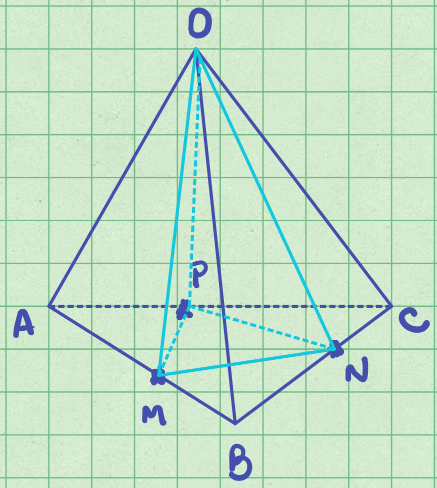
Dúvidas: 14 ; Red $\longrightarrow$ Vet ; 12 ; 19 (2a parte)

18b ; 21b ; 27c ; 25

14



$$V_{OABC} = \frac{1}{6} \left| [\vec{OA}, \vec{OB}, \vec{OC}] \right|$$

$$V_{OMNP} = \frac{1}{6} \left| [\vec{OM}, \vec{ON}, \vec{OP}] \right|$$

$$\frac{V_{OABC}}{V_{OMNP}} = ?$$

M, N, P dividem $\vec{AB}$, $\vec{BC}$, $\vec{CA}$ na razão 2 $\therefore$

$$\begin{cases} \vec{AM} = 2\vec{MB}, & \vec{MB} = 1/3\ \vec{AB} \\ \vec{BN} = 2\vec{NC}, & \vec{NC} = 1/3\ \vec{BC} \\ \vec{CP} = 2\vec{PA}, & \vec{PA} = 1/3\ \vec{CA} \end{cases}$$

$$\vec{OM} = \vec{OA} + \vec{AM}$$
$$\vec{OM} = \vec{OB} + \vec{BM} \quad (+)$$
$$2\vec{OM} = \vec{OA} + \vec{OB} + \vec{AM} + \vec{BM}$$
$$2\vec{OM} = \vec{OA} + \vec{OB} + 2\vec{MB} - \vec{MB}$$
$$2\vec{OM} = \vec{OA} + \vec{OB} + \vec{MB}$$
$$2\vec{OM} = \vec{OA} + \vec{OB} + \frac{1}{3}\vec{AB}$$
$$2\vec{OM} = \vec{OA} + \vec{OB} + \frac{1}{3}(\vec{OB} - \vec{OA})$$
$$2\vec{OM} = \frac{2}{3}\vec{OA} + \frac{4}{3}\vec{OB}$$
$$\vec{OM} = \frac{1}{3}\vec{OA} + \frac{2}{3}\vec{OB}$$

$$\vec{ON} = \vec{OB} + \vec{BN}$$
$$\vec{ON} = \vec{OC} + \vec{CN} \quad (+)$$
$$\vdots$$
$$\vec{ON} = \frac{1}{3}\vec{OB} + \frac{2}{3}\vec{OC}$$

$$\vec{OP} = \vec{OA} + \vec{AP}$$
$$\vec{OP} = \vec{OC} + \vec{CP} \quad (+)$$
$$\vdots$$
$$\vec{OP} = \frac{1}{3}\vec{OC} + \frac{2}{3}\vec{OA}$$

Assim:

$$[\vec{OM}, \vec{ON}, \vec{OP}] = \left[\frac{1}{3}\vec{OA} + \frac{2}{3}\vec{OB}, \frac{1}{3}\vec{OB} + \frac{2}{3}\vec{OC}, \frac{1}{3}\vec{OC} + \frac{2}{3}\vec{OA} \right]$$

$$\vdots$$

$$\frac{1}{3}[\vec{OA}, \vec{OB}, \vec{OC}]$$

Então:

$$\frac{V_{OABC}}{V_{OMNP}} = \frac{\cancel{1/6}\,|[\vec{OA}, \vec{OB}, \vec{OC}]|}{\cancel{1/6}\,|[\vec{OM}, \vec{ON}, \vec{OP}]|} = \frac{|[\vec{OA}, \vec{OB}, \vec{OC}]|}{1/3\,|[\vec{OA}, \vec{OB}, \vec{OC}]|}$$

$$\frac{V_{OABC}}{V_{OMNP}} = 3$$

Reduzidas $\longrightarrow$ Vetorial

$$r: \begin{cases} y = 2x - 3 \\ z = -x + 4 \end{cases}$$

Atribuir valores para x e encontrar pontos $\in r$:

$x = 0$: $y = -3$, $z = 4$ $\therefore A(0, -3, 4)$

$x = 1$: $y = -1$, $z = 3$ $\therefore B(1, -1, 3)$

$\vec{v} = \vec{AB} = B - A = (1, 2, -1)$ vetor diretor

A ou B + $\vec{v}$ $\Rightarrow$ Eq. vetorial!

E. vet. : $P = A + t\vec{v}$

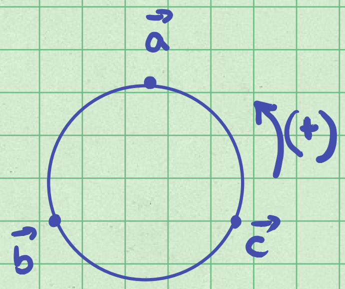
$(x, y, z) = (0, -3, 4) + t(1, 2, -1)$, $t \in \mathbb{R}$

12 $\{\vec{a}, \vec{b}, \vec{c}\}$ $\quad \vec{a} \parallel \vec{u}$, $k > 0$

$\vec{b} = \alpha \vec{u} + \beta \vec{v}$, $\quad x_b = x > 0$

$\vec{u} = (1,1,1)$

$\vec{v} = (0,1,2)$



Base Ortonormal $\begin{cases} \text{vet. Unit.} \\ \text{2 a 2} \perp \end{cases}$

$\vec{a} = k(1,1,1)$

$\vec{a} = (k,k,k)$, $\quad |\vec{a}| = 1$

$\sqrt{3k^2} = 1 \quad k = \pm \dfrac{1}{\sqrt{3}} = \pm \dfrac{\sqrt{3}}{3}$

$\vec{a} = \left(\dfrac{\sqrt{3}}{3}, \dfrac{\sqrt{3}}{3}, \dfrac{\sqrt{3}}{3} \right)$

$\vec{b} = \alpha \vec{u} + \beta \vec{v}$

$\vec{b} = \alpha(1,1,1) + \beta(0,1,2)$

$\vec{b} = (\alpha, \alpha + \beta, \alpha + 2\beta)$ //

$\vec{a} \perp \vec{b}$ (ortonormal) : $\vec{a} \cdot \vec{b} = \vec{0}$

$\left(\dfrac{\sqrt{3}}{3}, \dfrac{\sqrt{3}}{3}, \dfrac{\sqrt{3}}{3} \right) \cdot (\alpha, \alpha+\beta, \alpha+2\beta) = 0$

$\dfrac{\sqrt{3}}{3} (1,1,1) \cdot (\alpha, \alpha+\beta, \alpha+2\beta) = 0$

$\dfrac{\sqrt{3}}{3} (\alpha + \alpha + \beta + \alpha + 2\beta) = 0$

$\therefore (3\alpha + 3\beta) = 0 \longrightarrow \beta = -\alpha$ //

$\therefore \vec{b} = (\alpha, 0, -\alpha)$

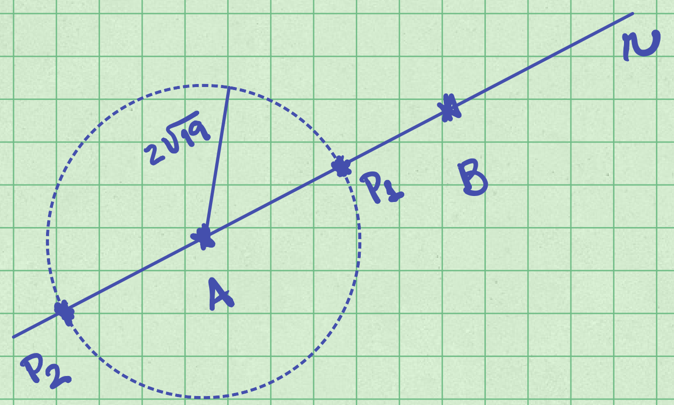
$|\vec{b}| = 1 \quad \therefore \sqrt{\alpha^2 + \alpha^2} = 1 \quad \therefore \alpha\sqrt{2} = \pm 1$

