Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

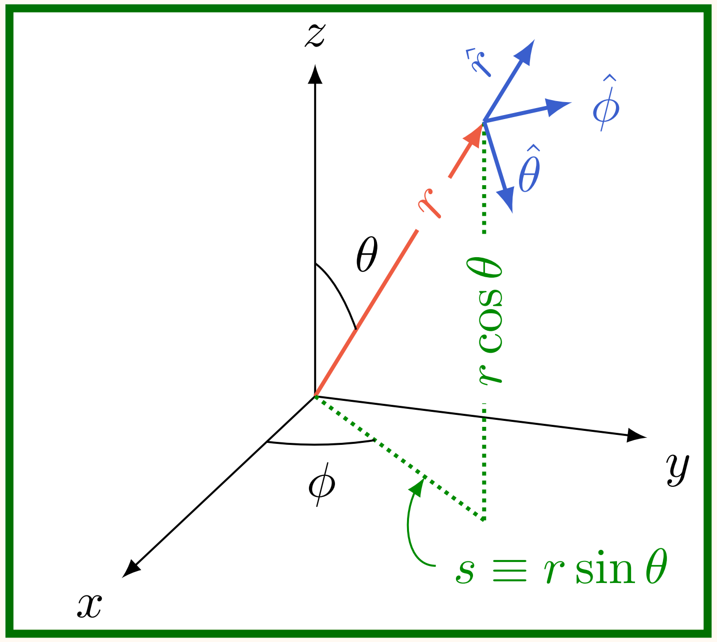
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

Aula de 9 de junho

Métodos especiais

Coordenadas esféricas



$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

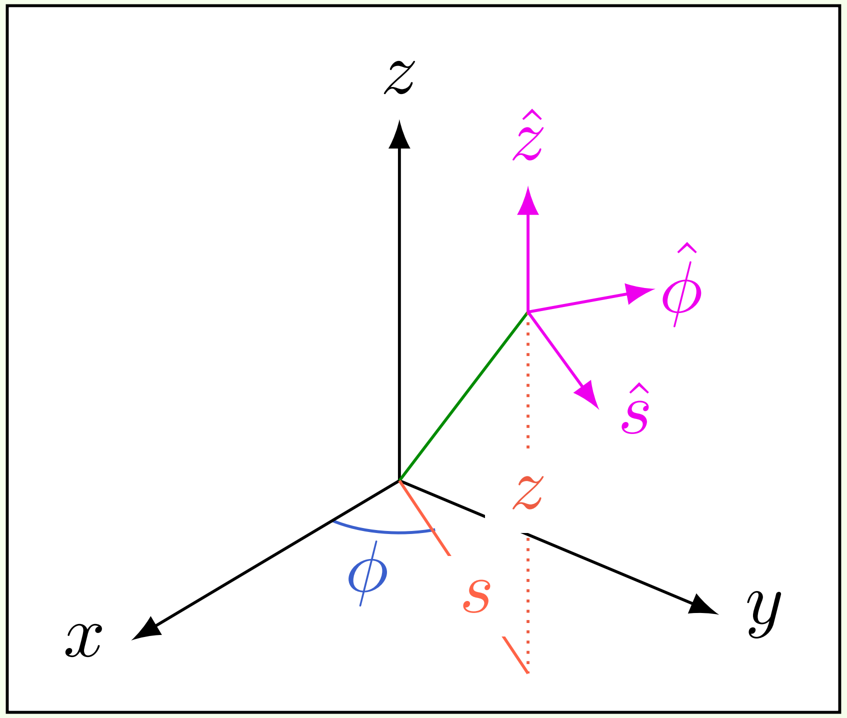
$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$

$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Coordenadas cilíndricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}s\,\hat{s} + s\,\mathrm{d}\phi\,\hat{\phi} + \mathrm{d}z\,\hat{z}$$



$$\vec{\nabla} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{s}\frac{\partial (s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla}\times\vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

