Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

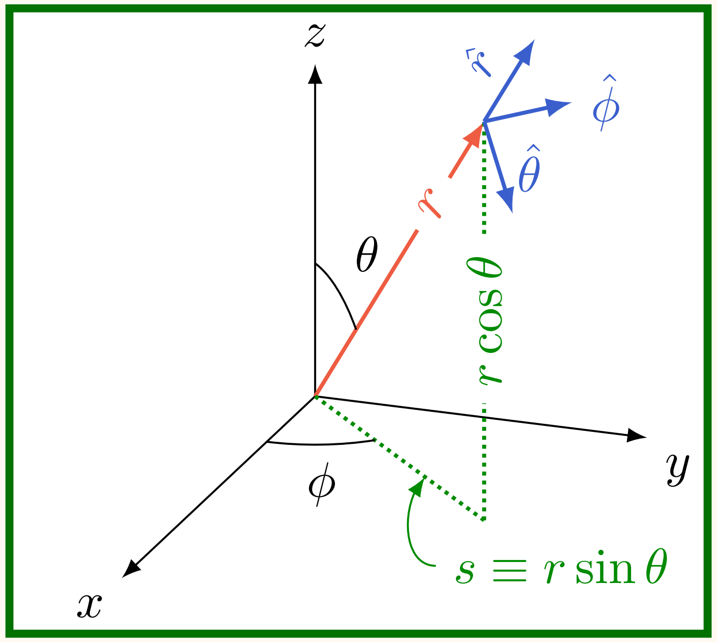
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

Aula de 7 de junho
Métodos especiais

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2}\frac{\partial (r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$\vec{\nabla} \times \vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{r}$$

$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\, v_\theta\right) - \frac{\partial v_r}{\partial \theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Coordenadas cilíndricas

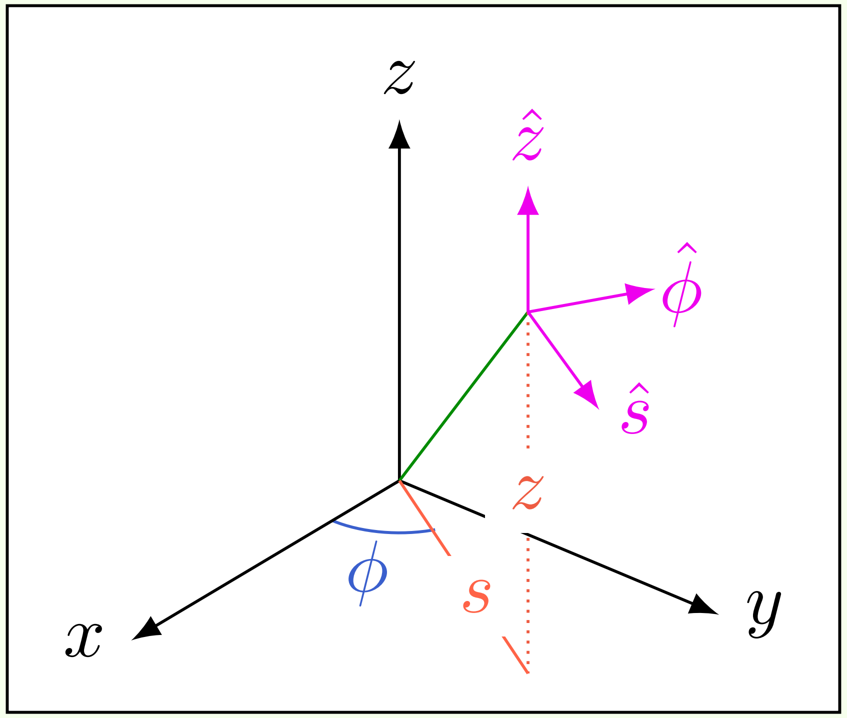
$$d\vec{\ell} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$$

$$\vec{\nabla} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s}\frac{\partial (s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

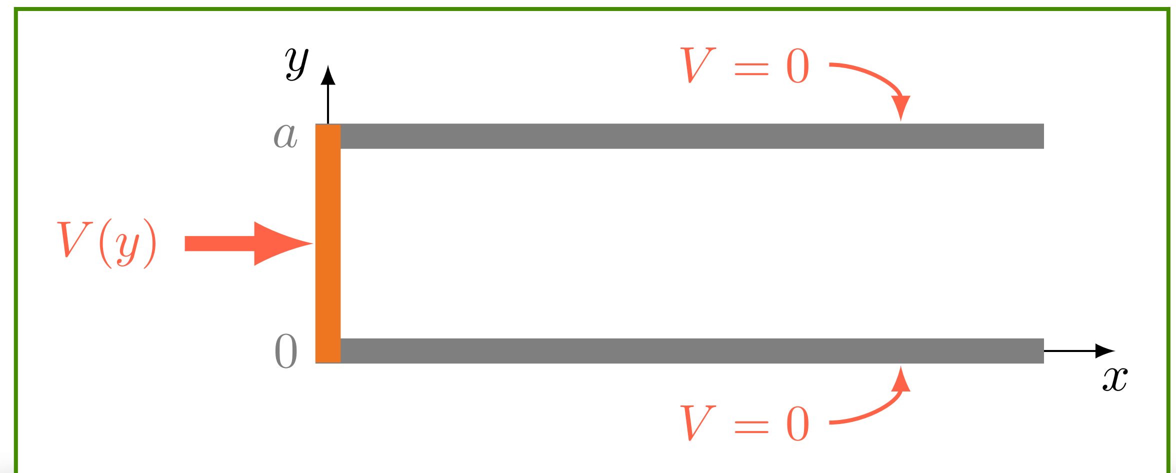
$$\vec{\nabla} \times \vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$



Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$



Simetria plana

$$V(x,y) = \sum_{n=1}^{\infty} V_n \exp\left(-\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$

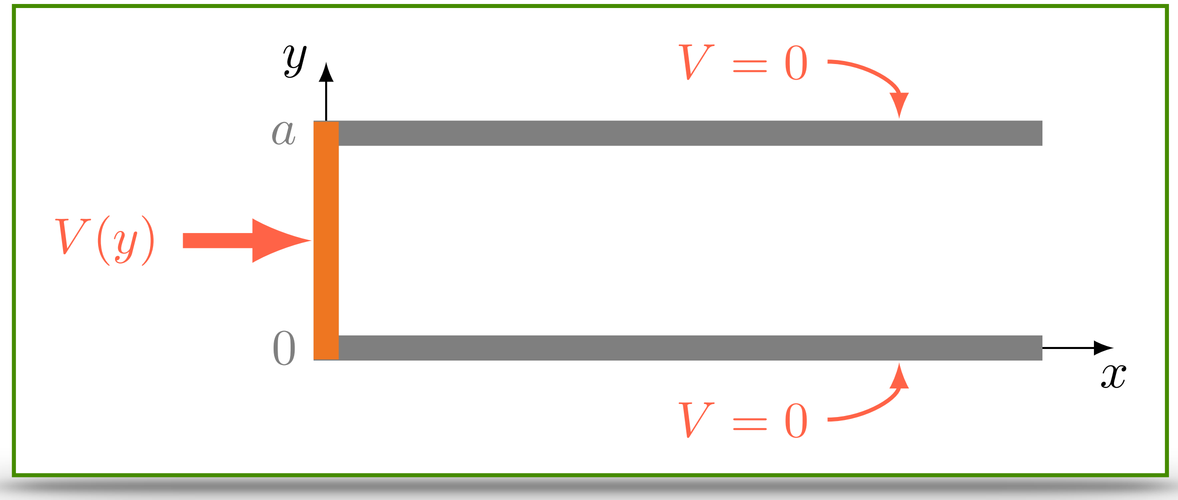
PERMITE IMPOR CONDIÇÕES DE CONTORNO

- $V(x, y=0) = 0$ [POIS $\text{sen}\left(\frac{n\pi y}{a}\right) = 0$]
- $V(x, y=a) = 0$ [POIS $\sin\left(\frac{n\pi y}{a}\right) = 0$]
- $V(x=0, y) = V(y)$ [$\Rightarrow V(y) = \sum_{n=1}^{\infty} V_n \sin\left(\frac{n\pi y}{a}\right)$]
- $V(x \to \infty, y) = 0$ [POIS $\exp\left(-\frac{n\pi x}{a}\right) \to 0$]

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$V(x,y) = \sum_{n=1}^{\infty} V_n \exp\left(-\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$



$$V_m = \frac{2}{a} \int_0^a V(0,y) \sin\left(\frac{m\pi y}{a}\right) \, \mathrm{d}y$$

GRAÇAS À ORTOGONALIDADE ENTRE AS FUNÇÕES $\sin\left(\frac{n\pi y}{a}\right)$

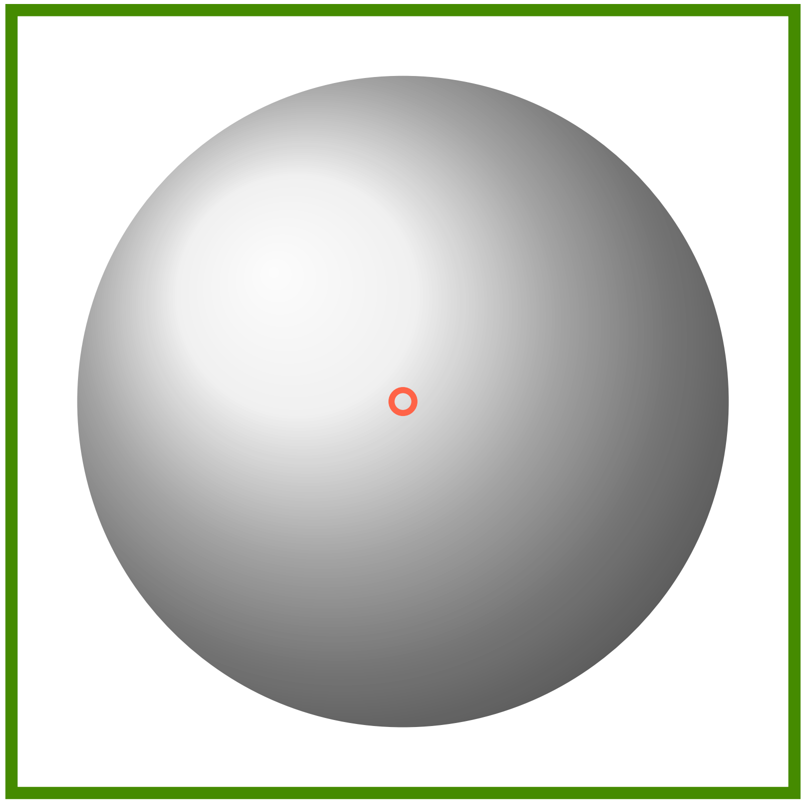
Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

