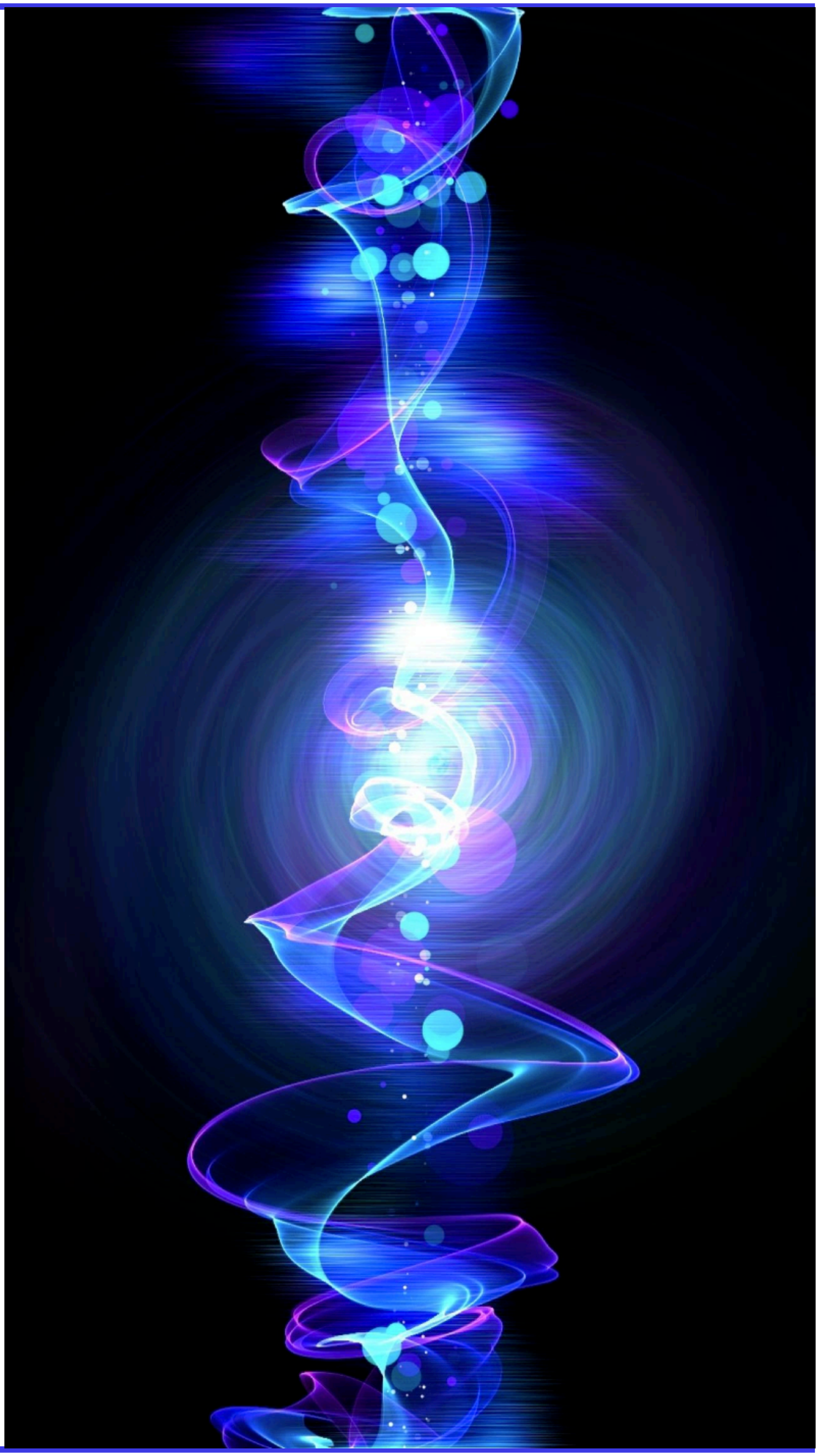
Topics in radiation

⚡ What is "radiation"
⚡ The closed lines of radiation fields
⚡ Larmor formula
⚡ The angular momentum of radiation



What <u>is</u> radiation?

- The idea behind radiation is that it corresponds to the ***flow of something*** (energy, momentum, etc.) that is carried along some direction, but is not "attached" to its source.