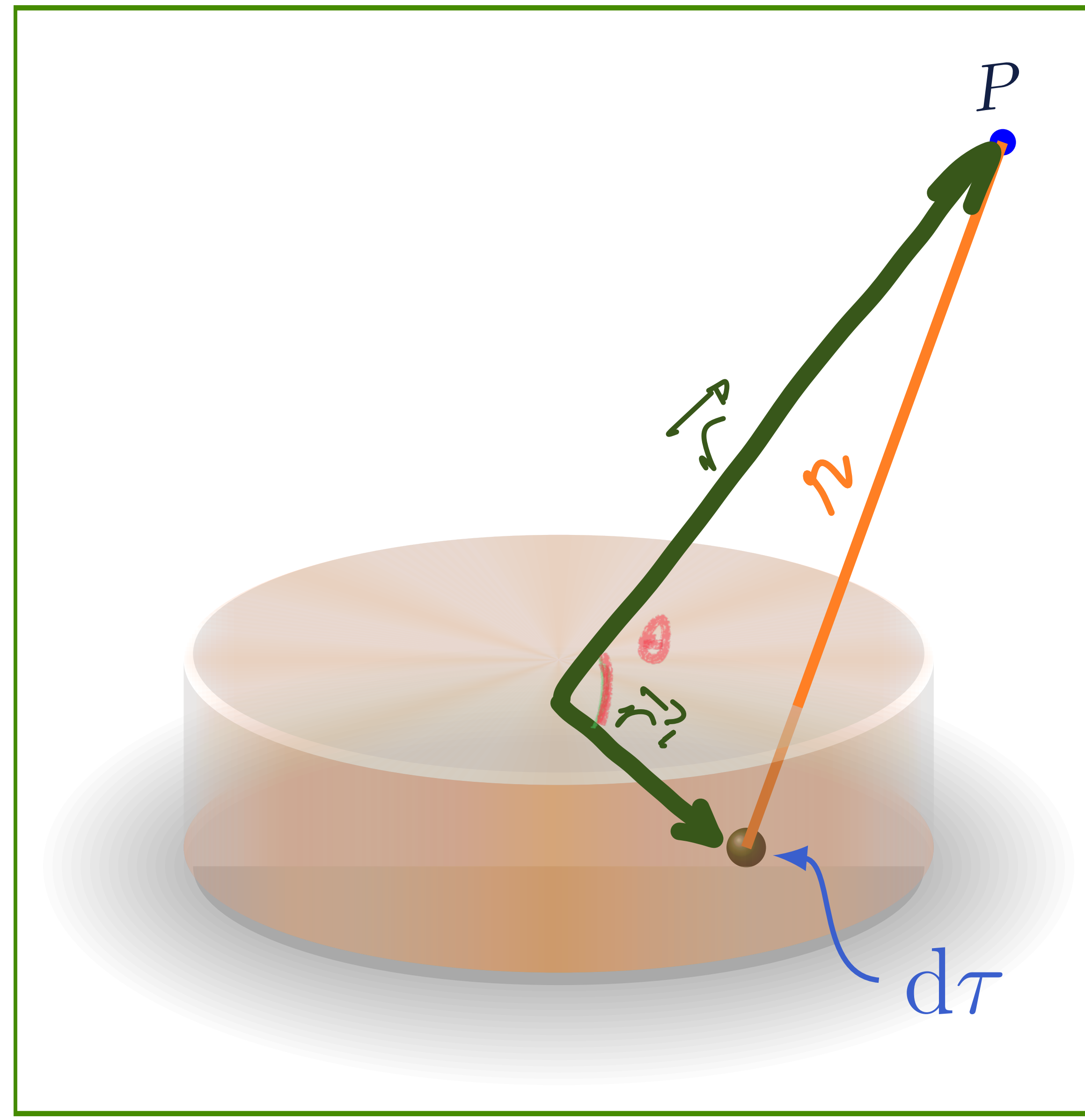
$$\nabla^2 V = 0$$

Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{r}\,d\tau$$

LEI DOS COSSENOS

$$r^2 = r^2 + r'^2 - 2rr'\cos\theta$$

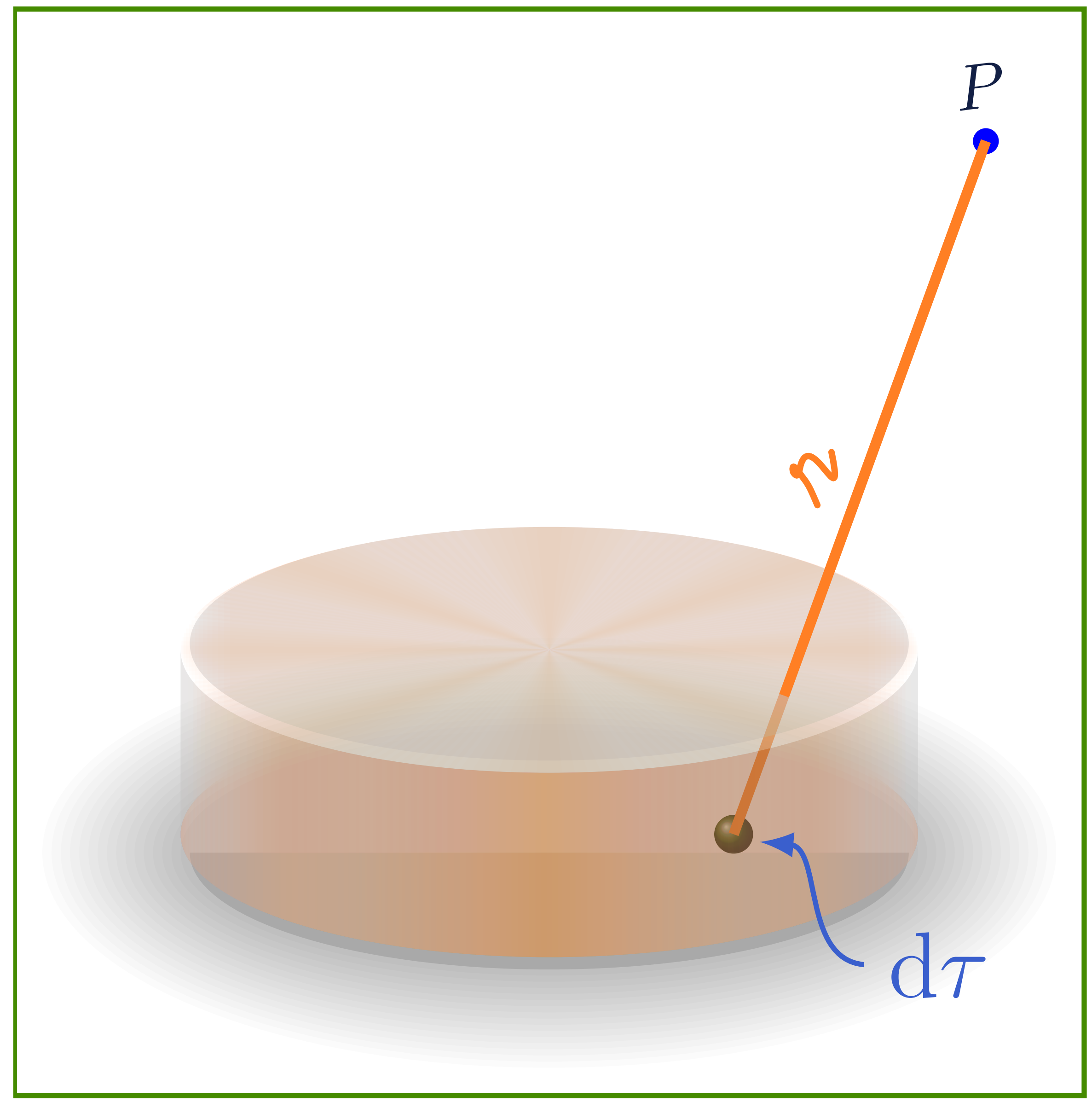


$$\nabla^2 V = 0$$

Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\, \mathrm{d}\tau$$

$$\frac{1}{\mathscr{r}} = (r^2 + r'^2 - 2rr'\cos\theta')^{-1/2}$$



$$\nabla^2 V = 0$$

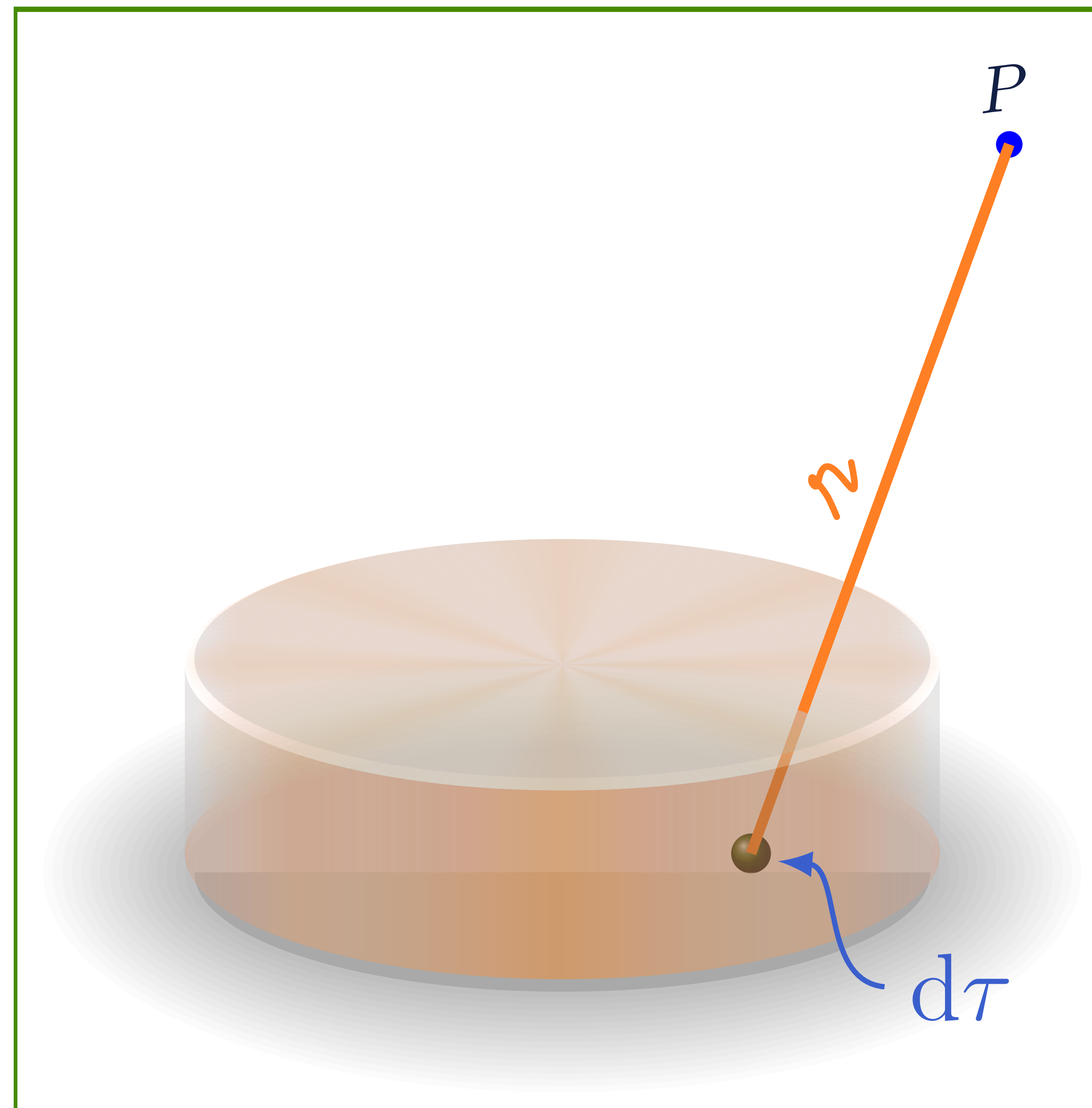
Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\,\mathrm{d}\tau$$

$$\frac{1}{\mathscr{r}} = \left(r^2 + r'^2 - 2rr'\cos\theta'\right)^{-1/2}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left[1 - 2\frac{r'}{r}\cos\theta' + \left(\frac{r'}{r}\right)^2\right]^{-1/2}$$

EXPANDIR EM SÉRIE DE TAYLOR



$$\nabla^2 V = 0$$

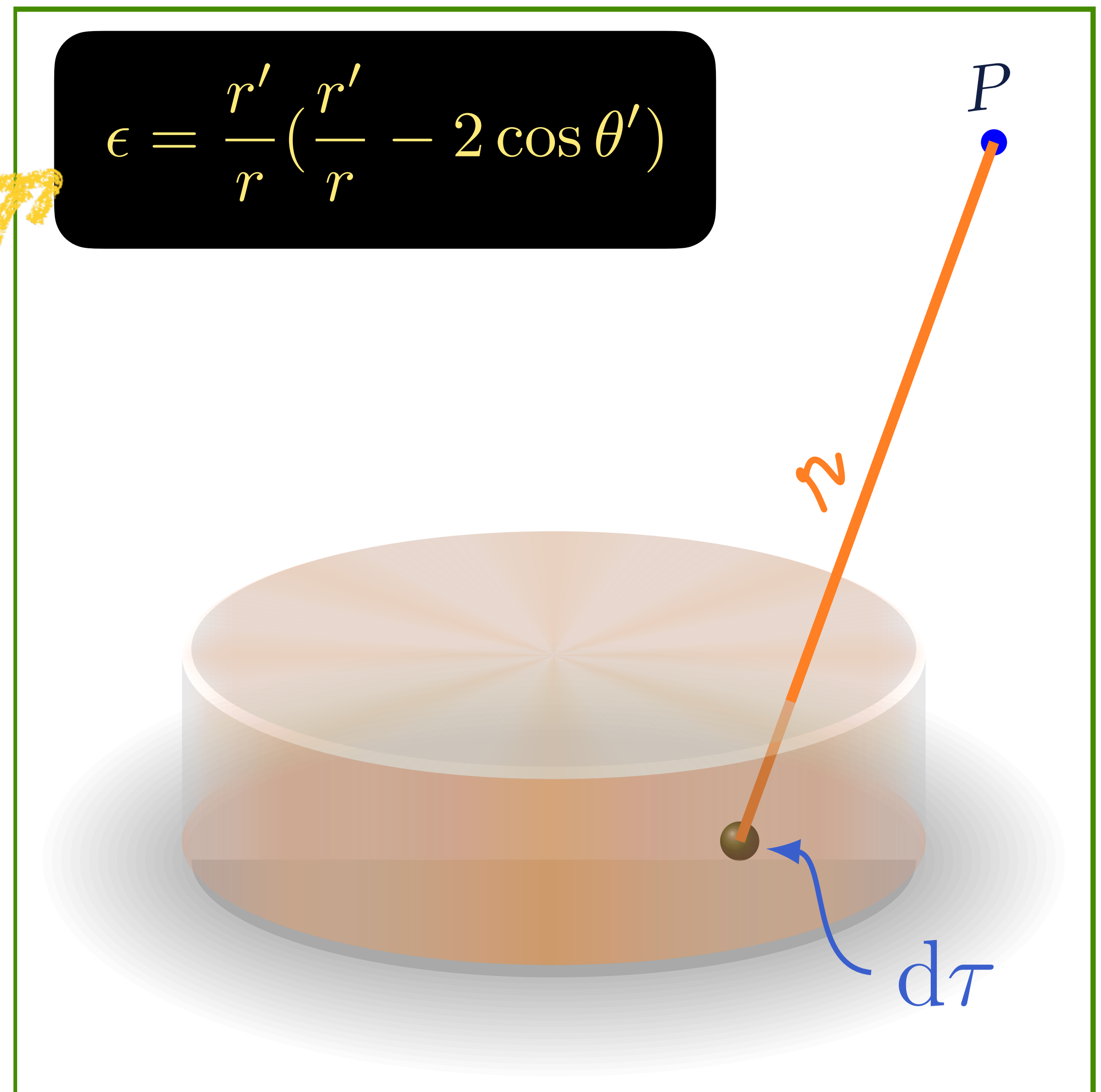
Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\,\mathrm{d}\tau$$

$$\frac{1}{\mathscr{r}} = (r^2 + r'^2 - 2rr'\cos\theta')^{-1/2}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left[1 - 2\frac{r'}{r}\cos\theta' + \left(\frac{r'}{r}\right)^2\right]^{-1/2}$$

