Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

16 de junho de 2021

Dielétricos

Polarização

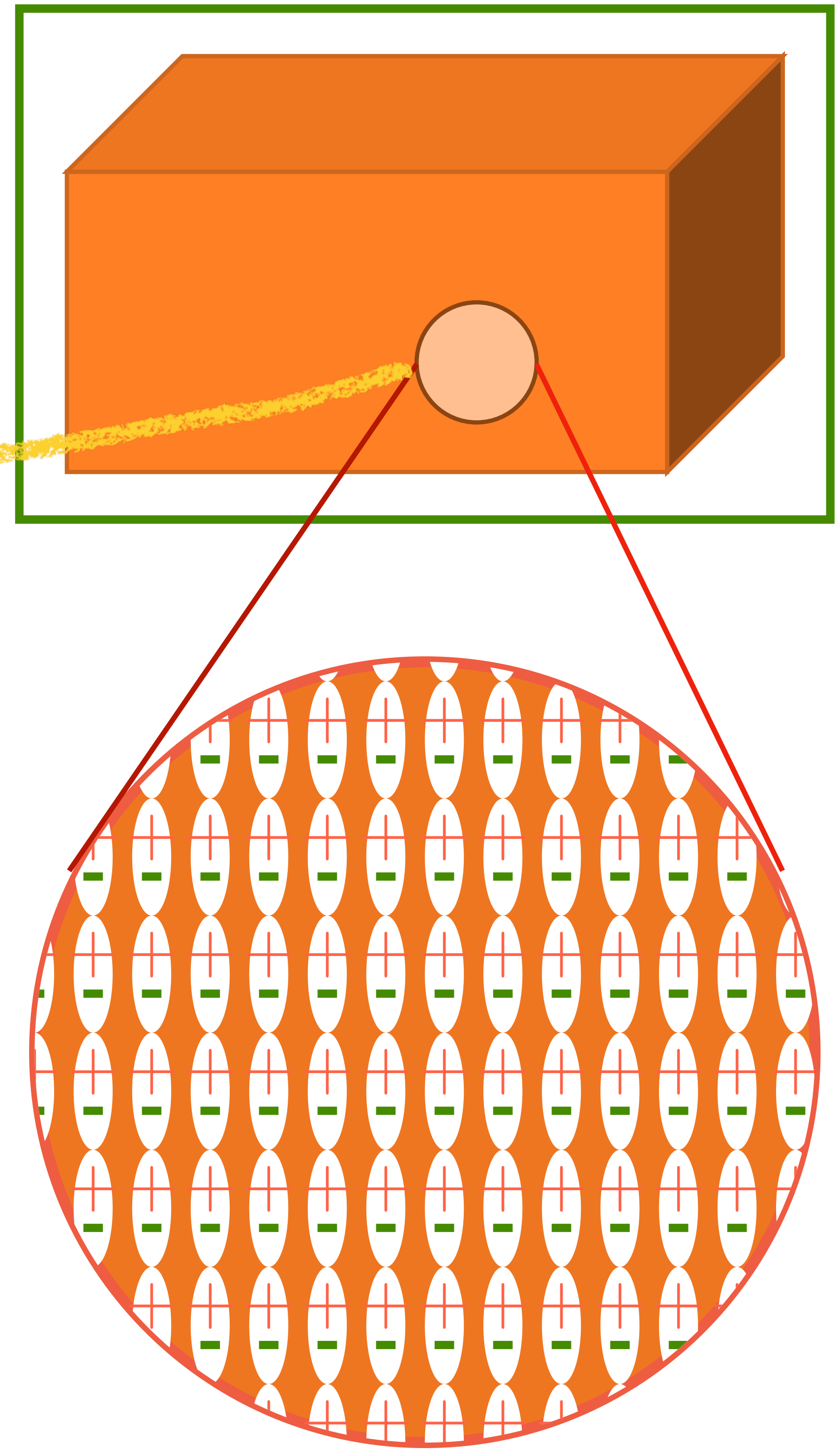
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\vec{P} = \frac{\sum_j \vec{p_j}}{\Delta\tau}$$

(Polarização)

PEQUENO VOLUME $\Delta\tau$
CONTÉM MUITOS DIPOLOS

VETOR POLARIZAÇÃO
DEPENDE DA
POSIÇÃO
É UM CAMPO, PORTANTO



Polarização

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$\vec{P} = \frac{\sum_j \vec{p_j}}{\Delta\tau}$$

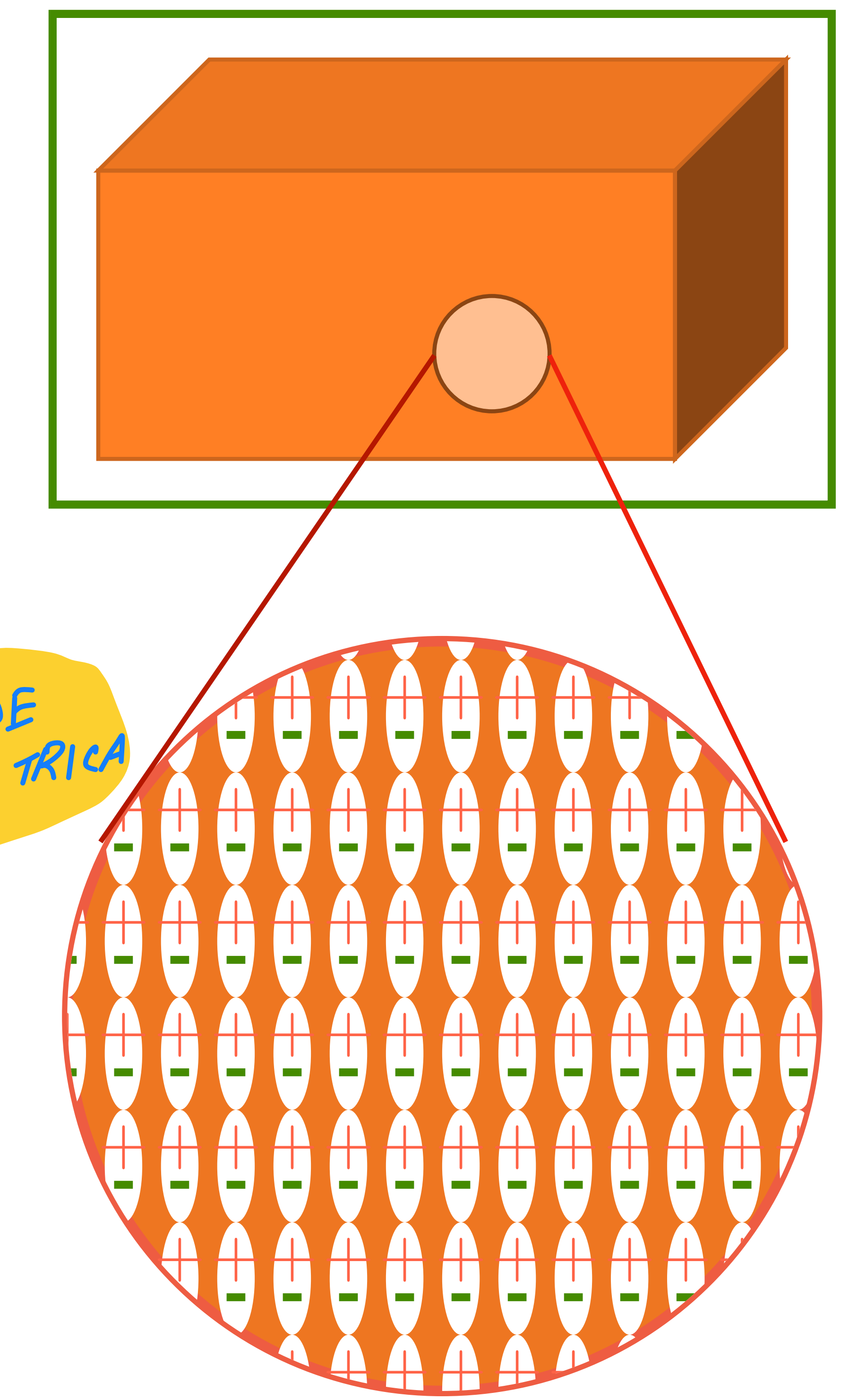
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\mathscr{r}} \cdot \vec{P}(\vec{r'})}{\mathscr{r}^2} \, d\tau'$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \oint_{\mathcal{S}} \frac{\sigma_b}{\mathscr{r}} \, dA' + \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\rho_b}{\mathscr{r}} \, d\tau'$$