Simetria esférica

$$V(r, \theta, \phi) = R(r)\Theta(\theta)$$

PARA GARANTIR QUE CONDIÇÕES DE CONTORNO SEJAM SATISFEITAS

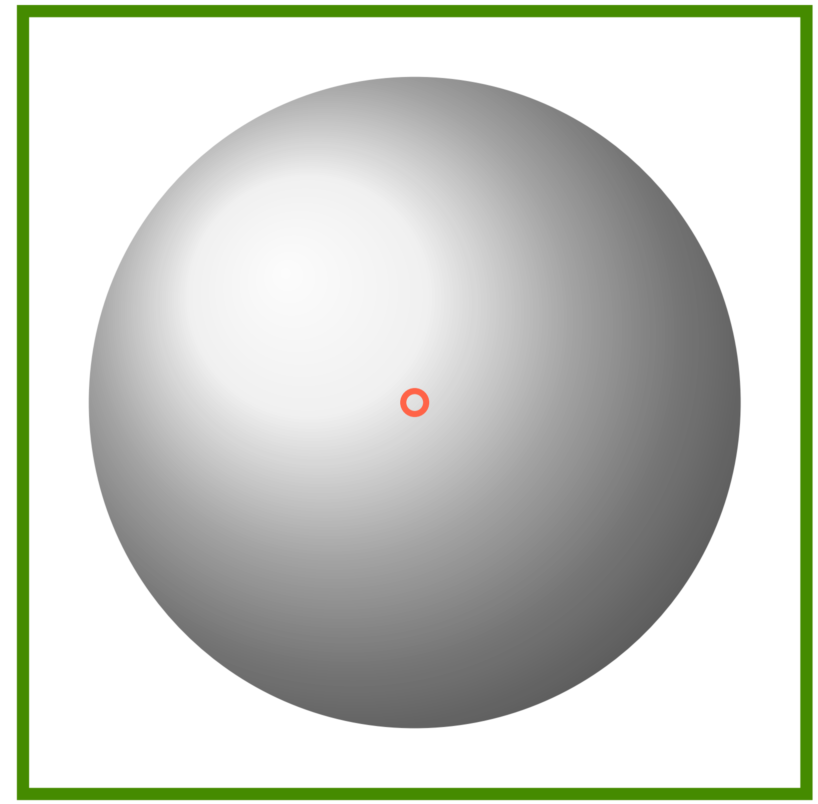


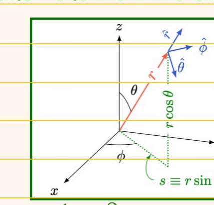
Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

Simetria esférica

$$V(r,\theta,\phi) = R(r)\Theta(\theta)$$





Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

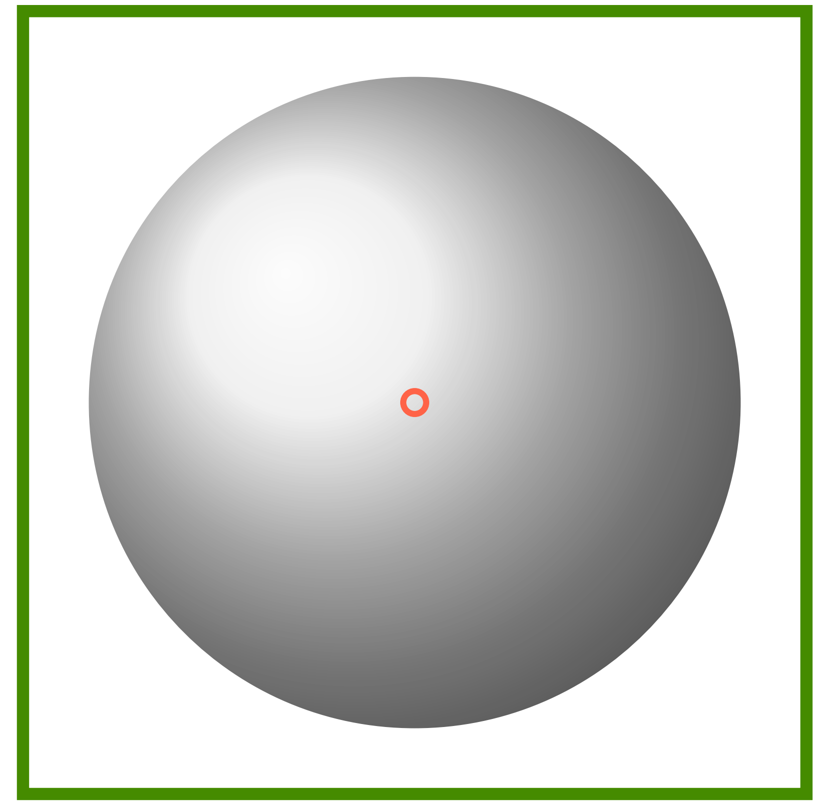
$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\, v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Equação de Laplace
Separação de variáveis

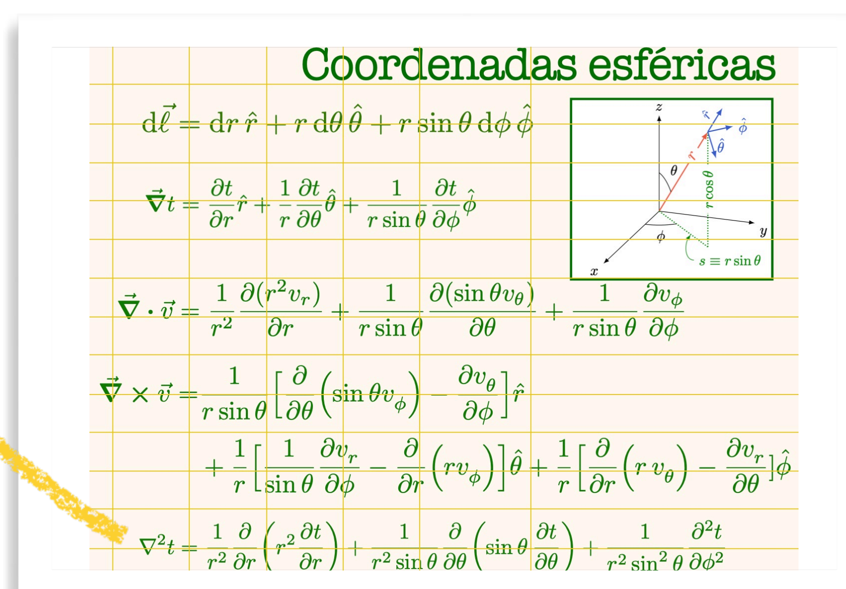
$$\nabla^2 V = 0$$



Simetria esférica

$$V(r,\theta,\phi) = R(r)\Theta(\theta)$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$



$$\nabla^2 V = 0$$

Equação de Laplace
Separação de variáveis

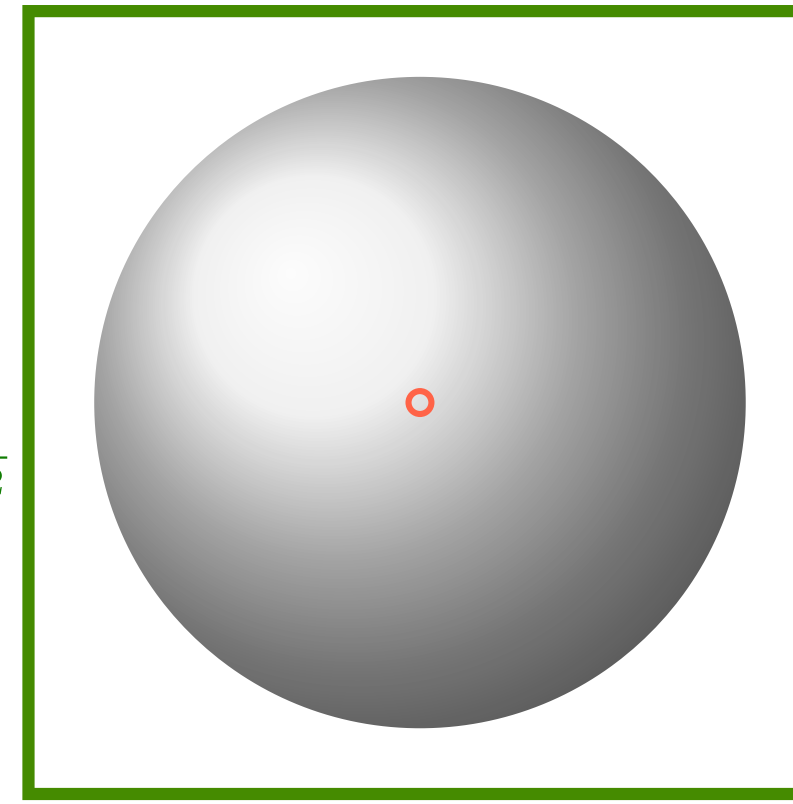
Simetria esférica

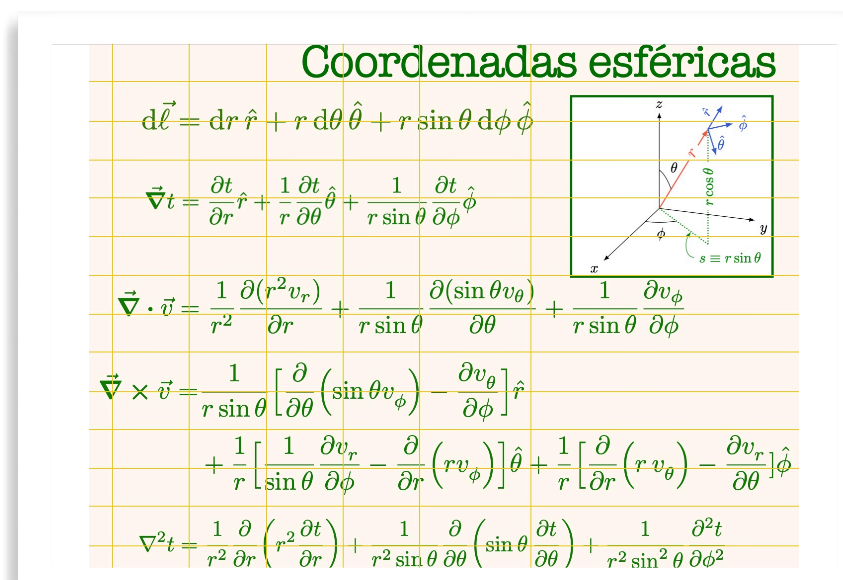
$$V(r,\theta,\phi) = R(r)\Theta(\theta)$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

$$\Theta(\theta)\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + R(r)\frac{1}{r^2\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