- In EM, the power that crosses some area $d\vec{A} = r^2 d^2\Omega\,\hat{r}$ is determined by the Poynting vector:

$$dP = \langle \vec{S} \cdot d\vec{A} \rangle_t = \epsilon_0 \left[\langle \vec{E} \times \vec{B} \rangle_t \cdot \hat{r} \right] r^2\, d^2\Omega$$

- We can expand the power per solid angle as a series over the distance r from the source:

$$\lim_{r\to\infty} \frac{dP}{d^2\Omega} = P_0(\theta,\varphi) + \frac{1}{r} P_1(\theta,\varphi) + \ldots$$

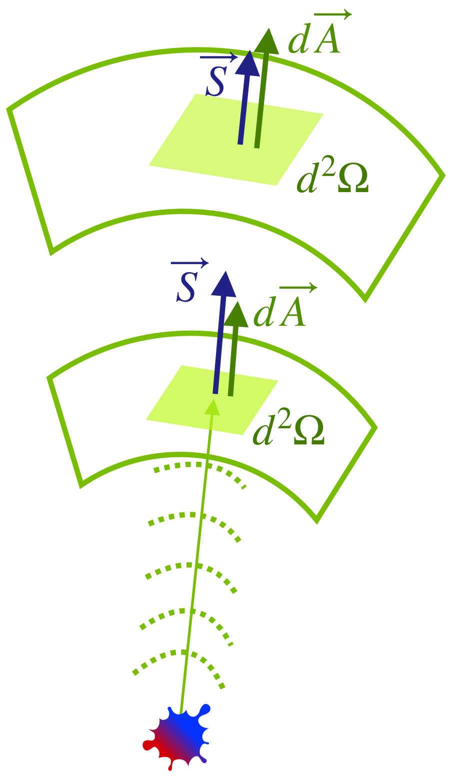
- For "true" radiation, the term P_0 is non-zero: there is a part of the fields which are "detached" from the source, and which are able to carry that energy and momentum out to infinity.

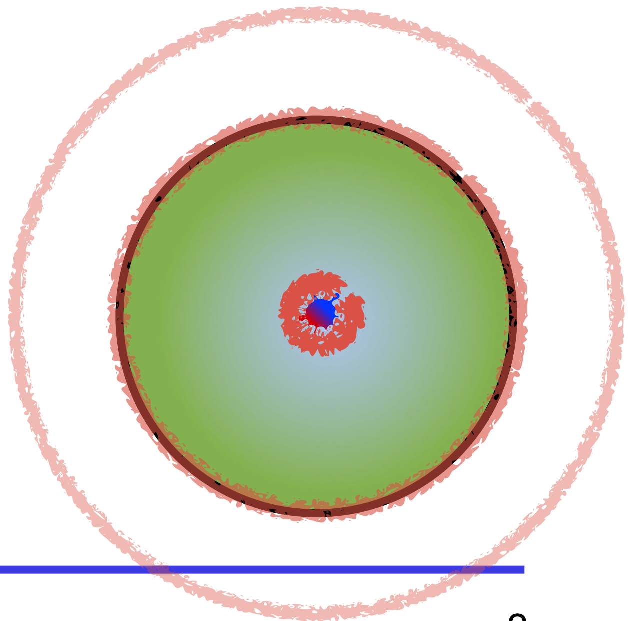
- Always remember that there is "price" to pay for the ejected energy and momentum: the work done on (or by) the system that generated those fields. In order to see that, remember the Poynting theorem:

$$\frac{\partial \rho_{EM}}{\partial t} + \vec{\nabla} \cdot \vec{S} + \vec{J} \cdot \vec{E} = 0 \quad ,$$ and Integrating that over any volume we obtain, using the Divergence theorem:

$$\frac{\partial U_{EM}}{\partial t} + \oint d\vec{A} \cdot \vec{S} = - \int d^3x\, \vec{J} \cdot \vec{E}$$

- Now, consider a surface far away from the source, and a disturbance that lasts for a short time Δt.
  - ➡ While the disturbance does its work on the charges/currents, the energy is changing, but there is no energy transfer through that surface (the fields could not yet reach that far away surface).
  - ➡ Later, there is no more work being done, but the Poynting vector takes that energy away from the volume.
  - ➡ Finally, there is neither work being done, nor radiation crossing the surface, and the energy is constant again — but with a lower value.