ϵ



$$\nabla^2 V = 0$$

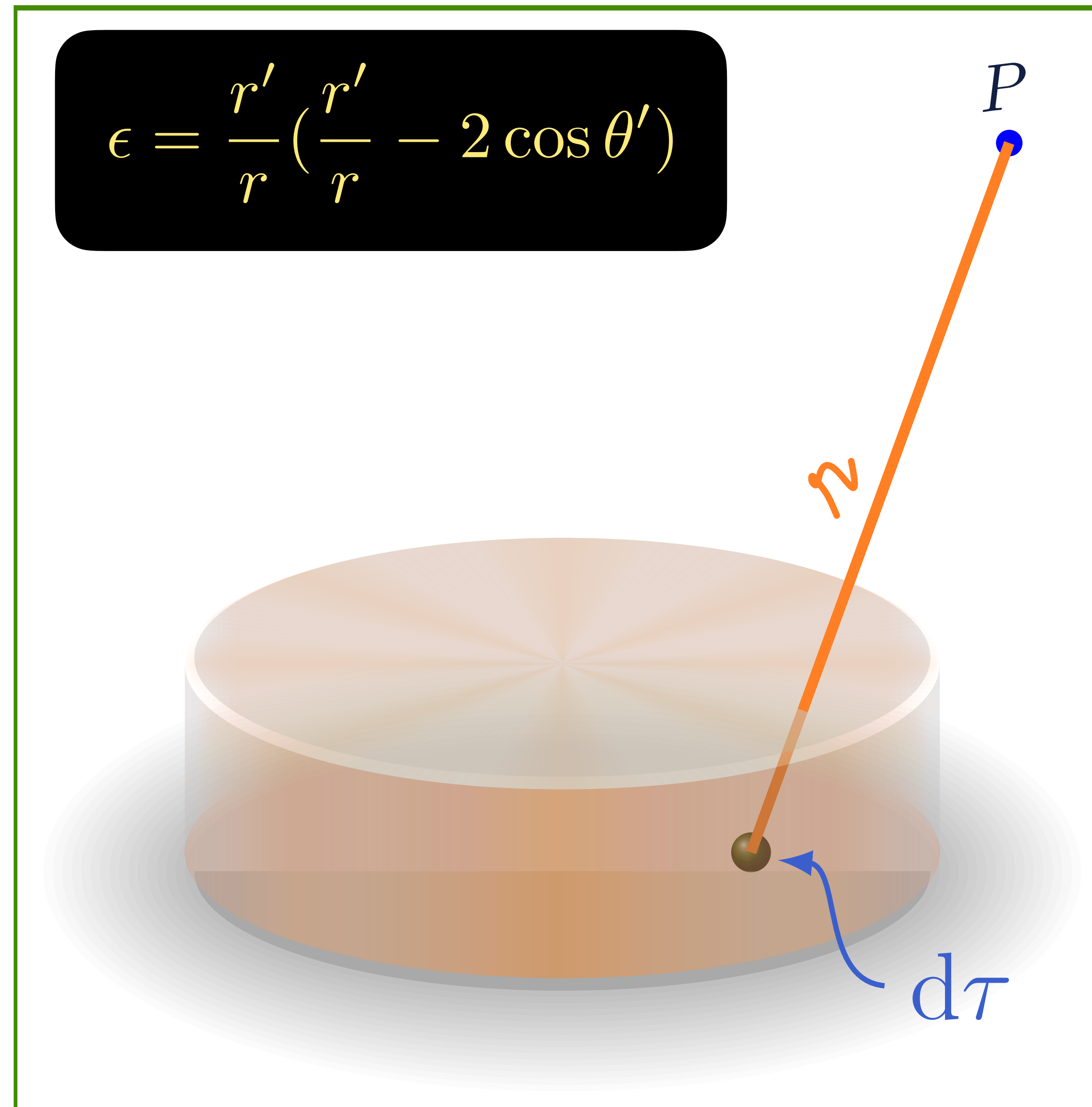
Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\, \mathrm{d}\tau$$

$$\frac{1}{\mathscr{r}} = (r^2 + r'^2 - 2rr'\cos\theta')^{-1/2}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left[1 - 2\frac{r'}{r}\cos\theta' + \left(\frac{r'}{r}\right)^2\right]^{-1/2}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}(1+\epsilon)^{-1/2}$$



$$\nabla^2 V = 0$$

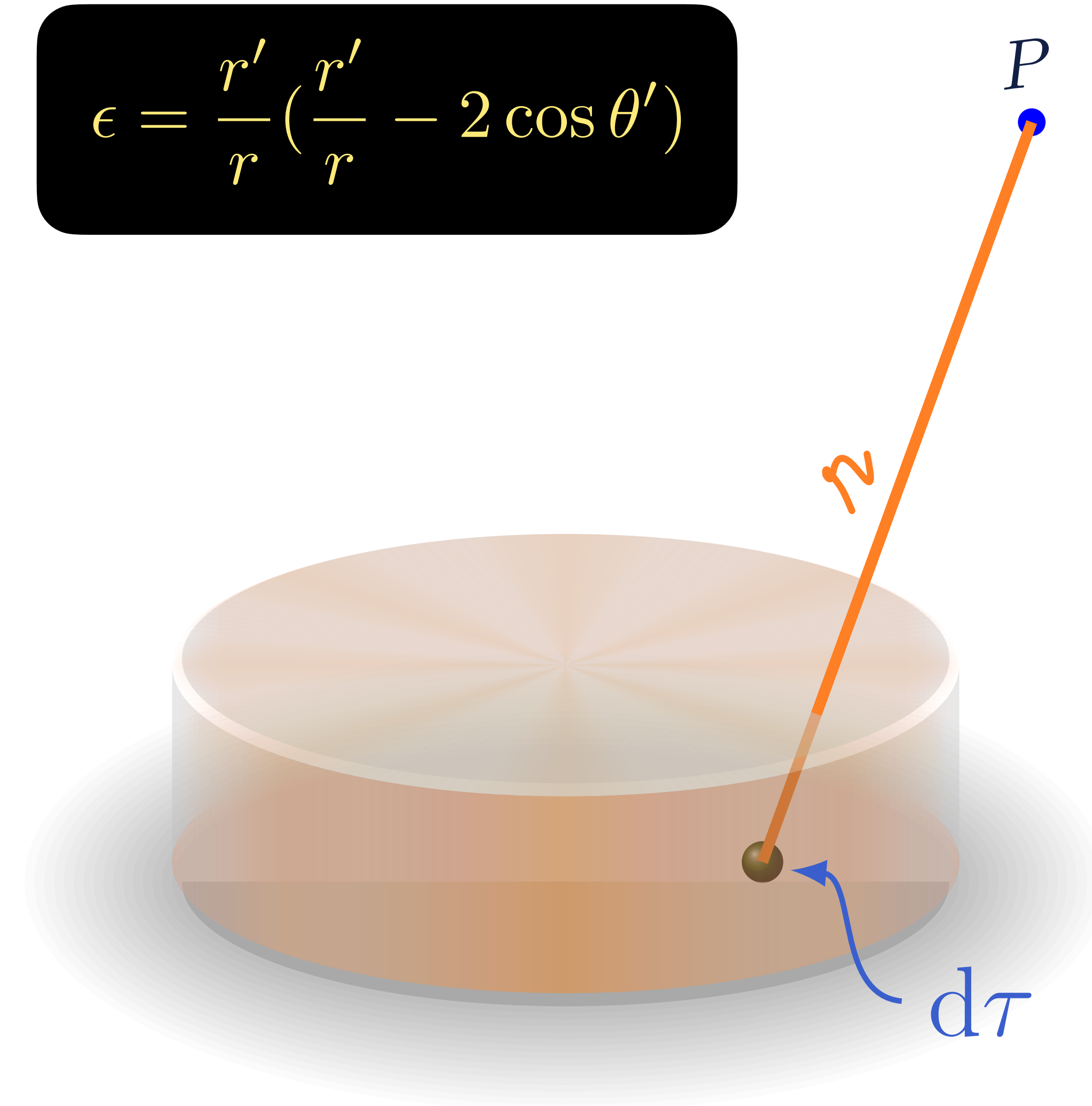
Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\, \mathrm{d}\tau$$

$$\frac{1}{\mathscr{r}} = (r^2 + r'^2 - 2rr'\cos\theta')^{-1/2}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left[1 - 2\frac{r'}{r}\cos\theta' + \left(\frac{r'}{r}\right)^2\right]^{-1/2}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}(1+\epsilon)^{-1/2}$$



$$\epsilon = \frac{r'}{r}\left(\frac{r'}{r} - 2\cos\theta'\right)$$

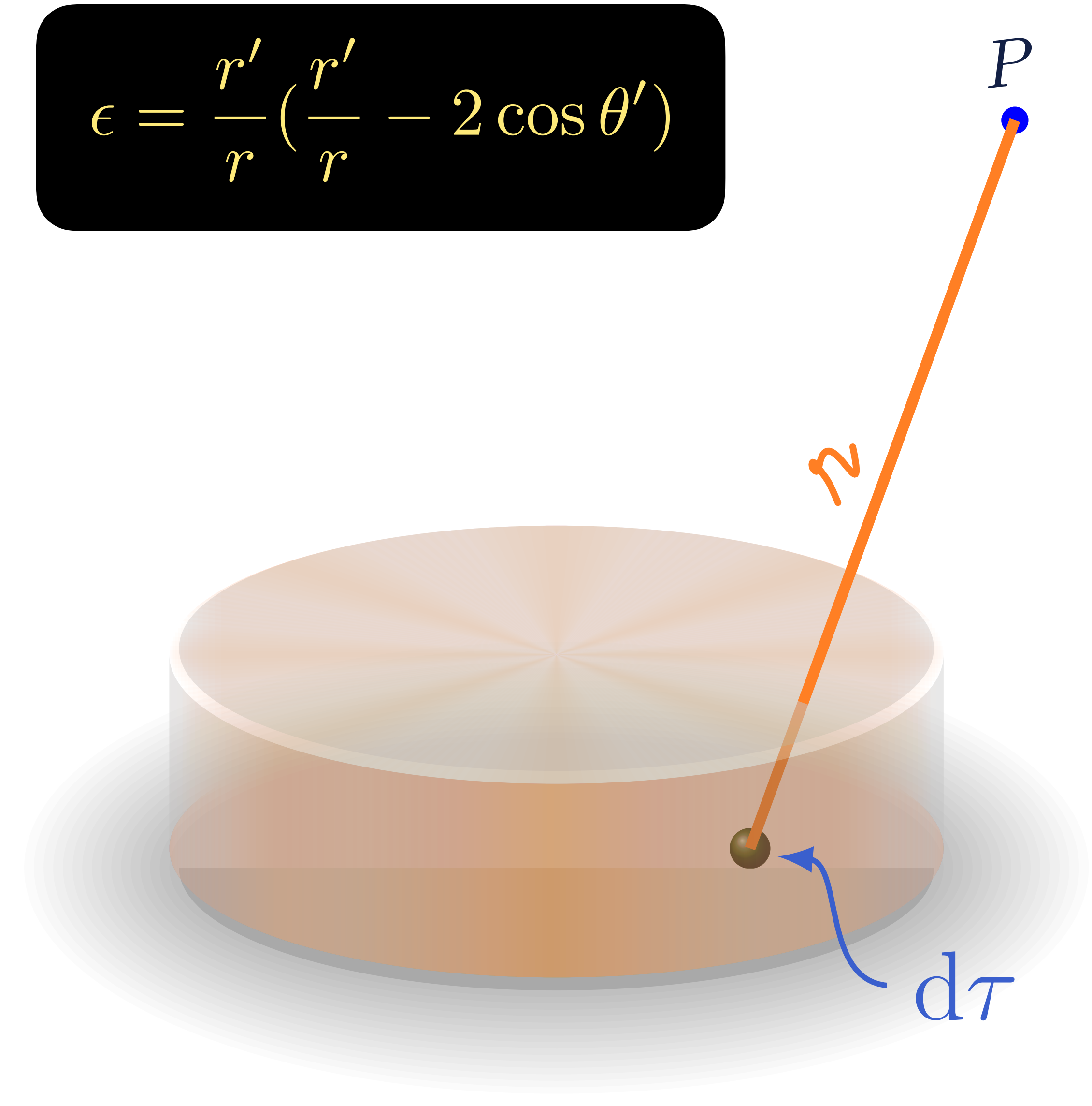
$$(1+b)^n = 1 + nb + \frac{n(n-1)}{2}b^2 + \frac{n(n-1)(n-2)}{6}b^3 + \ldots$$

