(a) $\lim_{x\longrightarrow 0} \frac{tg(4x)}{x}$

$$\lim_{x\to 0} \frac{sen(x)}{x} = 1$$

$$\lim_{x\to 0} \frac{tg(4x)}{x} = \frac{sen(4x)}{cos(4x)} \cdot \frac{1}{x} =$$

$$= \lim_{x\to 0} \frac{4 \cdot sen(4x)}{4 \cdot x} \quad \frac{1}{cos(4x)}$$

$4x = u$

$x \to 0$

$u \to 0$

$$= \lim_{u\to 0} \left[(4) \left(\frac{sen(u)}{u}\right) \cdot \left(\frac{1}{cos(u)}\right) \right] =$$

$$= 4 \cdot \underbrace{\lim_{u\to 0} \left(\frac{sen(u)}{u}\right)}_{1} \cdot \underbrace{\lim_{u\to 0} \frac{1}{cos(u)}}_{1} =$$

$$4 \cdot 1 \cdot 1 = 4$$

(c) $\lim_{x\longrightarrow 0} \frac{sen(3x)}{sen(4x)}$

$$= \lim_{x\to 0} \frac{\frac{sen(3x)}{x}}{\frac{sen(4x)}{x}} =$$

$$= \lim_{x\to 0} 3 \underbrace{\frac{sen(3x)}{3x}}_{1} \cdot \frac{1}{4 \cdot \underbrace{\frac{sen(4x)}{4 \cdot x}}_{1}} = \frac{3}{4}$$

(a) $\lim_{x \longrightarrow +\infty} \left(1 + \frac{2}{x}\right)^{x+1}$

$\frac{2}{x} = \frac{1}{u}$

$\Downarrow$

$x \to +\infty \Rightarrow u \to +\infty$

$\frac{x}{2} = u \Rightarrow \boxed{x = 2u}$

$\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^{x} = e$

$\lim_{u \to \infty} \left(1 + \frac{1}{u}\right)^{2u+1} = \underbrace{\lim_{u \to \infty} \left[\left(1 + \frac{1}{u}\right)^{u}\right]^{2}}_{e^2} \cdot \underbrace{\lim_{u \to \infty} \left(1 + \frac{1}{u}\right)}_{1}$

$= e^2$

(c) $\lim_{x \longrightarrow 0} (1 + 2x)^{\frac{1}{x}}$

$\frac{1}{x}$, $x \to 0^+ \Rightarrow \frac{1}{x} \to +\infty$

$x \to 0^- \Rightarrow \frac{1}{x} \to -\infty$

$\frac{1}{x} = u \Rightarrow 2x = \frac{2}{u}$

$\lim_{u \to \pm\infty} \left(1 + \frac{2}{u}\right)^{u} =$

$\frac{2}{u} = \frac{1}{v}$

$u = 2v$

$= \lim_{v \to \pm\infty} \left[\left(1 + \frac{1}{v}\right)^{v}\right]^{2} = e^2$

$$\lim_{x\to 0} \frac{a^x - 1}{x} = \ln(a)$$

(d) $\lim\limits_{x \longrightarrow 0} \dfrac{e^{2x}-1}{x}$

$2x = u \Rightarrow \quad x = \dfrac{u}{2} \Rightarrow$

$\Rightarrow \quad \dfrac{1}{x} = \dfrac{2}{u}$

$x \to 0 \Rightarrow u \to 0$

$$= \lim_{u\to 0} \frac{e^u - 1}{u} \cdot 2 =$$

$$\ln(e) \cdot 2 = 2$$

(e) $\lim\limits_{x \longrightarrow 0} \dfrac{e^{x^2}-1}{x}$ $= \lim\limits_{x\to 0} \left(\dfrac{e^{x^2} - 1}{x^2} \cdot \right) x =$

$U = x^2$ $= \lim\limits_{u \to 0} \left(\dfrac{e^u - 1}{u} \right) \cdot \left(\pm \sqrt{u} \right) = 0$

$x \to 0$

$\Downarrow$

$u \to 0$

1 0

(g) $\lim\limits_{x \longrightarrow 0} \dfrac{8^x - 3^x}{x}$ $= \lim\limits_{x \to 0} \left(\dfrac{8^x - 1}{x} - \left(\dfrac{3^x - 1}{x} \right) \right) =$

$$= \ln(8) - \ln(3) = \ln\left(\frac{8}{3}\right)$$

$$\ln(e^a) = a$$

(i) $\lim\limits_{x\to 0} \dfrac{e^{ax}-e^{bx}}{sen(ax)-sen(bx)}, \quad a,b\neq 0$

$$\lim_{x\to 0} \frac{e^{ax}-1-e^{bx}+1}{sen(ax)-sen(bx)} = \lim_{x\to 0} \frac{\dfrac{e^{ax}-1}{x} - \dfrac{(e^{bx}-1)}{x}}{\dfrac{a}{a}\dfrac{sen(ax)}{x} - \dfrac{b}{b}\dfrac{sen(bx)}{x}} =$$

$$= \frac{a-b}{a-b} = 1$$

(j) $\lim\limits_{x\to -\infty} \left(1-\dfrac{9}{x}\right)^{x+3}$

$$-\frac{9}{x} = \frac{1}{u} \Rightarrow x = -9u$$

$$\lim_{u\to +\infty} \left(1+\frac{1}{u}\right)^{-9u+3} =$$

$$= \lim_{u\to +\infty} \left(1+\frac{1}{u}\right)^{-9u} \cdot \lim_{u\to +\infty} \cancel{\left(1+\frac{1}{u}\right)^{3}} = 1$$

$$= \lim_{u\to +\infty} \frac{1}{\left[\left(1+\frac{1}{u}\right)^{u}\right]^{9}} \cdot 1 =$$

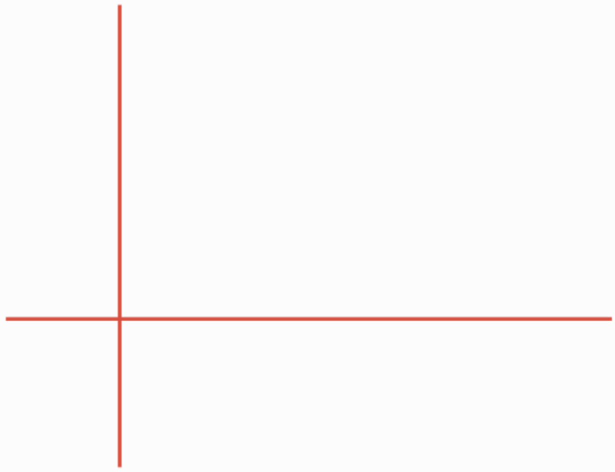
$$= \frac{1}{e^9}$$

(f) $f(x) = \begin{cases} 2x, & \text{se} \quad x \le 1, \\ 1, & \text{se} \quad x > 1. \end{cases}$

$\lim\limits_{x \to 1^-} f(x) = 2$ $\qquad$ $\lim\limits_{x \to 1^+} f(x) = 1$

$f(1) = 2$

Não é contínua



(g) $f(x) = \begin{cases} \frac{x^2-4}{x-2}, & \text{se} \quad x \neq 2, \\ 4, & \text{se} \quad x = 2. \end{cases}$

$f(2) = 4$

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{(x-2)(x+2)}{x-2} = 4$$