DENSIDADE SUPERFICIAL

DENSIDADE VOLUMÉTRICA

CARGAS DE POLARIZAÇÃO



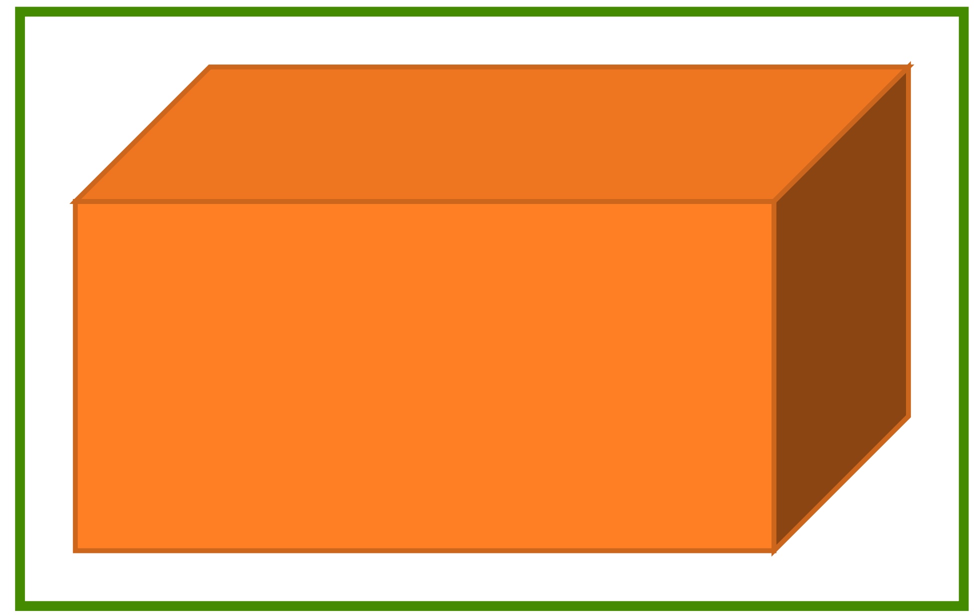
Vetor deslocamento

CARGAS LIVRES

$$\rho = \rho_f + \rho_b$$

CARGAS DE POLARIZAÇÃO

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$$



$$\vec{\nabla} \cdot \vec{D} = \rho_f$$

LEI DE GAUSS PARA DIELÉTRICOS

$$\int_A \vec{D} \cdot \hat{n} \, \mathrm{d}A = Q_f$$

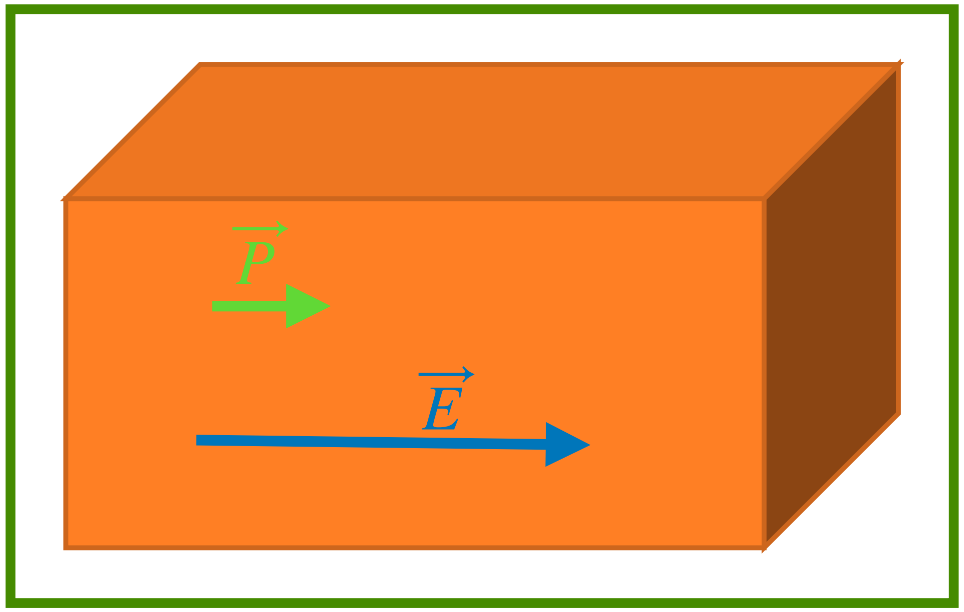
FORMA INTEGRAL

$$\rho_b \equiv -\vec{\nabla} \cdot \vec{P}$$

Meios lineares

$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$

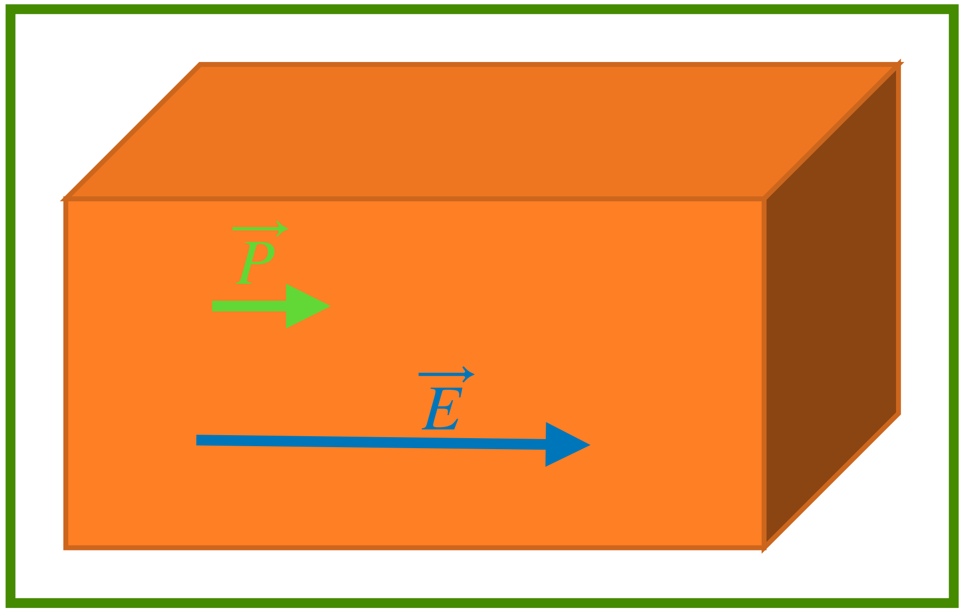
Susceptibilidade



Meios lineares

$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$

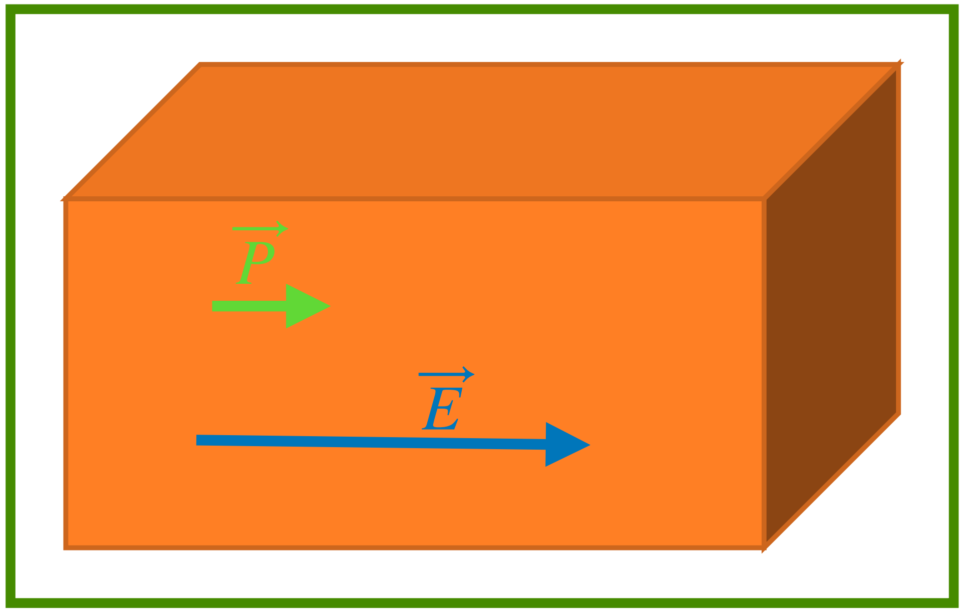
Susceptibilidade



$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$$

Meios lineares

$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$



Susceptibilidade

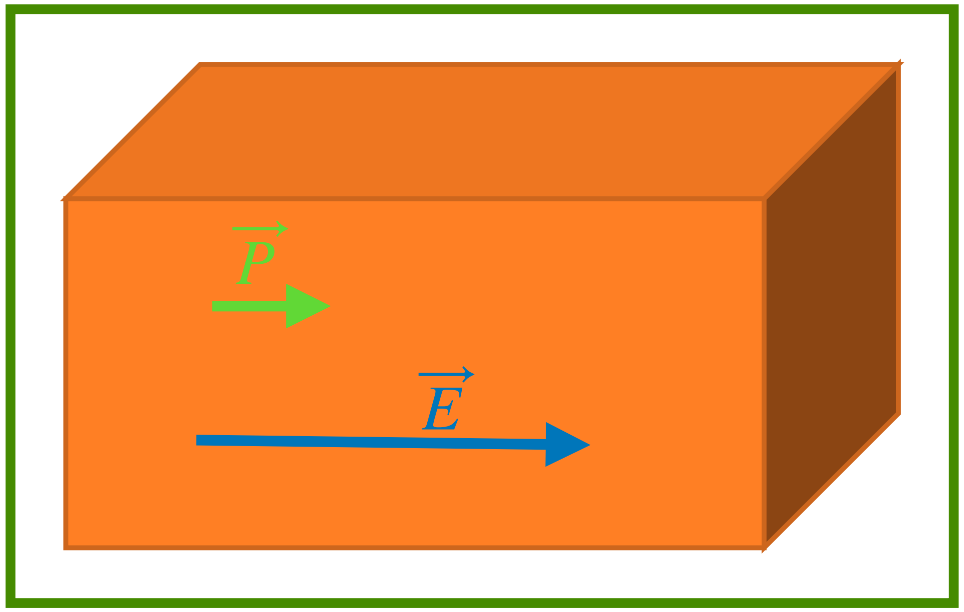
$$\vec{D} = \epsilon_0 \vec{E} + \chi_e \epsilon_0 \vec{E}$$

$$\vec{D} \equiv \epsilon_0 \vec{E} + \vec{P}$$

Meios lineares

$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$

Susceptibilidade



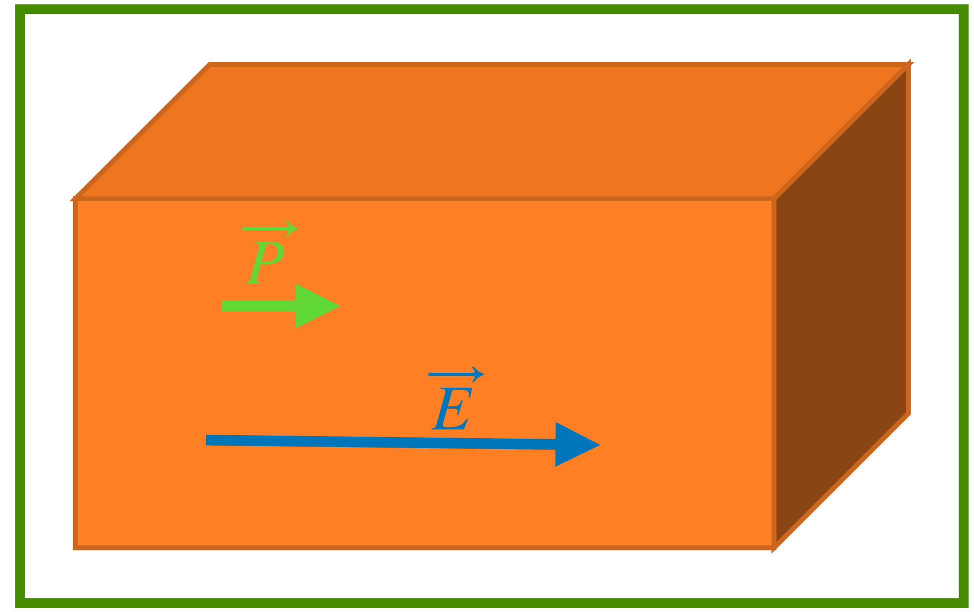
$$\vec{D} = \epsilon_0 \vec{E} + \chi_e \epsilon_0 \vec{E}$$

$$\vec{D} = \underbrace{(1 + \chi_e)\epsilon_0}_{\epsilon} \vec{E}$$

Meios lineares

$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$

Susceptibilidade



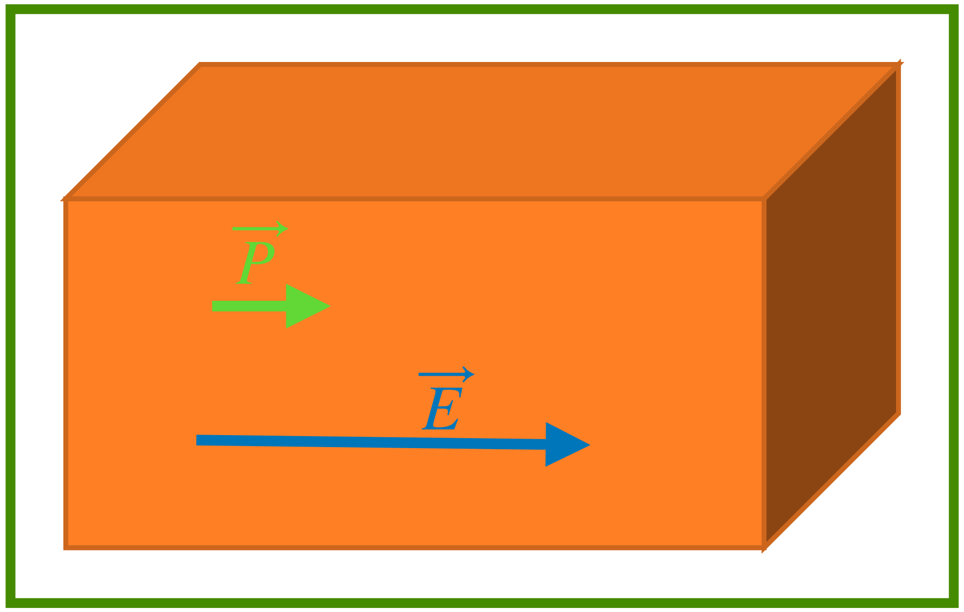
$$\vec{D} = \epsilon_0 \vec{E} + \chi_e \epsilon_0 \vec{E}$$

$$\vec{D} = (1 + \chi_e)\epsilon_0 \vec{E}$$

$$\epsilon = (1 + \chi_e)\epsilon_0$$

Permissividade

Meios lineares



$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$

Susceptibilidade

$$\vec{D} = \epsilon_0 \vec{E} + \chi_e \epsilon_0 \vec{E}$$

$$\vec{D} = (1 + \chi_e) \epsilon_0 \vec{E}$$

$$\epsilon = (1 + \chi_e) \epsilon_0$$

Permissividade

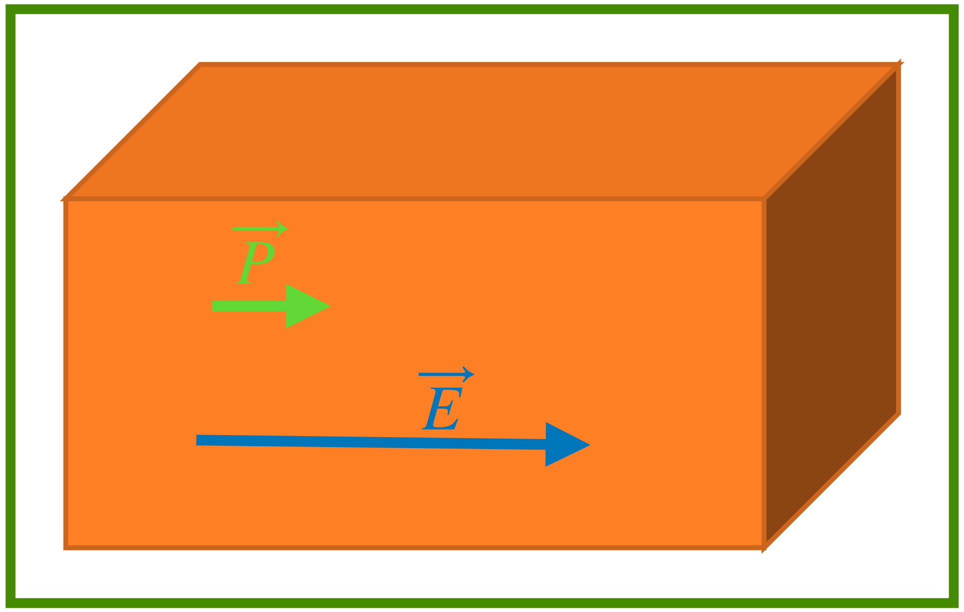
$$\epsilon_r = 1 + \chi_e$$

Constante dielétrica

Meios lineares

$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$

Susceptibilidade



$$\vec{D} = \epsilon_0 \vec{E} + \chi_e \epsilon_0 \vec{E}$$

$$\vec{D} = (1 + \chi_e)\epsilon_0 \vec{E}$$

$$\epsilon = (1 + \chi_e)\epsilon_0$$

Permissividade

$$\epsilon_r = 1 + \chi_e$$

Constante dielétrica

$$\vec{D} = \epsilon \vec{E} = \epsilon_r \epsilon_0 \vec{E}$$

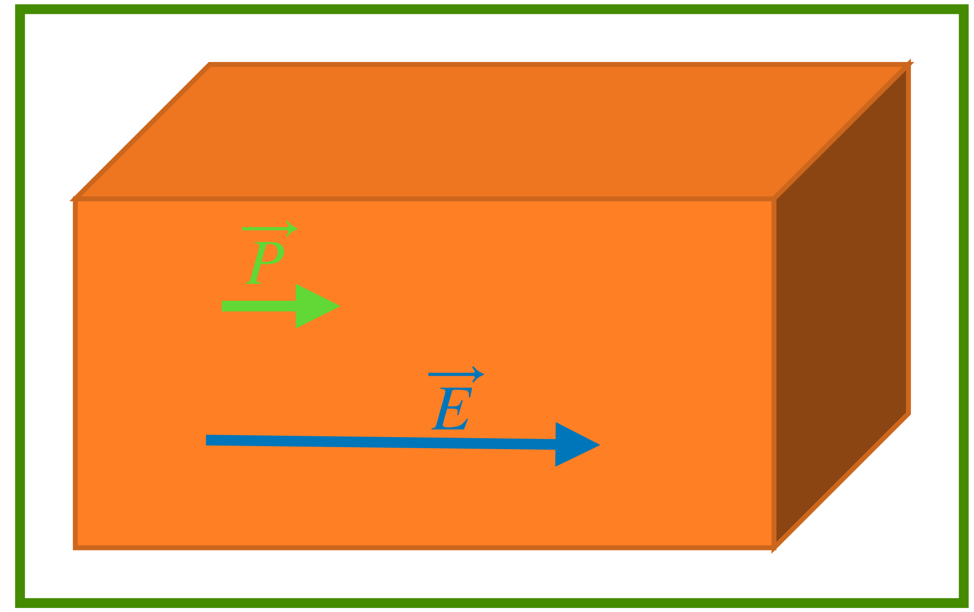
Meios lineares

$$\vec{P} = \chi_e \epsilon_0 \vec{E}$$

Susceptibilidade

$$\vec{D} = \epsilon \vec{E} = \epsilon_r \epsilon_0 \vec{E}$$

$$\epsilon_r = 1 + \chi_e$$



ϵ_r ?

