Aula 19 — Interação Luz-Matéria (Limite quântico): Jaynes-Cummings, Estados "vestidos" e emissão espontânea...

Considere o Hamiltoniano completo

$$\hat{H} = \hat{H}_A + \hat{H}_C + \hat{H}_{Int}$$

Átomo: $\hat{H}_A = \sum_i E_i |i\rangle\langle i| = \sum_i \hbar\omega_i |i\rangle\langle i| = E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2| + \cdots$

CAMPO: $\hat{H}_C = \frac{1}{2}\sum_k (a_k^\dagger a_k + 1/2)$

Interação: $\hat{H}_{Int} = \underbrace{-e\,\vec{r}\cdot\vec{E}}_{\text{semi-clássica}}$ (op. dipolo elétrico)

↳ p/ "quantizar" uso campo $\vec{E}$ quantizado

P/ simplificar, vamos considerar átomo 2 níveis interagindo com um único modo do campo EM.

$$\hat{H}_A = E_1|1\rangle\langle 1| + E_2|2\rangle\langle 2| = \frac{1}{2}\hbar\omega_0\big(\underbrace{|2\rangle\langle 2| - |1\rangle\langle 1|}_{\hat{\sigma}_z \begin{pmatrix}1&0\\0&-1\end{pmatrix}}\big) = \frac{1}{2}\hbar\omega_0\hat{\sigma}_z$$

$\to \hbar\begin{pmatrix}\omega_0 & 0\\ 0 & -\omega_0\end{pmatrix}$: $\omega_0 = (E_2 - E_1)/\hbar$

$\begin{pmatrix}E_2 & 0\\ 0 & E_1\end{pmatrix}$

$|1\rangle \Rightarrow |g\rangle$
$|2\rangle \Rightarrow |e\rangle$

↳ $\hat{H}_A = \frac{1}{2}\hbar\omega_0(|e\rangle\langle e| - |g\rangle\langle g|)$ ⇐

Na interação de dipolo ($\hat{\mu} = -e\hat{\vec{r}}$) ⤳ caso geral $\hat{\vec{\mu}} = e\sum_{j=1}^{N_{elétrons}} \vec{r}_j$

$\langle i|\hat{\mu}|j\rangle =$

p/ átomo 2 níveis: $\{|e\rangle, |g\rangle\} \to |g\rangle\langle g|;\ |g\rangle\langle e|;\ |e\rangle\langle g|;\ |e\rangle\langle e|$

$$e\hat{\vec{r}} = \big(\underbrace{|e\rangle\langle g| + |g\rangle\langle e|}_{\hat{\sigma}_x = \begin{pmatrix}0&1\\1&0\end{pmatrix}}\big) = \sigma_x = (\sigma^+ + \sigma^-)$$

O Campo:

$$\vec{E} \to \hat{\vec{E}} = \underbrace{\sum_k \mathcal{E}_k \hat{e}_k \hat{a}_k \cdot e^{i\omega_k t} + h.c.}_{} = \sum_k \mathcal{E}_k\hat{e}_k\underbrace{(\hat{a}_k + \hat{a}_k^\dagger)}_{}$$

↳ $\hat{H}_{Int} = \hbar\sum_k g_k(\hat{\sigma}^+ + \hat{\sigma}^-)(\hat{a}_k + \hat{a}_k^\dagger)$

↑ const. acoplamento campo átomo

$\hat{H}_{Int} = -e\,\hat{\vec{r}}\cdot\hat{\vec{E}}$

Combinando todos termos

$$\boxed{\hat{H} = \frac{1}{2}\hbar\omega_0\hat{\sigma}_z + \sum_k \hbar\omega_k(a_k^\dagger a_k) + \sum_k \hbar g_k(\sigma^+ + \sigma^-)(\hat{a}_k + \hat{a}_k^\dagger)}$$

