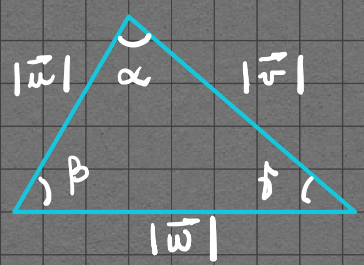
P1 - Turma PN

1. $\vec{u}\cdot\vec{v} - \vec{u}\cdot\vec{w} - \vec{v}\cdot\vec{w} = ?$ (1)

$\vec{u} + \vec{v} + \vec{w} = \vec{0}$

$|\vec{u}| = \sqrt{2}$, $|\vec{v}| = \sqrt{3}$, $|\vec{w}| = 5$



$\vec{v_1}\cdot\vec{v_2} = |\vec{v_1}|\,|\vec{v_2}|\cos\theta$

$\theta = \measuredangle_{\vec{v_1},\vec{v_2}}$

Do triângulo :

$|\vec{u}|^2 = |\vec{v}|^2 + |\vec{w}|^2 - 2\,\underbrace{|\vec{v}||\vec{w}|\cos\gamma}_{\vec{v}\cdot\vec{w}}$

$2 = 3 + 25 - 2(\vec{v}\cdot\vec{w})$

$\vec{v}\cdot\vec{w} = 13$ //

$|\vec{v}|^2 = |\vec{u}|^2 + |\vec{w}|^2 - 2\,\underbrace{|\vec{u}||\vec{w}|\cos\beta}_{\vec{u}\cdot\vec{w}}$

$3 = 2 + 25 - 2(\vec{u}\cdot\vec{w})$

$\vec{u}\cdot\vec{w} = 12$ //

$|\vec{w}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2\,\underbrace{|\vec{u}||\vec{v}|\cos\alpha}_{\vec{u}\cdot\vec{v}}$

$25 = 2 + 3 - 2(\vec{u}\cdot\vec{v})$

$\vec{u}\cdot\vec{v} = -10$ //

Substituindo os valores encontrados em (1) :

$\vec{u}\cdot\vec{v} - \vec{u}\cdot\vec{w} - \vec{v}\cdot\vec{w} = -10 - 12 - 13$

$\vec{u}\cdot\vec{v} - \vec{u}\cdot\vec{w} - \vec{v}\cdot\vec{w} = -35$

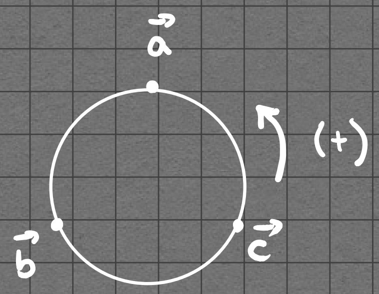
② $\{\vec a, \vec b, \vec c\}$

$\vec a \parallel \vec u$, $k > 0$

$\vec b = \alpha \vec u + \beta \vec v$, $x_b > 0$

$\vec u = (1,1,1)$; $\vec v = (0,1,2)$

BASE DE SENTIDO POSITIVO:



BASE ORTONORMAL $\begin{cases} \text{Vetores Unitários: } |\ \ | = 1 \\ \text{Dois a Dois Ortogonais} \end{cases}$

$\vec a \parallel \vec u \longrightarrow \vec a = k(1,1,1) = (k,k,k)$ //

$|\vec a| = 1 \quad \therefore \quad \sqrt{3k^2} = 1$ ou $k = \pm \dfrac{1}{\sqrt3}$

Mas $k > 0$, então : $\vec a = \left(\dfrac{\sqrt3}{3}, \dfrac{\sqrt3}{3}, \dfrac{\sqrt3}{3}\right)$ //

$\vec b = \alpha \vec u + \beta \vec v \longrightarrow \vec b = \alpha(1,1,1) + \beta(0,1,2)$

$\vec b = (\alpha, \alpha + \beta, \alpha + 2\beta)$ //

Mas $\vec a \perp \vec b$: $\vec a \cdot \vec b = \vec 0$

$\left(\dfrac{\sqrt3}{3}, \dfrac{\sqrt3}{3}, \dfrac{\sqrt3}{3}\right) \cdot (\alpha, \alpha+\beta, \alpha+2\beta) = 0$

$\dfrac{\sqrt3}{3}(1,1,1) \cdot (\alpha, \alpha+\beta, \alpha+2\beta) = 0$

$\dfrac{\sqrt3}{3}(\alpha + \alpha + \beta + \alpha + 2\beta) = 0$

$\therefore (3\alpha + 3\beta) = 0 \longrightarrow \beta = -\alpha$ //

$\therefore \vec b = (\alpha, 0, -\alpha)$

$|\vec b| = 1 \quad \therefore \quad \sqrt{2\alpha^2} = 1$ ou $\alpha = \pm 1/\sqrt2$

Mas $x_b > 0$, então : $\vec{b} = \left(\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2} \right)$

A base é de sentido positivo, portanto:

$$\vec{c} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \sqrt{3}/3 & \sqrt{3}/3 & \sqrt{3}/3 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 \end{vmatrix}$$

$$= \frac{\sqrt{3}}{3} \cdot \frac{\sqrt{2}}{2} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{vmatrix}$$

$$= \frac{\sqrt{6}}{6} \left(-\vec{i} + 2\vec{j} - \vec{k} \right)$$

Portanto: $\vec{c} = \left(-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{3}, -\frac{\sqrt{6}}{6} \right)$

OBS: o vetor resultante do produto vetorial de dois vetores unitários também é unitário.