GASES

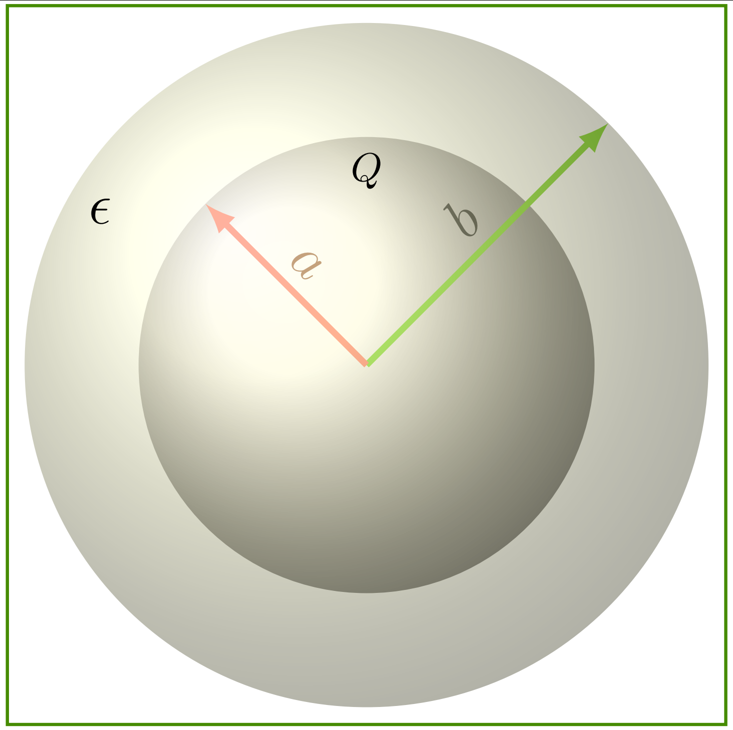
Material	Dielectric Constant	Material	Dielectric Constant
Vacuum	1	Benzene	2.28
Helium	1.000065	Diamond	5.7
Neon	1.00013	Salt	5.9
Hydrogen	1.00025	Silicon	11.8
Argon	1.00052	Methanol	33.0
Air (dry)	1.00054	Water	80.1
Nitrogen	1.00055	Ice (-30° C)	99
Water vapor (100° C)	1.00587	$KTaNbO_3$ (0° C)	34,000

ϵ_r

Pratique o que aprendeu

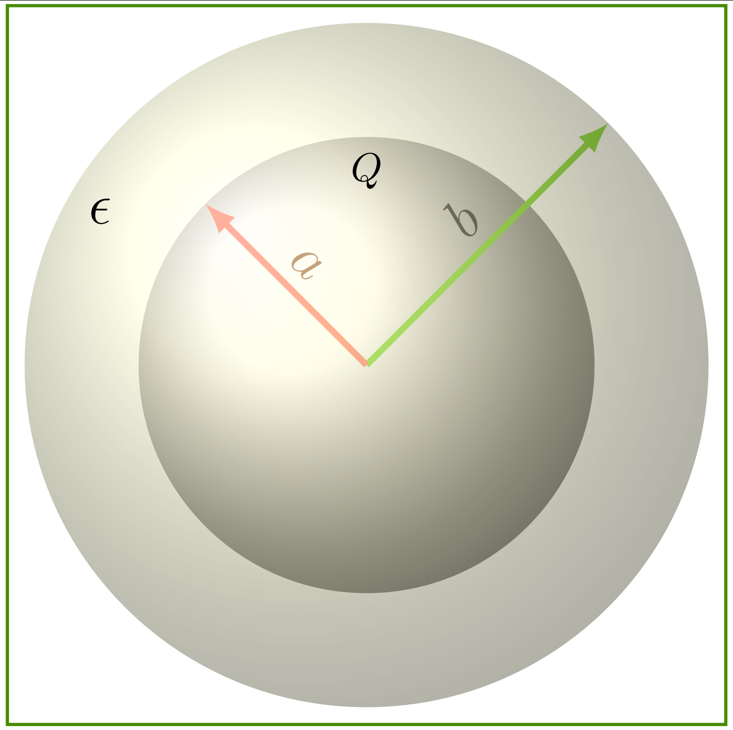
$\vec{E} = ?$

DENTRO E FORA
DO DIELÉTRICO



Pratique o que aprendeu

$$\vec{E} = ?$$

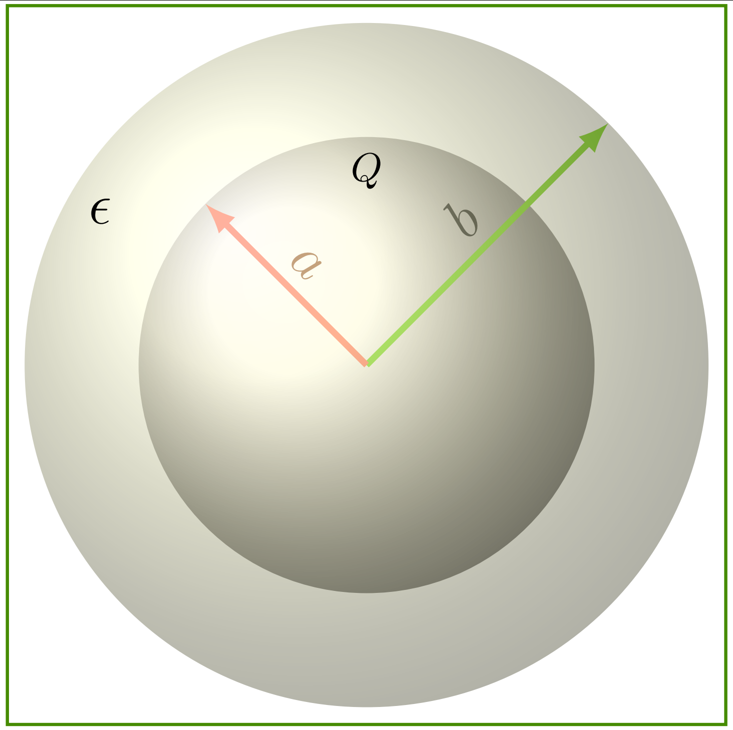


$$\vec{D} = \epsilon \vec{E} = \epsilon_r \epsilon_0 \vec{E}$$

MAIS FÁCIL ENCONTRAR $\vec{D}$, PRIMEIRO

Pratique o que aprendeu

$$\vec{E} = ?$$



$$\int_{\mathcal{A}} \vec{D} \cdot \hat{n} \, \mathrm{d}A = 0 \qquad (r < a)$$

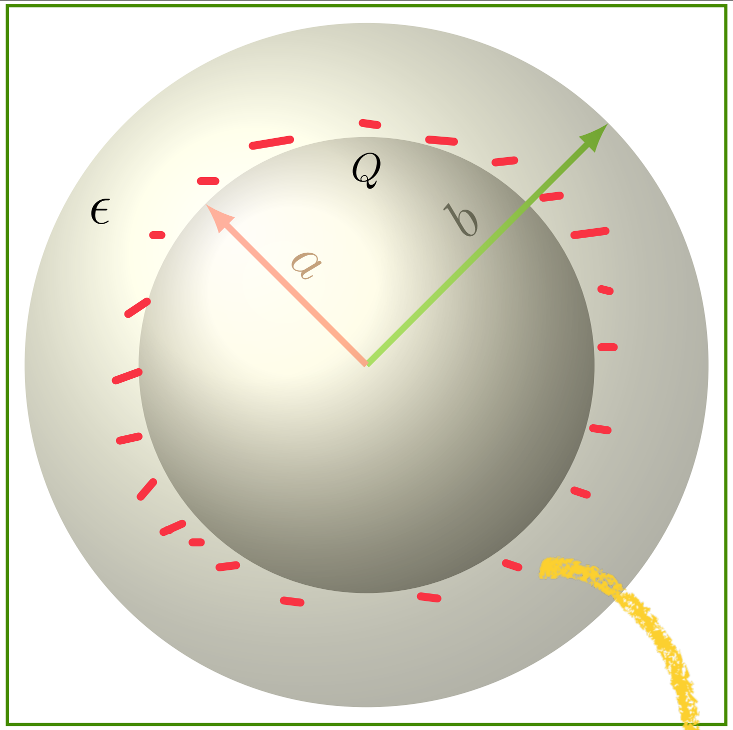
POR SIMETRIA, $\vec{D} \parallel \hat{n}$

$\Rightarrow \int \vec{D} \cdot \hat{n} \, dA = D \int dA = 4\pi R^2 D$

$\Rightarrow D = 0$ $\Rightarrow E = 0$

$$\vec{D} = \epsilon \vec{E} = \epsilon_r \epsilon_0 \vec{E}$$

Pratique o que aprendeu



$$\vec{E} = ?$$

$$\int_A \vec{D} \cdot \hat{n} \, \mathrm{d}A = Q \qquad (a < r < b)$$

$$D \cdot 4\pi R^2 = Q$$

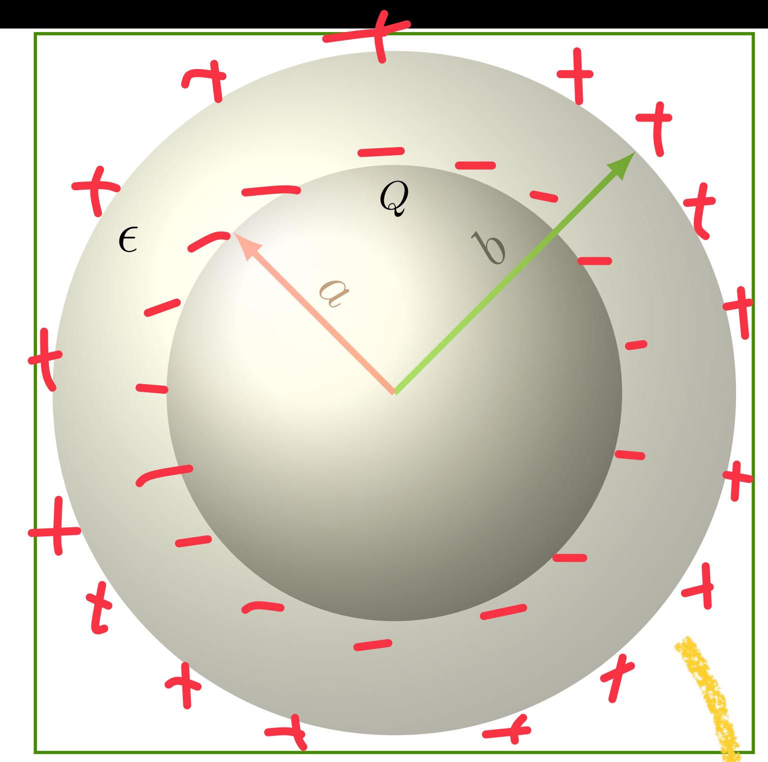
$$D = \frac{1}{4\pi} \frac{Q}{R^2} \qquad (a < r < b)$$

$$E = \frac{1}{4\pi\epsilon} \frac{Q}{R^2}$$

$$\vec{D} = \epsilon \vec{E} = \epsilon_r \epsilon_0 \vec{E}$$

σ_b REDUZ CAMPO ELÉTRICO DENTRO DO DIELÉTRICO

Pratique o que aprendeu



$$\vec{E} = ?$$

$$\int_{\mathcal{A}} \vec{D} \cdot \hat{n} \, \mathrm{d}A = Q \qquad (b < r)$$

$$D \cdot 4\pi R^2 = Q$$

$$D = \frac{1}{4\pi} \frac{Q}{R^2} \qquad (b < r)$$

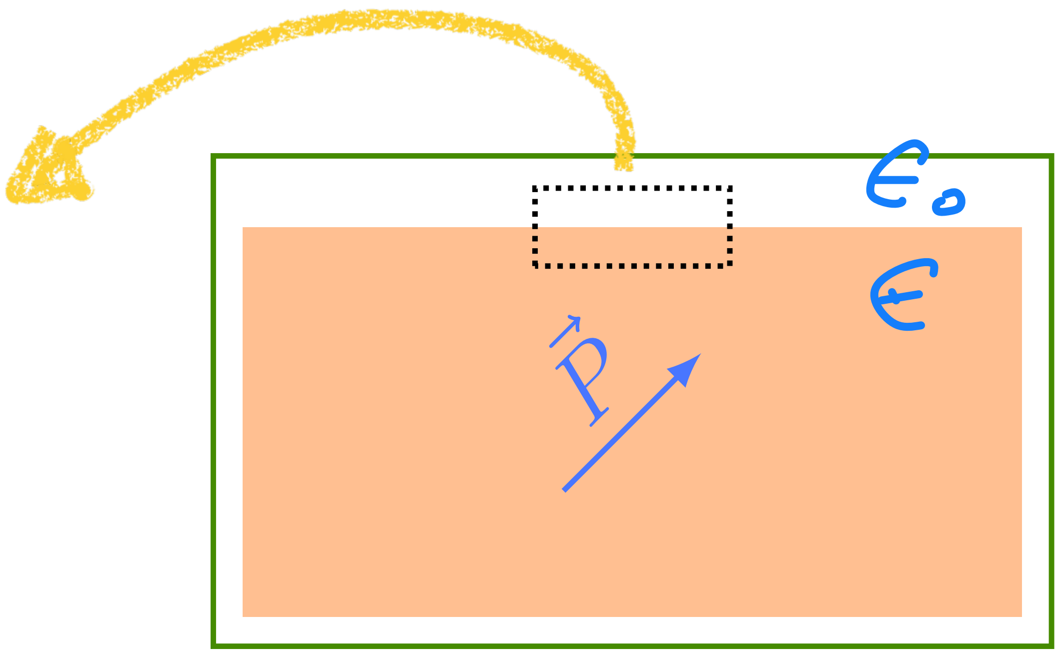
$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R^2}$$

$$\vec{D} = \epsilon\vec{E} = \epsilon_r \epsilon_0 \vec{E}$$

CAMPOS DAS DUAS CARGAS SUPERFICIAIS σ_b SE CANCELAM

Condições de contorno

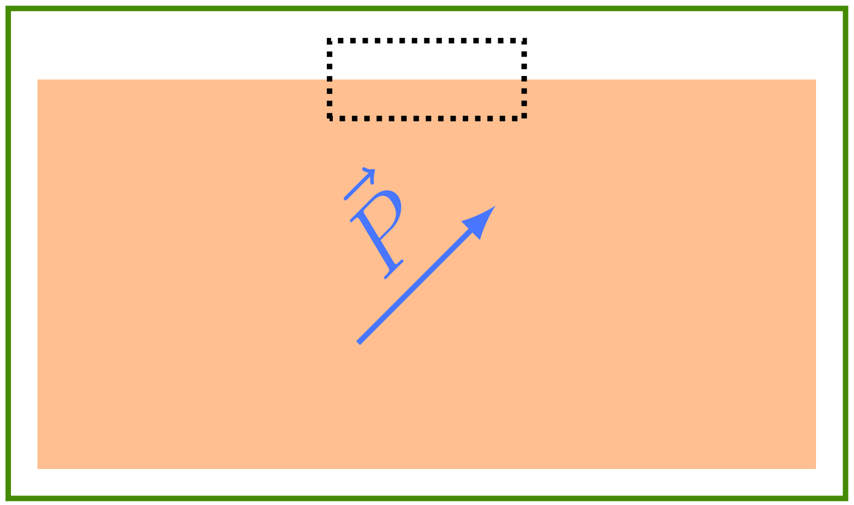
USAREMOS GAUSS E STOKES



Condições de contorno

Gauss

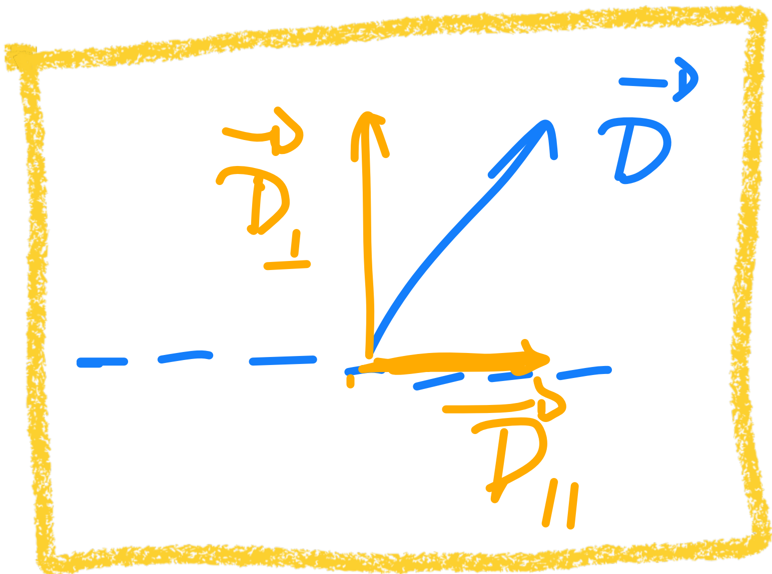
$$\int_A \vec{D} \cdot \hat{n} \, \mathrm{d}A = 0$$

