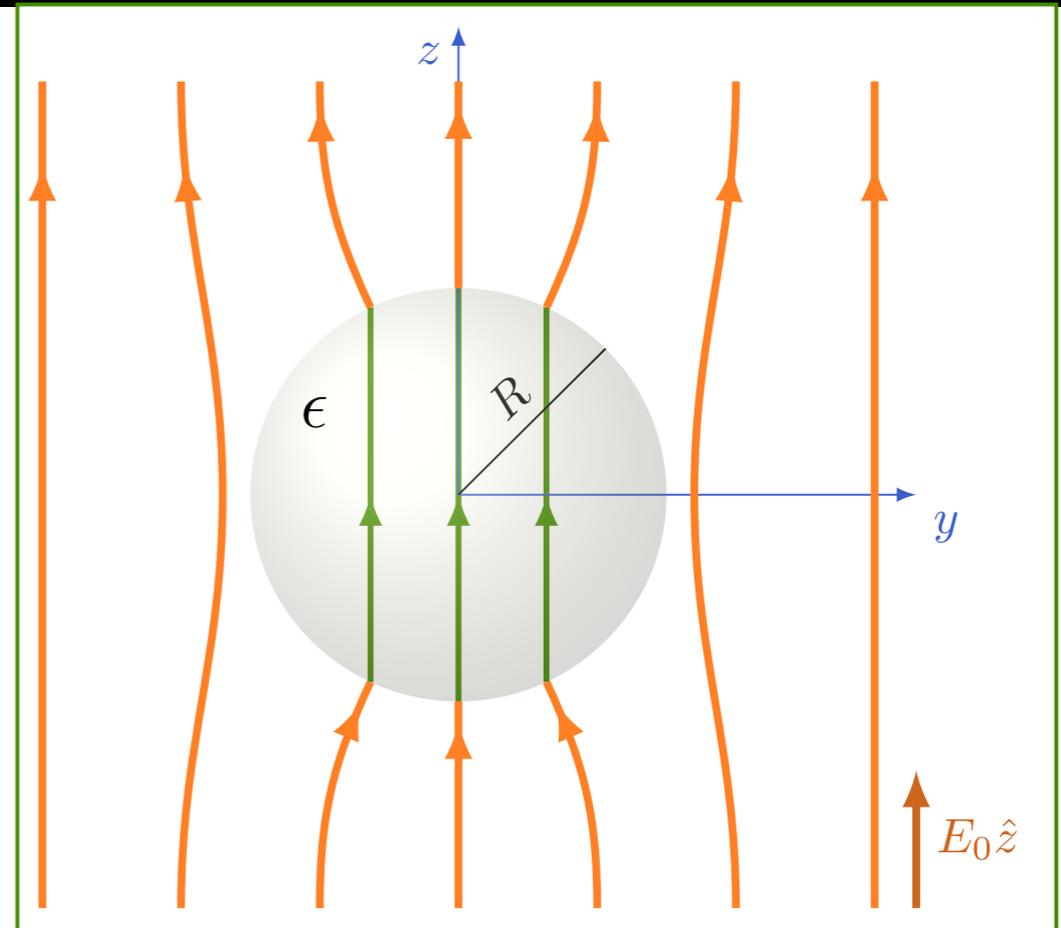
# Pratique o que aprendeu

$$V_f(r) = -E_0 r \cos \theta + \sum_{\ell} \frac{B_{\ell}}{r^{\ell+1}} P_{\ell}(\cos \theta)$$

$$V_d(R) = V_f(R)$$

$$\epsilon \frac{\partial V_d}{\partial n} \Big|_R = \epsilon_0 \frac{\partial V_f}{\partial n} \Big|_R$$

$$V_d(r) = \sum_{\ell} A_{\ell} r^{\ell} P_{\ell}(\cos \theta)$$



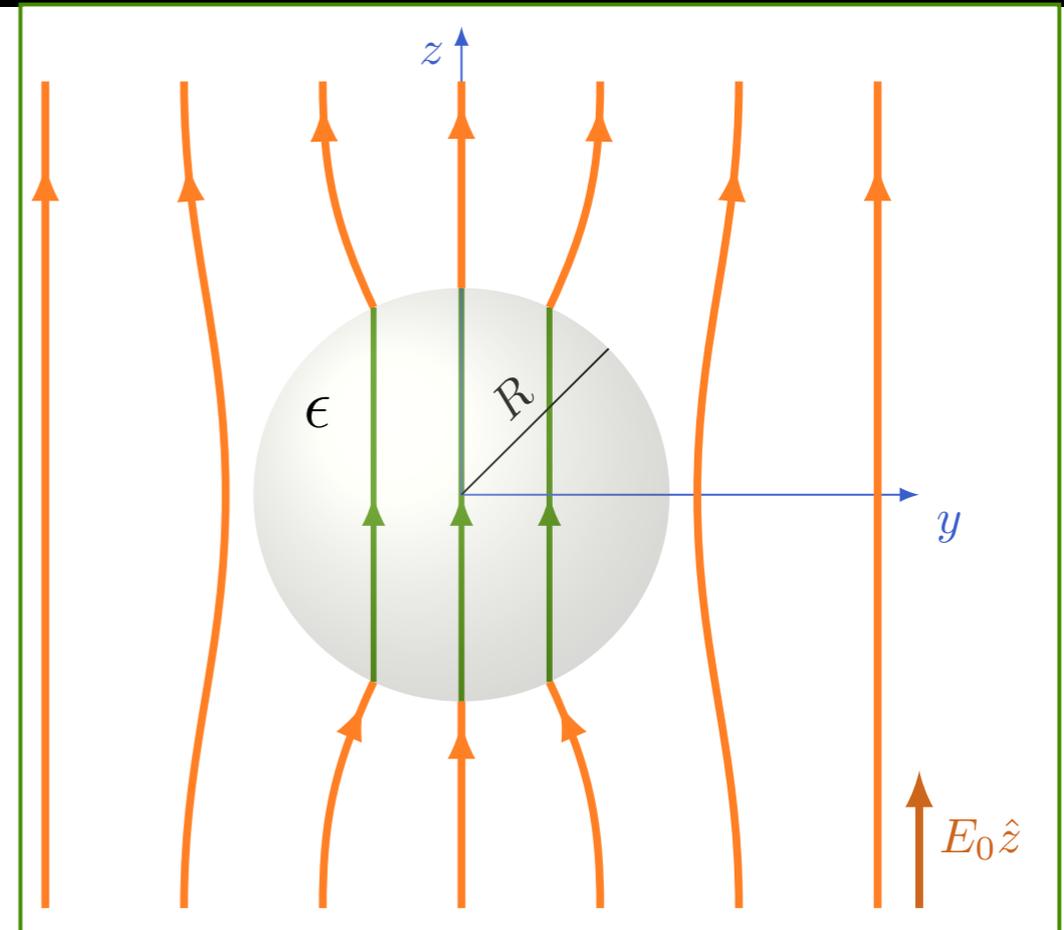
# Pratique o que aprendeu

$$V_f(r) = -E_0 r \cos \theta + \sum_{\ell} \frac{B_{\ell}}{r^{\ell+1}} P_{\ell}(\cos \theta)$$