Larmor's formula

- An interesting expressions for the emitted power from a radiating source is ***Larmor's formula***. In Lecture 15 we came across a result used for the Jefimenko expressions:

$$\vec{E}(t,\vec{x}) \;=\; 0 + \frac{1}{4\pi\epsilon_0}\int d^3x' \left[\frac{\Delta\hat{x}}{\Delta x^2}\rho(t'=t_{Ret}) + \frac{\Delta\hat{x}}{c\,\Delta x}\left[\frac{\partial\rho}{\partial t'}\right]_{t'=t_{Ret}} - \frac{1}{c^2}\frac{1}{\Delta x}\frac{\partial\vec{J}}{\partial t'}\bigg|_{t'=t_{Ret}} \right]$$

$$\vec{B}(t,\vec{x}) \;=\; \frac{\mu_0}{4\pi}\int d^3x' \left[\vec{J}(t'=t_{Ret})\times\frac{\Delta\hat{x}}{\Delta x^2} - \frac{\Delta\hat{x}}{c\,\Delta x}\times\left(\frac{\partial\vec{J}}{\partial t'}\right)_{t'=t_{Ret}} \right]$$

- If we neglect the terms $\sim 1/\Delta x^2$ (in other words, if we assume that we are in the radiation zone), then $\Delta x \to r$, $\Delta\hat{x} \to \hat{r}$ and we get:

$$\vec{E}_{rad} \;\simeq\; \frac{1}{4\pi\epsilon_0\,c\,r}\int d^3x' \left[\hat{r}\left(\frac{\partial\rho}{\partial t'}\right)_{t'=t_{Ret}} - \frac{1}{c}\left(\frac{\partial\vec{J}}{\partial t'}\right)_{t'=t_{Ret}} \right]$$

$$\vec{B}_{rad} \;\simeq -\frac{\mu_0}{4\pi\,c}\frac{\hat{r}}{r}\times\int d^3x' \left(\frac{\partial\vec{J}}{\partial t'}\right)_{t'=t_{Ret}}$$

- Using the continuity equation and the identity $\vec{A}\times(\vec{A}\times\vec{B}) = \vec{A}(\vec{A}\cdot\vec{B}) - \vec{B}(\vec{A}\cdot\vec{A})$, we can derive another useful expression for the electric field:

$$\vec{E}_{rad} \;\simeq\; \frac{\mu_0}{4\pi}\frac{\hat{r}}{r}\times\left[\hat{r}\times\int d^3x'\,\frac{\partial\vec{J}(\vec{x}',t'=t-r/c)}{\partial t'}\right]$$

Larmor's formula

- Consider a point charge moving along some time-dependent trajectory $\vec{x}_q(t)$, velocity $\vec{v}_q = \dot{\vec{x}}_q$, and acceleration $\vec{a}_q = \dot{\vec{v}}_q$. Furthermore, let's assume that the particle's position is always close to the origin, $|\vec{x}_q| \sim 0$.
- This particle has a current density:

$$\vec{J} = q\,\vec{v}_q\,\delta[\vec{x} - \vec{x}_q(t)]$$

- Substituting this into the expression we just found for the electric field and neglecting terms $\sim v_q/c$ we get:

$$\vec{E}_{rad} \simeq \frac{\mu_0\, q}{4\pi\, r}\,\hat{r} \times \left[\hat{r} \times \vec{a}_q(t_{Ret})\right] \simeq \frac{\mu_0\, q}{4\pi\, r}\,\vec{a}_\perp(t_{Ret}) \quad ,$$

where $\vec{a}_\perp$ is the component of the acceleration perpendicular to the direction $\hat{r}$.

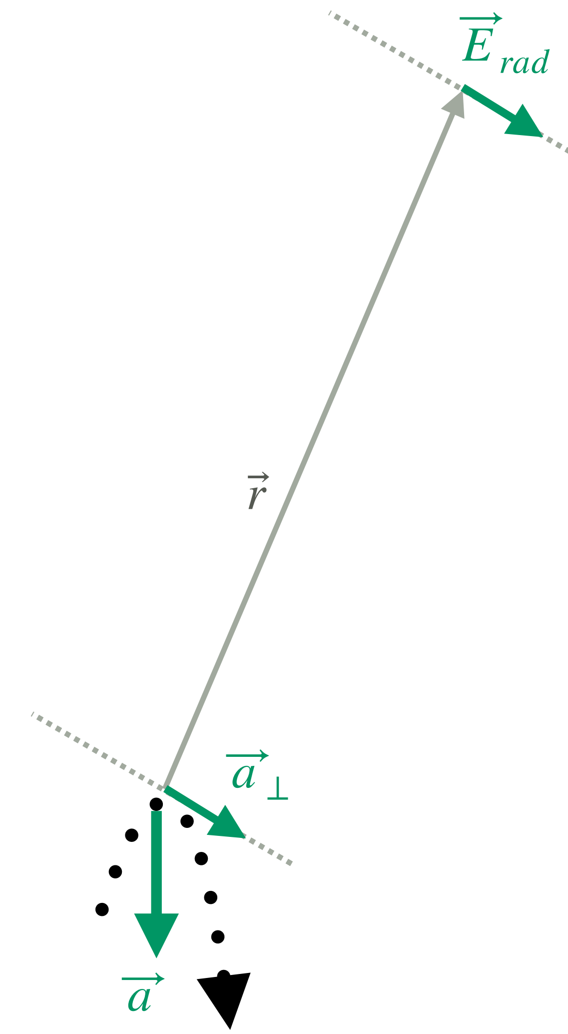
The magnetic field is perpendicular to both $\vec{E}_{rad}$ and $\hat{r}$: $\vec{B}_{rad} = \hat{r} \times \vec{E}/c$.