$R\Theta$ $\qquad$ $R\Theta$





Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$

$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial\phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Equação de Laplace
Separação de variáveis

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Simetria esférica

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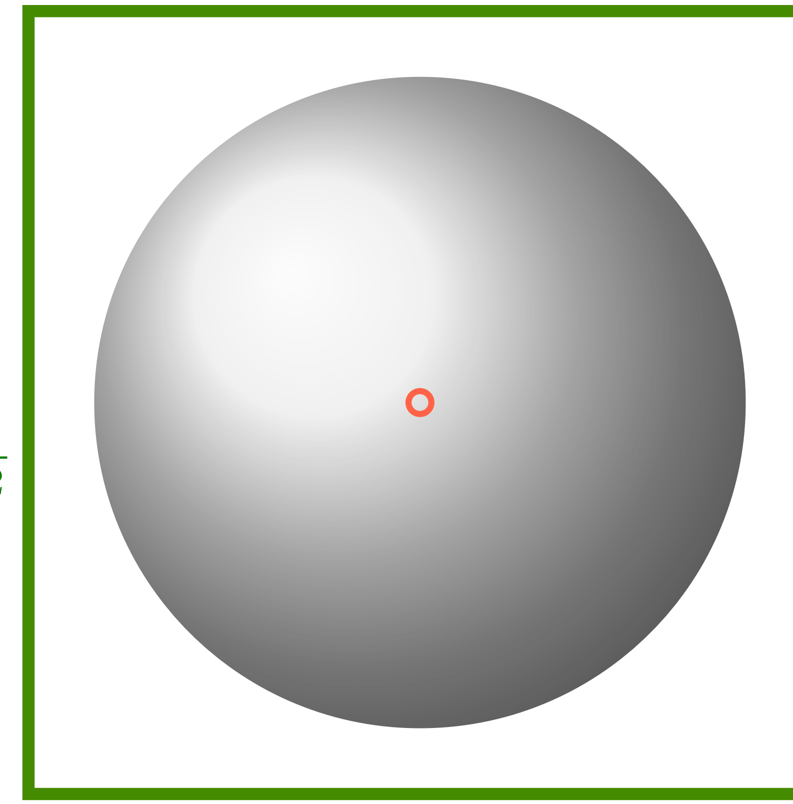
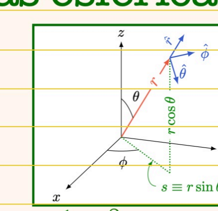
$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$



$$\underbrace{\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right)}_{\ell(\ell+1)} + \underbrace{\frac{1}{\Theta}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right)}_{-\ell(\ell+1)} = 0$$



Coordenadas esféricas

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$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial\phi}\hat{\phi}$$

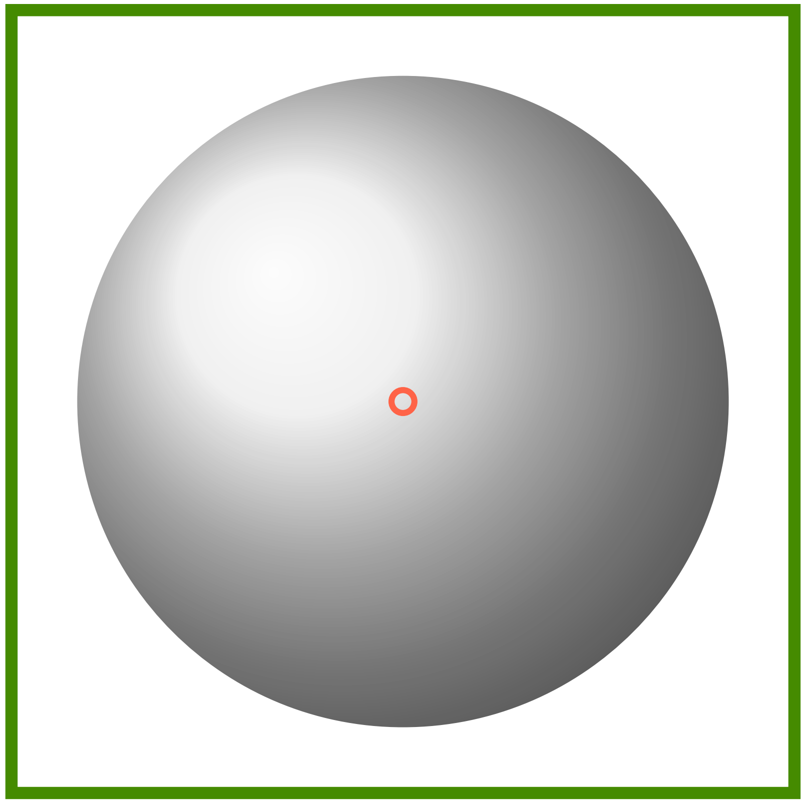
$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\, v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\, v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$



Simetria esférica

$$V(r,\theta,\phi) = R(r)\Theta(\theta)$$

$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \frac{1}{\Theta}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) = \ell(\ell+1)R$$ ⇒ PROCURAR SOLUÇÃO $R(r) = r^s$

$$\frac{d}{dr}\left(s\, r^{s+1}\right) = \ell(\ell+1)\, r^s$$

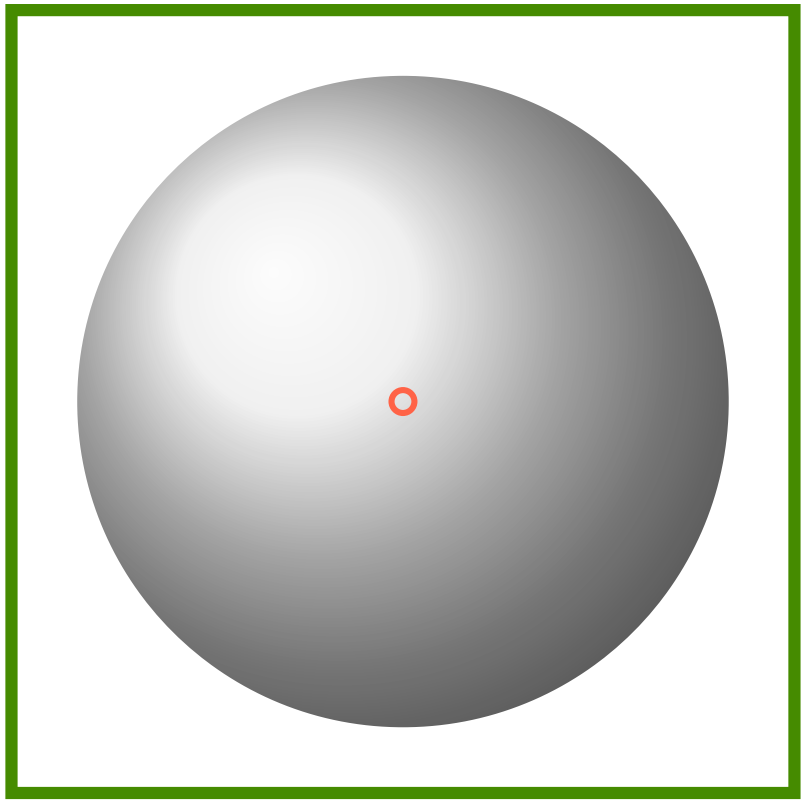
$$s(s+1)\, r^s = \ell(\ell+1)\, r^s$$

EQ. 2º GRAU PARA s

DUAS SOLUÇÕES: $s = \ell$, $s = -\ell-1$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$



Simetria esférica

$$V(r,\theta,\phi) = R(r)\Theta(\theta)$$

$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \frac{1}{\Theta}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) = \ell(\ell+1)R$$

$$\Rightarrow R(r) = Ar^{\ell} + \frac{B}{r^{\ell+1}}$$

$s = \ell$ $\quad$ $s = -\ell - 1$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

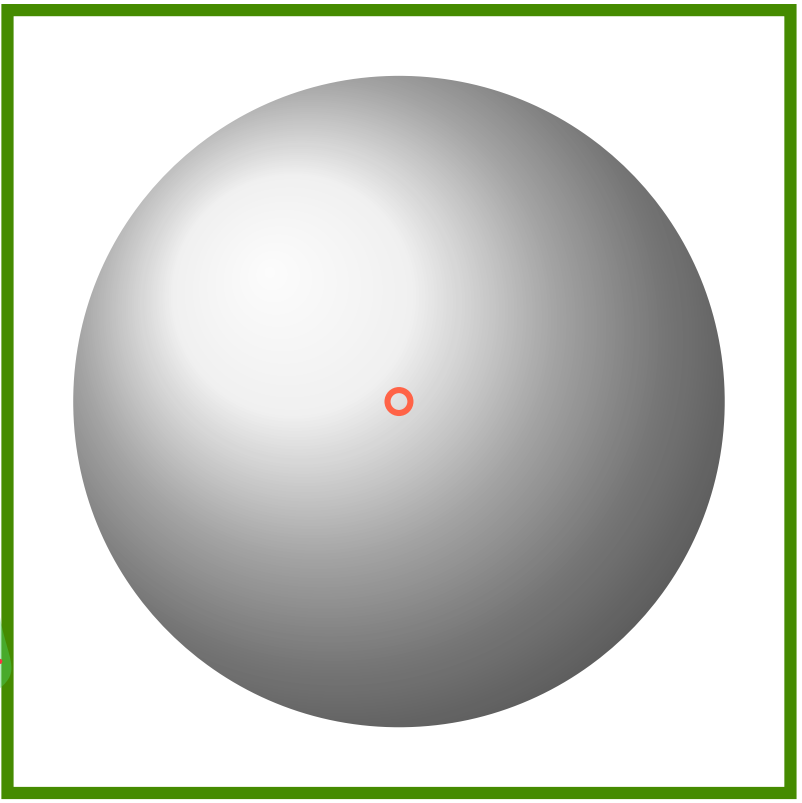
Simetria esférica

$$V(r,\theta,\phi) = R(r)\Theta(\theta)$$

$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \frac{1}{\Theta}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = -\ell(\ell+1)\Theta$$