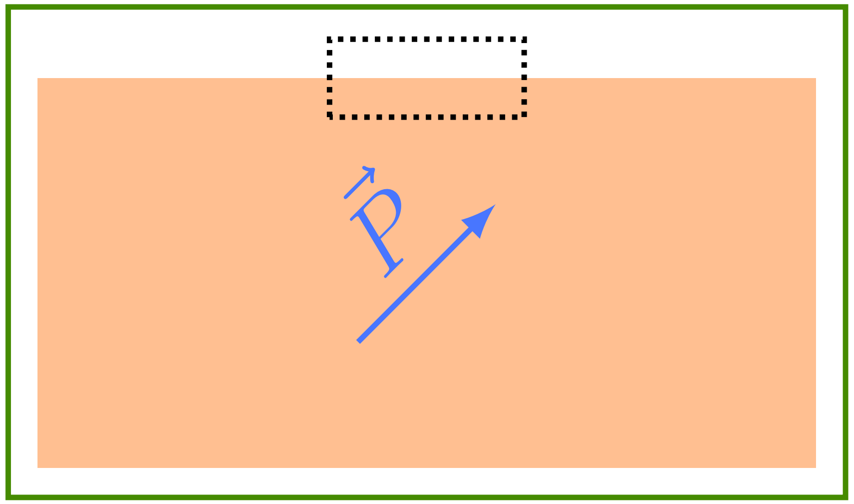


Condições de contorno

$$\int_{\mathcal{A}} \vec{D} \cdot \hat{n} \; \mathrm{d}A = 0$$

$$D_{a\perp} A - D_{b\perp} A = 0$$





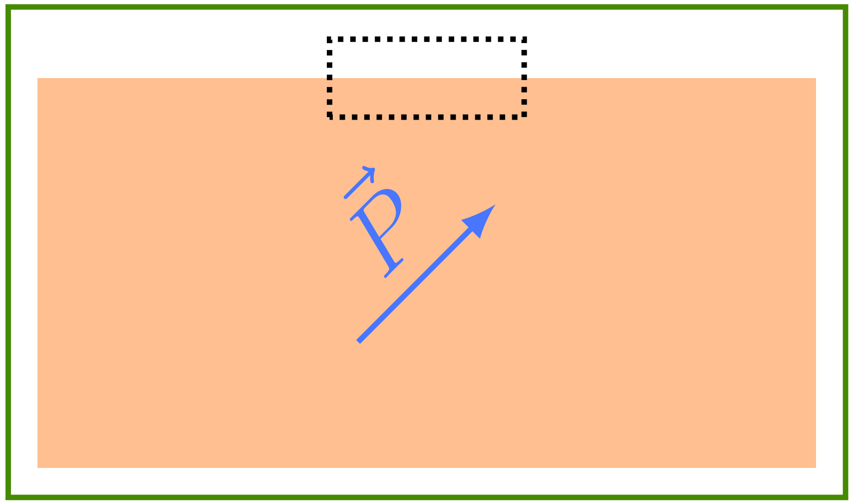
$$\vec{D} = \epsilon \vec{E} = \epsilon_r \epsilon_0 \vec{E}$$

Condições de contorno

$$\int_{\mathcal{A}} \vec{D} \cdot \hat{n} \; \mathrm{d}A = 0$$

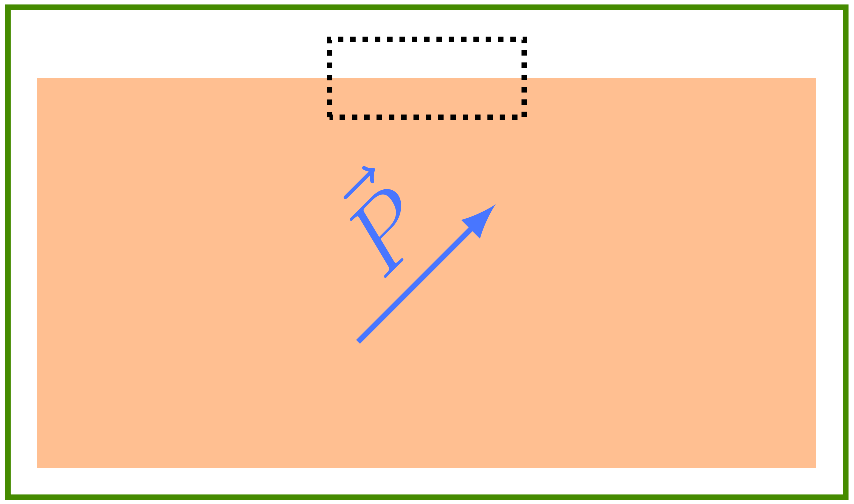
$$D_{a\perp} A - D_{b\perp} A = 0$$

$$D_{a\perp} = D_{b\perp}$$



Condições de contorno

$$\int_{\mathcal{A}} \vec{D} \cdot \hat{n} \; \mathrm{d}A = 0$$



$$D_{a\perp} A - D_{b\perp} A = 0$$

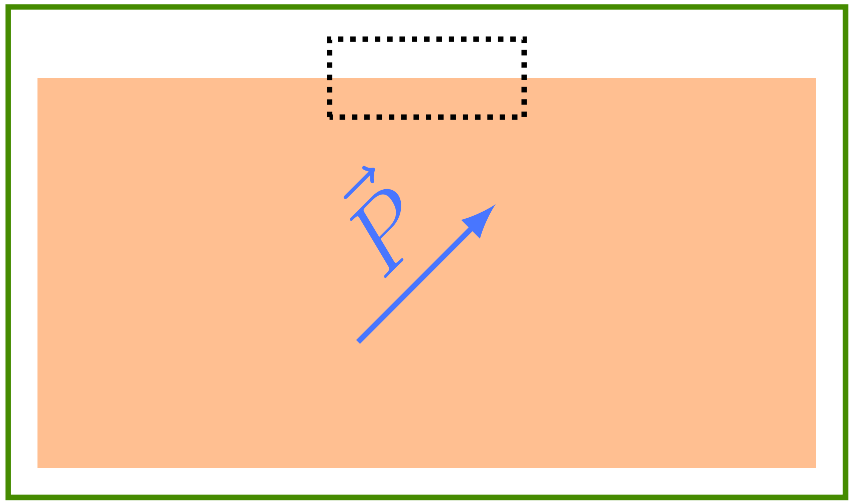
$$D_{a\perp} = D_{b\perp}$$

$$\oint \vec{E} \cdot \mathrm{d}\vec{\ell} = 0$$

STOKES

Condições de contorno

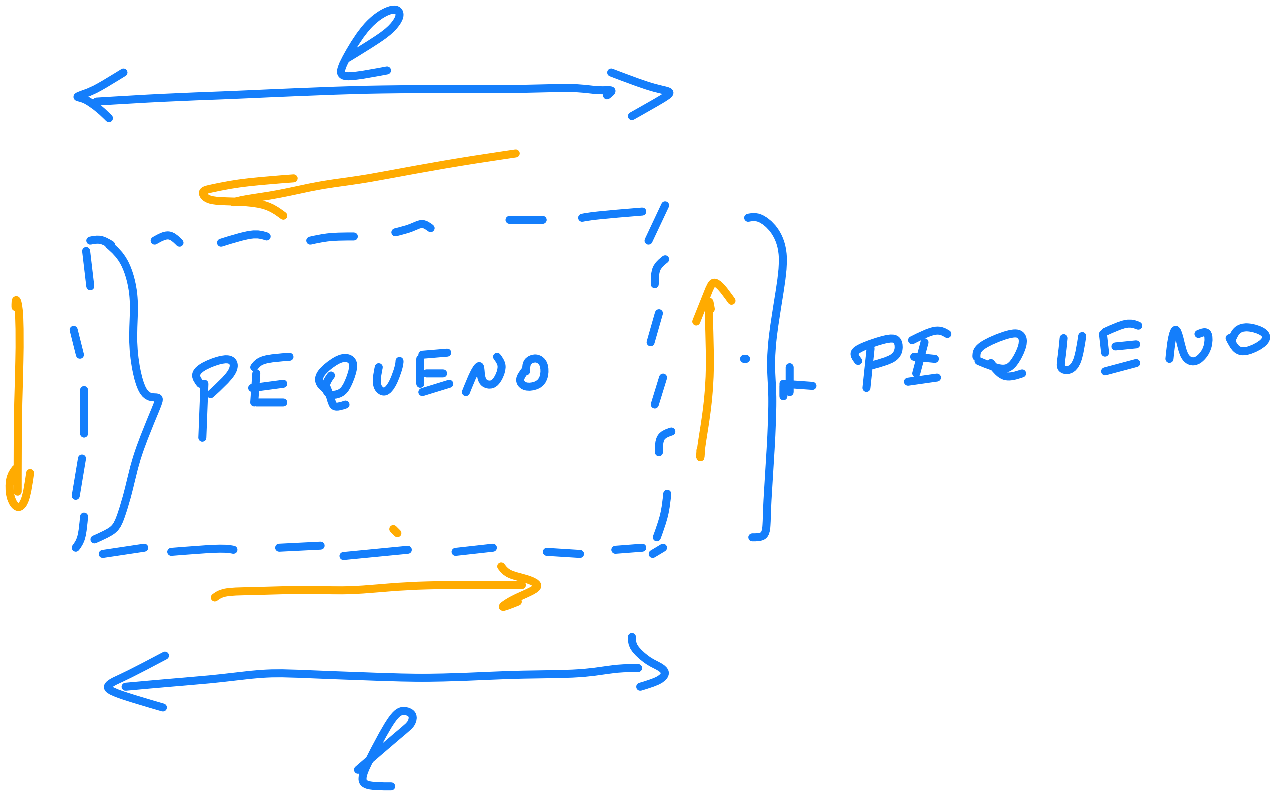
$$\int_{\mathcal{A}} \vec{D} \cdot \hat{n} \; \mathrm{d}A = 0$$



$$D_{a\perp} A - D_{b\perp} A = 0$$

$$D_{a\perp} = D_{b\perp}$$

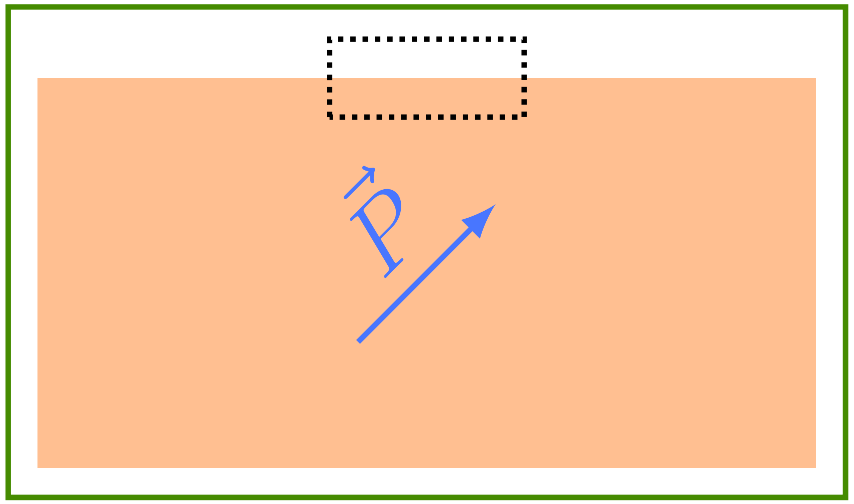
$$\oint \vec{E} \cdot \mathrm{d}\vec{\ell} = 0$$