- This means that the ***power*** radiated by this moving particle is given by:

$$\frac{dP}{d\Omega} = \frac{r^2}{\mu_0 c}\,|\vec{E}_{rad}|^2 = \frac{r^2}{\mu_0 c}\left[\left(\frac{\mu_0 q}{4\pi\, r}\right)^2 \vec{a}^{\,2}(t_{Ret})\,\sin^2\theta\right]$$

$$= \frac{\mu_0\, q^2}{16\pi^2 c}\,\vec{a}^{\,2}(t_{Ret})\,\sin^2\theta$$

This is basically Larmor's formula: it tells us that the radiation from moving particles is directly given by the acceleration — at the retarded times!



Radiation and field line reconnections

- One of the implications from the discussion above is that the fields become "detached" from the sources. We can see that in terms of the field lines of radiative fields ***closing in on themselves*** at some point in time.
- To see that, consider the dipole field. Let's take the vector potential we found in an earlier class (Lecture 15):

$$\vec{A}_\omega \to -\frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d^3x'\,\vec{x}'\,\vec{\nabla}'\cdot\vec{J}_\omega(\vec{x}') \;=\; \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int d^3x'\,\vec{x}'\,\frac{\partial\rho_\omega(\vec{x}')}{\partial t}$$

$$\Rightarrow \quad \vec{A}(t,\vec{x}) \;=\; \frac{\mu_0}{4\pi}(-i\omega)\frac{e^{i(kr-\omega t)}}{r}\,\vec{p}_0$$

- Now, we can use the Ampère law to write the Electric field as:

$$\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} = \vec{\nabla}\times\vec{B} = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \vec{\nabla}^2\vec{A}$$

- Now, suppose that the dipole is oriented in the z direction, so $\vec{A} = A_z\hat{z}$ and we have:

$$\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} = \vec{\nabla}(\partial_z A_z) - \hat{z}\,\vec{\nabla}^2 A_z = \frac{\mu_0\,p_0}{4\pi}(-i\omega)\left[\vec{\nabla}\left(\partial_z\frac{e^{i(kr-\omega t)}}{r}\right) - \hat{z}\,\vec{\nabla}^2\left(\frac{e^{i(kr-\omega t)}}{r}\right)\right]$$

- It is clear that the time derivative of $\partial\vec{E}/\partial t = (-i\omega)\vec{E}$. Now, due to axial symmetry, it is convenient to use cylindrical coordinates. We obtain:

$$\vec{E} = \frac{\mu_0\,c^2\,p_0}{4\pi}\left[\hat{\rho}\,\partial_\rho\partial_z\left(\frac{e^{i(kr-\omega t)}}{r}\right) - \hat{z}\,\frac{1}{\rho}\partial_\rho\left(\rho\,\partial_\rho\frac{e^{i(kr-\omega t)}}{r}\right)\right]$$

- This means that we can write the field in terms of the following expression:

$$\vec{E} = \frac{p_0}{4\pi\epsilon_0}\,\frac{1}{\rho}\left(\hat{\rho}\,\partial_z f - \hat{z}\,\partial_\rho f\right) \quad , \quad \text{where} \;\; f = \rho\,\partial_\rho\left(\frac{e^{i(kr-\omega t)}}{r}\right)$$

Radiation and field line reconnections

- Let's write this result again here:

$$\vec{E} = \frac{p_0}{4\pi\epsilon_0}\frac{1}{\rho}\left(\hat{\rho}\,\partial_z f - \hat{z}\,\partial_\rho f\right) \quad , \quad f = \rho\partial_\rho\left(\frac{e^{i(kr-\omega t)}}{r}\right) \to \rho\partial_\rho\left[\frac{\cos(kr-\omega t)}{r}\right]$$