$$\nabla^2 V = 0$$

Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\,\mathrm{d}\tau$$

$$\epsilon = \frac{r'}{r}\left(\frac{r'}{r} - 2\cos\theta'\right)$$



$$\frac{1}{\mathscr{r}} = (r^2 + r'^2 - 2rr'\cos\theta')^{-1/2}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left[1 - 2\frac{r'}{r}\cos\theta' + \left(\frac{r'}{r}\right)^2\right]^{-1/2}$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}(1+\epsilon)^{-1/2}$$

$$(1+\epsilon)^{-1/2} = 1 - \frac{1}{2}\epsilon + \frac{3}{8}\epsilon^2 - \frac{5}{16}\epsilon^3 + \ldots$$

$$(1+b)^n = 1 + nb + \frac{n(n-1)}{2}b^2 + \frac{n(n-1)(n-2)}{6}b^3 + \ldots$$

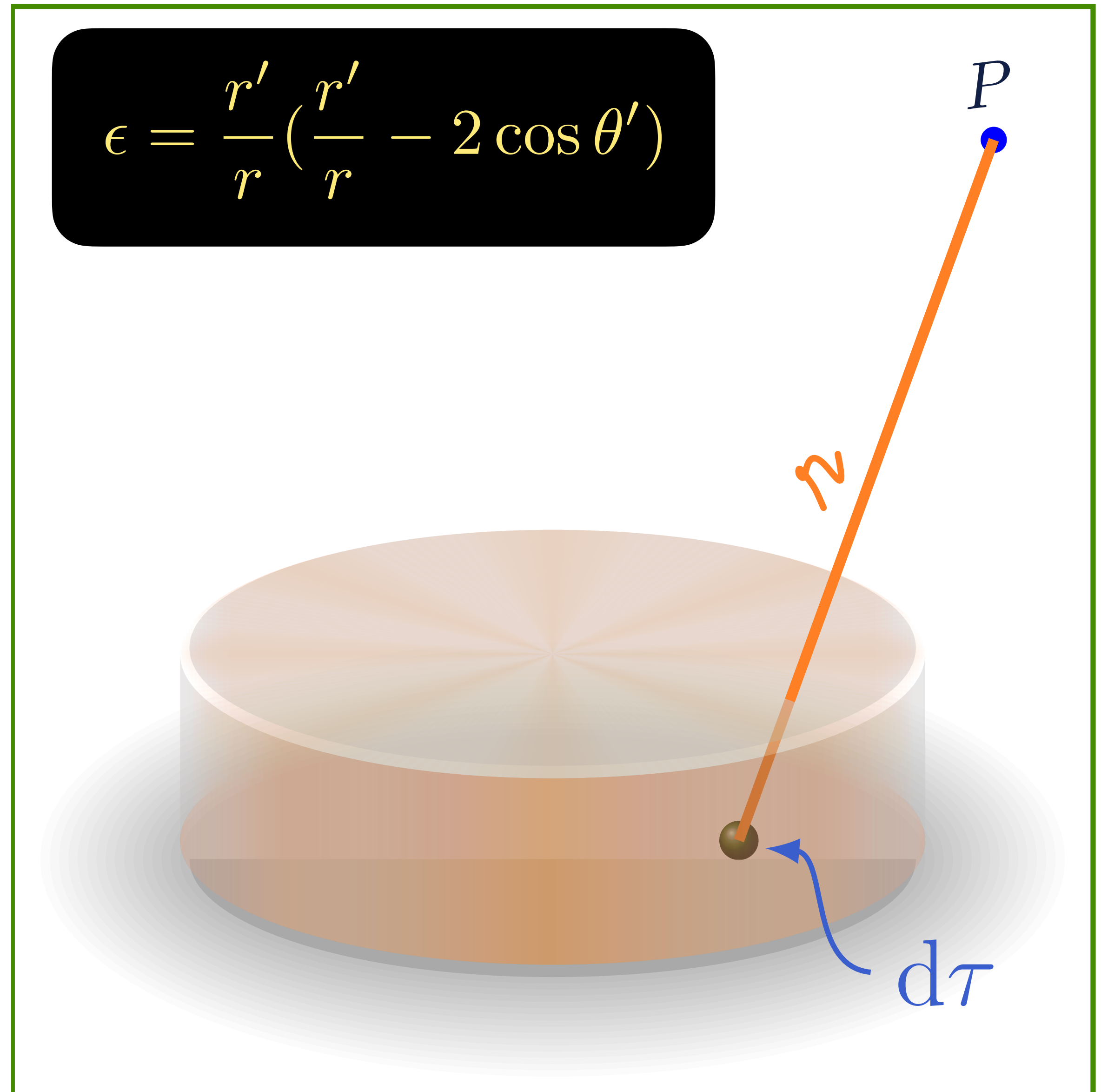
$$\nabla^2 V = 0$$

Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\, \mathrm{d}\tau$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left[1 + \left(\frac{r'}{r}\right)(\cos\theta') + \ldots\right]$$

ATÉ 1ª ORDEM
(VÊM MAIS TERMOS, NAS PRÓXIMAS TELAS)



$$\nabla^2 V = 0$$

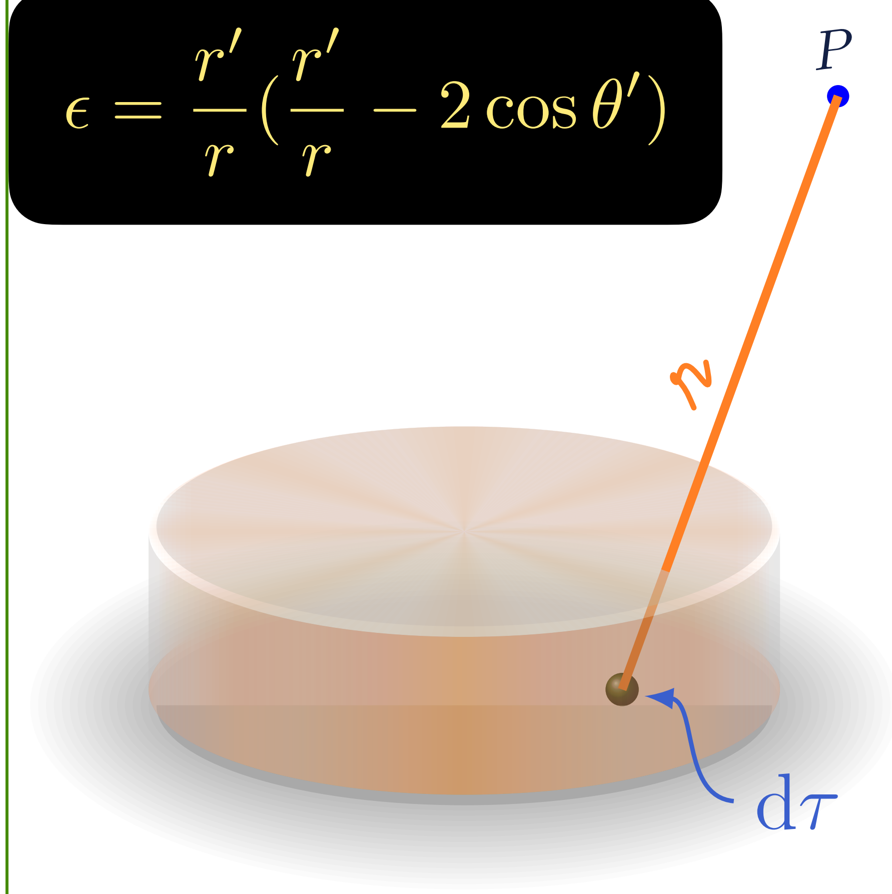
Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\, d\tau$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\Big[1 + \Big(\frac{r'}{r}\Big)(\cos\theta') + \Big(\frac{r'}{r}\Big)^2 \Big(\frac{3\cos^2\theta' - 1}{2}\Big) + \ldots\Big]$$

ATE' 2ª ORDEM

$$\epsilon = \frac{r'}{r}\Big(\frac{r'}{r} - 2\cos\theta'\Big)$$



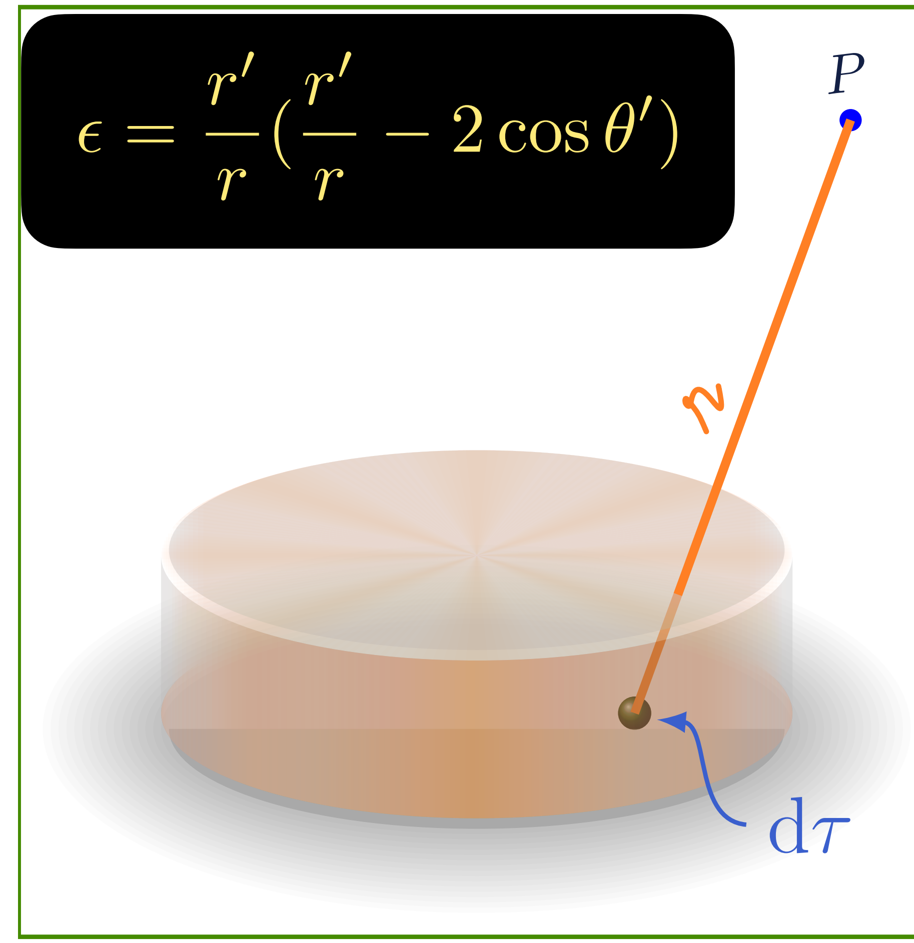
$$\nabla^2 V = 0$$

Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\, d\tau'$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\left[1 + \left(\frac{r'}{r}\right)\underbrace{(\cos\theta')}_{P_1(\cos\theta')} + \left(\frac{r'}{r}\right)^2 \underbrace{\left(\frac{3\cos^2\theta' - 1}{2}\right)}_{P_2(\cos\theta')} + \underbrace{\ldots}_{\left(\frac{r'}{r}\right)^3 P_3(x)}\right]$$