3 $r: \begin{cases} 3x + y + 6z + 13 = 0 \\ 9x + 3y + 5z = 0 \end{cases}$ $r = \alpha_1 \cap \alpha_2$

$s: \begin{cases} x = 10 \\ 4x + y - z - 9 = 0 \end{cases}$ $s = \alpha_1 \cap \alpha_2$

$\exists\ I = r \cap s$?

$r = \alpha_1 \cap \alpha_2$

$$\begin{cases} 3x + y + 6z + 13 = 0 \\ 9x + 3y + 5z = 0 \end{cases}$$ cujà solução é $r: \begin{cases} z = -13/6 \\ y = -3x \end{cases}$

$s = \alpha_1 \cap \alpha_2$

$$\begin{cases} x = 10 \\ 4x + y - z - 9 = 0 \end{cases}$$ cujà solução é $s: \begin{cases} x = 10 \\ y - z = -31 \end{cases}$

ESTUDO DA POSIÇÃO RELATIVA ENTRE r e s:

$r: \begin{cases} z = -13/6 \\ y = -3x \end{cases}$ $\quad x = 0:\ y = 0,\ z = -13/6 \quad (A)$

$x = 1:\ y = -3,\ z = -13/6 \quad (B)$

$\vec{v}_r = \vec{AB} = (1, -3, 0)$ $\quad A(0, 0, -13/6) \in r$

$s: \begin{cases} x = 10 \\ y - z = -31 \end{cases}$ $\quad y = 0:\ x = 10,\ z = 31 \quad (C)$

$y = 1:\ x = 10,\ z = 32 \quad (D)$

$\vec{v}_s = \vec{CD} = (0, 1, 1)$ $\quad C(10, 0, 31) \in s$

$\vec{AC} = C - A = (10, 0, 173/6)$ (vetor formado por um ponto de r e outro de s)

(I) $[\vec{v}_r, \vec{v}_s, \vec{AC}] = ?$ $\begin{cases} \text{Reversas } \neq 0 \\ \text{Coplanares } = 0 \end{cases}$

$$\begin{vmatrix} 1 & -3 & 0 \\ 0 & 1 & 1 \\ 10 & 0 & \frac{173}{6} \end{vmatrix} = \frac{173}{6} - 30 = -\frac{7}{6} \neq 0$$

Como r e s são retas reversas, $\nexists\ I = r \cap s$.

4) $A(3,2,-1) \in \alpha$

α : simult. ortogonal α_1 e α_2 $\quad \alpha = ?$

$\vec{n} \perp \alpha$, $|\vec{n}| = 14$, $\theta = \measuredangle \vec{n}, \vec{i}$ ... agudo

$\alpha_1: x + 2y + 5 = 0 \longrightarrow \vec{n_1} = (1,2,0)$

$\alpha_2: x + 4y + 3z + 7 = 0 \longrightarrow \vec{n_2} = (1,4,3)$

$$\vec{n} \parallel \vec{n_1} \times \vec{n_2} : \quad \vec{n} = k \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 1 & 4 & 3 \end{vmatrix} = k(6\vec{i} - 3\vec{j} + 2\vec{k})$$

$$\vec{n} = k(6,-3,2) = (6k,-3k,2k)$$

$|\vec{n}| = 14$

$\sqrt{\vec{n} \cdot \vec{n}} = 14$

$\sqrt{36k^2 + 9k^2 + 4k^2} = 14 \quad (\ )^2$

$49k^2 = 196$

$k^2 = 4$

$k = \pm 2 \quad ?$

Como $\measuredangle \vec{n}, \vec{i}$ ... agudo, $\vec{n} \cdot \vec{i} > 0$:

$$(6k,-3k,2k) \cdot (1,0,0) > 0$$

$$6k > 0 \quad \therefore \quad k > 0 \longrightarrow k = 2$$

Assim: $\vec{n} = (12,-6,4)$

$$\left.\begin{array}{l} A(3,2,-1) \in \alpha \\ \vec{n} = (12,-6,4) \perp \alpha \end{array}\right\} \quad P(x,y,z) \in \alpha \iff \vec{AP} \perp \vec{n}$$

$\vec{AP} \cdot \vec{n} = 0$, $\quad \vec{AP} = (x-3, y-2, z+1)$

$(x-3, y-2, z+1) \cdot (12,-6,4) = 0$

$12x - 36 - 6y + 12 + 4z + 4 = 0$

$12x - 6y + 4z - 20 = 0$ ou $\alpha: 6x - 3y + 2z - 10 = 0$