TRABALHAR COM θ NÃO É BOM



Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

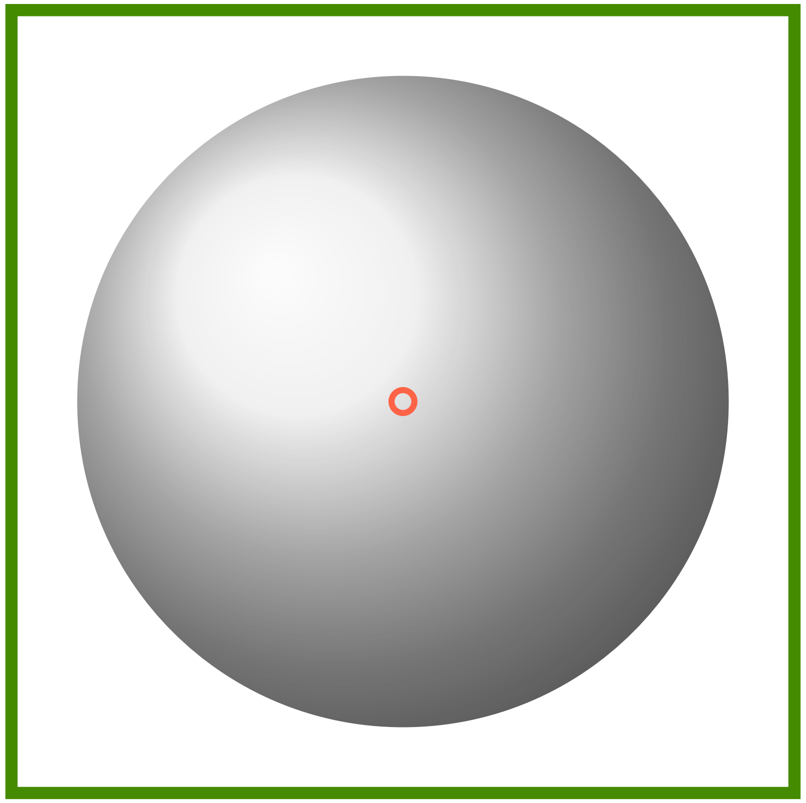
Simetria esférica

$$V(r,\theta,\phi) = R(r)\Theta(\theta)$$

$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \frac{1}{\Theta}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

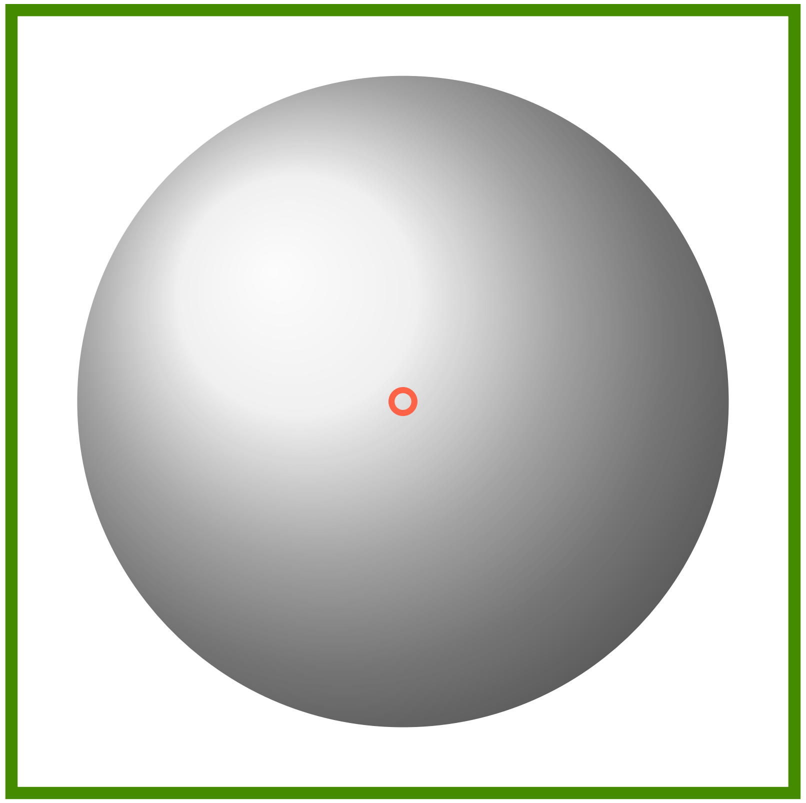
$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = -\ell(\ell+1)\Theta$$

$$u \equiv \cos\theta$$



Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$



Simetria esférica

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$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \frac{1}{\Theta}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = -\ell(\ell+1)\Theta$$

$u \equiv \cos\theta \qquad \Rightarrow \sin\theta\,\mathrm{d}\theta = -\,\mathrm{d}\cos\theta$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

Simetria esférica

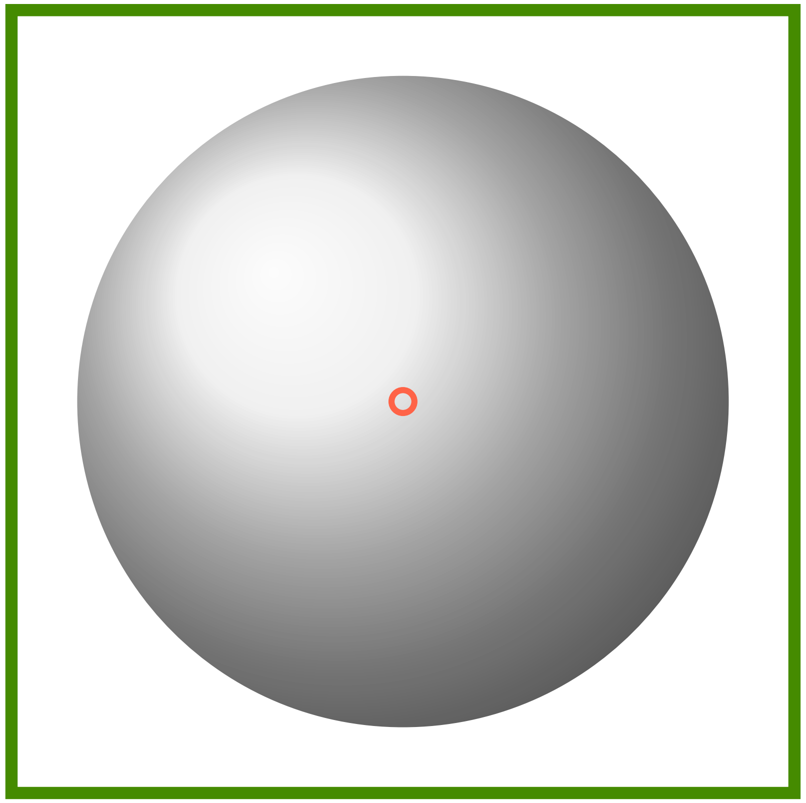
$$V(r,\theta,\phi) = R(r)\Theta(\theta)$$

$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \frac{1}{\Theta}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = -\ell(\ell+1)\Theta$$

$$u \equiv \cos\theta \qquad \Rightarrow \sin\theta\,\mathrm{d}\theta = -\,\mathrm{d}\cos\theta$$

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{\sin^2\theta}{\sin\theta}\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = -\ell(\ell+1)\Theta$$



$$\nabla^2 V = 0$$

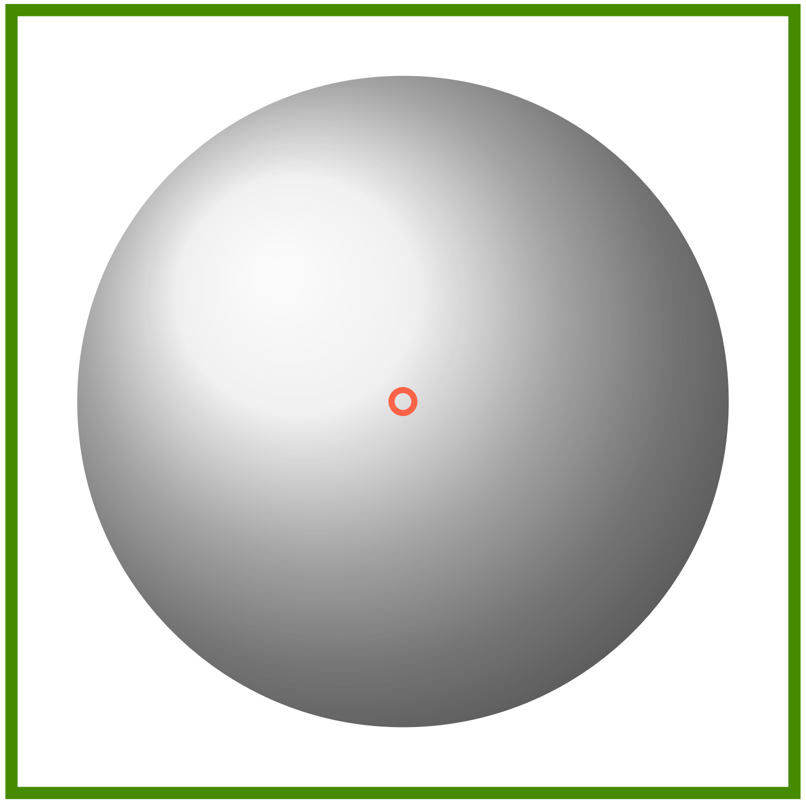
Equação de Laplace
Separação de variáveis

Simetria esférica

$$V(r,\theta,\phi) = R(r)\Theta(\theta)$$

$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \frac{1}{\Theta}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = -\ell(\ell+1)\Theta$$



MAIS SIMPLES!

$$u \equiv \cos\theta \qquad \Rightarrow \sin\theta\,\mathrm{d}\theta = -\,\mathrm{d}\cos\theta$$

$$\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{\sin^2\theta}{\sin\theta}\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = -\ell(\ell+1)\Theta \quad \Rightarrow \quad \frac{\mathrm{d}}{\mathrm{d}u}\left((1-u^2)\frac{\mathrm{d}\Theta}{\mathrm{d}u}\right) = -\ell(\ell+1)\Theta$$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