$\alpha = \pm 1/\sqrt{2} = \pm \sqrt{2}/2$

$\therefore \ \vec{b} = \left(\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2} \right)$ //

$\vec{c} = \vec{a} \times \vec{b} \quad \ldots \quad \vec{c}$ unitário

19



$P_1(x_1, y_1, z_1)$

$P_2(x_2, y_2, z_2)$

$\overrightarrow{AP_i} \parallel \vec{v}$

$|\overrightarrow{AP_i}| = 2\sqrt{19}$

$i = 1, 2$

$\left.\begin{array}{l} A(1,2,3) \\ P_1(x_1, y_1, z_1) \end{array}\right\} \ \overrightarrow{AP_1} = (x_1 - 1, y_1 - 2, z_1 - 3)$

$\vec{v} = (-3, 1, -3) \ \ldots \ \vec{v} \parallel r$

$\overrightarrow{AP_1} \parallel \vec{v}$

$\overrightarrow{AP_1} = k\vec{v}$

$(x_1 - 1, y_1 - 2, z_1 - 3) = k(-3, 1, -3)$

$\begin{cases} x_1 - 1 = -3k \\ y_1 - 2 = k \\ z_1 - 3 = -3k \end{cases} \qquad \therefore \ \overrightarrow{AP_1} = (-3k, k, -3k)$ //

Mas:

$|\overrightarrow{AP_1}| = 2\sqrt{19}$

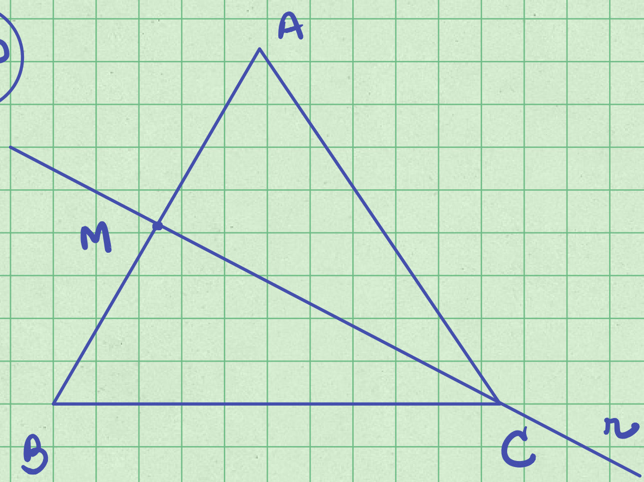
$\sqrt{9k^2 + k^2 + 9k^2} = 2\sqrt{19}$

$\sqrt{19k^2} = 2\sqrt{19}$

$k = 2$

$\begin{cases} x_1 = -5 \\ y_1 = 4 \\ z_1 = -3 \end{cases}$

18b



$A(3,6,-7)$

$B(-5,2,3)$

$C(4,-7,-6)$

$$M\left(\frac{x_a+x_b}{2}, \frac{y_a+y_b}{2}, \frac{z_a+z_b}{2}\right)$$

$M(-1,4,-2)$

⋮

$$\left.\begin{matrix} M \\ C \end{matrix}\right\} \vec{v} \longrightarrow \text{Eqs. Param.}$$

21b

$$\left.\begin{matrix} A(1,0,1) \\ B(2,1,-1) \\ C(1,-1,0) \end{matrix}\right\}$$

3 pontos são colineares?

$\vec{AB} = B - A = (1,1,-2)$

$\vec{AC} = C - A = (0,-1,-1)$

$\exists K \in \mathbb{R} \,/\, \vec{AB} = K\vec{AC}$? NÃO//

Logo, A, B, C não são colineares e formam um plano!