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$$V_d(r) = \sum_{\ell} A_{\ell} r^{\ell} P_{\ell}(\cos \theta)$$



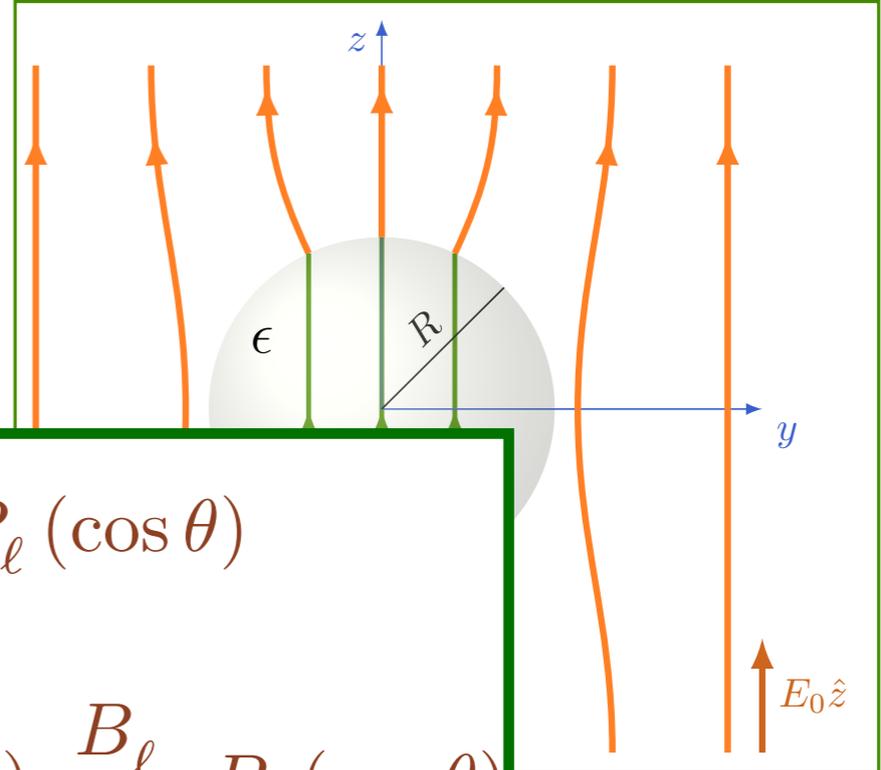
$$r = R \Rightarrow \sum_{\ell} A_{\ell} R^{\ell} P_{\ell}(\cos \theta) = -E_0 R \cos \theta + \sum_{\ell} \frac{B_{\ell}}{R^{\ell+1}} P_{\ell}(\cos \theta)$$

$$\epsilon_r \sum_{\ell} \ell A_{\ell} R^{\ell-1} P_{\ell}(\cos \theta) = -E_0 \cos \theta - \sum_{\ell} (\ell + 1) \frac{B_{\ell}}{R^{\ell+2}} P_{\ell}(\cos \theta)$$

# Pratique o que aprendeu

$$V_f(r) = -E_0 r \cos \theta + \sum_{\ell} \frac{B_{\ell}}{r^{\ell+1}} P_{\ell}(\cos \theta)$$

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$$\ell = 1 \Rightarrow A_1 R = -E_0 R + \frac{B_1}{R^2}$$

$$\epsilon_r A_1 = -E_0 - 2 \frac{B_1}{R^3}$$

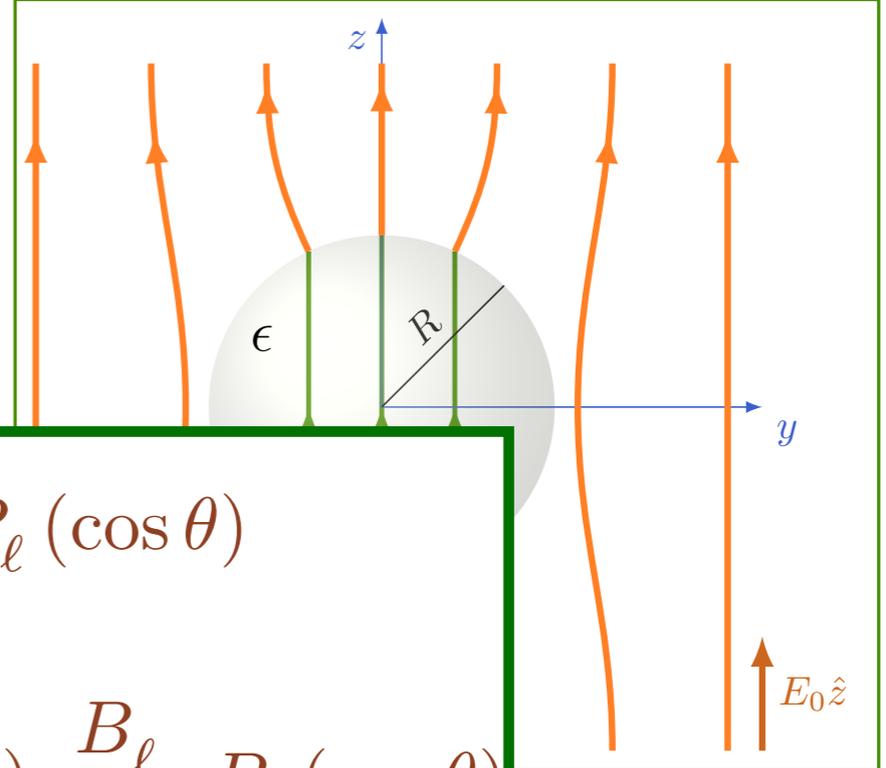
$$\Rightarrow A_1 = -\frac{3}{\epsilon_r + 2} E_0$$

$$B_1 = \frac{\epsilon_r - 1}{\epsilon_r + 2} R^3 E_0$$

# Pratique o que aprendeu

$$V_f(r) = -E_0 r \cos \theta + \sum_{\ell} \frac{B_{\ell}}{r^{\ell+1}} P_{\ell}(\cos \theta)$$