Simetria esférica

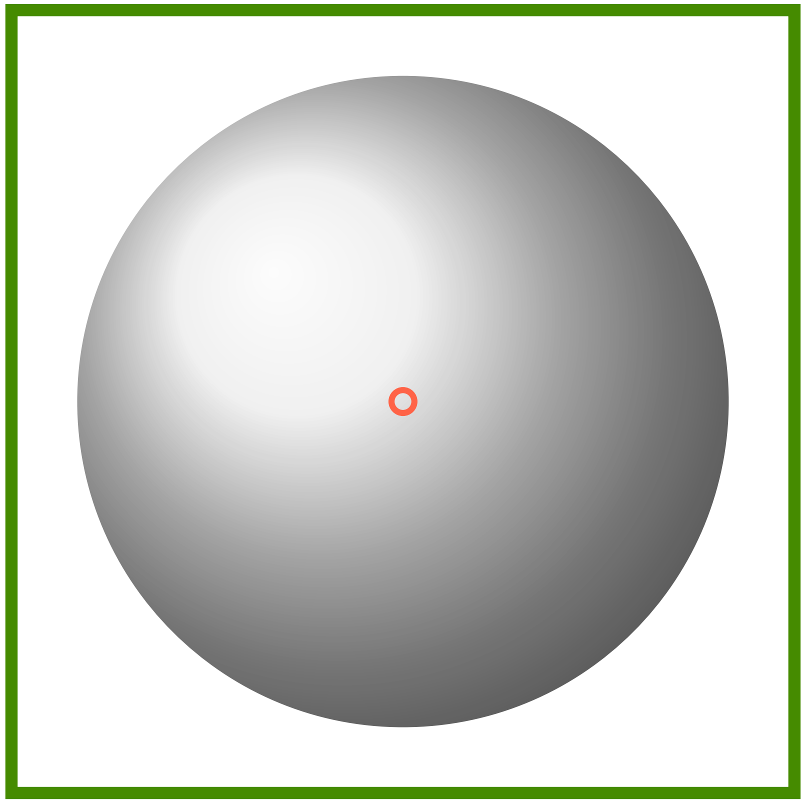
$$V(r,\theta,\phi) = R(r)\Theta(\theta)$$

$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \frac{1}{\Theta}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = -\ell(\ell+1)\sin\theta\,\Theta$$

$$\Theta(\theta) = P_\ell(u) \qquad (u = \cos\theta,\ \ell = 0, 1, \ldots)$$

$$\Rightarrow \frac{\mathrm{d}}{\mathrm{d}u}\left((1-u^2)\frac{\mathrm{d}\Theta}{\mathrm{d}u}\right) = -\ell(\ell+1)\Theta$$



$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = \frac{3x^2-1}{2}$$

$$P_3(x) = \frac{5x^3-3x}{2}$$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

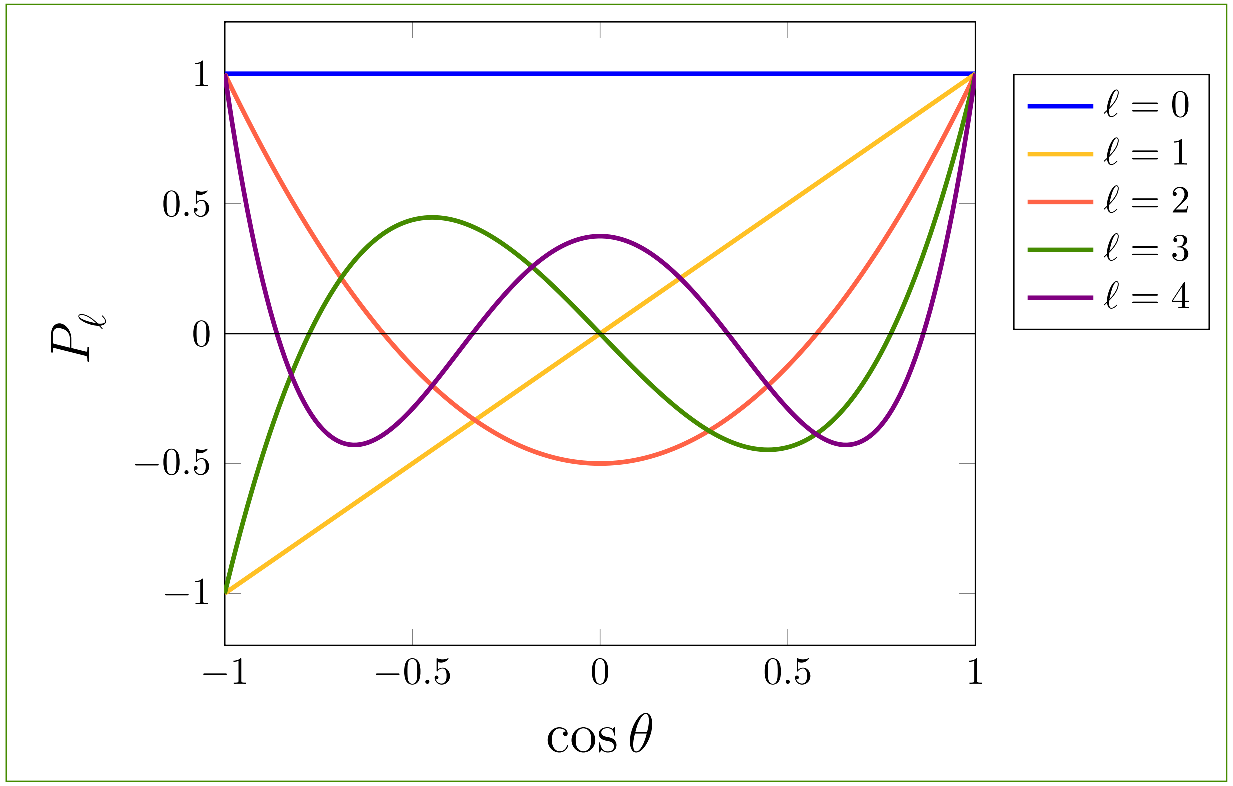
Simetria esférica

$$V(r,\theta,\phi) = R(r)\Theta(\theta)$$

$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \frac{1}{\Theta}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = -\ell(\ell+1)\sin\theta\,\Theta$$

$$\Theta(\theta) = P_\ell(u) \qquad (u = \cos\theta,\ \ell = 0, 1, \ldots)$$



$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = \frac{3x^2 - 1}{2}$$

$$P_3(x) = \frac{5x^3 - 3x}{2}$$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

Simetria esférica

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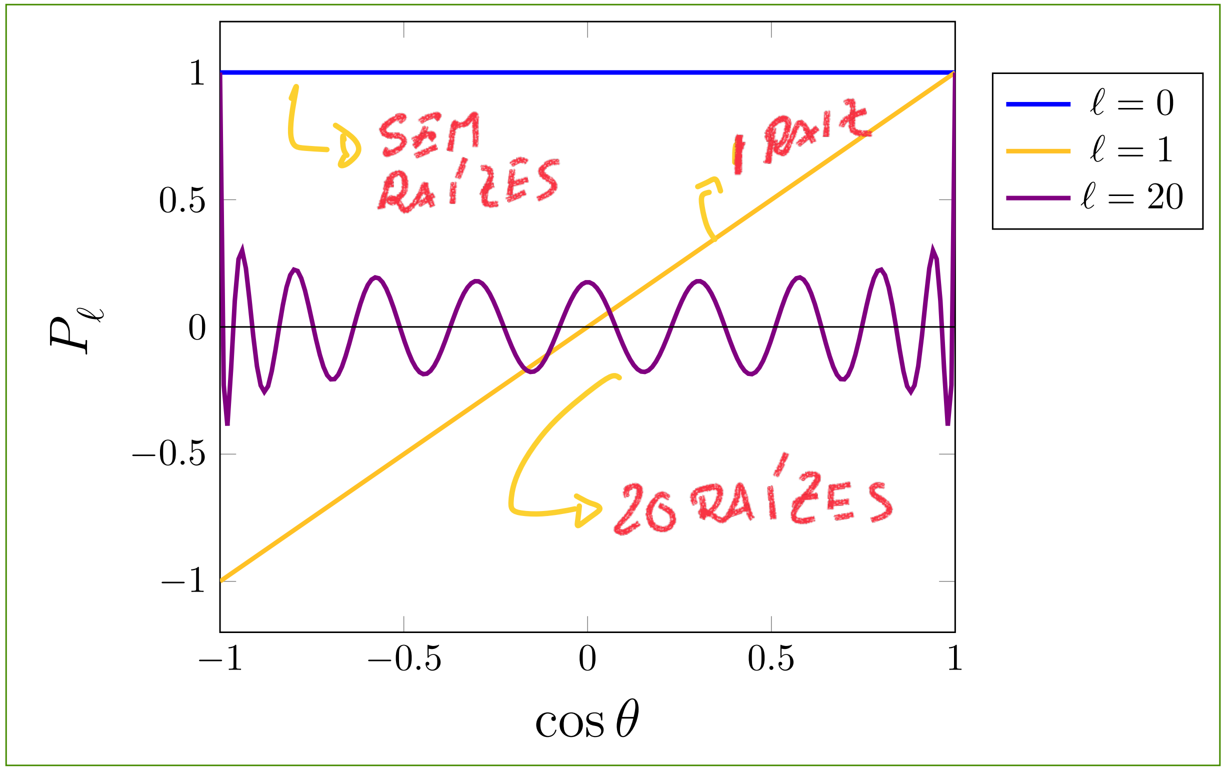