$\vec{n} \parallel \vec{AB} \times \vec{AC} \Rightarrow \vec{n} = K(\vec{AB} \times \vec{AC})$

A, B ou C $+ \ \vec{n}$: $\vec{AP} \cdot \vec{n} = 0$

⋮

α

27c

$$r: \begin{cases} 3x + y + 6z + 13 = 0 \\ 9x + 3y + 5z = 0 \end{cases} \quad r = \alpha_1 \cap \alpha_2$$

$$s: \begin{cases} x = 10 \\ 4x + y - z - 9 = 0 \end{cases} \quad s = \alpha_1 \cap \alpha_2$$

$r = \alpha_1 \cap \alpha_2$

$$\begin{cases} 3x + y + 6z + 13 = 0 \\ 9x + 3y + 5z = 0 \end{cases} \longrightarrow y = -13 - 3x - 6z$$

$$9x + 3(-13 - 3x - 6z) = 0$$

$$-18z = 39$$

$$z = -13/6$$

$$y = -13 - 3x - 6\left(\frac{-13}{6}\right)$$

$$y = -3x$$

$$r: \begin{cases} z = -13/6 \\ y = -3x \end{cases}$$

$s = \alpha_1 \cap \alpha_2$

$$\begin{cases} x = 10 \\ 4x + y - z - 9 = 0 \end{cases}$$

$$40 + y - z - 9 = 0$$

$$y - z = -31$$

$$s: \begin{cases} x = 10 \\ y - z = -31 \end{cases}$$

$r \cap s \longrightarrow$ **ANTES**: fazer o estudo da posição relativa entre r e s

$$r: \begin{cases} z = -13/6 \\ y = -3x \end{cases}$$

$x = 0$: $y = 0$, $z = -13/6$ (A)

$x = 1$: $y = -3$, $z = -13/6$ (B)

$$\vec{v_r} = \vec{AB} = (1, -3, 0)$$

$$A(0, 0, -13/6) \in r$$

$$s: \begin{cases} x = 10 \\ y - z = -31 \end{cases}$$

$y = 0$: $x = 10$, $z = 31$ (C)

$y = 1$: $x = 10$, $z = 32$ (D)

$$\vec{v_s} = \vec{CD} = (0, 1, 1)$$

$$C(10, 0, 31) \in s$$

$\vec{AC} = C - A = (10, 0, 173/6)$

ESTUDO DA POSIÇÃO RELATIVA

1 $[\vec{v_r}, \vec{v_s}, \vec{AC}] = ?$ $\begin{cases} \text{Reversas } \neq 0 \\ \text{Coplanares } = 0 \end{cases}$

$$\begin{vmatrix} 1 & -3 & 0 \\ 0 & 1 & 1 \\ 10 & 0 & \frac{173}{6} \end{vmatrix} = \frac{173}{6} - 30 = -\frac{7}{6} \neq 0$$

Como r e s são retas reversas, $\nexists\ I = r \cap s$.

25 $\vec{w_1} \parallel r : \begin{cases} \frac{x-1}{2} = y - 9 \\ z = 18 \end{cases}$

$\vec{u} = \vec{w_1} + \vec{w_2}$

$\vec{w_2} \parallel \alpha : \begin{cases} x = 1 + h \\ y = 1 + t \\ z = h - t \end{cases}, h, t \in \mathbb{R}$

$\vec{v_r} = (2, 1, 0) \longrightarrow \vec{w_1} \parallel \vec{v_r} \therefore \vec{w_1} = k\vec{v_r} = (2k, k, 0)$

$\left.\begin{array}{l} \vec{u_\alpha} = (1, 0, 1) \\ \vec{v_\alpha} = (0, 1, -1) \end{array}\right\}$ $\vec{w_2} = \alpha\vec{u_\alpha} + \beta\vec{v_\alpha} = \alpha(1,0,1) + \beta(0,1,-1)$

$\vec{w_2} = (\alpha, \beta, \alpha - \beta)$

$\vec{u} = (1, 2, 4)$

$\vec{u} = \vec{w_1} + \vec{w_2}$

$(1,2,4) = (2k, k, 0) + (\alpha, \beta, \alpha - \beta)$ $\begin{cases} k = -5 \\ \alpha = 11 \\ \beta = 7 \end{cases}$