$$V_d(r) = \sum_{\ell} A_{\ell} r^{\ell} P_{\ell}(\cos \theta)$$



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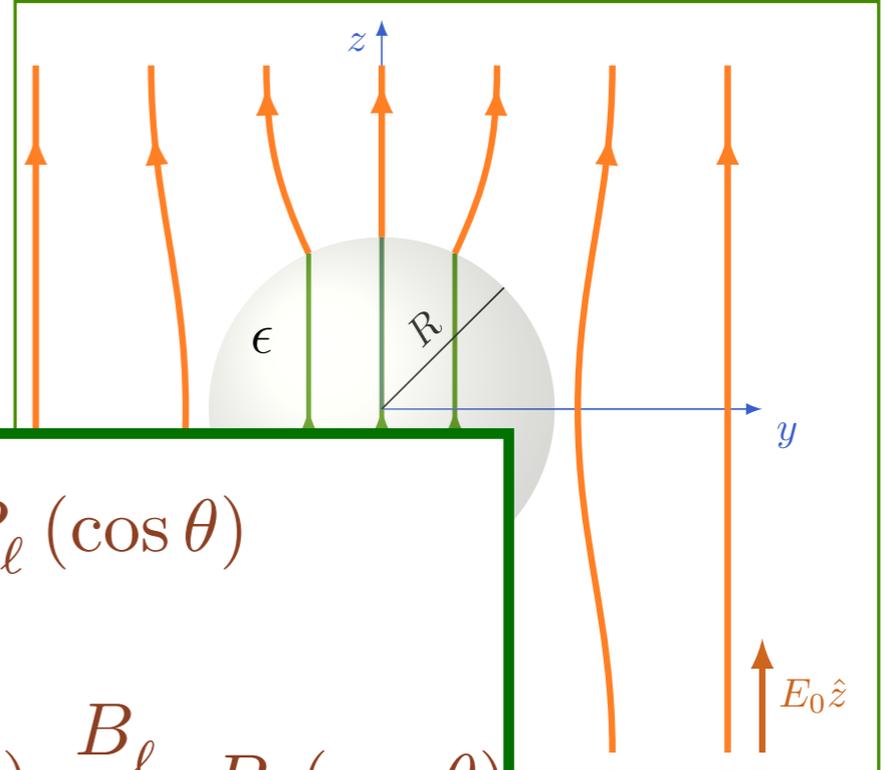
$$\ell \neq 1 \Rightarrow A_{\ell} R^{\ell} = \frac{B_{\ell}}{R^{\ell+2}}$$

$$\epsilon_r \ell A_{\ell} R^{\ell-1} = -(\ell + 1) \frac{B_{\ell}}{R^{\ell+2}}$$

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$$V_f(r) = -E_0 r \cos \theta + \sum_{\ell} \frac{B_{\ell}}{r^{\ell+1}} P_{\ell}(\cos \theta)$$

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$$\epsilon_r \ell A_{\ell} R^{\ell-1} = -(\ell + 1) \frac{B_{\ell}}{R^{\ell+2}}$$

$$\Rightarrow A_{\ell} = B_{\ell} = 0$$

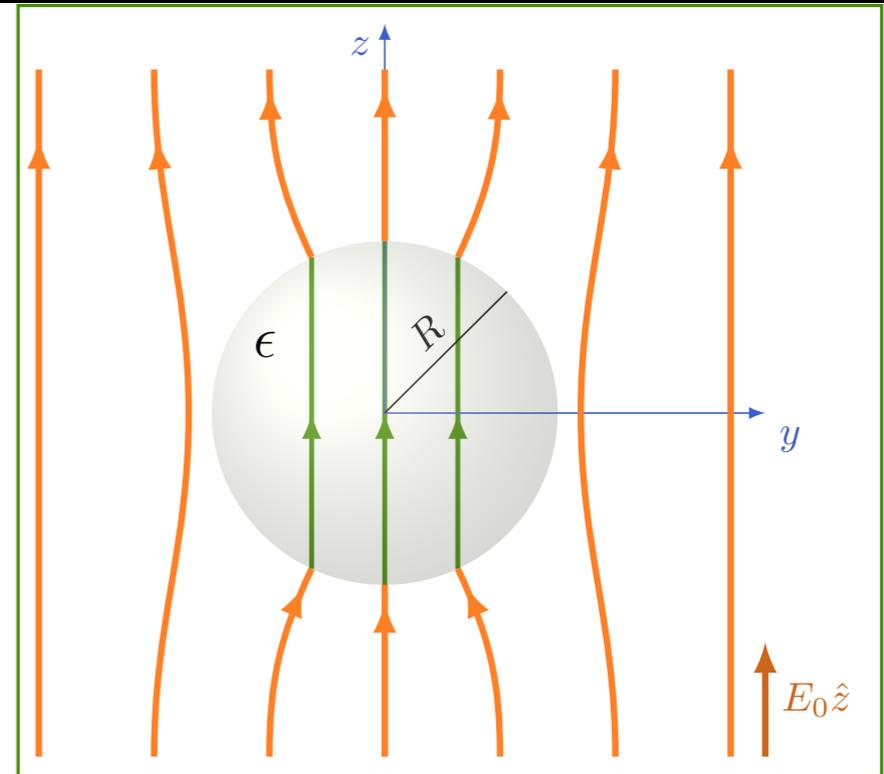
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$$V_f(r) = -E_0 r \cos \theta + \sum_{\ell} \frac{B_{\ell}}{r^{\ell+1}} P_{\ell}(\cos \theta)$$

$$V_d(r) = \sum_{\ell} A_{\ell} r^{\ell} P_{\ell}(\cos \theta)$$

$$\Rightarrow A_1 = -\frac{3}{\epsilon_r + 2} E_0$$

$$B_1 = \frac{\epsilon_r - 1}{\epsilon_r + 2} R^3 E_0$$



$$\Rightarrow A_{\ell} = B_{\ell} = 0$$

$$V_d(r) = -\frac{3}{\epsilon_r + 2} E_0 r \cos \theta = -\frac{3}{\epsilon_r + 2} E_0 z$$

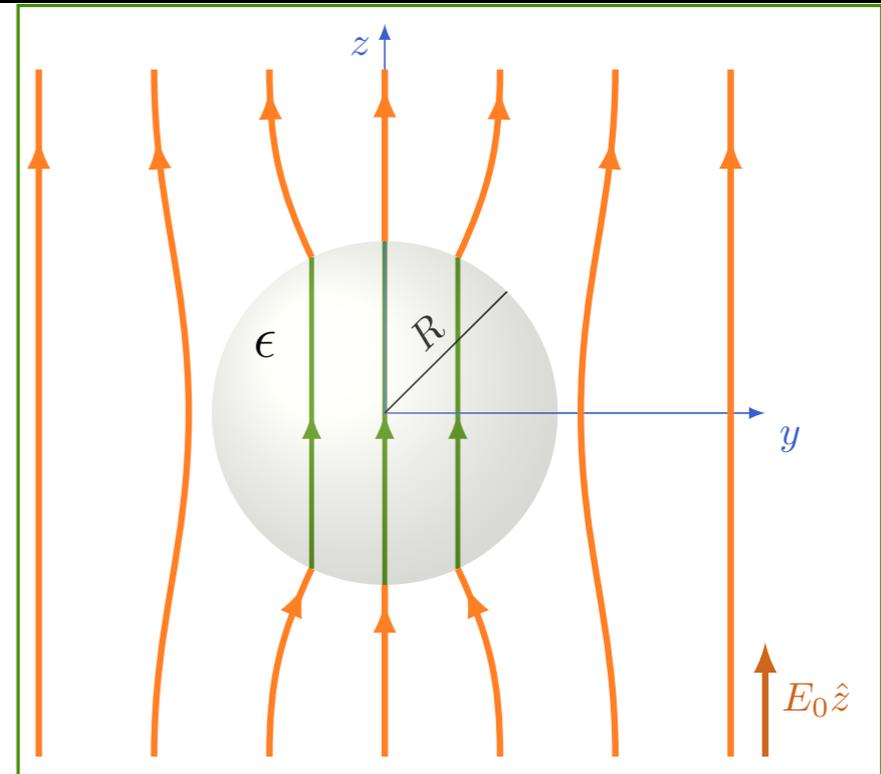
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$$V_f(r) = -E_0 r \cos \theta + \frac{\epsilon_r - 1}{\epsilon_r + 2} \frac{E_0 R^3 \cos \theta}{r^2}$$

# Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

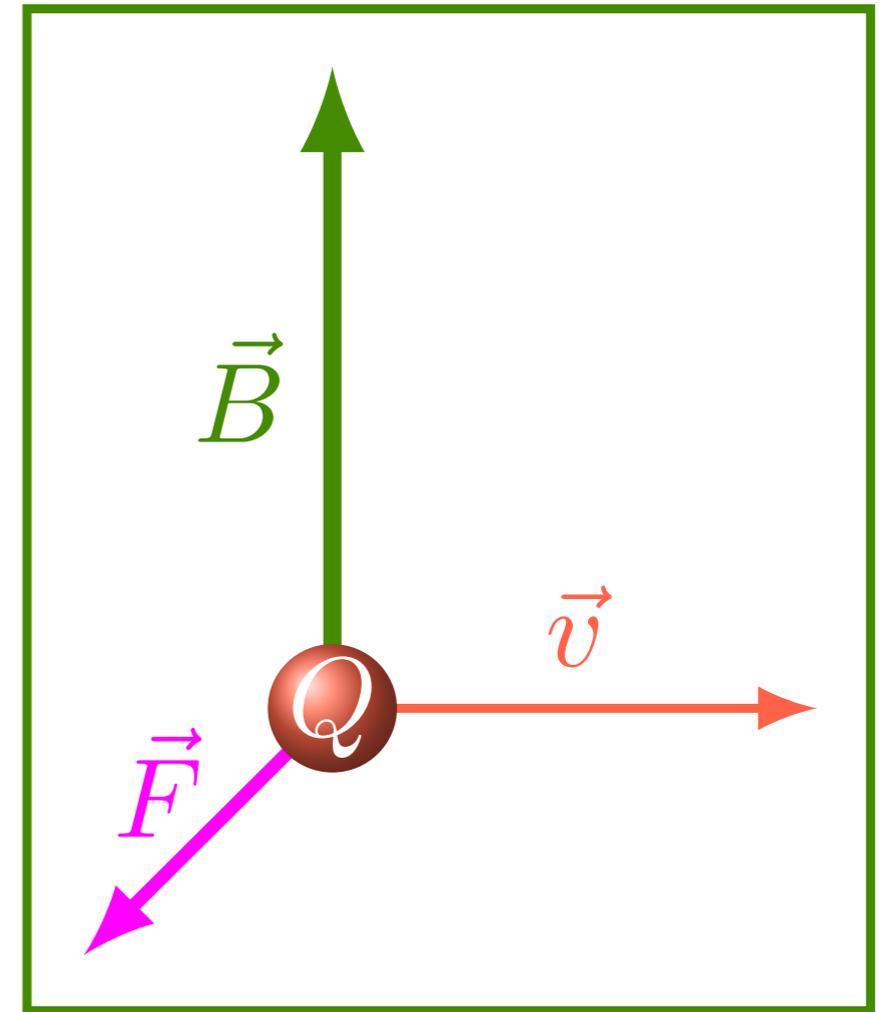
$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