$$-E_{a\|}\ell + E_{b\|}\ell = 0$$

Condições de contorno

$$\int_{\mathcal{A}} \vec{D} \cdot \hat{n} \; \mathrm{d}A = 0$$



$$D_{a\perp} A - D_{b\perp} A = 0$$

$$D_{a\perp} = D_{b\perp}$$

$$\oint \vec{E} \cdot \mathrm{d}\vec{\ell} = 0$$

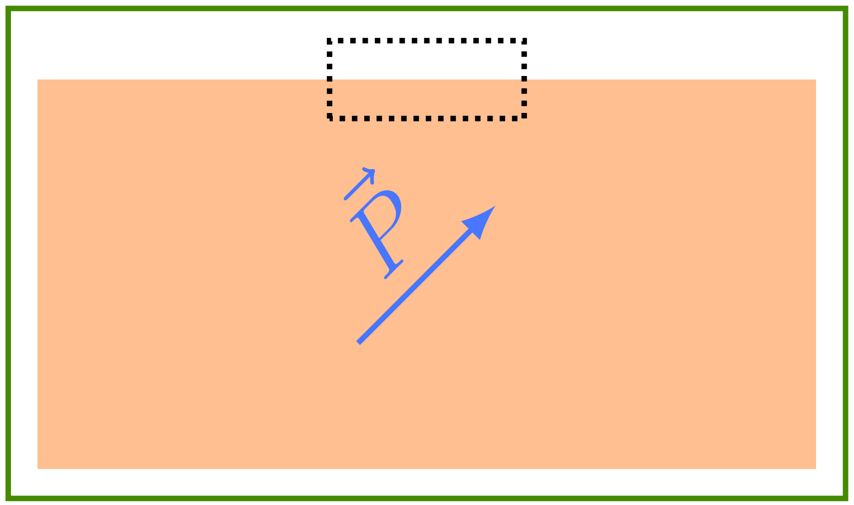
$$-E_{a\|} \ell + E_{b\|} \ell = 0$$

$$E_{a\|} = E_{b\|}$$

Condições de contorno

$$D_{a\perp} = D_{b\perp}$$

$$D_{a\|} - P_{a\|} = D_{b\|} - P_{b\|}$$

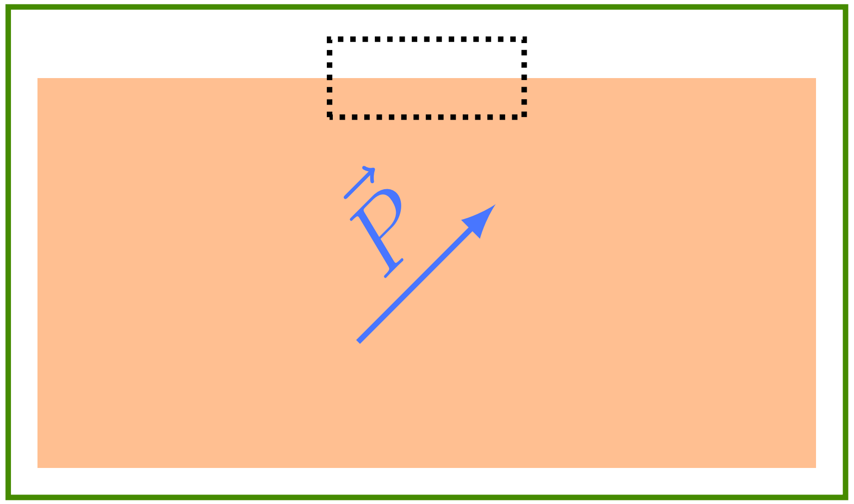


Condições de contorno

$$D_{a\perp} = D_{b\perp}$$

$$D_{a\|} - P_{a\|} = D_{b\|} - P_{b\|}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{P} = -\rho_b$$



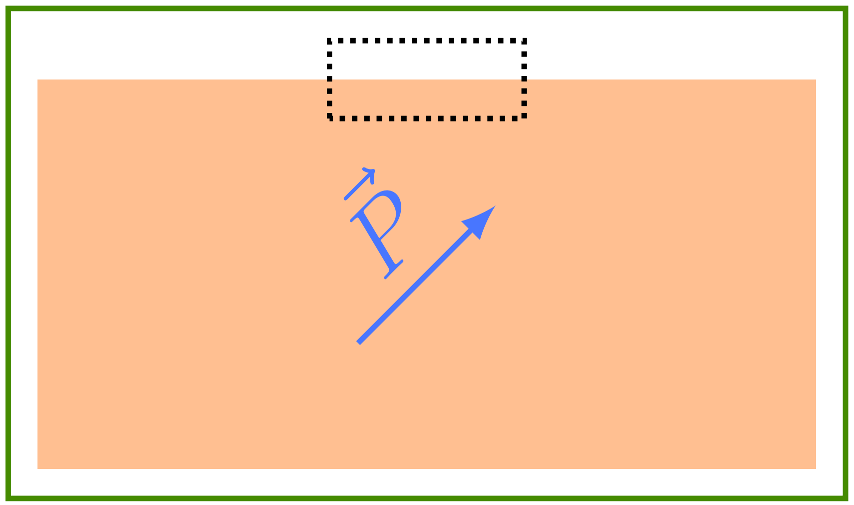
Condições de contorno

$$D_{a\perp} = D_{b\perp}$$

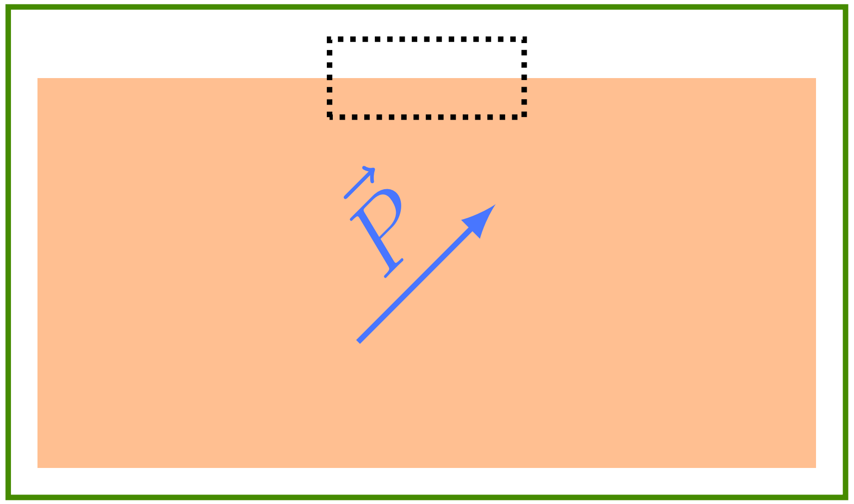
$$D_{a\|} - P_{a\|} = D_{b\|} - P_{b\|}$$

$$\vec{\nabla} \cdot \vec{P} = -\rho_b$$

$$\rho_b = -\vec{\nabla} \cdot (\chi_e \epsilon_0 \vec{E})$$



Condições de contorno



$$D_{a\perp} = D_{b\perp}$$

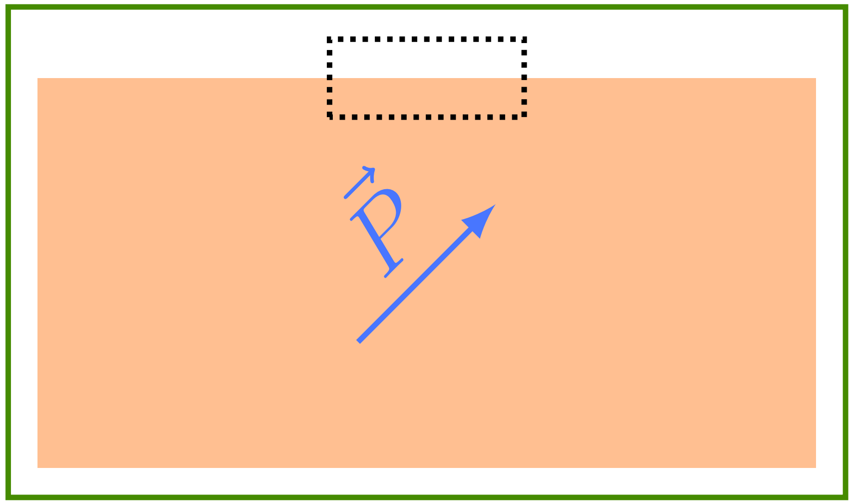
$$D_{a\parallel} - P_{a\parallel} = D_{b\parallel} - P_{b\parallel}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{P} = -\rho_b$$

$$\rho_b = -\vec{\boldsymbol{\nabla}} \cdot (\chi_e \epsilon_0 \vec{E}) = -\vec{\boldsymbol{\nabla}} \cdot (\chi_e \epsilon_0 \frac{\vec{D}}{\epsilon})$$

MELHOR TRABALHAR COM $\vec{D}$

Condições de contorno



$$D_{a\perp} = D_{b\perp}$$

$$D_{a\|} - P_{a\|} = D_{b\|} - P_{b\|}$$

$$\vec{\nabla} \cdot \vec{P} = -\rho_b$$

$$\rho_b = -\vec{\nabla} \cdot (\chi_e \epsilon_0 \vec{E}) = -\vec{\nabla} \cdot (\chi_e \epsilon_0 \frac{\vec{D}}{\epsilon})$$

$$\rho_b = -\vec{\nabla} \cdot (\frac{\chi_e}{1+\chi_e} \vec{D})$$

$\vec{\nabla} \cdot \vec{D} = \rho_f$

$$\rho_b = -\frac{\chi_e}{1+\chi_e} \rho_f$$

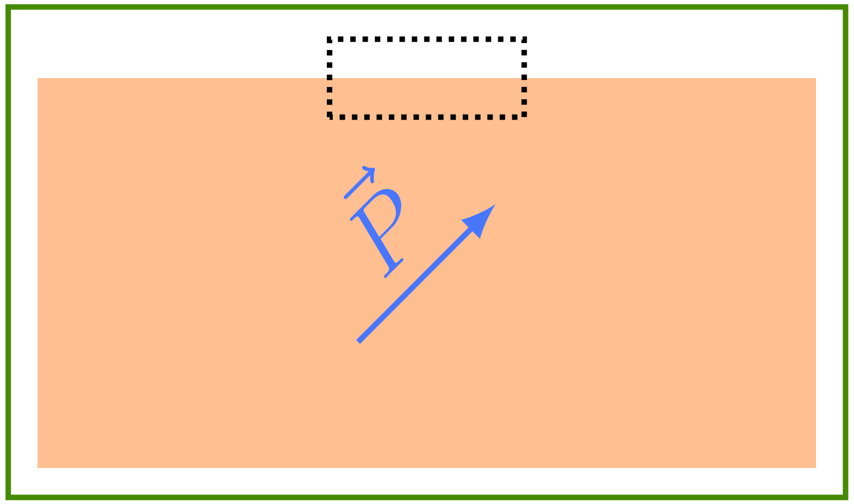
SE $\rho_f = 0$, NÃO HAVERÁ CARGA DIPOLAR VOLUMÉTRICA (PODE HAVER σ_b)

Condições de contorno

$$D_{a\perp} = D_{b\perp}$$

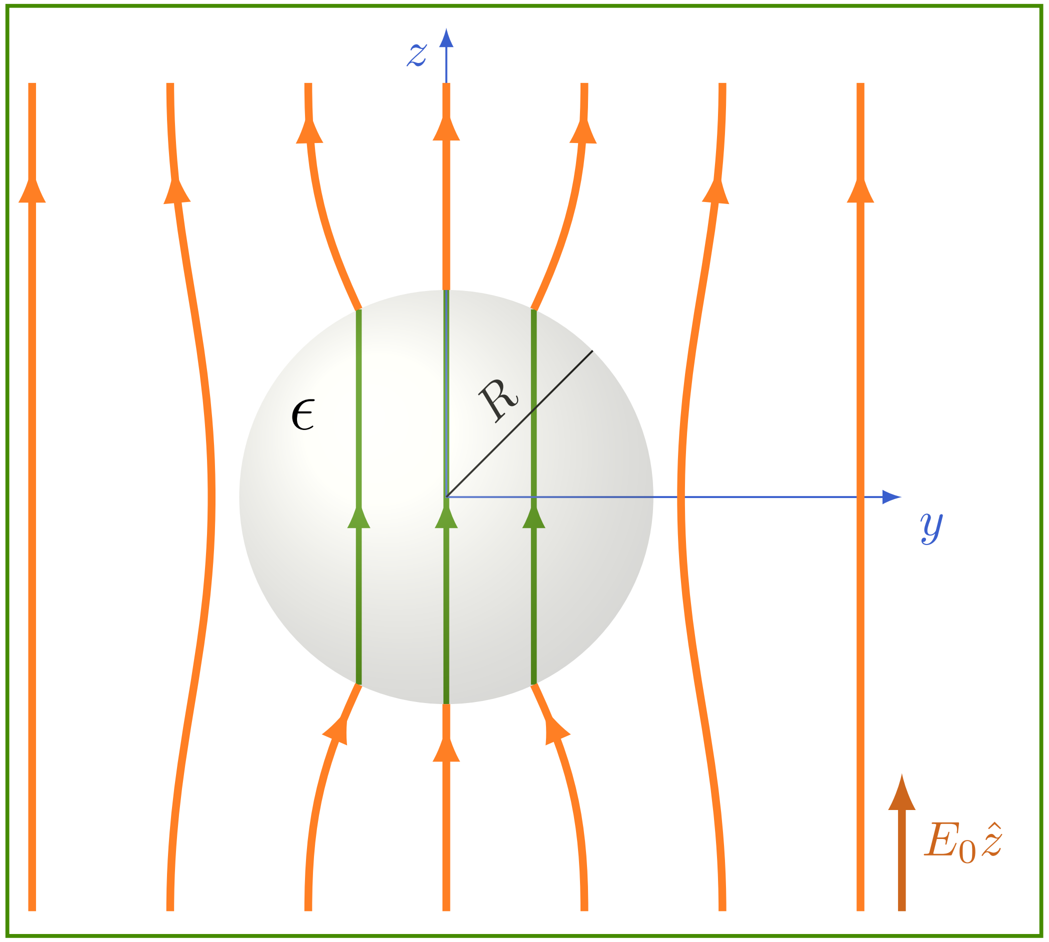
$$D_{a\|} - P_{a\|} = D_{b\|} - P_{b\|}$$

$$V_b = V_a$$



Pratique o que aprendeu

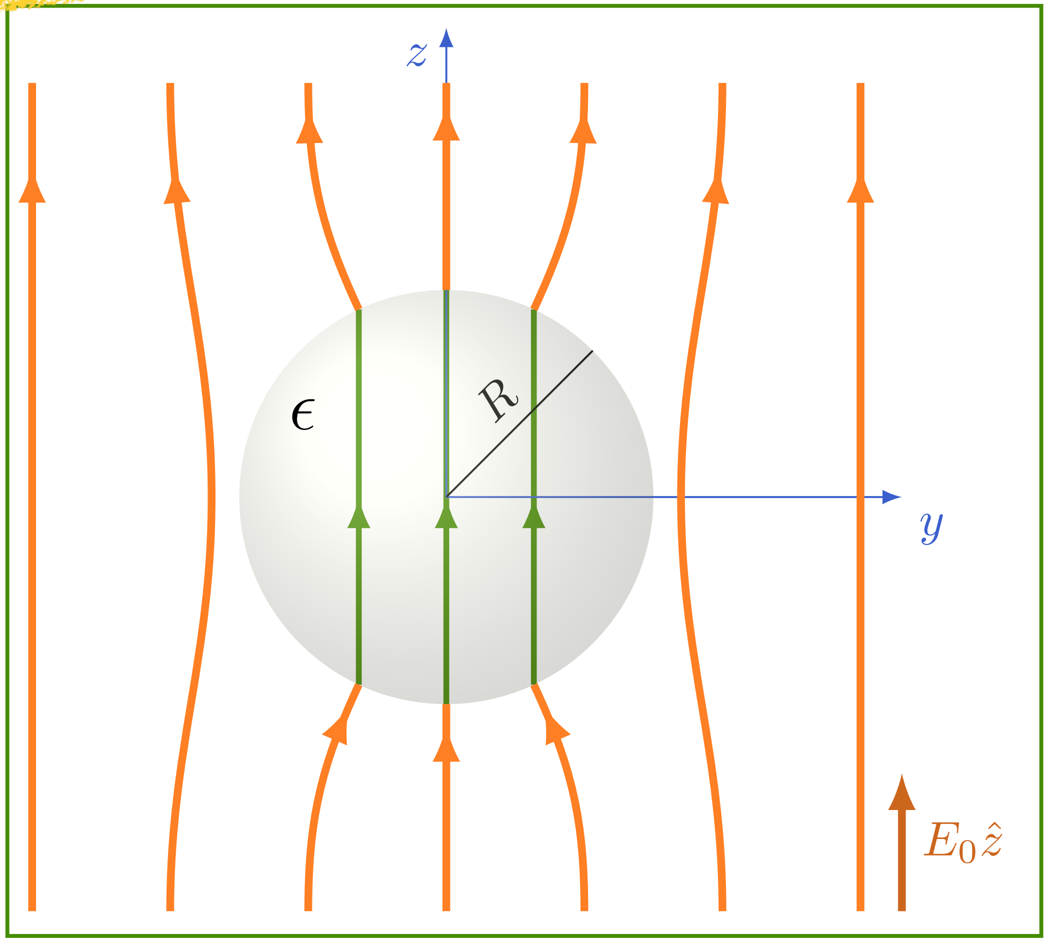
$$D_{a\perp} = D_{b\perp} \qquad D_{a\|} - P_{a\|} = D_{b\|} - P_{b\|} \qquad V_b = V_a$$