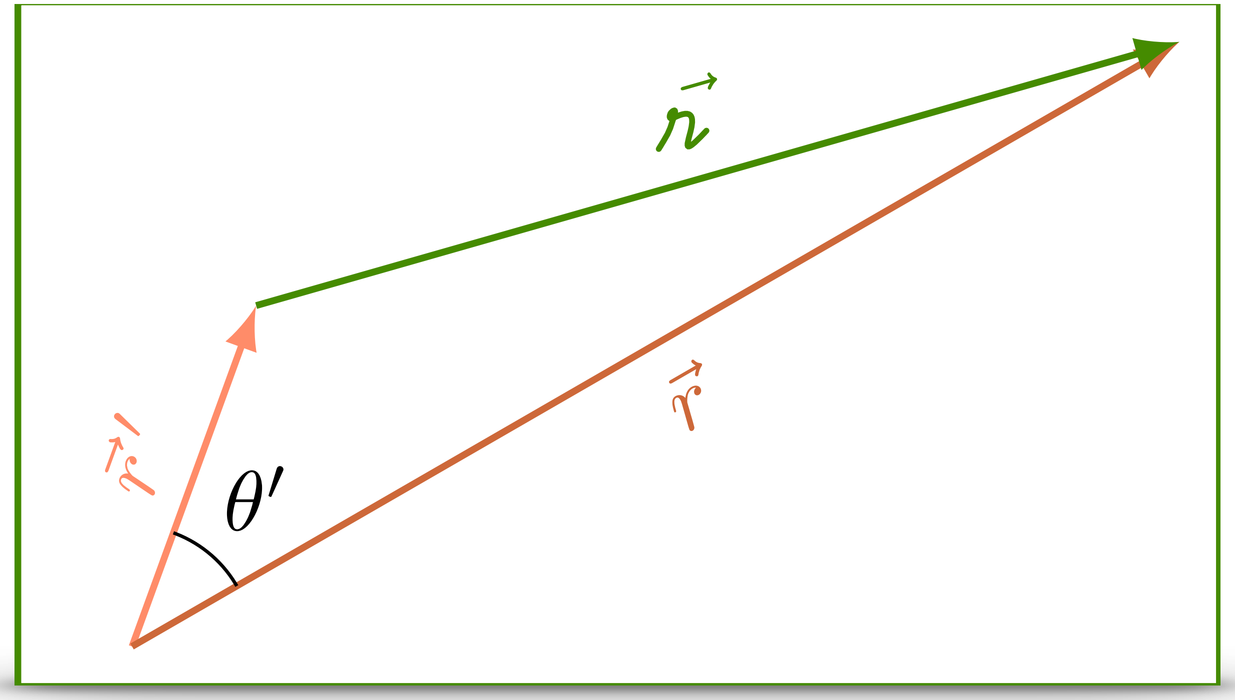
$$\frac{1}{\mathscr{r}} = \frac{1}{r}\sum_{\ell=0}^{\infty}\left(\frac{r'}{r}\right)^{\ell} P_\ell(\cos\theta')$$



$$\nabla^2 V = 0$$

Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\,d\tau'$$



$$\frac{1}{\mathscr{r}} = \frac{1}{r}\sum_{\ell=0}^{\infty}\left(\frac{r'}{r}\right)^{\ell} P_\ell(\cos\theta')$$

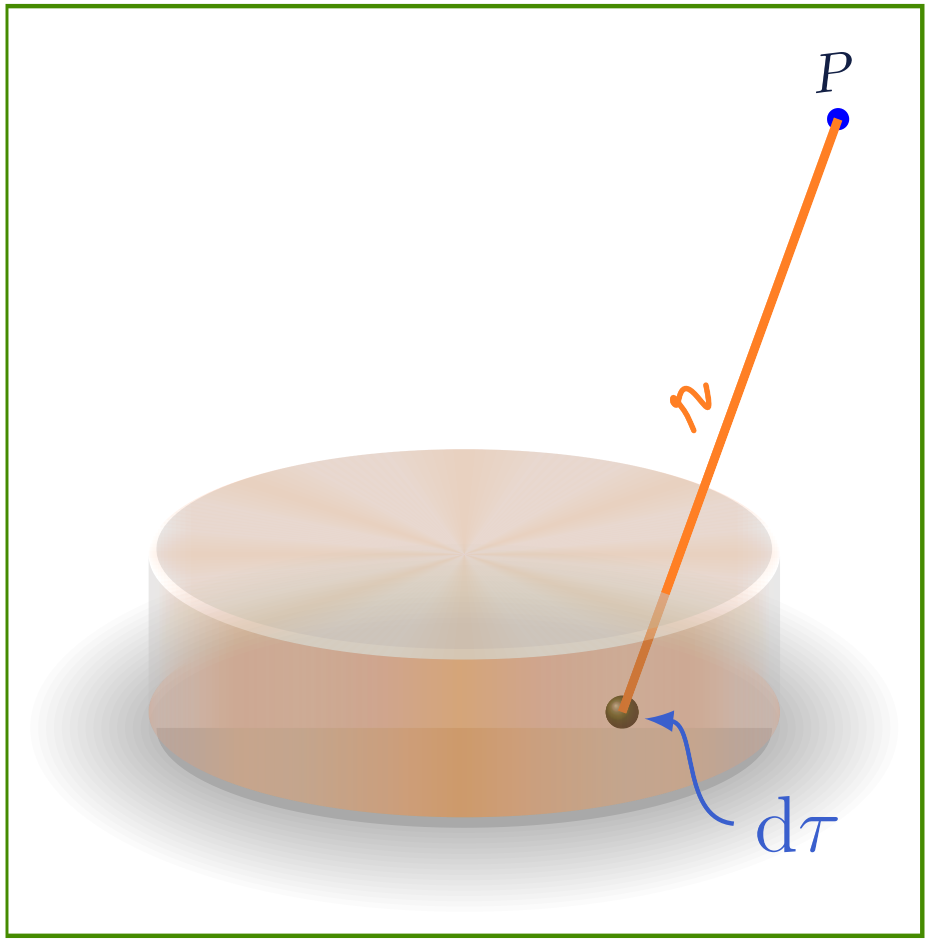
↳ VÁLIDO PARA $r' < r$

$$\nabla^2 V = 0$$

Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\,\mathrm{d}\tau'$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\sum_{\ell=0}^{\infty}\left(\frac{r'}{r}\right)^{\ell} P_\ell(\cos\theta')$$



$$\nabla^2 V = 0$$

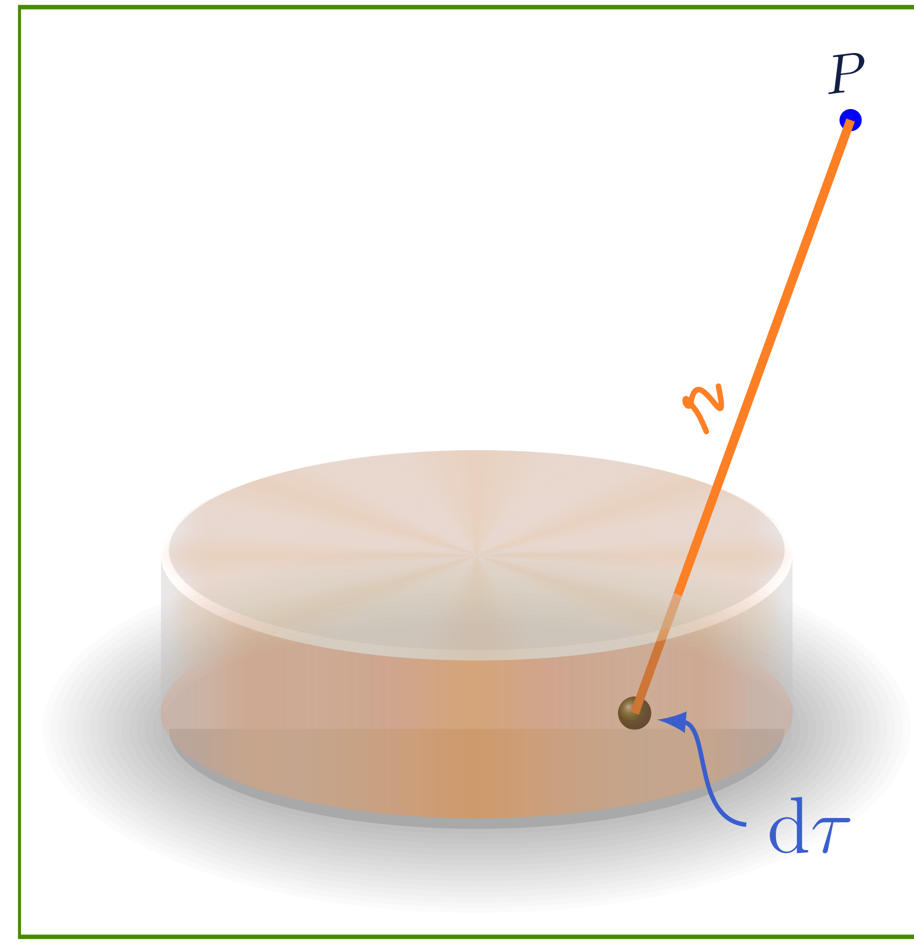
Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\, \mathrm{d}\tau'$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\sum_{\ell=0}^{\infty}\left(\frac{r'}{r}\right)^{\ell} P_\ell(\cos\theta')$$

↳ $r^{\ell+1}$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int (r')^n P_n(\cos\theta')\rho(\vec{r}\,')\, \mathrm{d}\tau'$$