21 de junho de 2021  
Magnetismo

# Força de Lorentz

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$$

REGRA  
DA MÃO DIREITA

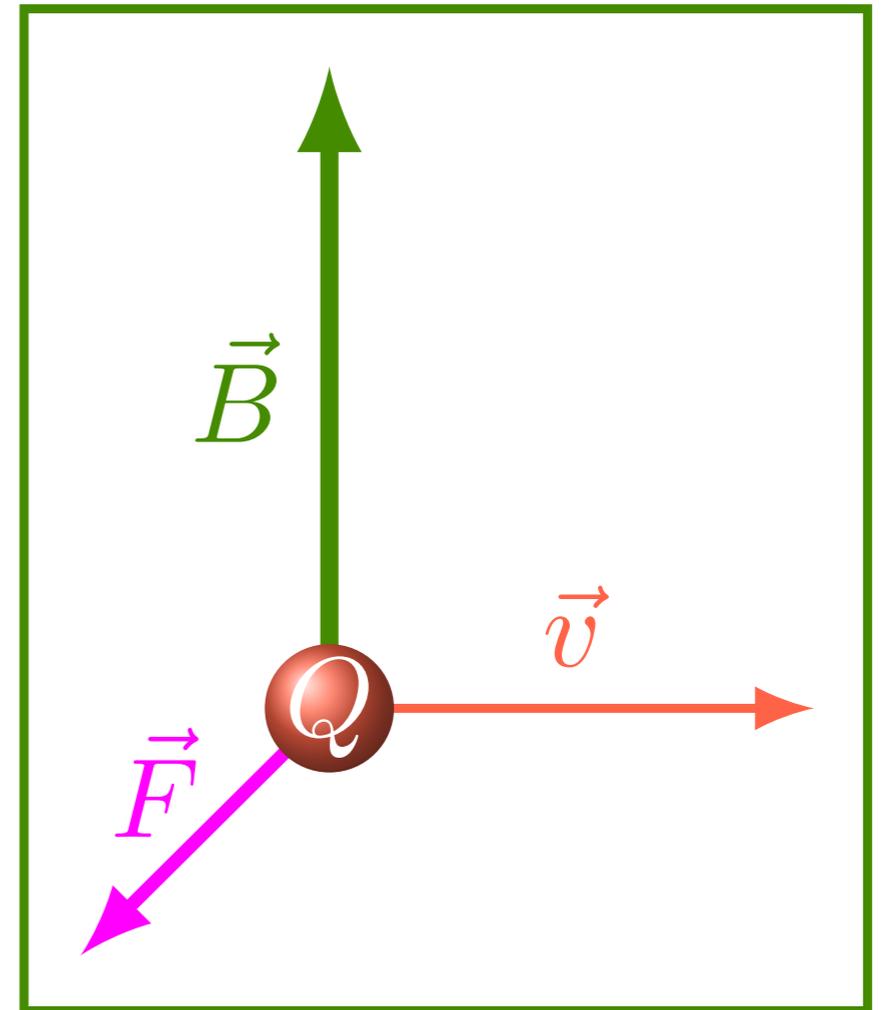


# Força de Lorentz

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$$

Força magnética não realiza trabalho

AS VEZES, PARECE REALIZAR

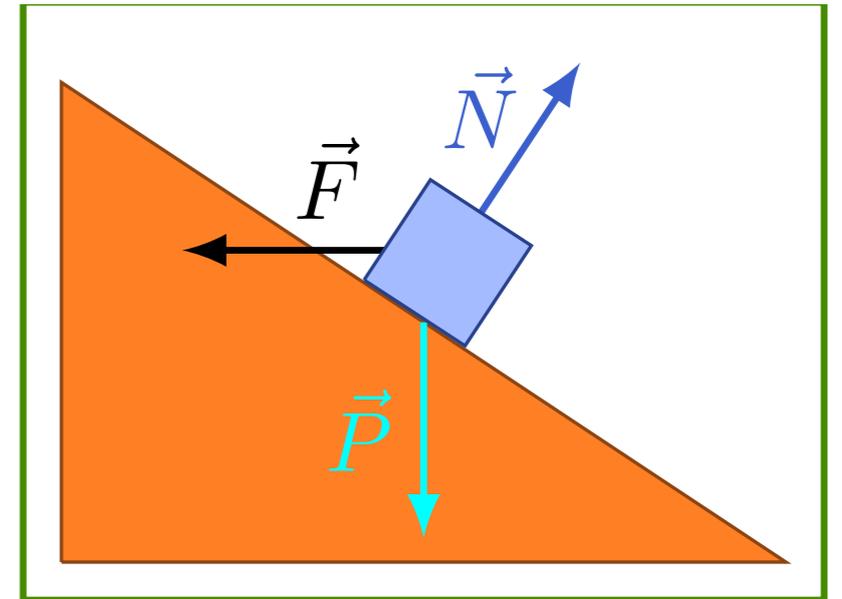


# Força de Lorentz

ANALOGIA

$$\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})$$

Força normal não realiza trabalho,  
MAS PARECE REALIZAR

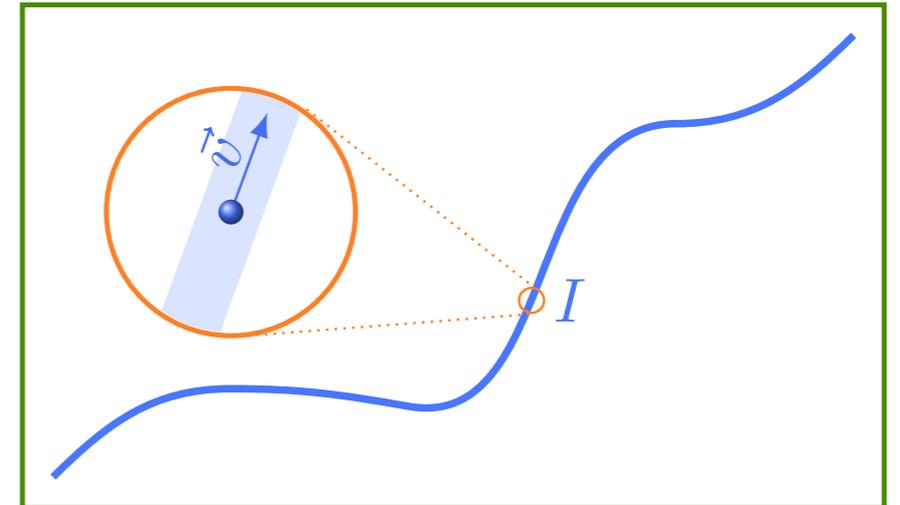


QUEM EMPURRA O BLOCO  
PARA CIMA É A NORMAL,  
MAS QUEM REALIZA TRABALHO  
É A FORÇA  $\vec{F}$

# Corrente elétrica

$$\vec{I} = \lambda \vec{v}$$

↳ VETOR CORRENTE

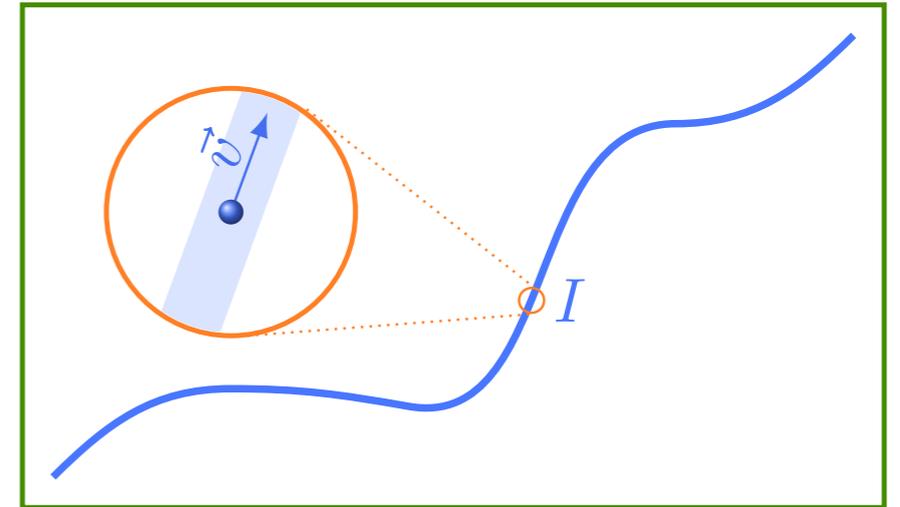


# Corrente elétrica

$$\vec{I} = \lambda \vec{v}$$

$$\vec{F} = \int \vec{v} \times \vec{B} \, dq$$

$\underbrace{\hspace{1.5cm}}_{dq = \lambda dl}$