Pratique o que aprendeu

$$\epsilon_b \frac{\partial V_b}{\partial n} = \epsilon_a \frac{\partial V_a}{\partial n} \qquad V_b = V_a$$

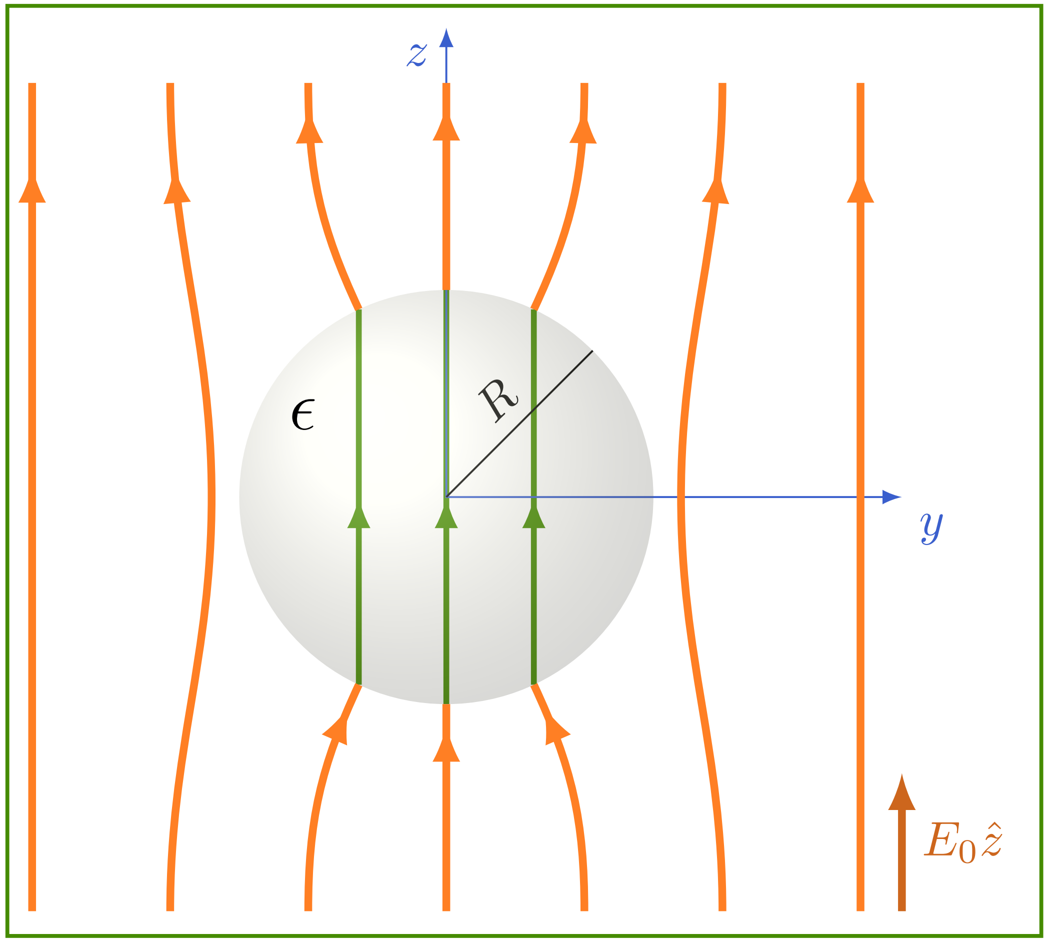
CAMPO PERPENDICULAR



Pratique o que aprendeu

$$\epsilon_b \frac{\partial V_b}{\partial n} = \epsilon_a \frac{\partial V_a}{\partial n} \qquad V_b = V_a$$

Condições de contorno



Pratique o que aprendeu

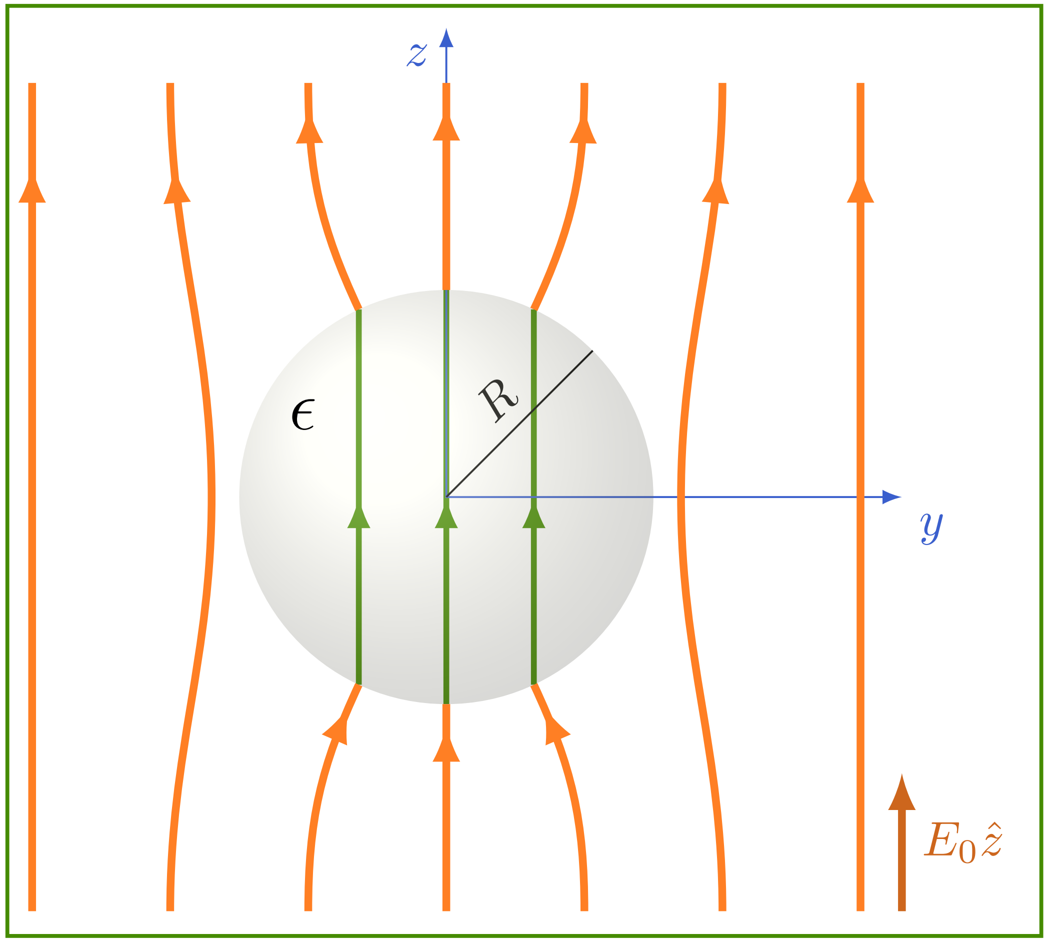
$$\epsilon_b \frac{\partial V_b}{\partial n} = \epsilon_a \frac{\partial V_a}{\partial n} \qquad V_b = V_a$$

Condições de contorno

$$V(r \to \infty) = -E_0 r \cos\theta$$

$$V_d(R) = V_f(R)$$

$$\epsilon \frac{\partial V_d}{\partial n}\bigg|_R = \epsilon_0 \frac{\partial V_f}{\partial n}\bigg|_R$$



Pratique o que aprendeu

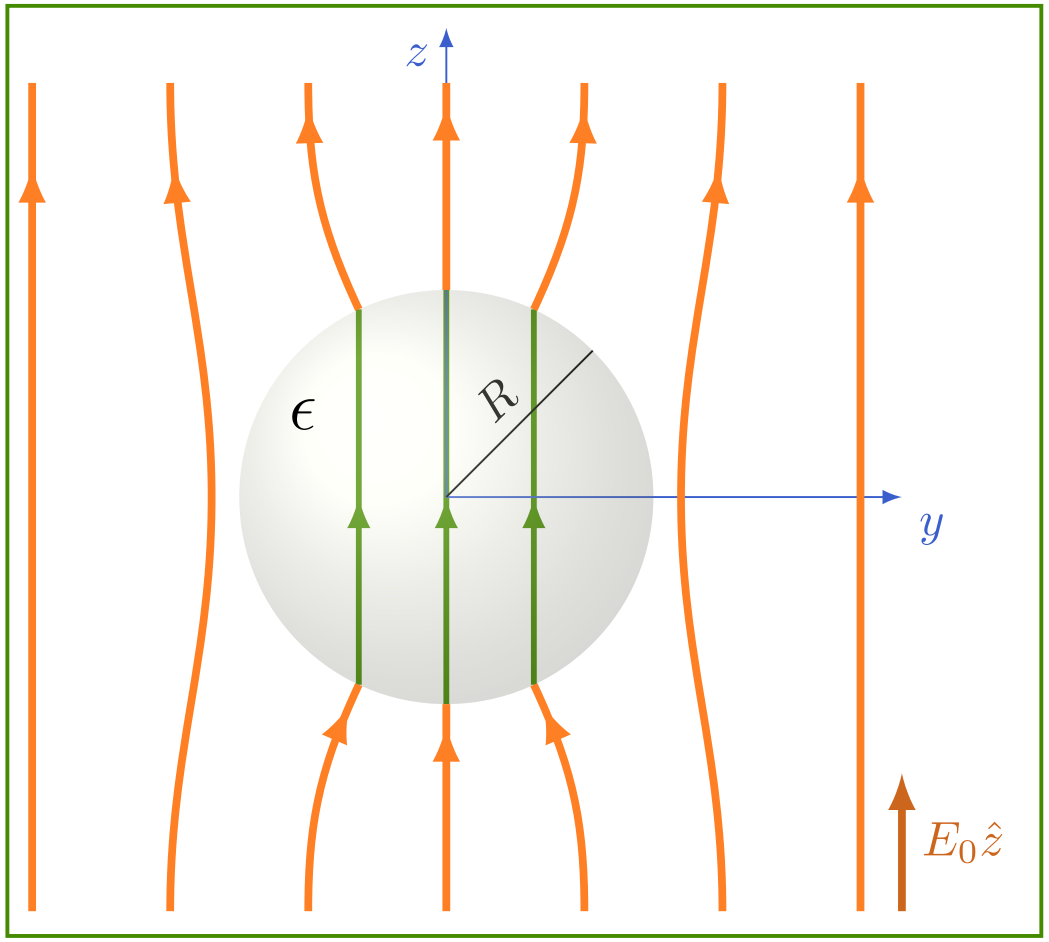
$$\epsilon_b \frac{\partial V_b}{\partial n} = \epsilon_a \frac{\partial V_a}{\partial n} \qquad V_b = V_a$$

Condições de contorno

$$V(r \to \infty) = -E_0 r \cos\theta$$

$$V_d(R) = V_f(R)$$

$$\epsilon \frac{\partial V_d}{\partial n}\Big|_R = \epsilon_0 \frac{\partial V_f}{\partial n}\Big|_R$$