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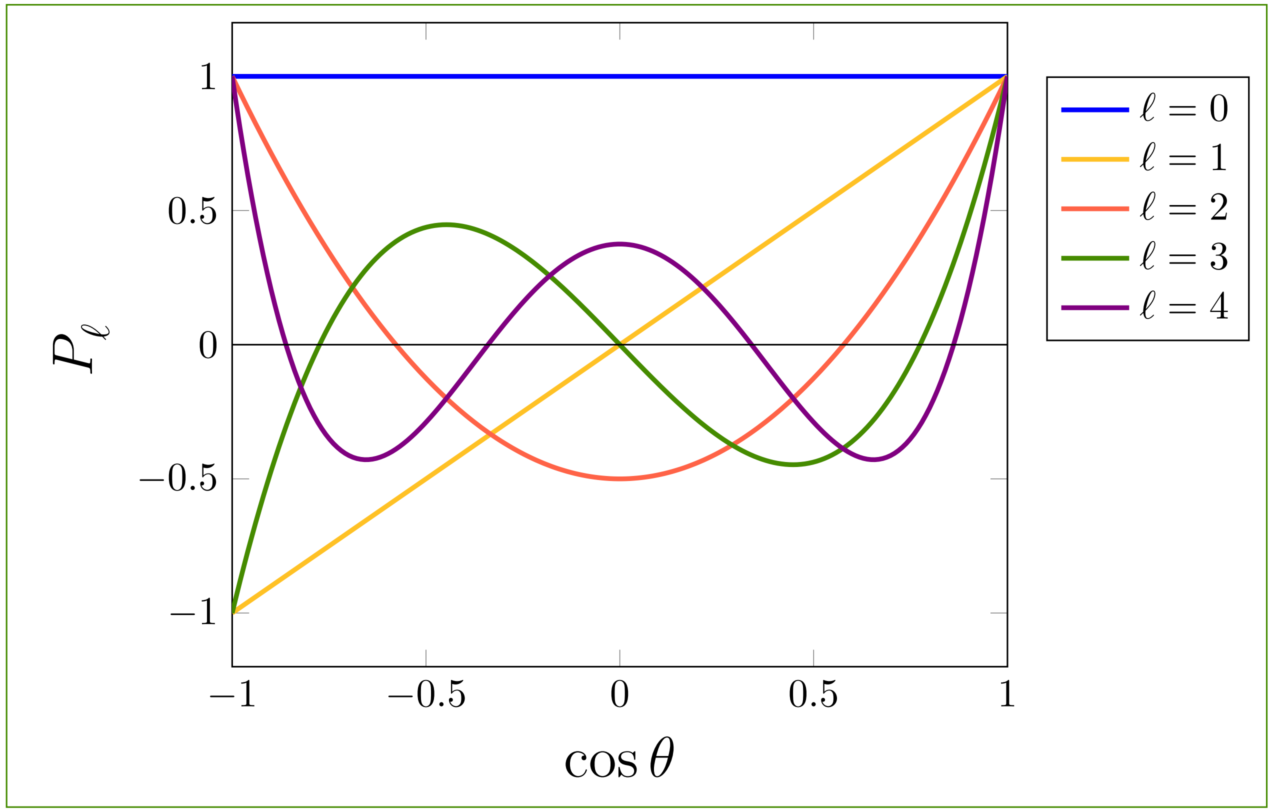


$$P_\ell(x) = \frac{1}{2^\ell \ell!}\left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^\ell (x^2-1)^\ell$$

FÓRMULA DE RODRIGUES

NÃO SÃO NORMALIZADAS

$$P_0(x) = 1$$

$$P_1(x) = x$$

$$P_2(x) = \frac{3x^2-1}{2}$$

$$P_3(x) = \frac{5x^3-3x}{2}$$

$P_\ell(1) = 1$

$$\int_{-1}^{1} P_\ell(x) P_{\ell'}(x)\, \mathrm{d}x = \frac{2}{2\ell+1}\delta_{\ell,\ell'}$$

ORTOGONAIS, COMO AS FUNÇÕES DE FOURIER

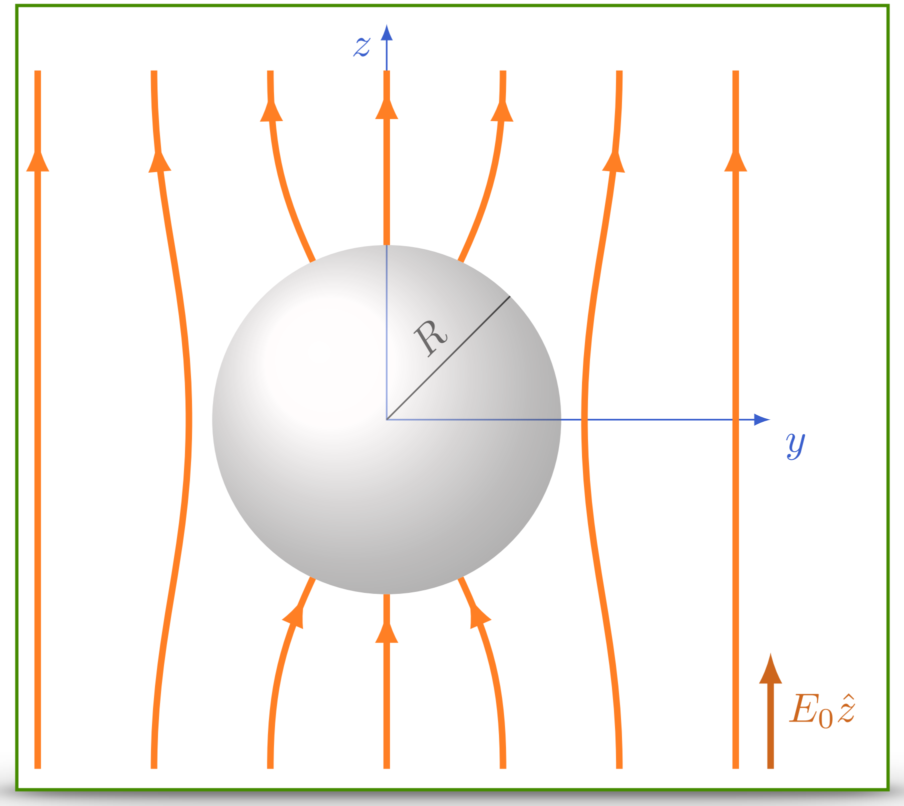
Pratique o que aprendeu

$$\nabla^2 V = 0$$

$\vec{E} = E_0 \hat{z}$ (LONGE DA ESFERA)

$$V(r,\theta) = -E_0 r\cos\theta \qquad (r \gg R)$$

$$V(R,\theta) = 0$$



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Pratique o que aprendeu

$$\nabla^2 V = 0$$

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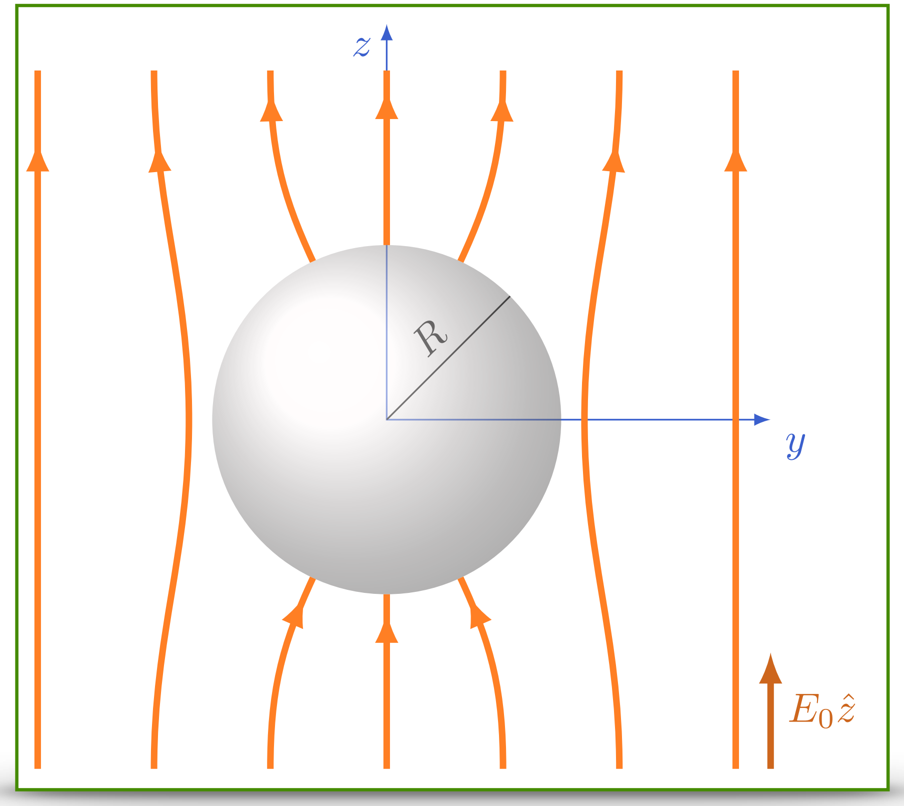
$$V(R,\theta) = 0$$