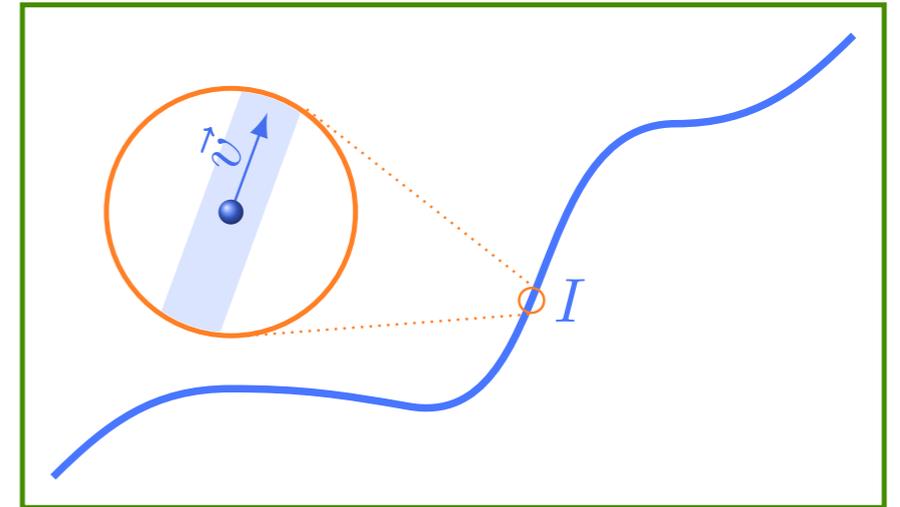
# Corrente elétrica

$$\vec{I} = \lambda \vec{v}$$

$$\vec{F} = \int \vec{v} \times \vec{B} dq$$

$$\vec{F} = \int \vec{v} \times \vec{B} \lambda d\ell$$





# Corrente elétrica

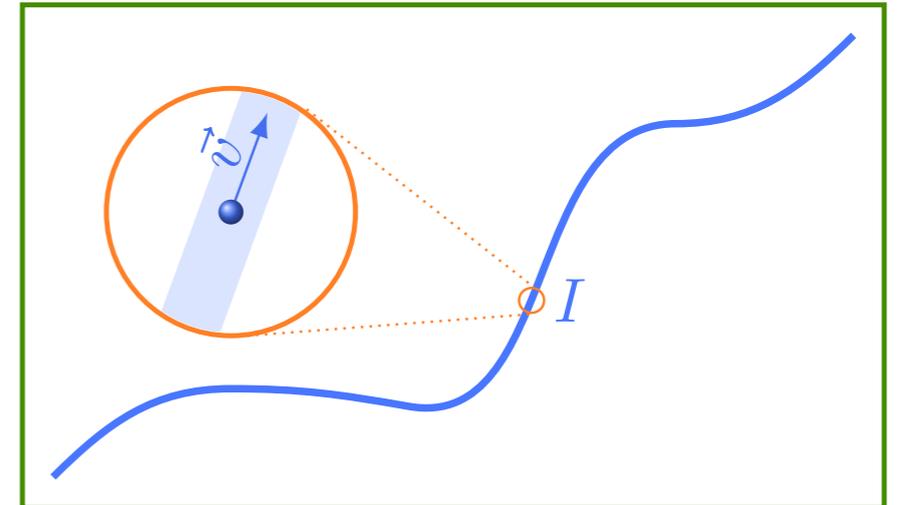
$$\vec{I} = \lambda \vec{v}$$

$$\vec{F} = \int \vec{v} \times \vec{B} dq$$

$$\vec{F} = \int \vec{v} \times \vec{B} \lambda dl$$

$$\vec{F} = \int \vec{I} \times \vec{B} dl$$

$\vec{I} \rightarrow \vec{I} d\vec{l}$



# Corrente elétrica

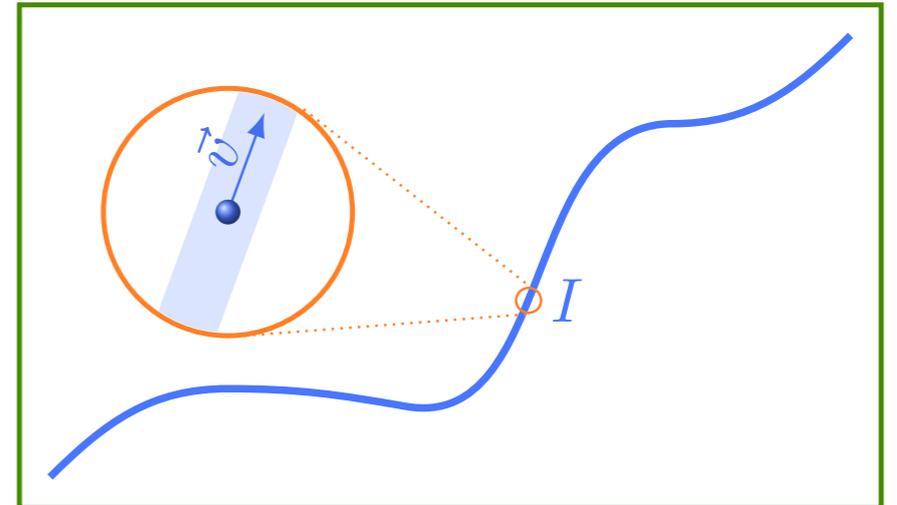
$$\vec{I} = \lambda \vec{v}$$

$$\vec{F} = \int \vec{v} \times \vec{B} dq$$

$$\vec{F} = \int \vec{v} \times \vec{B} \lambda d\ell$$

$$\vec{F} = \int \vec{I} \times \vec{B} d\ell$$

$$\vec{F} = I \int d\vec{\ell} \times \vec{B}$$



# Pratique o que aprendeu

$$\vec{F} = I \int d\vec{\ell} \times \vec{B}$$

Qual a corrente  $I$  que equilibra o peso?