Pratique o que aprendeu

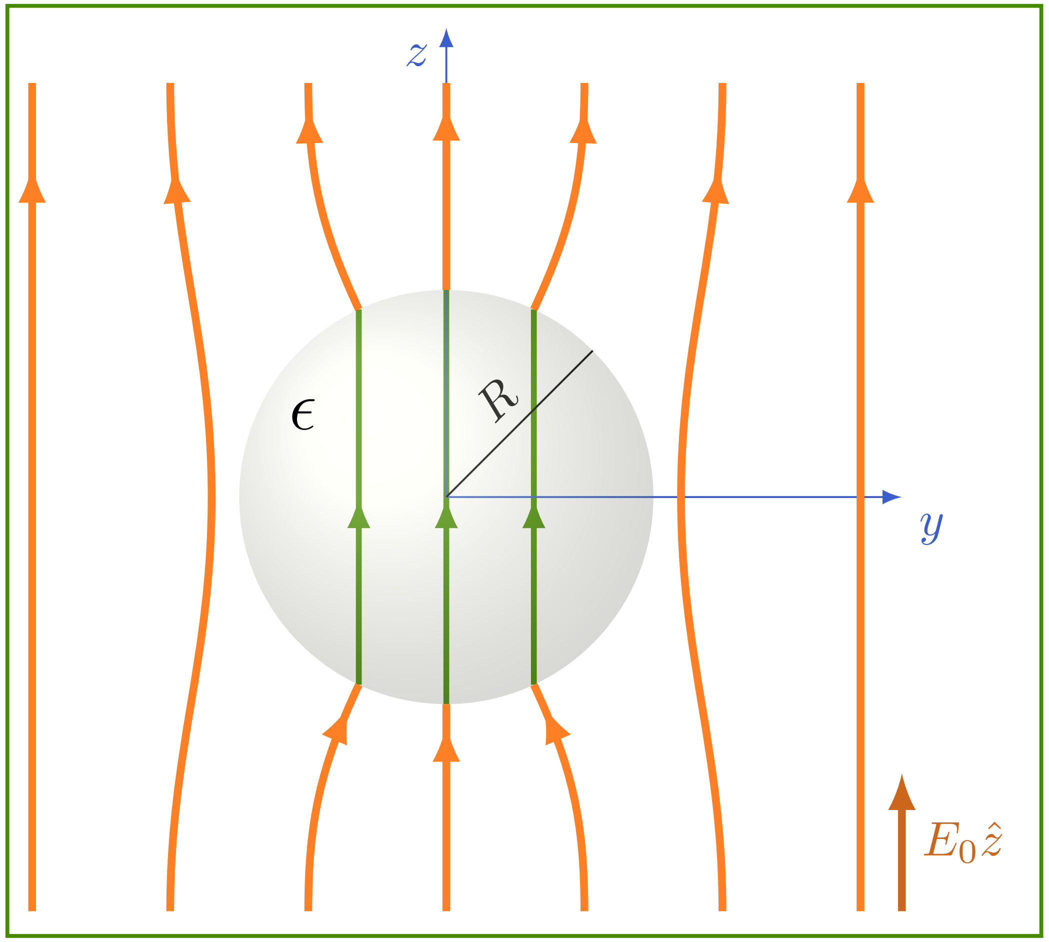
$$\epsilon_b \frac{\partial V_b}{\partial n} = \epsilon_a \frac{\partial V_a}{\partial n} \qquad V_b = V_a$$

Condições de contorno

$$V(r \to \infty) = -E_0 r \cos\theta$$

$$V_d(R) = V_f(R)$$

$$\epsilon \frac{\partial V_d}{\partial n}\Big|_R = \epsilon_0 \frac{\partial V_f}{\partial n}\Big|_R$$



DENTRO E FORA DA ESFERA

$$V(r) = \sum_\ell \left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right) P_\ell(\cos\theta)$$

$$r \to \infty \quad \Rightarrow \quad V_f(r) = -E_0 r \cos\theta + \sum_\ell \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$

FORA

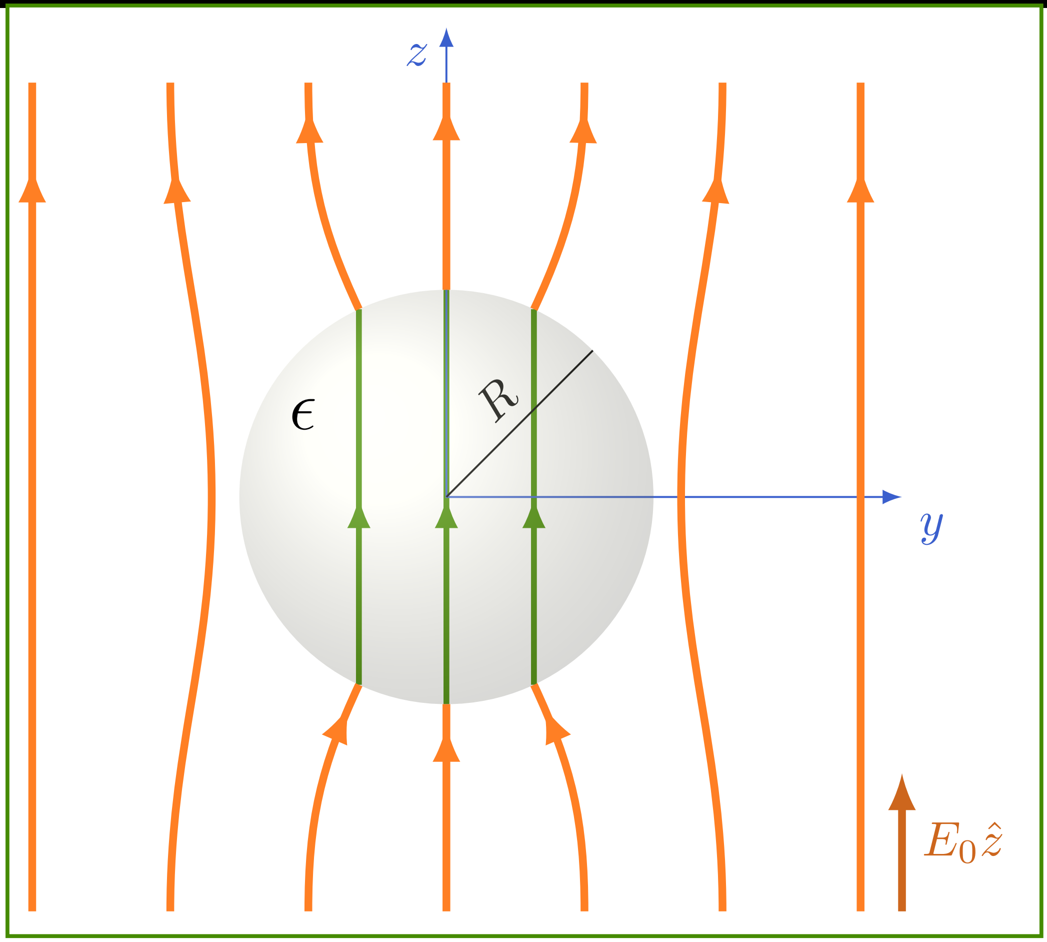
Pratique o que aprendeu

$$V_f(r) = -E_0 r\cos\theta + \sum_\ell \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$

$$V_d(R) = V_f(R)$$

$$\epsilon \frac{\partial V_d}{\partial n}\bigg|_R = \epsilon_0 \frac{\partial V_f}{\partial n}\bigg|_R$$

DENTRO FORA



Pratique o que aprendeu

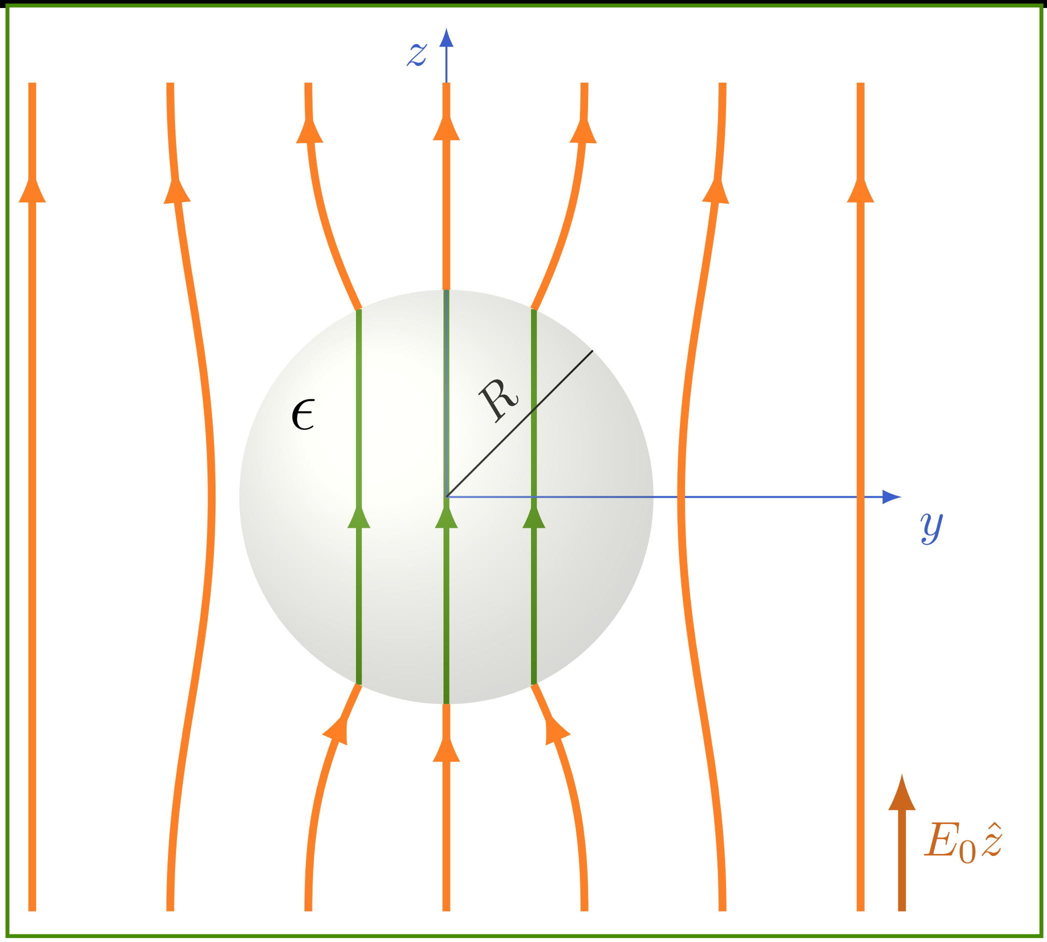
$$V_f(r) = -E_0 r\cos\theta + \sum_\ell \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$

$$V_d(R) = V_f(R)$$

$$\epsilon \frac{\partial V_d}{\partial n}\bigg|_R = \epsilon_0 \frac{\partial V_f}{\partial n}\bigg|_R$$

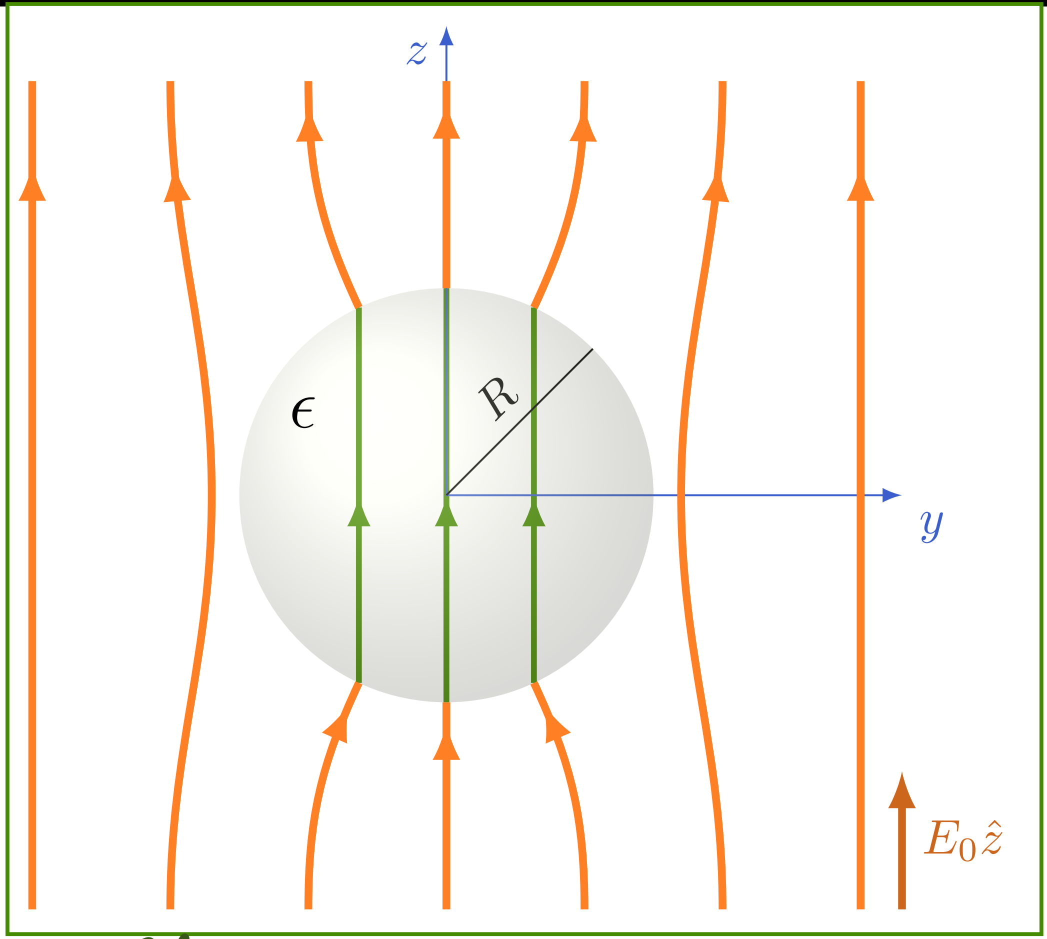
$$V_d(r) = \sum_\ell A_\ell r^\ell P_\ell(\cos\theta)$$

DENTRO



Pratique o que aprendeu

$$V_f(r) = -E_0 r\cos\theta + \sum_\ell \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$



$$V_d(R) = V_f(R)$$

$$\epsilon \frac{\partial V_d}{\partial n}\Big|_R = \epsilon_0 \frac{\partial V_f}{\partial n}\Big|_R$$

$$V_d(r) = \sum_\ell A_\ell r^\ell P_\ell(\cos\theta)$$

DENTRO

FORA

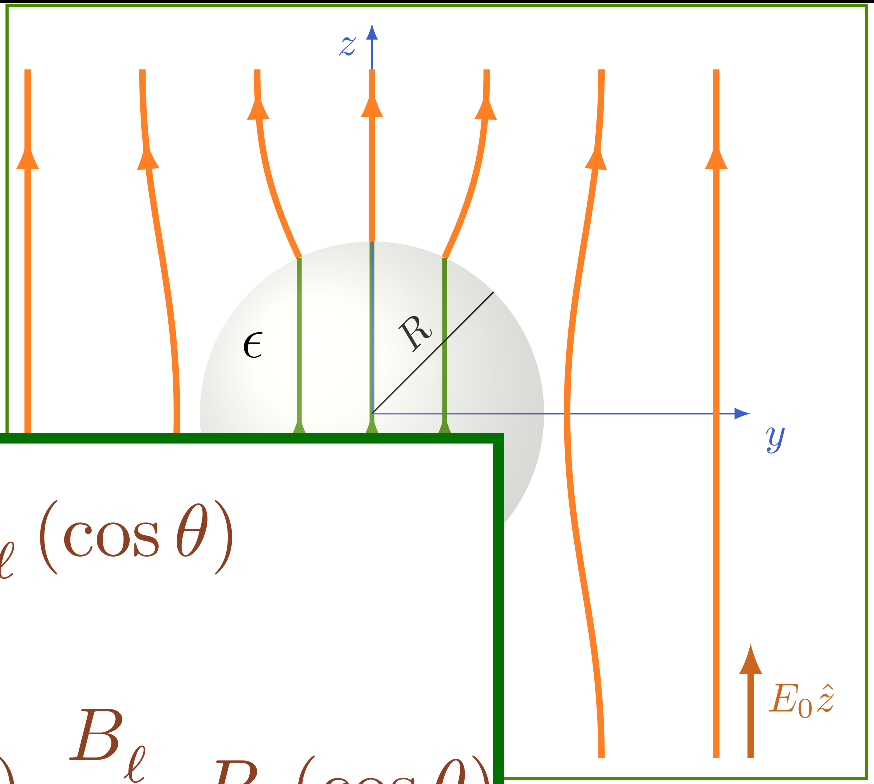
$$r = R \Rightarrow \quad \sum_\ell A_\ell R^\ell P_\ell(\cos\theta) = -E_0 R\cos\theta + \sum_\ell \frac{B_\ell}{R^{\ell+1}} P_\ell(\cos\theta)$$

$$\epsilon_r \sum_\ell \ell A_\ell R^{\ell-1} P_\ell(\cos\theta) = -E_0\cos\theta - \sum_\ell (\ell+1)\frac{B_\ell}{R^{\ell+2}} P_\ell(\cos\theta)$$