↑ Hamiltoniano de Rabi multi-modo

→ fazendo Aprox. "single-mode" p/ campo

- Átomo de 2 níveis + campo c/ 1 modo

$$\hat{H} = \frac{1}{2}\hbar\omega_0\hat{\sigma}_z + \hbar\omega a^\dagger a + g\hbar(\hat{\sigma}^+ + \hat{\sigma}^-)(a + a^\dagger)$$

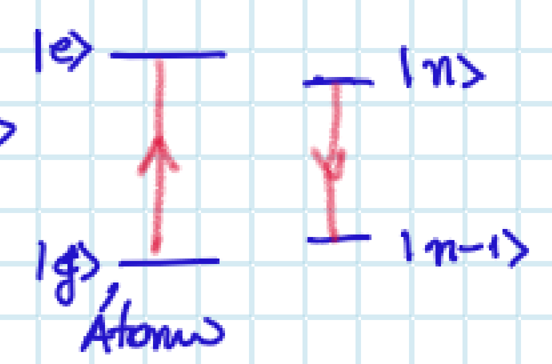
$$g\hbar(\sigma^+ a + \cancel{\sigma^+ a^+} + \cancel{\sigma^- a} + \sigma^- a^+) \Leftarrow$$

Olhando p/ o termo de interação átomo-campo na representação de interação

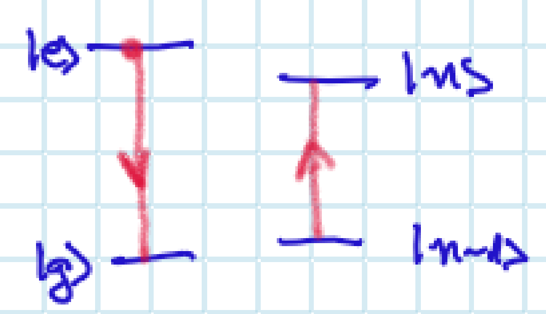
$$(\hat{H}_{int})_I = \hbar g\Big(\underbrace{\sigma^- a \,.\, e^{-i(\omega_0+\omega)t} + \sigma^+ a^+ e^{i(\omega_0+\omega)t}}_{\text{high-freq.}} + \Leftarrow \text{IGNORADOS}$$

$$\underbrace{\sigma^+\hat{a}\,.\,e^{i(\omega_0-\omega)t} + \sigma^- a^+ e^{-i(\omega_0-\omega)t}}_{\text{Low-freq}}\Big)$$