$$V(r,\theta) = R(r)\Theta(\theta)$$



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Pratique o que aprendeu

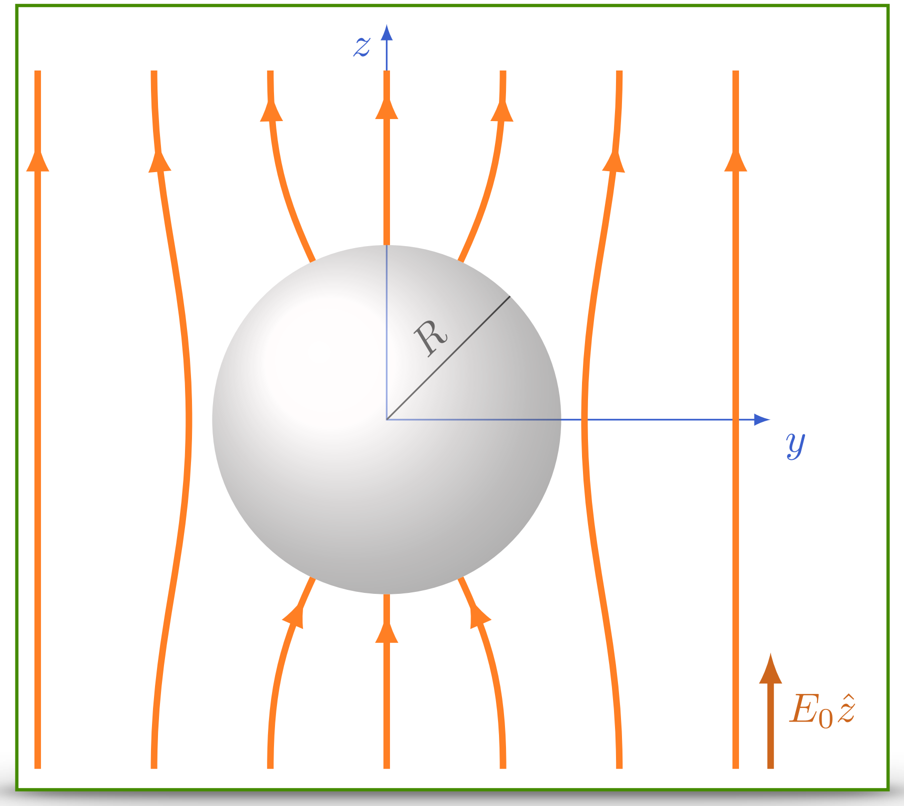
$$\nabla^2 V = 0$$

$$V(r,\theta) = -E_0 r\cos\theta \qquad (r \gg R)$$

$$V(R,\theta) = 0$$

$$V(r,\theta) = R(r)\Theta(\theta)$$

$$R_\ell(r) = Ar^\ell + \frac{B}{r^{\ell+1}}$$



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Pratique o que aprendeu

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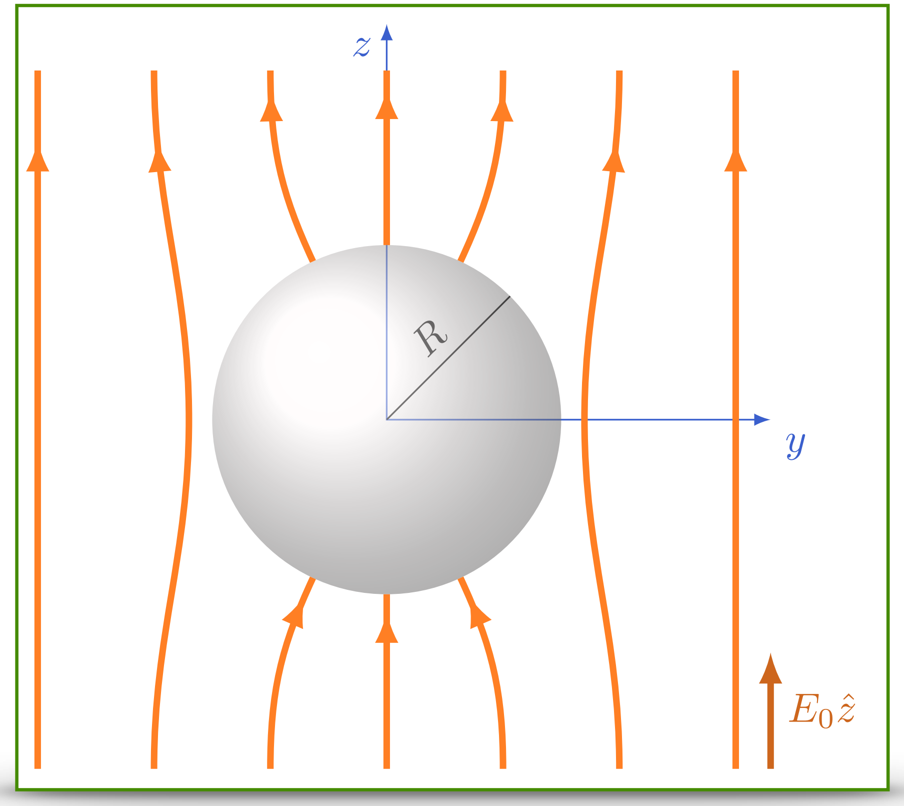
$$R_\ell(R) = 0 \quad \Rightarrow \quad AR^\ell = -\frac{B}{R^{\ell+1}}$$



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$$\nabla^2 V = 0$$

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$$V(R,\theta) = 0$$

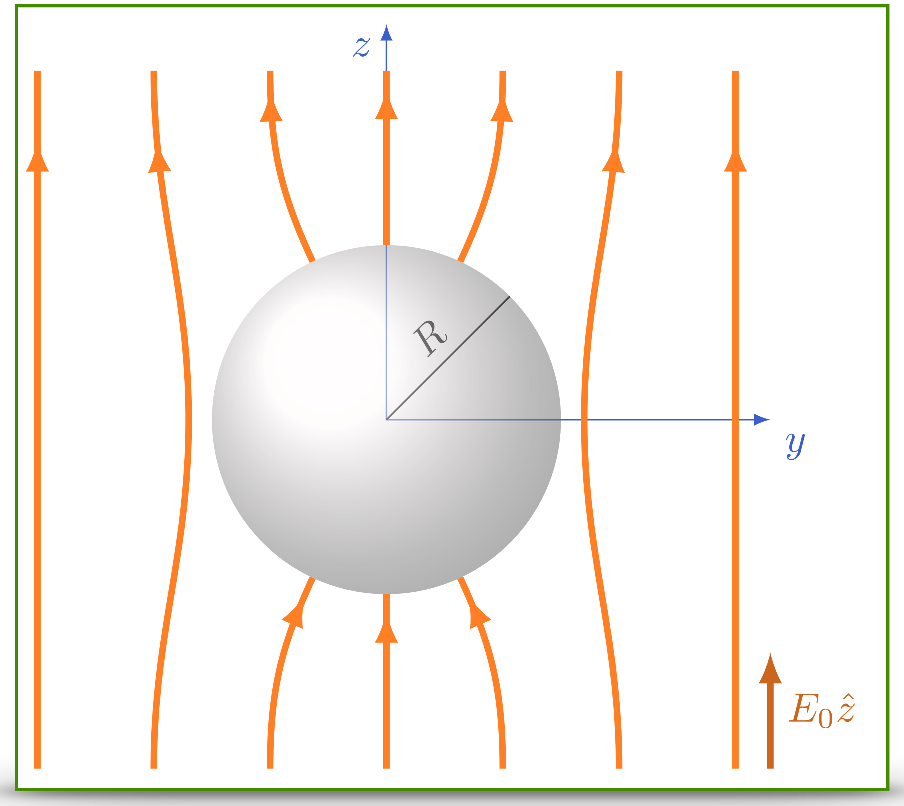
$$V(r,\theta) = R(r)\Theta(\theta)$$

$$R_\ell(r) = Ar^\ell + \frac{B}{r^{\ell+1}}$$

$$R_\ell(R) = 0 \quad \Rightarrow \quad AR^\ell = -\frac{B}{R^{\ell+1}}$$

$$R_\ell(r) = Ar^\ell \left[1 - \left(\frac{R}{r}\right)^{2\ell+1}\right]$$

0 ∀ℓ $r = R$



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Pratique o que aprendeu

$$\nabla^2 V = 0$$

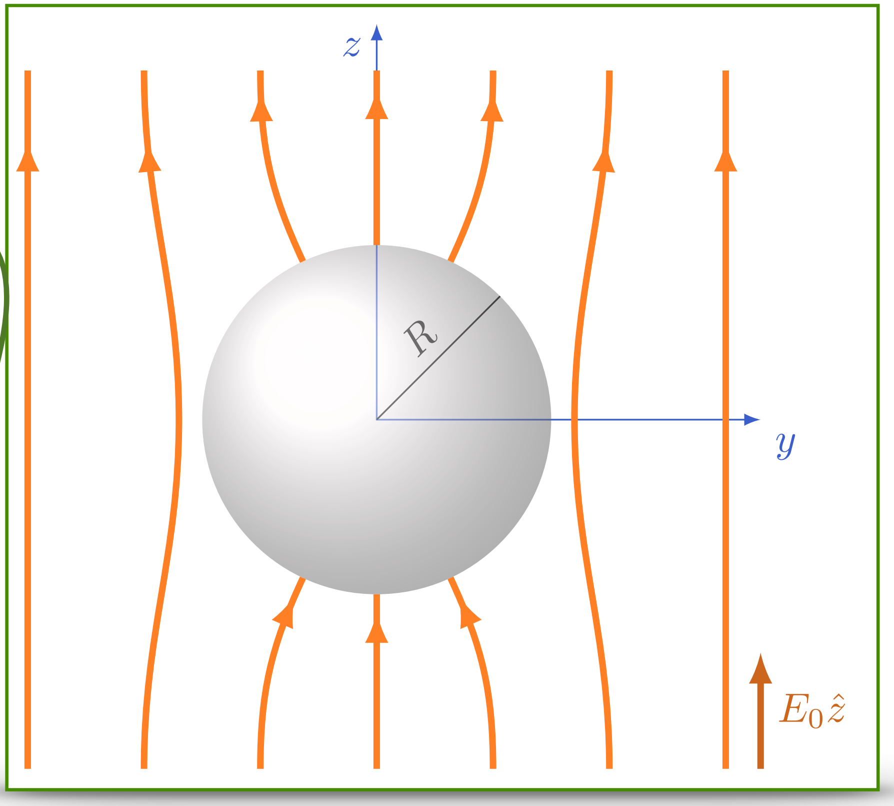
$$V(r,\theta) = -E_0 r\cos\theta \qquad (r \gg R)$$

$$V(R,\theta) = 0$$

$$R_\ell = A r^\ell\left(1 - \left(\frac{R}{r}\right)^{2\ell+1}\right)$$

$$V(r,\theta) = R(r)\Theta(\theta)$$

$$\Theta_\ell(\theta) = P_\ell(\cos\theta)$$



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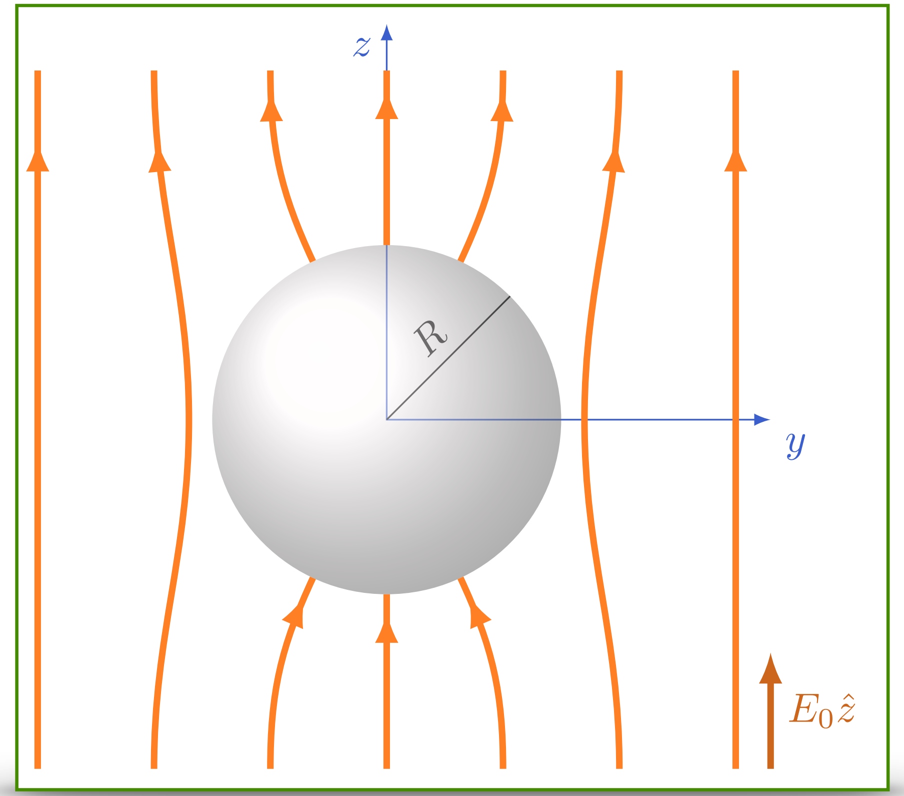
$$\Theta_\ell(\theta) = P_\ell(\cos\theta) \quad \Rightarrow \quad \ell = 1$$