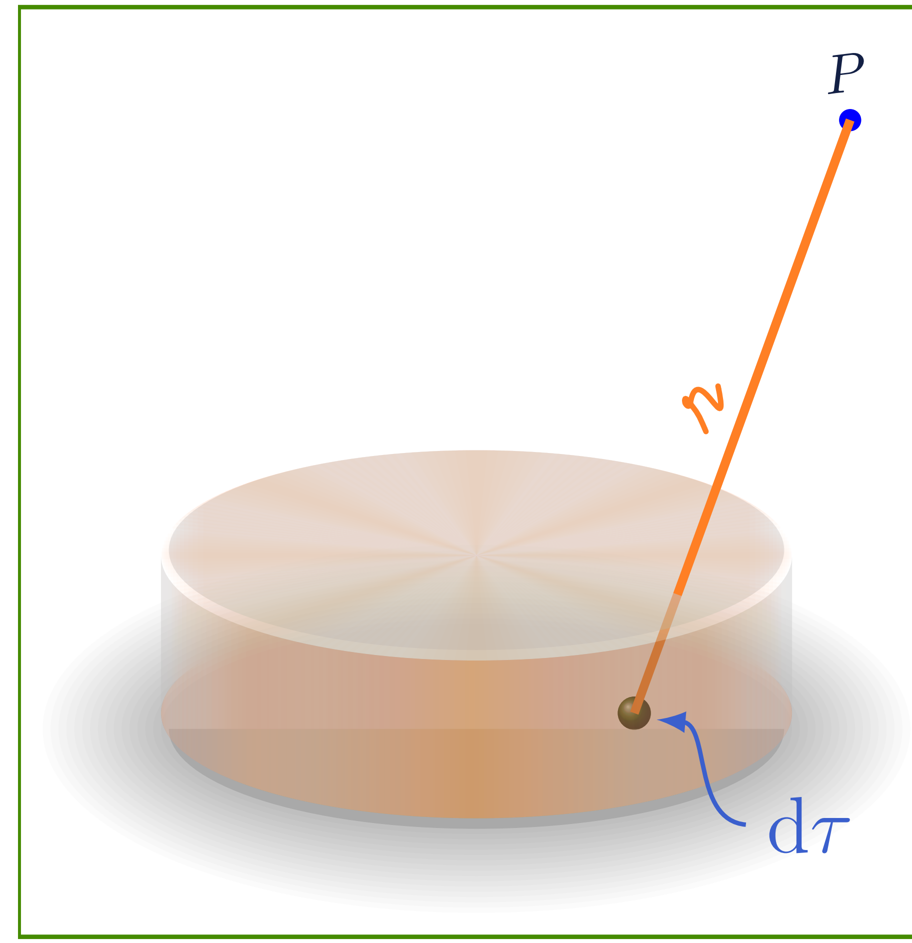


$$\nabla^2 V = 0$$

Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\, d\tau'$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\sum_{\ell=0}^{\infty}\left(\frac{r'}{r}\right)^{\ell} P_\ell(\cos\theta')$$



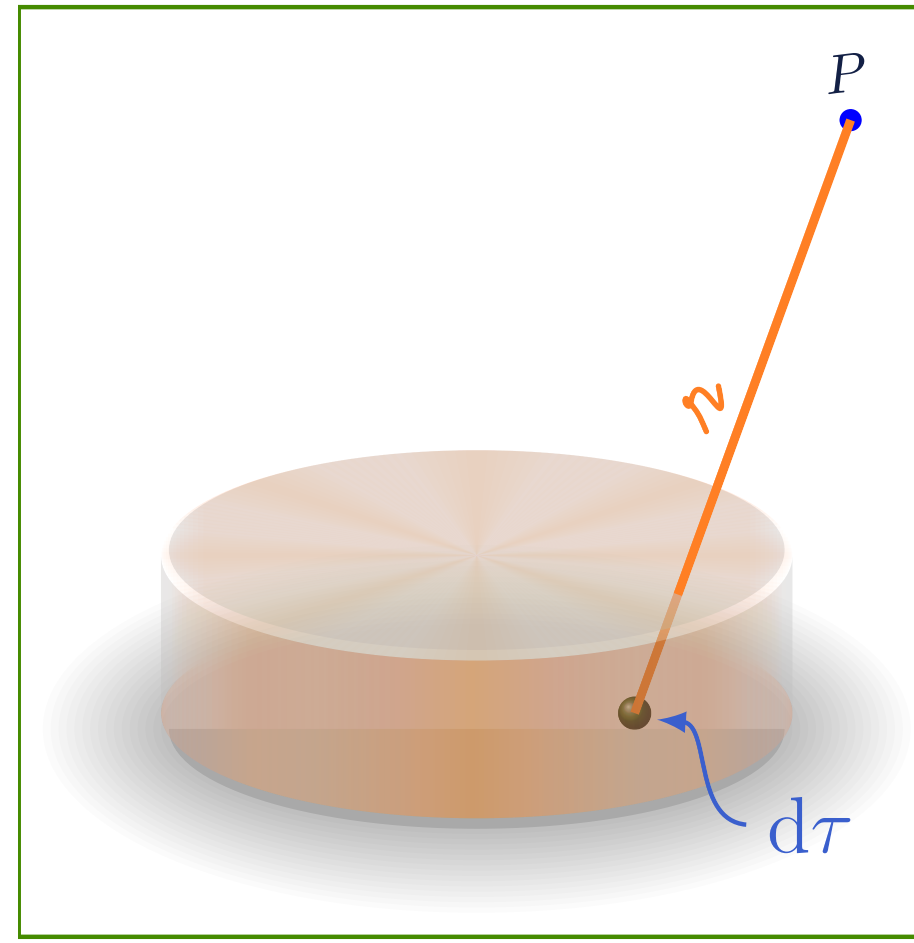
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int (r')^n P_n(\cos\theta')\rho(\vec{r'})\, d\tau'$$

$$n = 0 \quad \Rightarrow \quad \int (r')^n P_n(\cos\theta')\rho(\vec{r'})\, d\tau' = \int \rho(\vec{r'})\, d\tau'$$

$$\nabla^2 V = 0$$

Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\, d\tau'$$



$$\frac{1}{\mathscr{r}} = \frac{1}{r} \sum_{\ell=0}^{\infty} \left(\frac{r'}{r}\right)^{\ell} P_\ell(\cos\theta')$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \int (r')^n P_n(\cos\theta') \rho(\vec{r}\,')\, d\tau'$$

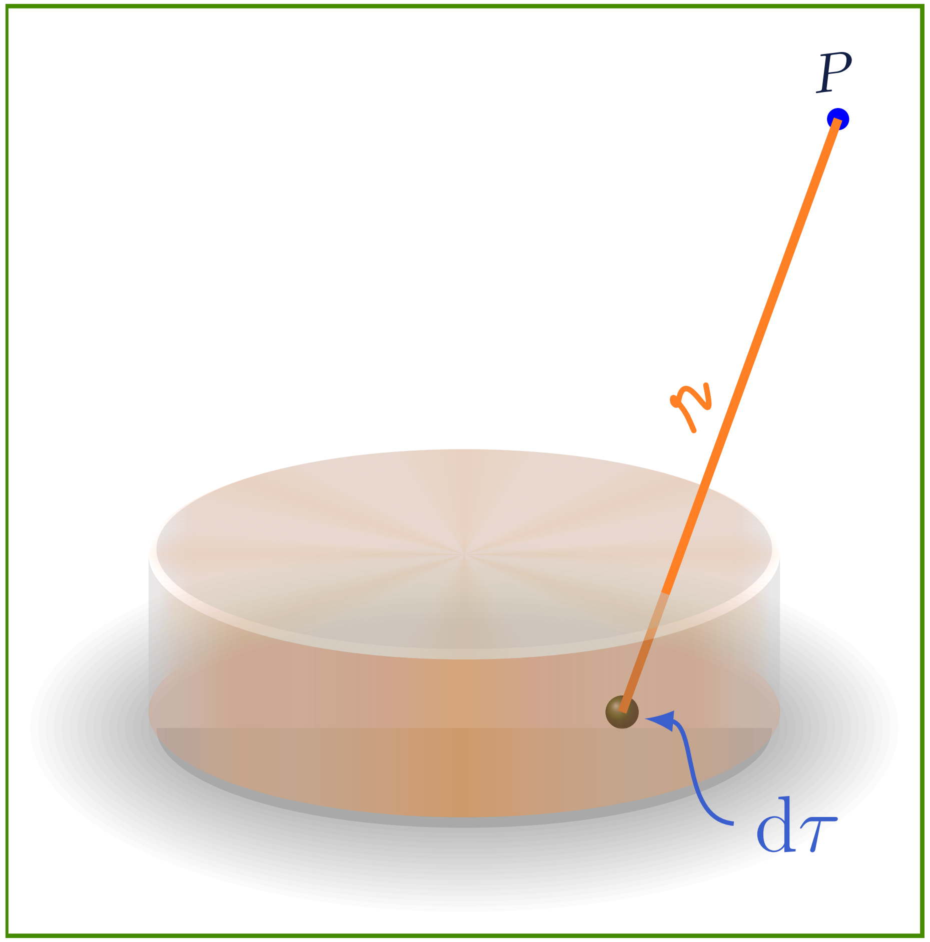
$$n = 0 \quad \Rightarrow \quad V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$ → MOMENTO DE MONOPOLO

$$\nabla^2 V = 0$$

Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\,\mathrm{d}\tau'$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\sum_{\ell=0}^{\infty}\left(\frac{r'}{r}\right)^{\ell} P_\ell(\cos\theta')$$



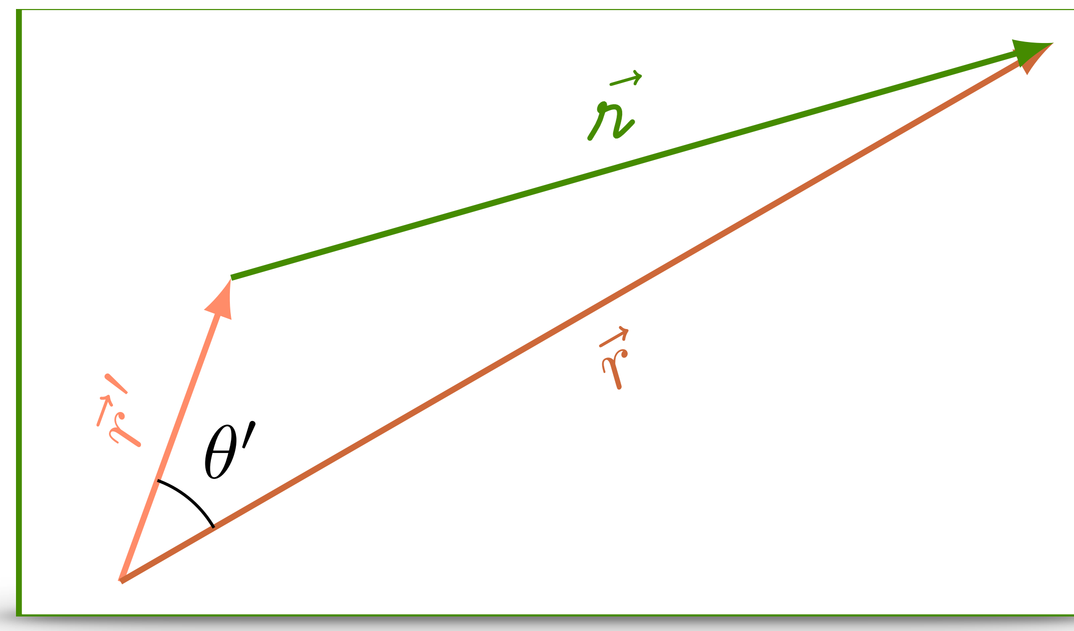
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int (r')^n P_n(\cos\theta')\rho(\vec{r'})\,\mathrm{d}\tau'$$