$\sigma^+\hat{a} \Rightarrow \sigma^+\otimes\hat{a} \Rightarrow$

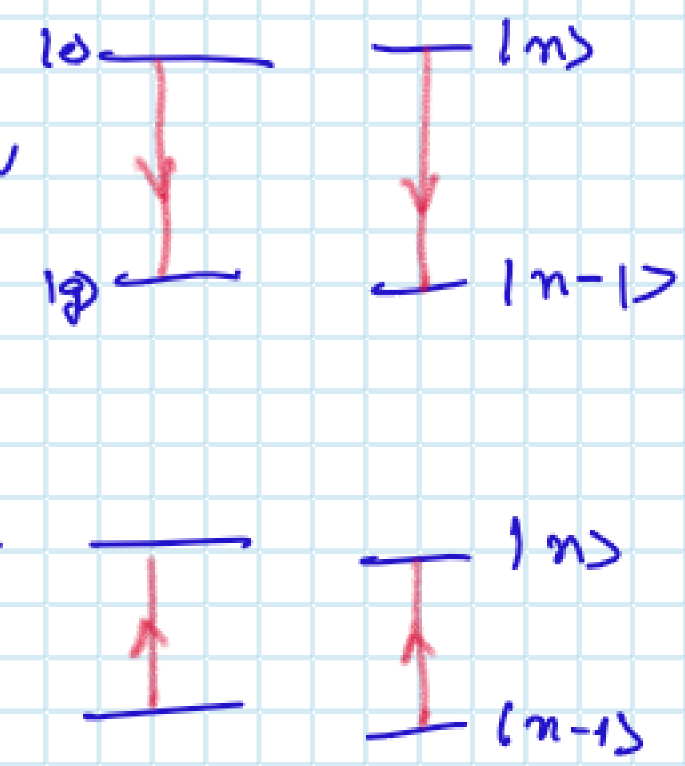


$\sigma^- a^+ \Rightarrow$



processos conservam energia

$\sigma^- a$



$\sigma^+ a^+$

"não físicos" ⇒ (i.e. NÃO conservam Energia)

Ignorando termos ñ físicos:

$\hat{\sigma}_z\otimes 1$ $\quad$ $1\times\hat{n}$ $\longrightarrow$ $|e\rangle|n-1\rangle$; $|g\rangle|n\rangle$

$$\boxed{\hat{H} = \frac{1}{2}\hbar\omega_0\hat{\sigma}_z + \hbar\omega a^\dagger a + g\hbar(\sigma^+ a + \sigma^- a^+)}$$

$|e, n-1\rangle$
$|g, n\rangle$

Hamiltoniano de Jaynes-Cummings

↳ se diagonizar o sistema completo obtém-se:

(energia) $\quad \boxed{E_{\pm,n} = \hbar\omega(n+1/2) \pm \frac{\hbar}{2}\underbrace{\sqrt{\Delta^2 + 4g^2(n+1)}}_{\Omega_R}}$; $\Delta = (\omega - \omega_0)$

(estados) $\quad \boxed{|\pm, n\rangle = \alpha|e\rangle|n-1\rangle + \beta|g\rangle|n\rangle}$

Freq. de Rabi generalizada
(*como vimos em teoria Perturb.)

Estado Átomo + fóton ≡ "dressed states"

• Emissão espontânea → Weisskopf-Wigner

↳ $\hat{H} = \frac{1}{2}\hbar\omega_0 + \sum_k \hbar\omega_k (a_k^\dagger a) + \sum_k \hbar g_k (\sigma^+ a_k e^{i(\omega_0-\omega_k)t} + \sigma^- \hat{a}_k^\dagger e^{-i(\omega_0-\omega)t})$

Assumindo o átomo inicialmente no estado excitado e campo no estado de vácuo -

↳ $|\psi(t)\rangle = \alpha(t)|e\rangle|0\rangle + \sum_k \beta_k(t)|g\rangle|1_k\rangle$

$|1_k\rangle = a_k^\dagger|0\rangle$

estado de 1 fóton no modo k.

Eq. Schrödinger

$$\dot{\alpha}(t) = -i\sum_k g_k e^{i(\omega_0-\omega_k)t}\beta_k(t) \qquad (1)$$

$$\rightarrow \dot{\beta}(t) = -i g_k e^{i(\omega_0-\omega_k)}\alpha(t) \qquad (2)$$

Subst. (2) em (1)

$$\dot{\alpha}(t) = -\sum_k |g_k|^2 \int_0^t dt' e^{-i(\omega_0-\omega_k)(t-t')}\alpha(t')$$

~ Aprox. 1: ⇒ Modos do vácuo formam um contínuo

$$\sum_k \rightarrow \int d^3k \cdot \rho(k) \rightsquigarrow \rho(k) = 2\left(\frac{L}{2\pi}\right)^3 \cdot k^2 dk\, d\phi \text{ sen}\theta$$

dens. de modo do campo

$$\dot{\alpha}(t) \propto \int_0^\infty d\omega_k\, \omega_k^3 \int_0^t dt' e^{-i(\omega_0-\omega_k)(t'-t)} \cdot \alpha(t')$$

proporcional

Aprox. 2: A integral só é importante q^do $\omega_k \approx \omega_0 \Rightarrow \omega_k^3 \to \omega_0^3$