Pratique o que aprendeu

$$V_f(r) = -E_0 r\cos\theta + \sum_\ell \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$

$$V_d(r) = \sum_\ell A_\ell r^\ell P_\ell(\cos\theta)$$



$$\sum_\ell A_\ell R^\ell P_\ell(\cos\theta) = -E_0 R\cos\theta + \sum_\ell \frac{B_\ell}{R^{\ell+1}} P_\ell(\cos\theta)$$

$$\epsilon_r \sum_\ell \ell A_\ell R^{\ell-1} P_\ell(\cos\theta) = -E_0\cos\theta - \sum_\ell (\ell+1)\frac{B_\ell}{R^{\ell+2}} P_\ell(\cos\theta)$$

$$\ell = 1 \quad \Rightarrow \quad A_1 R = -E_0 R + \frac{B_1}{R^2}$$

$$\epsilon_r A_1 = -E_0 - 2\frac{B_1}{R^3}$$

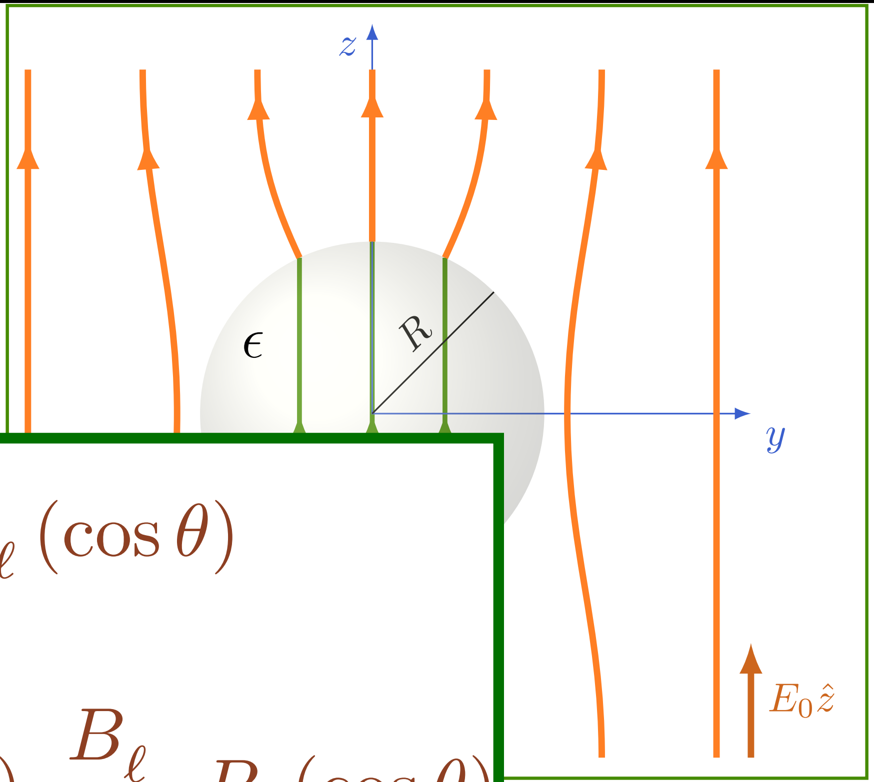
$$\Rightarrow \quad A_1 = -\frac{3}{\epsilon_r + 2} E_0$$

$$B_1 = \frac{\epsilon_r - 1}{\epsilon_r + 2} R^3 E_0$$

Pratique o que aprendeu

$$V_f(r) = -E_0 r\cos\theta + \sum_\ell \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$

$$V_d(r) = \sum_\ell A_\ell r^\ell P_\ell(\cos\theta)$$



$$\sum_\ell A_\ell R^\ell P_\ell(\cos\theta) = -E_0 R\cos\theta + \sum_\ell \frac{B_\ell}{R^{\ell+1}} P_\ell(\cos\theta)$$

$$\epsilon_r \sum_\ell \ell A_\ell R^{\ell-1} P_\ell(\cos\theta) = -E_0\cos\theta - \sum_\ell (\ell+1)\frac{B_\ell}{R^{\ell+2}} P_\ell(\cos\theta)$$

$$\ell \neq 1 \quad \Rightarrow \quad A_\ell R^\ell = \frac{B_\ell}{R^{\ell+2}}$$

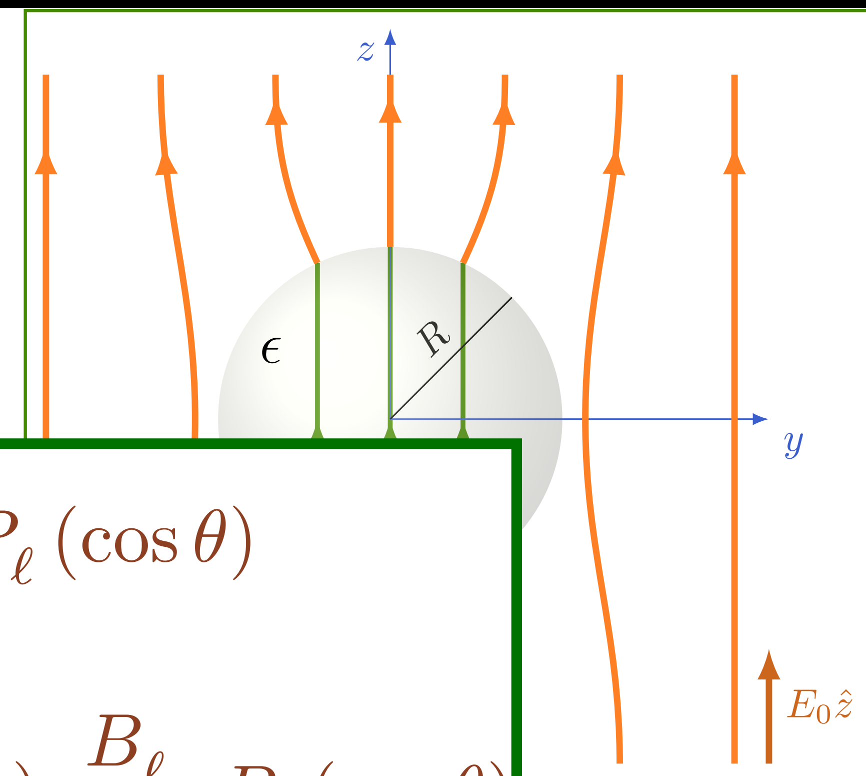
$$\epsilon_r \ell A_\ell R^{\ell-1} = -(\ell+1)\frac{B_\ell}{R^{\ell+2}}$$

INCOMPATÍVEIS PARA $A_\ell, B_\ell \neq 0$

Pratique o que aprendeu

$$V_f(r) = -E_0 r\cos\theta + \sum_\ell \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$

$$V_d(r) = \sum_\ell A_\ell r^\ell P_\ell(\cos\theta)$$



$$\sum_\ell A_\ell R^\ell P_\ell(\cos\theta) = -E_0 R\cos\theta + \sum_\ell \frac{B_\ell}{R^{\ell+1}} P_\ell(\cos\theta)$$

$$\epsilon_r \sum_\ell \ell A_\ell R^{\ell-1} P_\ell(\cos\theta) = -E_0\cos\theta - \sum_\ell (\ell+1)\frac{B_\ell}{R^{\ell+2}} P_\ell(\cos\theta)$$

$$\ell \neq 1 \quad \Rightarrow \quad A_\ell R^\ell = \frac{B_\ell}{R^{\ell+2}}$$

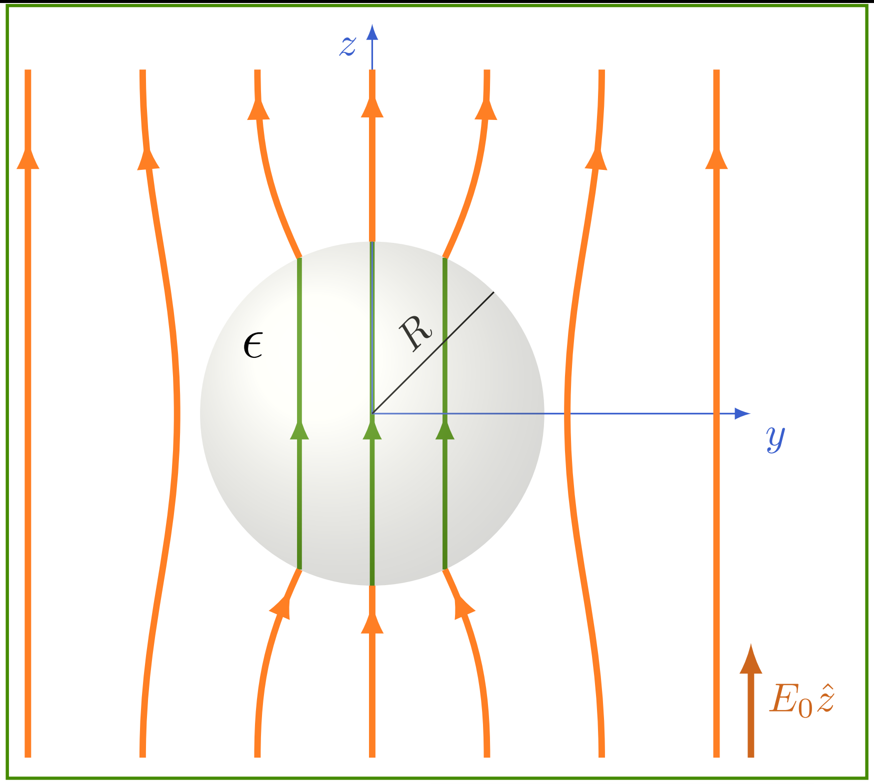
$$\epsilon_r \ell A_\ell R^{\ell-1} = -(\ell+1)\frac{B_\ell}{R^{\ell+2}}$$

$$\Rightarrow \quad A_\ell = B_\ell = 0$$

Pratique o que aprendeu

$$V_f(r) = -E_0 r\cos\theta + \sum_\ell \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$

$$V_d(r) = \sum_\ell A_\ell r^\ell P_\ell(\cos\theta)$$



$$\Rightarrow \quad A_1 = -\frac{3}{\epsilon_r + 2} E_0$$

$$B_1 = \frac{\epsilon_r - 1}{\epsilon_r + 2} R^3 E_0$$

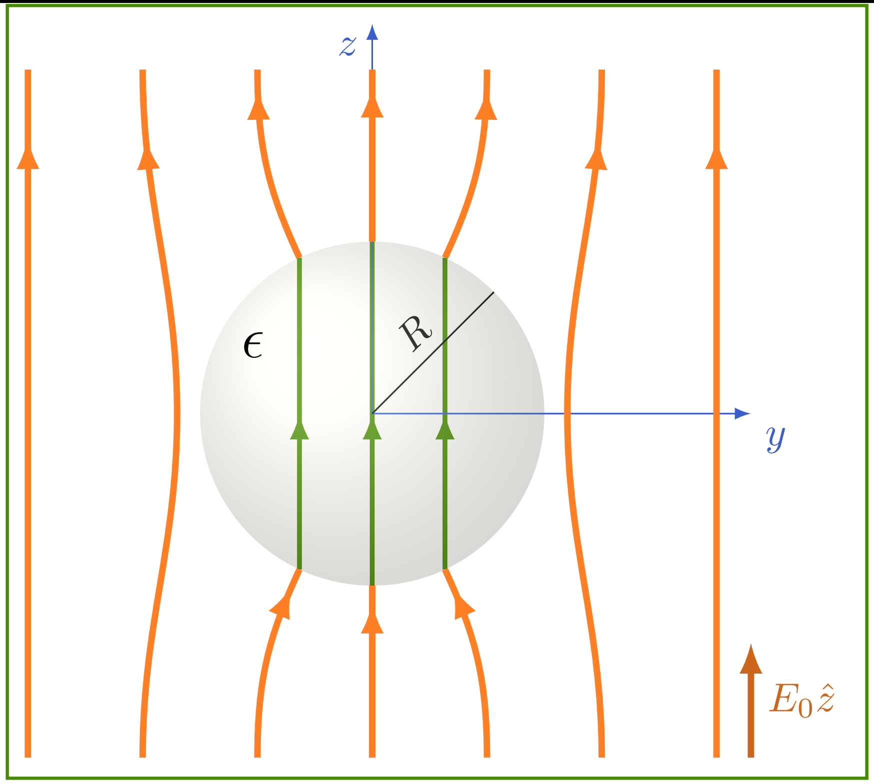
$$\Rightarrow \quad A_\ell = B_\ell = 0$$

$$V_d(r) = -\frac{3}{\epsilon_r + 2} E_0 r\cos\theta = -\frac{3}{\epsilon_r + 2} E_0 z$$

Pratique o que aprendeu

$$V_f(r) = -E_0 r\cos\theta + \sum_\ell \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$

$$V_d(r) = \sum_\ell A_\ell r^\ell P_\ell(\cos\theta)$$



$$\Rightarrow \quad A_\ell = B_\ell = 0$$

$$\Rightarrow \quad A_1 = -\frac{3}{\epsilon_r + 2}E_0$$

$$B_1 = \frac{\epsilon_r - 1}{\epsilon_r + 2}R^3 E_0$$

$$V_f(r) = -E_0 r\cos\theta + \frac{\epsilon_r - 1}{\epsilon_r + 2}\frac{E_0 R^3\cos\theta}{r^2}$$