- Now, notice that $\vec{\nabla} f = \hat{\rho}\partial_\rho f + \hat{z}\partial_z f$, so we get that

$$\vec{E} = \frac{p_0}{4\pi\epsilon_0}\frac{1}{\rho}\left(\hat{\varphi}\times\vec{\nabla} f\right) \quad , \quad \text{hence} \quad \vec{E}\cdot\vec{\nabla} f = 0$$

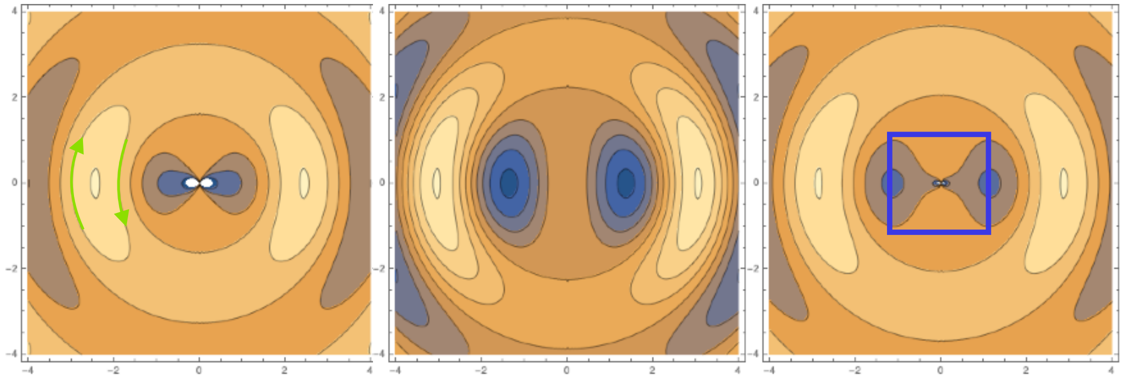
- This means that the field is always ***tangential*** to the surfaces of equal values of f, and the norm of the two are basically the same:

$$|\,\vec{E}\,|^2 = \left(\frac{\mu_0\, p_0}{4\pi}\right)^2 \frac{1}{\rho^2}\,|\,\vec{\nabla} f\,|^2$$

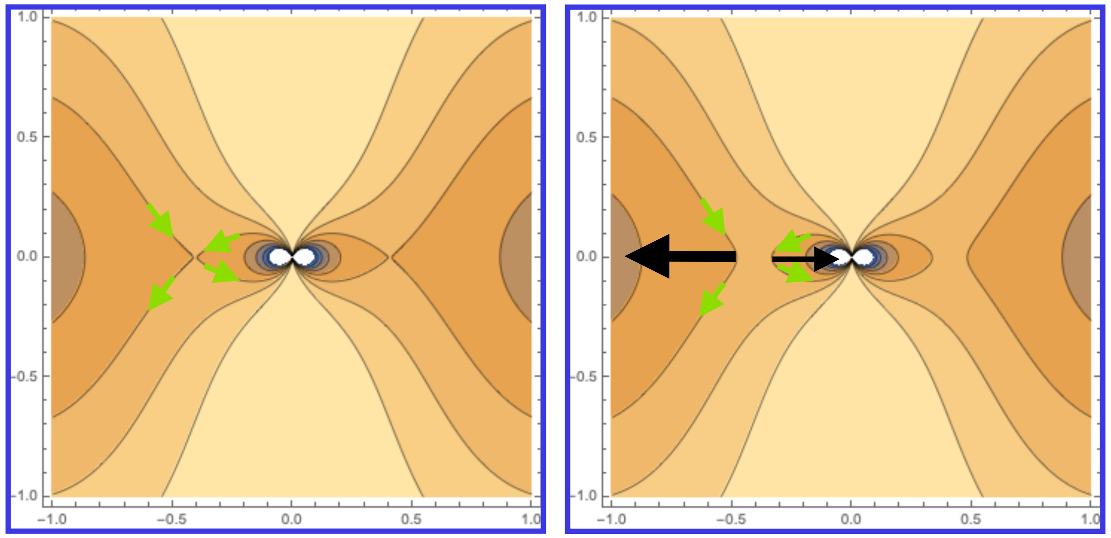
- What is interesting about this expression for the electric field is that we can regard the surfaces of constant f as the marking the places of equal field strength.