$$\dot{\alpha}(t) \propto -\omega_0^3 \int_0^t dt' \alpha(t') \underbrace{\int_{-\infty}^{\infty} d\omega_k e^{-i(\omega-\omega_k)(t'-t)}}_{2\pi\delta(t'-t)}$$

$$-\omega_0^3 \int_0^t dt' \alpha(t') \cdot 2\pi\delta(t'-t)$$

$$\dot{\alpha}(t) = \frac{d\alpha}{dt} = -2\pi\omega_0^3\alpha(t) \rightarrow \int\frac{d\alpha}{\alpha} = -\frac{\Gamma}{2}\int_0^t dt$$

$\boxed{|\alpha(t)|^2 \propto \exp(-\Gamma t)} \Longrightarrow \mathcal{P}(\omega_k) = \rho(\omega_k)\sum_{\text{Polariz}} \oint d\Omega |\beta_k|^2 \Rightarrow \boxed{\mathcal{P}(\omega_k) \propto \frac{1}{\left(\frac{\Gamma}{2}\right)^2 + (\omega_0-\omega_k)^2}}$

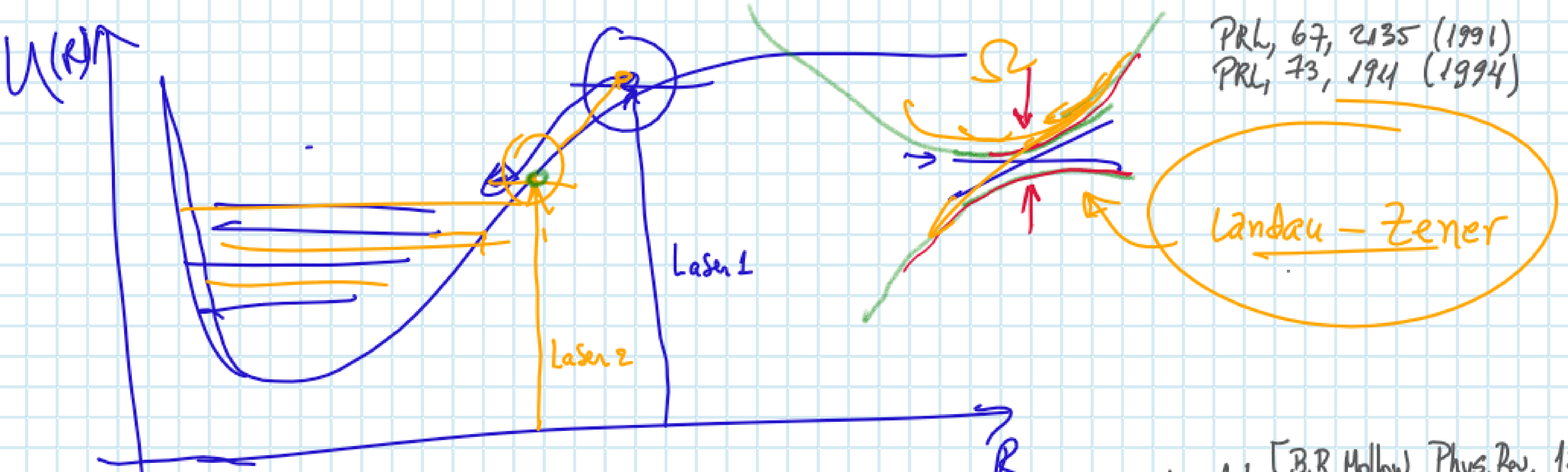
Âng. sólido

*espectro do fóton emitido

Lorentziana

taxa decaimento (exponencial!) ⇒ Tempo de vida! $(\tau = 1/\Gamma)$

Exemplo de Aplicação: "estados vestidos" + potenc. Molec. + fotocatálise



* Veja também o espectro (tripleto) de Mollow [B.R. Mollow, Phys. Rev. 188, 1969 (1969)] e "Light stark-shift" p/ mais efeitos relevantes de "dressed states" [Dalibard, Cohen-Tannoudji; JOSA B, 2, 1707 (1985)]