$n = 1 \Rightarrow$ $$\int (r')^n P_n(\cos\theta')\rho(\vec{r'})\,\mathrm{d}\tau' = \int (r'\cos\theta')\rho(\vec{r'})\,\mathrm{d}\tau'$$

$$\nabla^2 V = 0$$

Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\,\mathrm{d}\tau'$$



$$\frac{1}{\mathscr{r}} = \frac{1}{r}\sum_{\ell=0}^{\infty}\left(\frac{r'}{r}\right)^{\ell} P_\ell(\cos\theta')$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int (r')^n P_n(\cos\theta')\rho(\vec{r'})\,\mathrm{d}\tau'$$

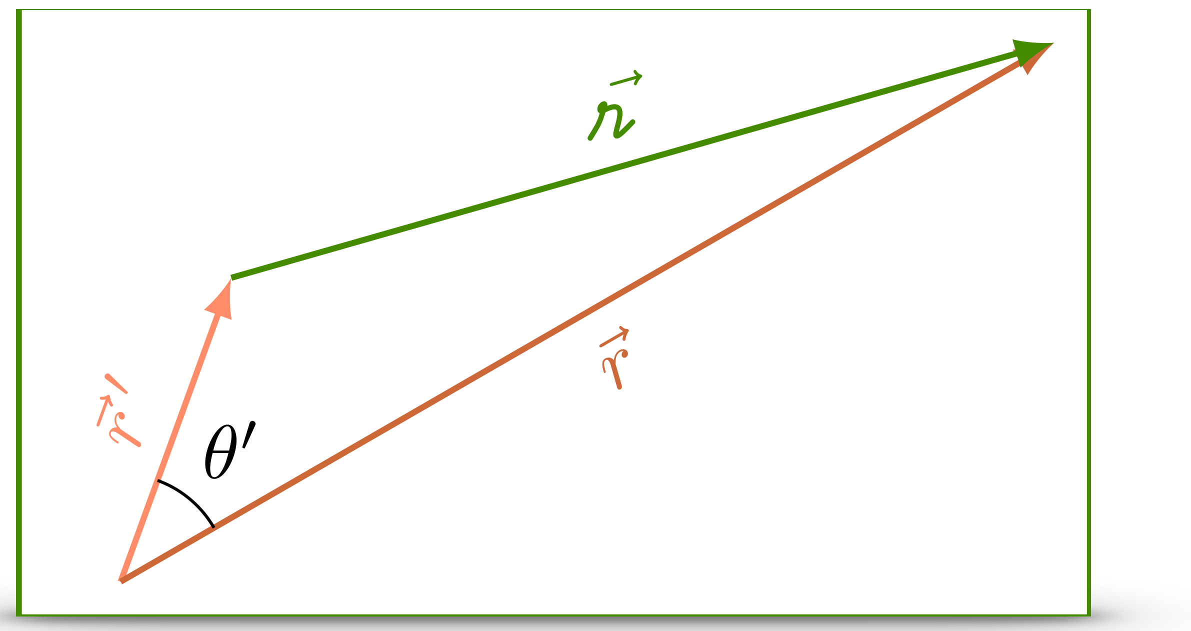
$$n = 1 \quad \Rightarrow \quad \int (r')^n P_n(\cos\theta')\rho(\vec{r'})\,\mathrm{d}\tau' = \int (r'\cos\theta')\rho(\vec{r'})\,\mathrm{d}\tau'$$

$$\cos\theta' = \hat{r}\cdot\hat{r}' \quad \Rightarrow \quad \int (r')^n P_n(\cos\theta')\rho(\vec{r'})\,\mathrm{d}\tau' = \hat{r}\cdot\int \vec{r'}\rho(\vec{r'})\,\mathrm{d}\tau'$$

$r'\cos\theta' = \hat{r}\cdot\vec{r}\,'$

$$\nabla^2 V = 0$$

Equação de Laplace
Expansão em multipolos



$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}\,')}{\mathscr{r}}\,d\tau'$$

$$\frac{1}{\mathscr{r}} = \frac{1}{r}\sum_{\ell=0}^{\infty}\left(\frac{r'}{r}\right)^{\ell} P_\ell(\cos\theta')$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int (r')^n P_n(\cos\theta')\rho(\vec{r}\,')\,d\tau'$$

$$n = 1 \quad \Rightarrow \quad V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$

MOMENTO DE DIPOLO

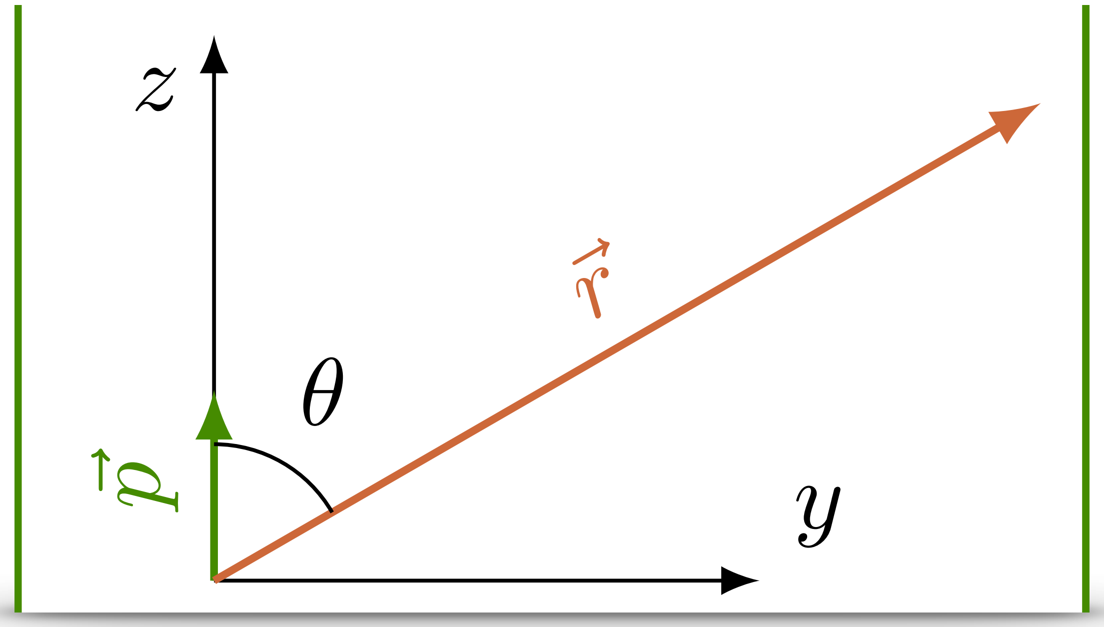
$$\nabla^2 V = 0$$

Equação de Laplace
Expansão em multipolos

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,')}{\mathscr{r}} \, \mathrm{d}\tau'$$

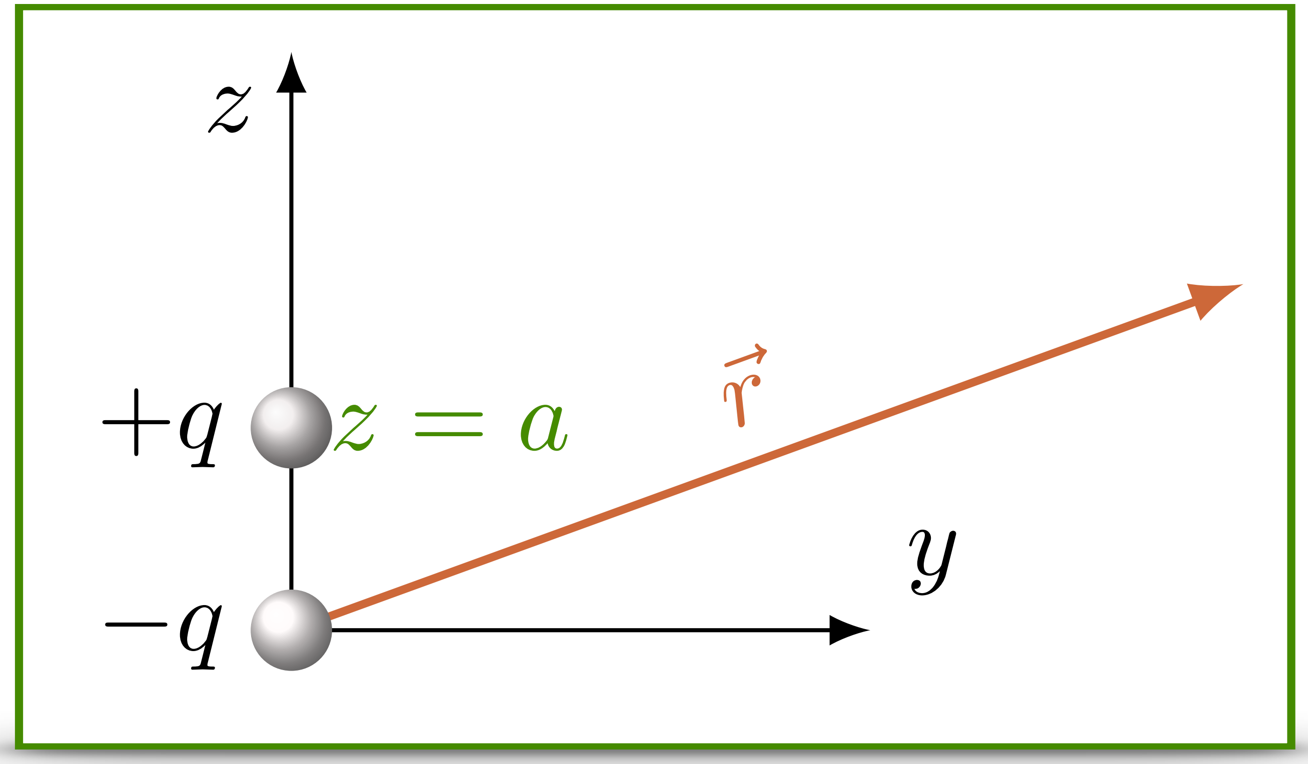
$$n = 1 \quad \Rightarrow \quad V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{p \cos\theta}{r^2}$$