(SATISFAZ A COND. CONTORNO)



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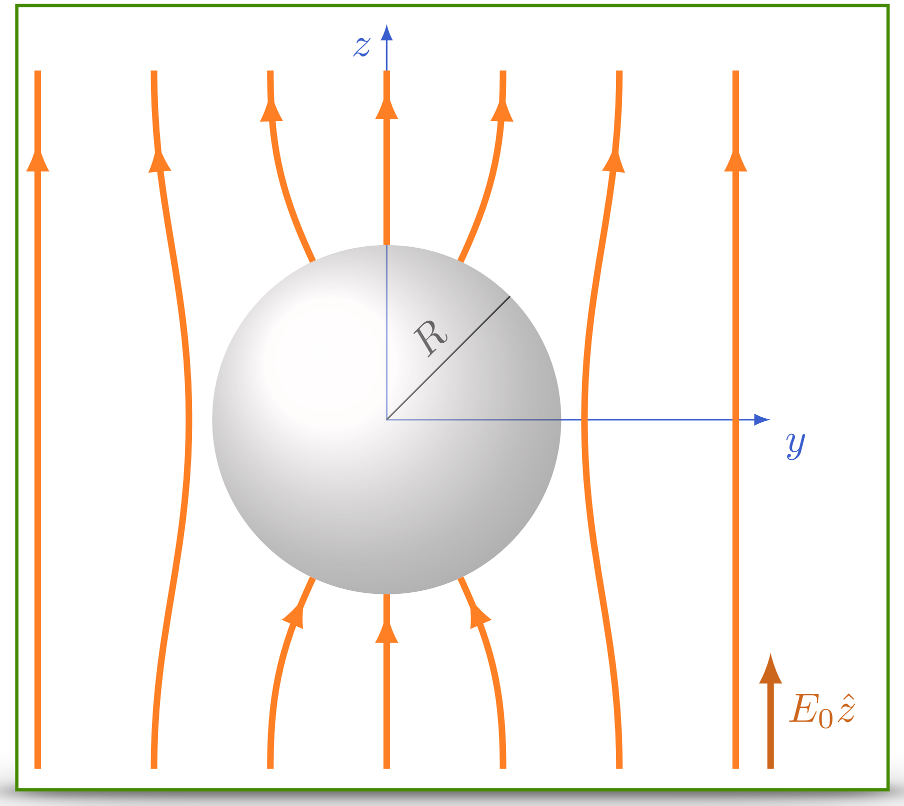

$$V(r,\theta) = R(r)\Theta(\theta)$$

$$\Theta_\ell(\theta) = P_\ell(\cos\theta) \quad \Rightarrow \quad \ell = 1$$

$$V(r,\theta) = AR_1(r)P_1(\cos\theta)$$

$$R_\ell(r) = Ar^\ell\left[1 - \left(\frac{R}{r}\right)^{2\ell+1}\right]$$

Pratique o que aprendeu

$$\nabla^2 V = 0$$

$$V(r,\theta) = -E_0 r\cos\theta \qquad (r \gg R)$$

$$V(R,\theta) = 0$$

$$V(r,\theta) = R(r)\Theta(\theta)$$

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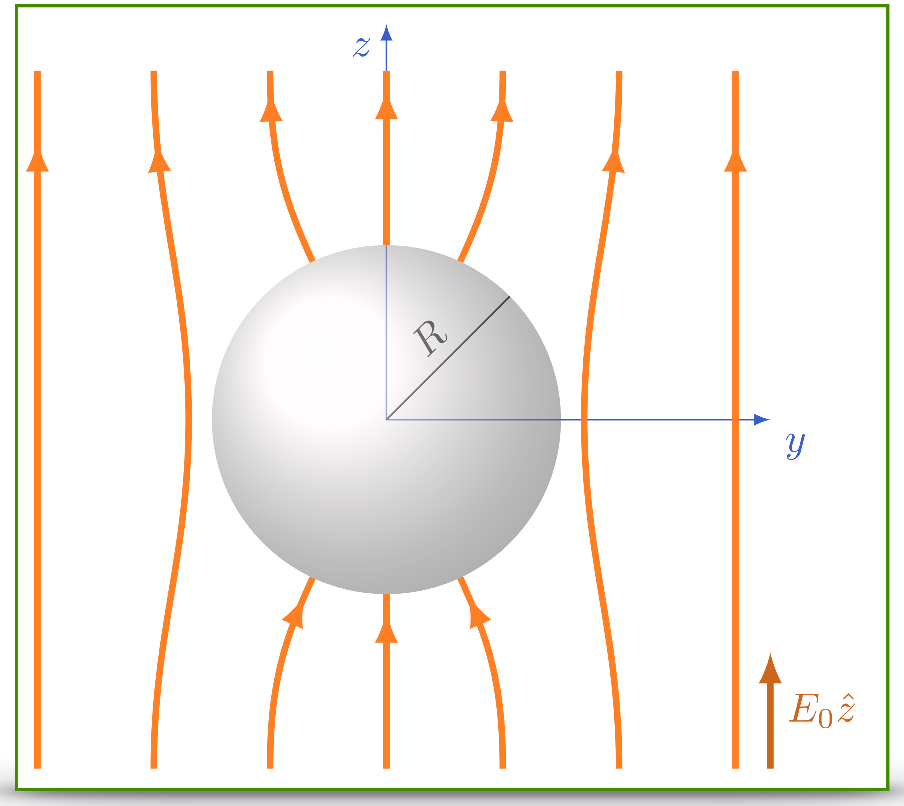
$$V(r,\theta) = AR_1(r)P_1(\cos\theta)$$

$$R_\ell(r) = Ar^\ell\left[1 - \left(\frac{R}{r}\right)^{2\ell+1}\right]$$

$$V(r,\theta) = -E_0 r\left[1 - \left(\frac{R}{r}\right)^3\right]\cos\theta$$

VAI PARA O ZERO $r \to \infty$



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Pratique o que aprendeu

$$V(r,\theta) = -E_0 r\cos\theta \qquad (r \gg R)$$

$$V(R,\theta) = 0$$

$$V(r,\theta) = R(r)\Theta(\theta)$$

$$\Theta_\ell(\theta) = P_\ell(\cos\theta) \quad \Rightarrow \quad \ell = 1$$

$$V(r,\theta) = AR_1(r)P_1(\cos\theta)$$

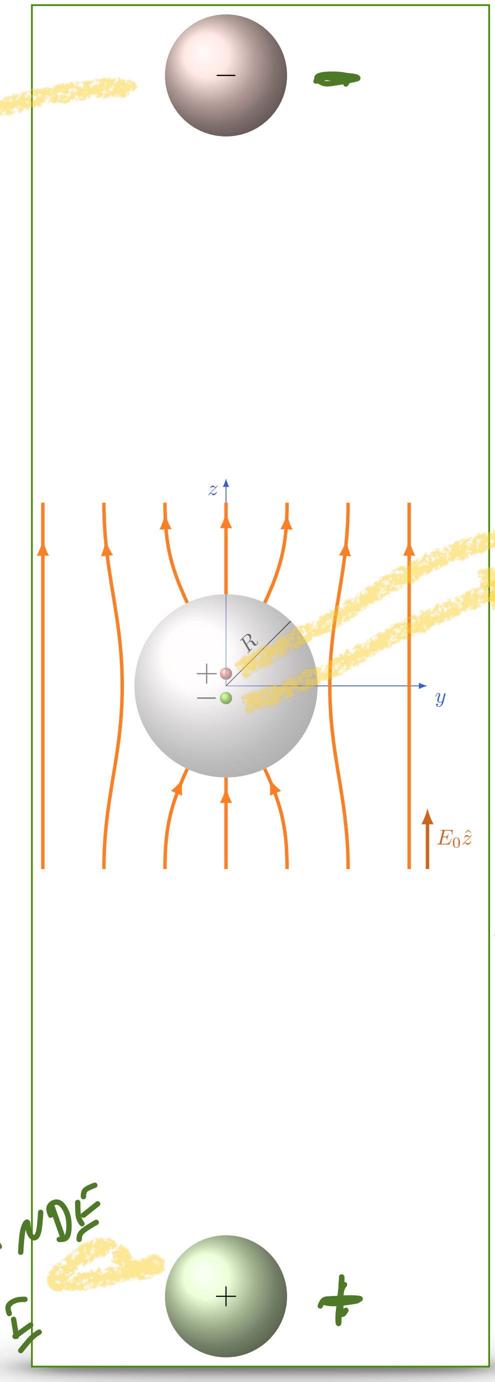
CAMPO DAS CARGAS IMAGENS

$$V(r,\theta) = -E_0 r\left[1 - \left(\frac{R}{r}\right)^3\right]\cos\theta$$

CAMPO DAS CARGAS DISTANTES

INTERPRETAÇÃO FÍSICA

CARGA MUITO GRANDE E DISTANTE



DUAS CARGAS IMAGENS PEQUENAS E PRÓXIMAS

CARGA MUITO GRANDE E DISTANTE