Sugestões de Leituras:

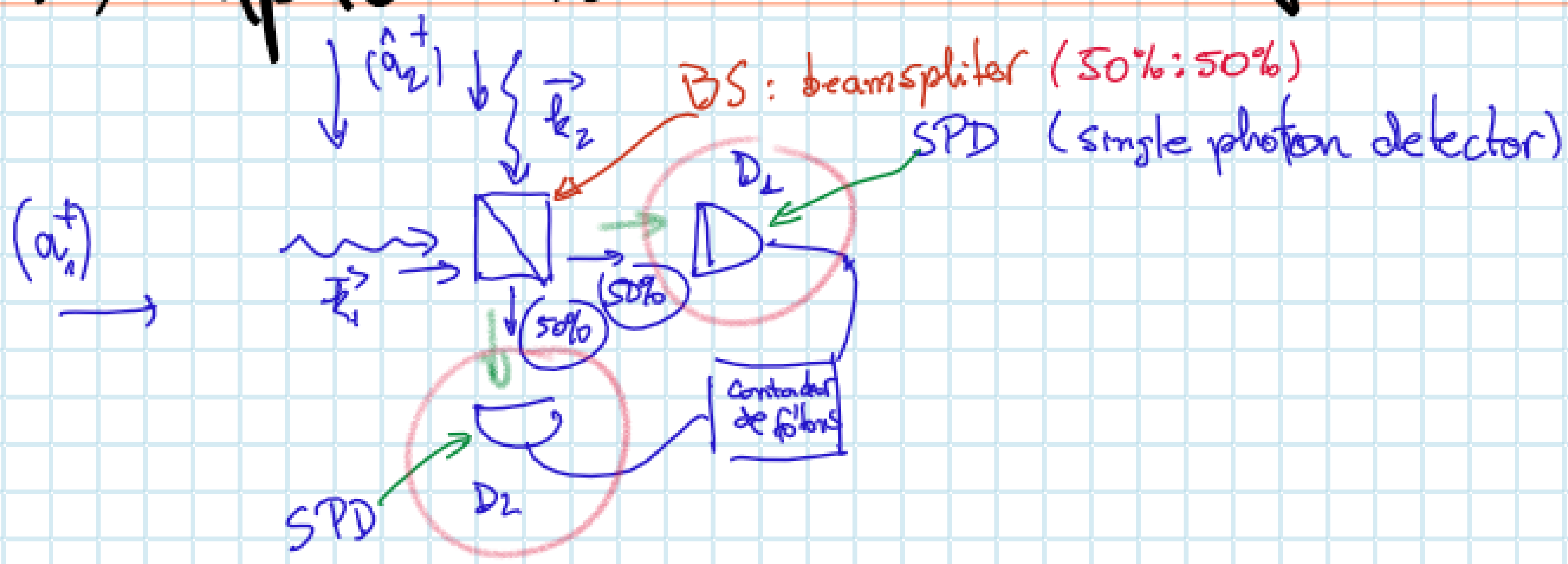
→ M. Scully & M. Zubairy – Quantum Optics (1997)

→ R. Loundon → The Quantum Theory of Light, 3rd ed. (2000)

→ J. Weiner & P.-T Ho – Light–Matter Intraction: Fund. & Applic.