$$\Delta \vec{\ell} \times \vec{B} = aB (\hat{x} \times -\hat{z}) = aB \hat{y}$$

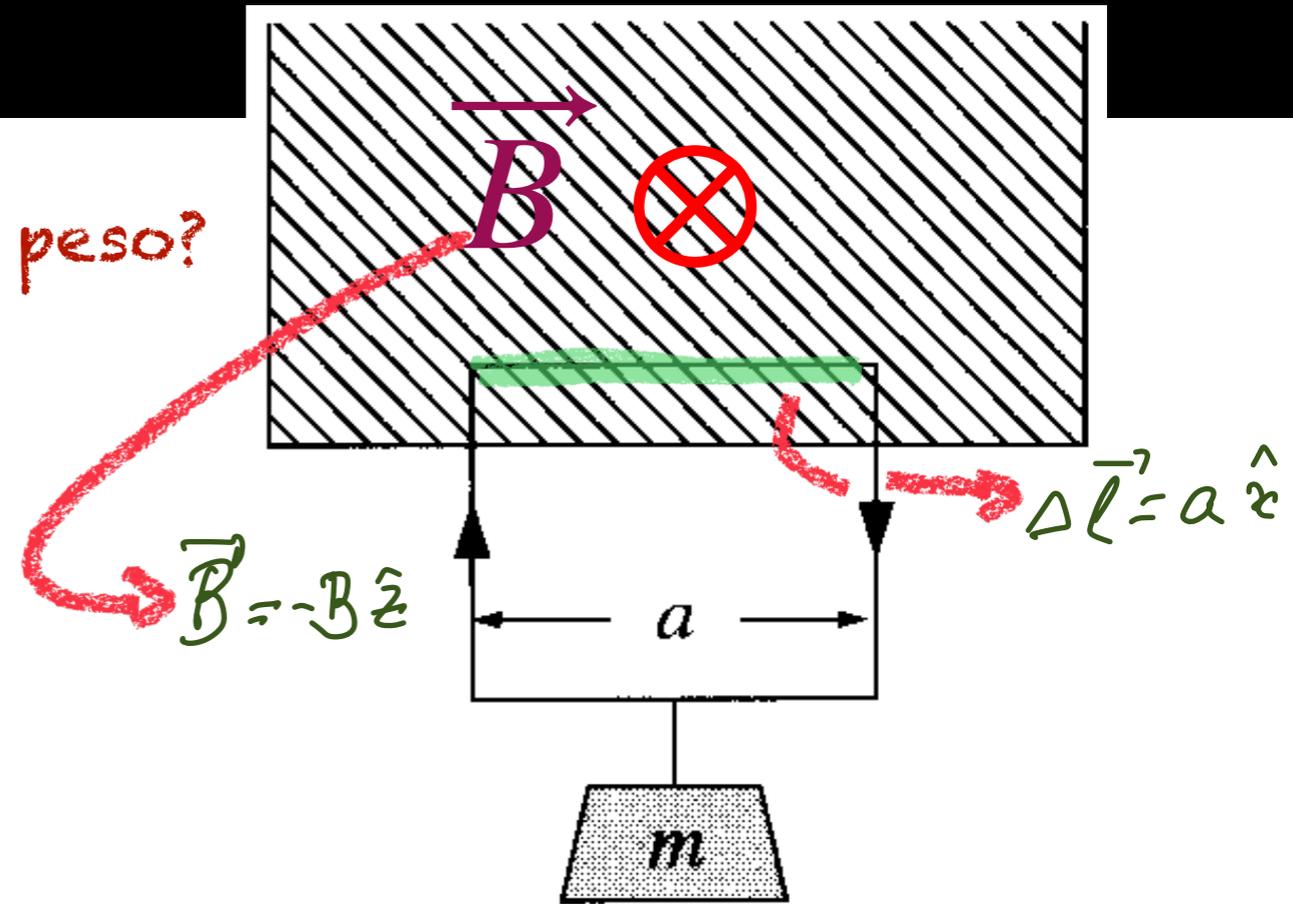
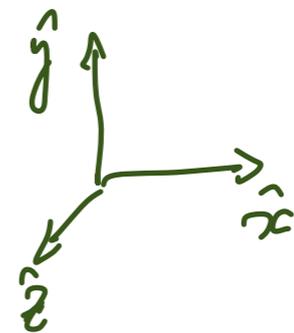


Figure 5.10



Pratique o que aprendeu

$$\vec{F} = I \int d\vec{\ell} \times \vec{B}$$

Qual a corrente  $I$  que equilibra o peso?

$$F = IaB \quad (\vec{F} = Iab \hat{y})$$

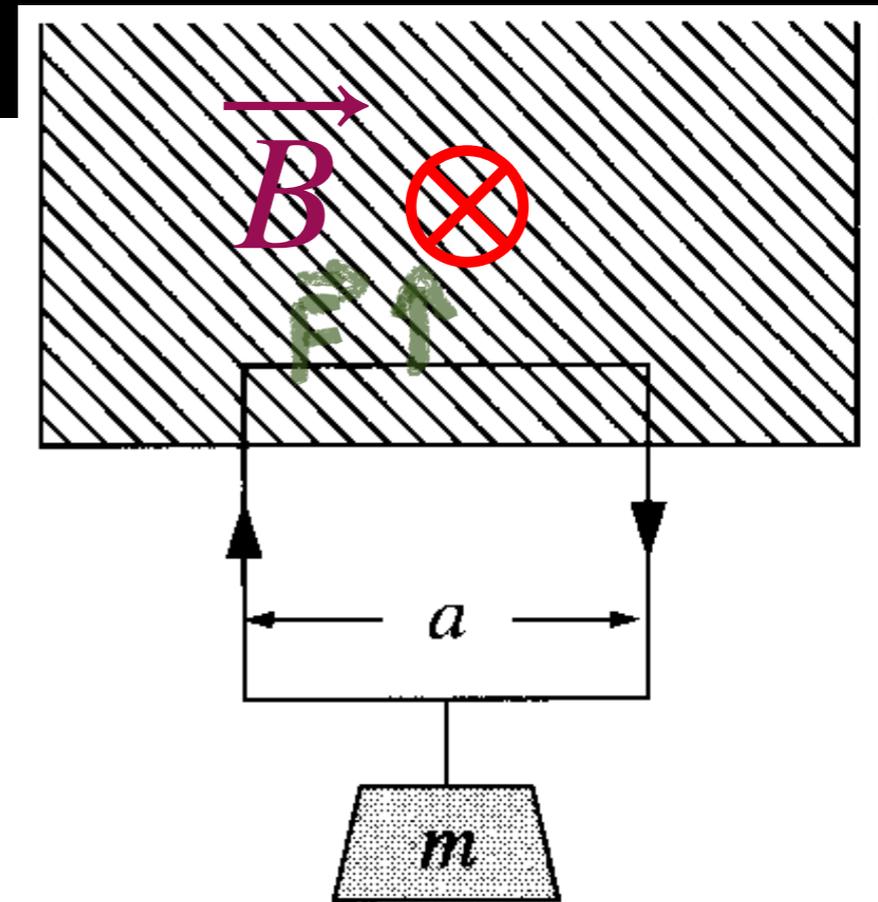
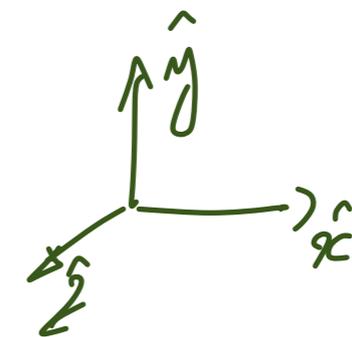


Figure 5.10



Pratique o que aprendeu

$$\vec{F} = I \int d\vec{\ell} \times \vec{B}$$

Qual a corrente  $I$  que equilibra o peso?