Pratique o que aprendeu

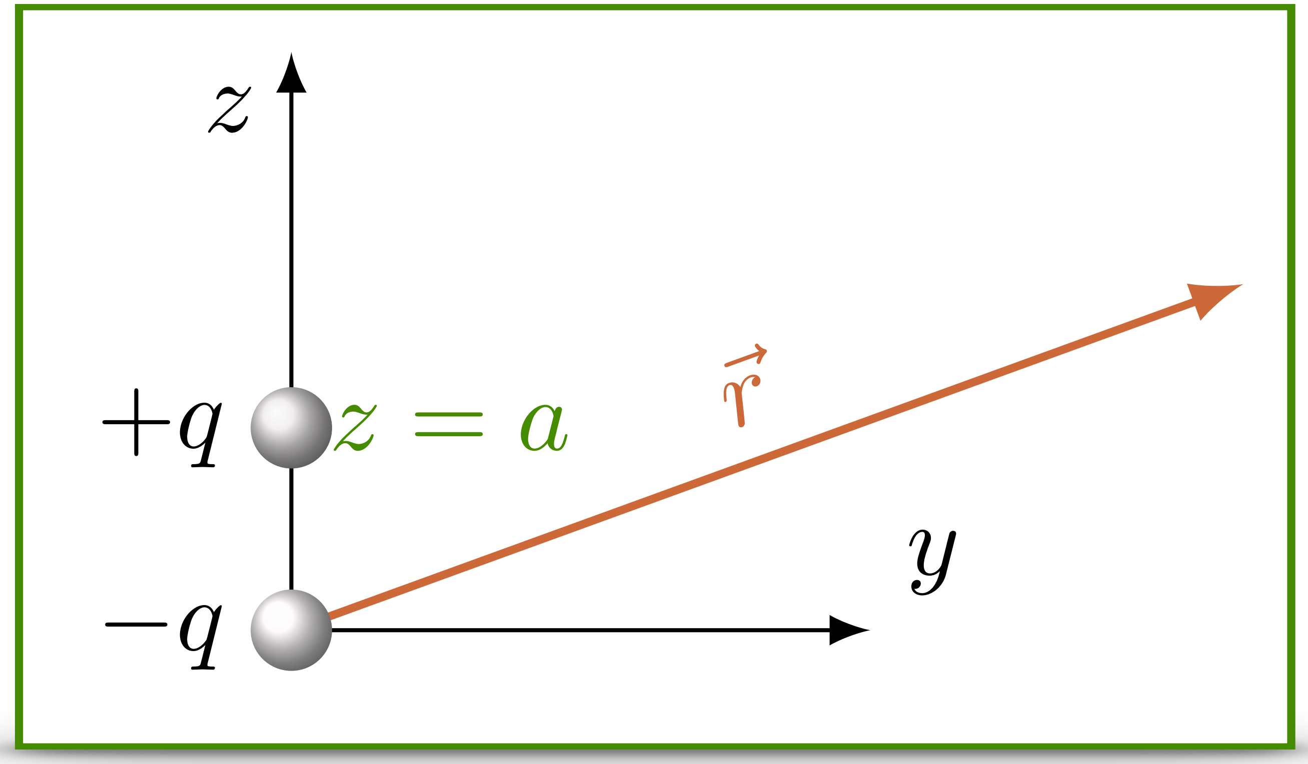
$$\vec{p} = \int \vec{r'} \rho(\vec{r'})\, \mathrm{d}\tau'$$



Pratique o que aprendeu

$$\vec{p} = \int \vec{r'} \rho(\vec{r'})\, \mathrm{d}\tau'$$

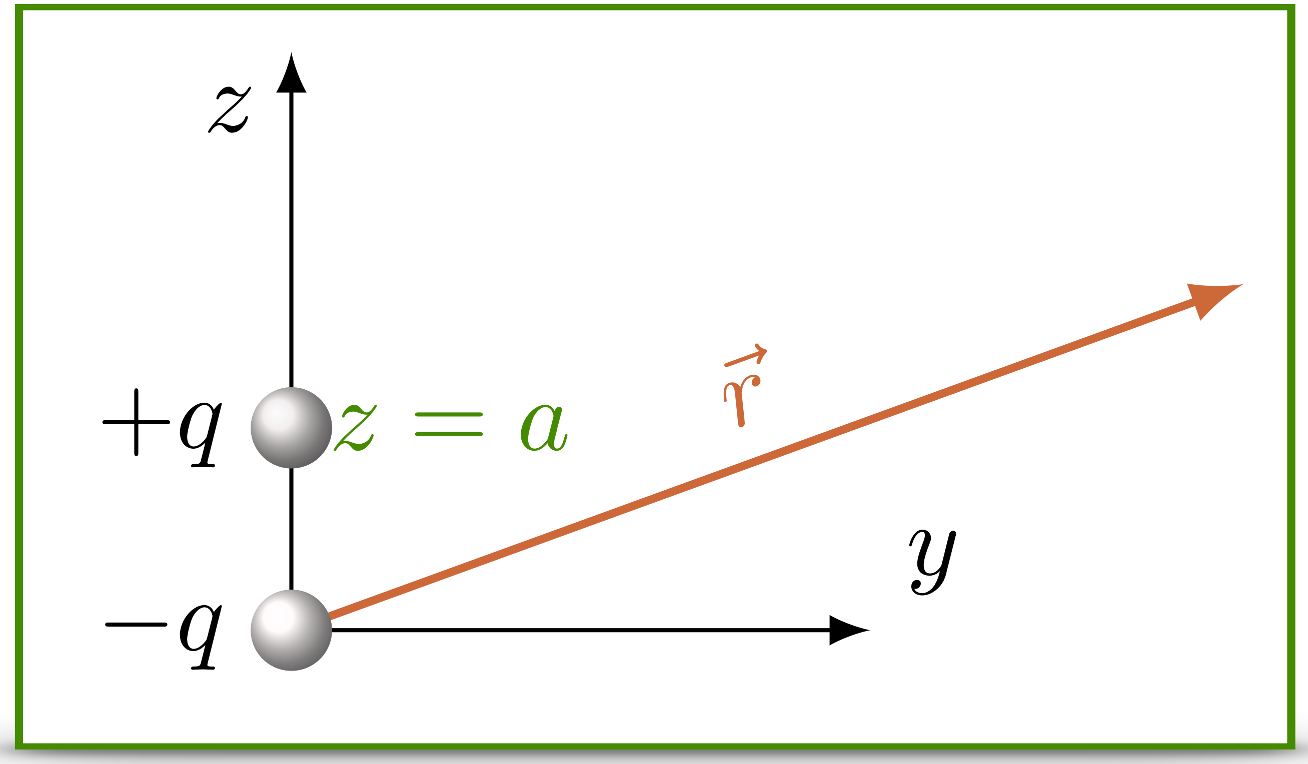
$$\rho(\vec{r'}) = q\delta^3(\vec{r'} - a\hat{z}) - q\delta^3(\vec{r'})$$



Pratique o que aprendeu

$$\vec{p} = \int \vec{r'} \rho(\vec{r'})\, \mathrm{d}\tau'$$

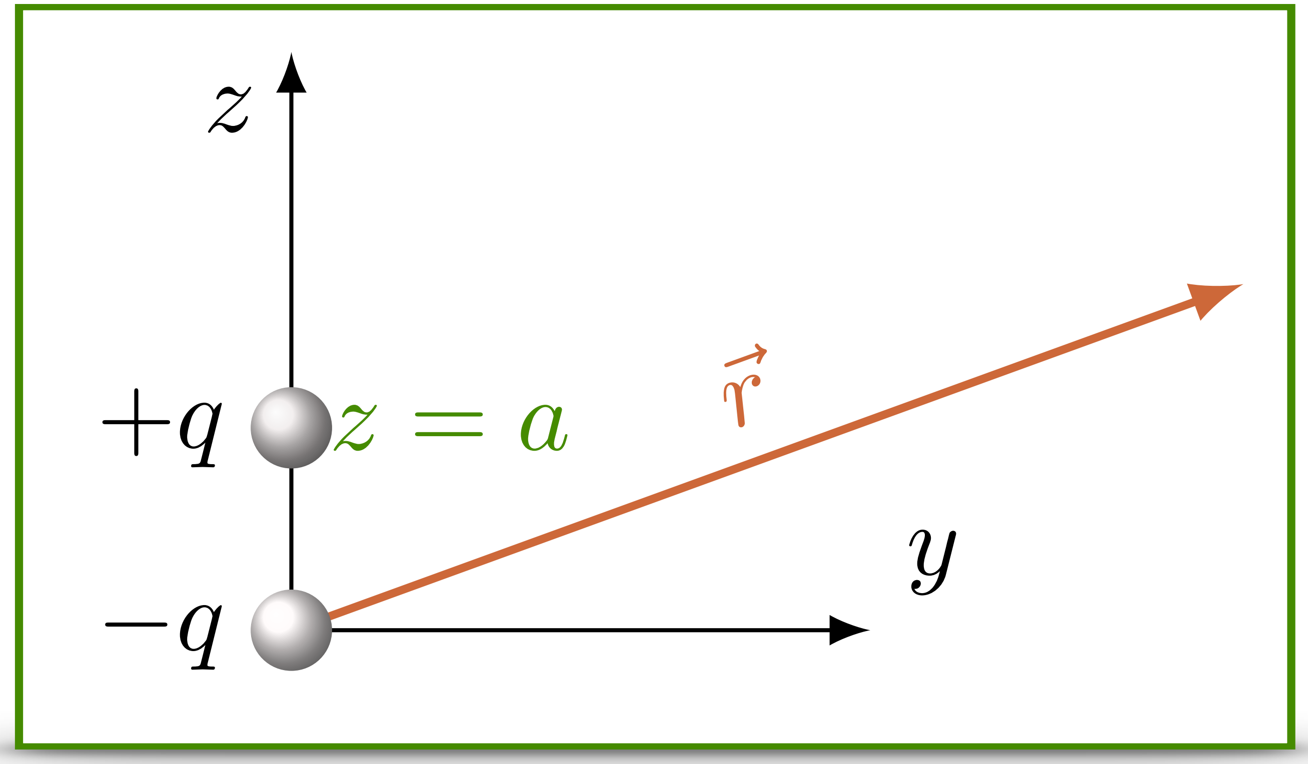
$$\rho(\vec{r'}) = q\delta^3(\vec{r'} - a\hat{z}) - q\delta^3(\vec{r'})$$



$$\vec{p} = q\int \vec{r'}\delta^3(\vec{r'} - a\hat{z})\, \mathrm{d}\tau' - q\int \vec{r'}\delta^3(\vec{r'})\, \mathrm{d}\tau'$$

Pratique o que aprendeu

$$\vec{p} = \int \vec{r'} \rho(\vec{r'})\, \mathrm{d}\tau'$$



$$\rho(\vec{r'}) = q\delta^3(\vec{r'} - a\hat{z}) - q\delta^3(\vec{r'})$$

$$\vec{p} = q\underbrace{\int \vec{r'}\delta^3(\vec{r'} - a\hat{z})\, \mathrm{d}\tau'}_{a\hat{z}} - q\underbrace{\int \vec{r'}\delta^3(\vec{r'})\, \mathrm{d}\tau'}_{0}$$

$$\vec{p} = q(a\hat{z} - 0) = qa\hat{z}$$