↝ * Veja outro exemplo interessante na próxima página! ⇒

Exemplo interessante - Efeito Hong-Ou-Mandel (HOM)



Inicial: $|\Psi_{in}\rangle = |\underbrace{1_{k_1}; 1_{k_2}}_{|k_1\rangle \otimes |k_2\rangle}\rangle = \underbrace{\hat{a}_1^+ \hat{a}_2^+}|00\rangle$

como será o meu estado final após o beamsplitter (divisor de feixe)

$$|\Psi_{out}\rangle = \underline{\hat{U}_{BS}}|\Psi_{in}\rangle$$

$$\boxed{\hat{U}_{BS} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}} \sim \begin{cases} \hat{a}_1^+ \to \frac{1}{\sqrt{2}}(\hat{a}_1^+ + \hat{a}_2^+) \\ \hat{a}_2^+ \to \frac{1}{\sqrt{2}}(\hat{a}_1^+ - \hat{a}_2^+) \end{cases}$$

$$|\Psi_{out}\rangle = \frac{1}{2}(\hat{a}_1^+ + \hat{a}_2^+)(\hat{a}_1^+ - \hat{a}_2^+)|00\rangle = \boxed{\frac{1}{2}\left((a_1^+)^2 - (a_2^+)^2\right)|00\rangle}$$

$$(\hat{a}_1^+ + \hat{a}_2^+)(\hat{a}_1^+ - \hat{a}_2^+) = \underbrace{a_1^+ a_1^+}_{(a_1^+)^2} \underbrace{- a_1^+ a_2^+ + a_2^+ a_1^+}_{[a_2^+, a_1^+] = 0} - \underbrace{a_2^+ a_2^+}$$

$|\Psi_{out}\rangle$

$$\to \hat{a}^{+2}|0\rangle = \hat{a}^+(\underbrace{a^+|0\rangle}) = a^+|1\rangle = \sqrt{2}|2\rangle$$

$$(a_1^+)^2|00\rangle = a_1^+ a_1^+ |00\rangle = a_1^+|10\rangle = \sqrt{2}|20\rangle$$

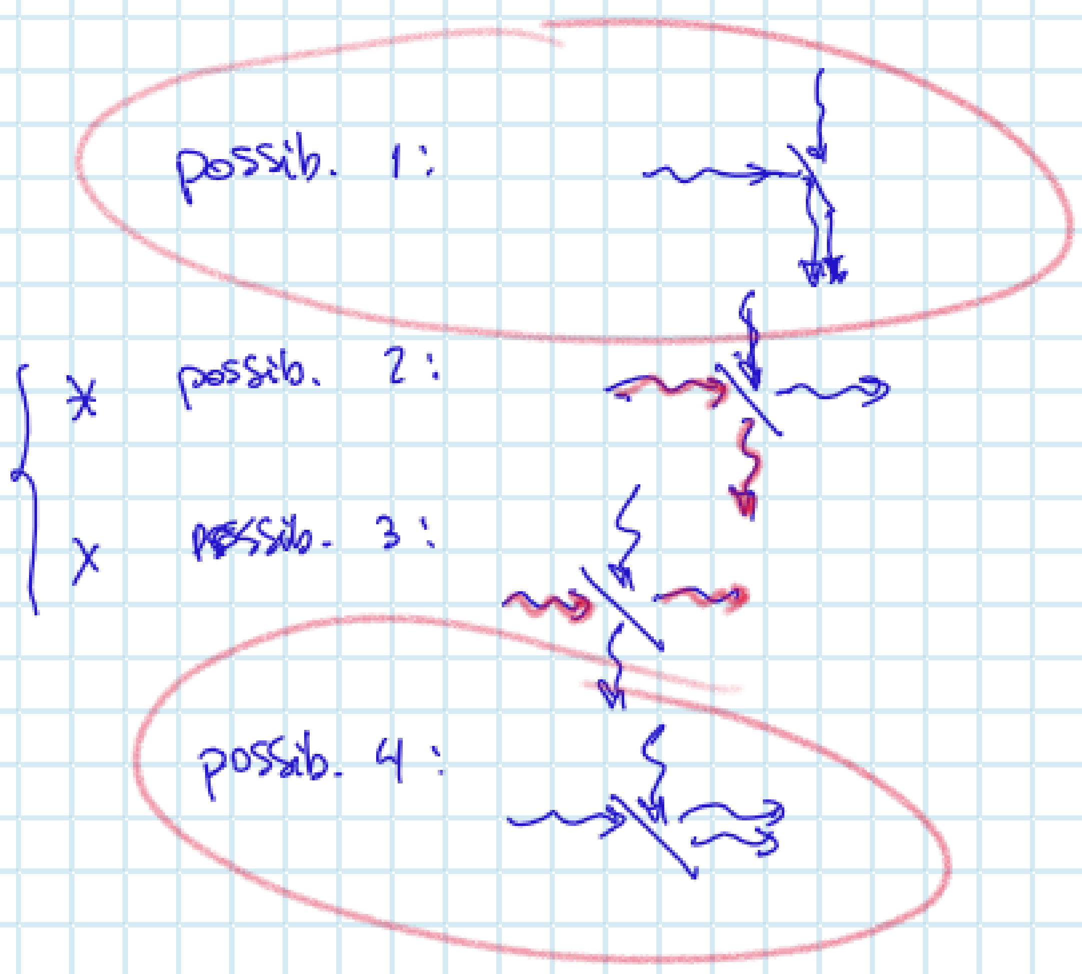
$$(a_2^+)^2|00\rangle = \sqrt{2}|02\rangle$$