$$F = I a B$$

$$F = mg$$

$$I = \frac{mg}{aB}$$

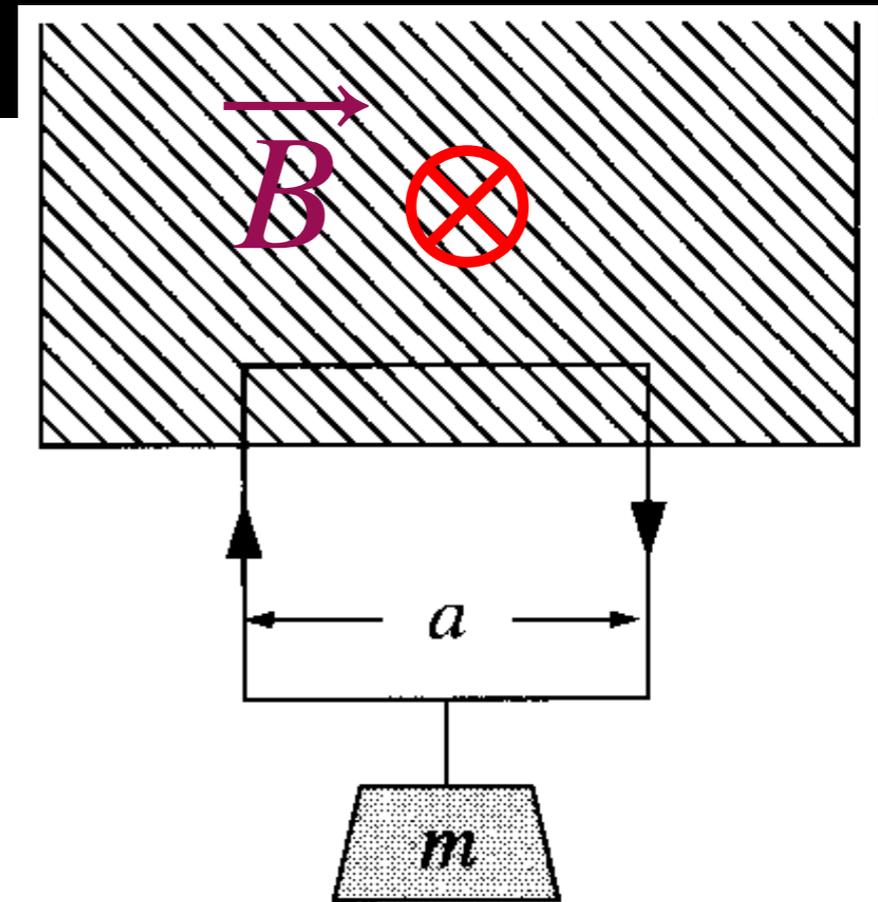


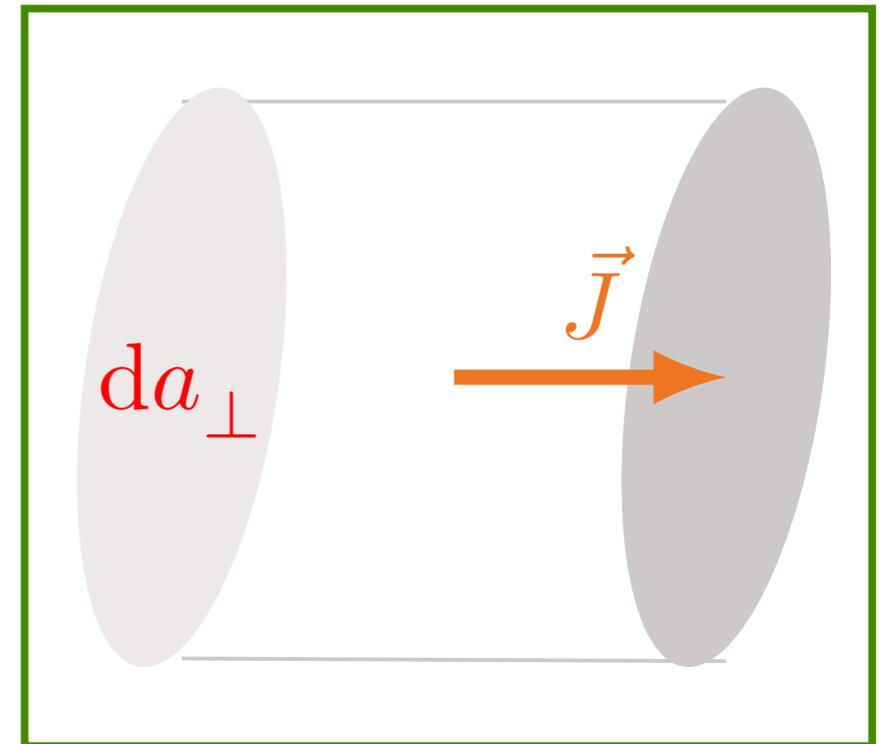
Figure 5.10

# Densidade de corrente

$$\vec{J} = \frac{d\vec{I}}{da_{\perp}}$$

$$\vec{J} = \rho\vec{v}$$

Força no fio



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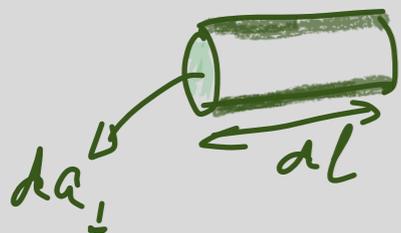
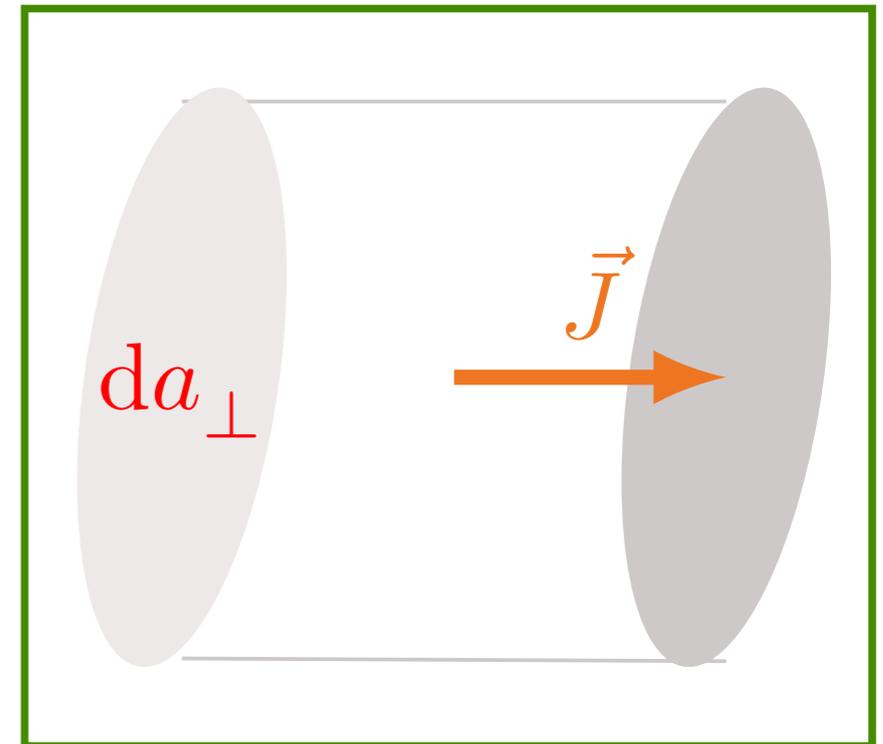
$$\vec{J} = \rho\vec{v}$$

Força no fio

$$\vec{F} = \int \vec{I} \times \vec{B} dl$$

$$dl da_{\perp} = d\vec{z}$$

$$\vec{I} dl = \vec{J} d\vec{z}$$



# Densidade de corrente

$$\vec{J} = \frac{d\vec{I}}{da_{\perp}}$$

$$\vec{J} = \rho\vec{v}$$

Força no fio

$$\vec{F} = \int \vec{I} \times \vec{B} \, d\ell$$

$$\vec{F} = \int \vec{J} \times \vec{B} \, d\tau$$