$$|\Psi_{out}\rangle = \frac{\sqrt{2}}{2}(|20\rangle - |02\rangle)$$

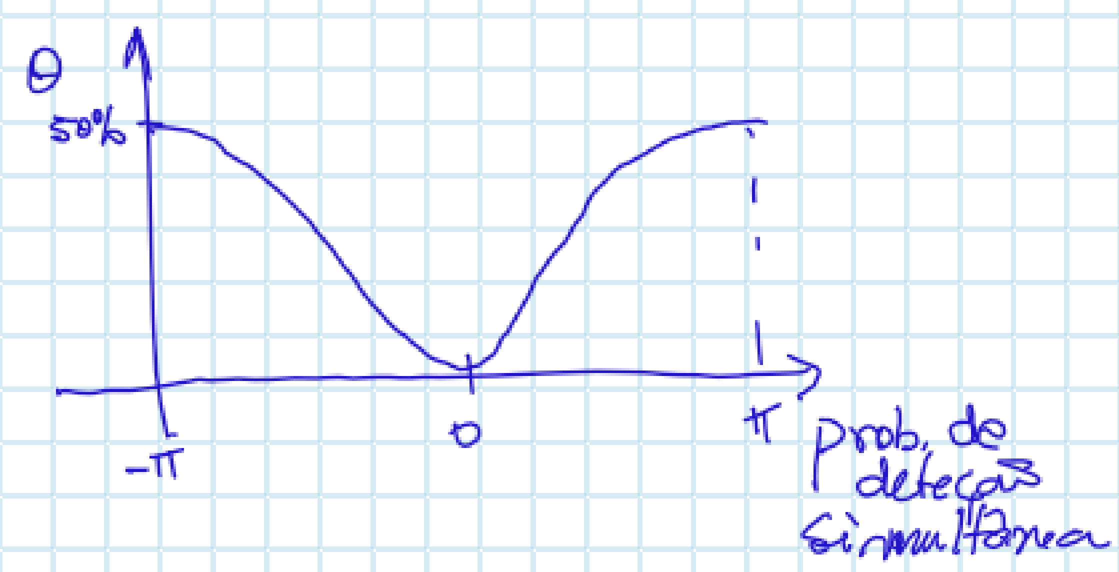
$$\langle\Psi_{out}|\Psi_{out}\rangle = \frac{2}{4}\left(\langle 20| - \langle 02|\right)\left(|20\rangle - |02\rangle\right)$$

$\underbrace{\langle 20|20\rangle}_{(1} + 1)$

$$= \frac{4}{4} = 1$$



Em termos de polarização



Sugestões de leitura:

- PRL 59, 2044 (1987)
- Nature, 556, 473 (2018)