Radiation and field line reconnections

- So, the surfaces of constant f as the marking the places of equal $|\vec{E}|$. But since that function has also a time dependence, the lines of constant f are "dynamic", they change with time — see the figures on the right, where the dipole is in the center.



- Moreover, the lines of equal values of f depend on the phase , $kr - \omega t$, so they "move" around, and every so often they "pinch off" at some place, as shown in this last figure. This means that the field lines at the "pinch" are connecting and separating: the field lines separate into lines that connect with the source, and lines that are detached from the source! Now, ***that's*** radiation!



The EM force, energy and momentum

- We saw in an earlier class that the stress-energy tensor for EM is given by:

$$T^{\alpha}{}_{\mu} = -\frac{1}{\mu_0}\left[F^{\alpha\nu}F_{\nu\mu} + \frac{1}{4}\,\delta^{\alpha}_{\mu}\,F^2\right] = \frac{1}{\mu_0}\left[F^{\alpha\nu}F_{\mu\nu} - \frac{1}{4}\,\delta^{\alpha}_{\mu}\,F^2\right]$$

- In particular, the space-space part of that tensor is given by:

$$T^{i}{}_{j} = \frac{1}{\mu_0}\left[\frac{1}{c^2}E^iE^j + B^iB^j - \frac{1}{2}\delta_{ij}\left(\frac{\vec{E}^2}{c^2} + \vec{B}^2\right)\right]$$

- Conservation of the electromagnetic stress-energy tensor means that:

$$\partial_\alpha T^{\alpha\mu} + f^\mu = 0$$

- Consider first the component $\mu = 0$. Using Maxwell's equations we obtain:

$$\partial_\alpha T^{0\alpha} + f^0 = 0 \quad \Rightarrow \quad \frac{\partial \rho_{EM}}{\partial t} + \vec{\nabla}\cdot\vec{S} + \vec{J}\cdot\vec{E} = 0 \quad ,$$

which describes the rate of change of energy in terms of the flux of momentum ($\vec{\nabla}\cdot\vec{S}$) and the work done to the system ($\vec{J}\cdot\vec{E}$).

- The spatial component, on the other hand, again takes a bit more algebra, but you can show that it yields:

$$\partial_\alpha T^{i\alpha} + f^i = 0 \quad \Rightarrow \quad \frac{1}{c^2}\frac{\partial S^i}{\partial t} - \partial_j T^{ij} + f^i = 0 \quad , \quad \text{where} \quad \vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \quad \text{is the Poynting vector .}$$

where $\vec{f}$ is the Lorentz force. This last equation describes momentum flux (since the density of momentum is $\vec{p} = \vec{S}/c^2$), including the force that is exerted on the system ($\vec{f} = \rho\vec{E} + \vec{J}\times\vec{B}$). Another way to write this equation is in terms of the ***total force and momentum*** of the particles in the system:

$$F^i = \frac{dP^i}{dt} = \int d^3x\left(-\frac{1}{c^2}\frac{\partial S^i}{\partial t} + \partial_j T^{ij}\right) \quad ,$$

and if you work out this expression using Maxwell's equations you obtain:

$$\vec{F} = \int d^3x\left(\rho\vec{E} + \vec{J}\times\vec{B}\right)$$

The EM force, energy and momentum

- But let's go back to our previous expression:

$$F^i = \frac{dP^i}{dt} = \int d^3x \left(-\frac{1}{c^2}\frac{\partial S^i}{\partial t} + \partial_j T^{ij} \right)$$

- Notice that we could have written:

$$\frac{d}{dt}\left(P^i + \int d^3x \frac{1}{c^2} S^i \right) = \int d^3x\, \partial_j T^{ij}$$

and since the total momentum is the integral over the volume of the momentum density (p^i), we have:

$$\int d^3x \frac{d}{dt}\left(p^i + \frac{1}{c^2} S^i \right) = \int d^3x\, \partial_j T^{ij}$$

- Therefore, the two terms above are the momentum density in the particles and in the EM field. We can now use the Divergence theorem to turn the integral of the right-hand-side into a surface integral, leading to:

$$\frac{d}{dt}\left(P^i + P^i_{EM}\right) = \int d^2S_j\, T^{ij}$$, where the EM momentum density is $\vec{p}_{EM} = \vec{S}/c^2$, and $P^i_{EM} = \int d^3x\, p^i_{EM}$.

- Clearly, if we take the surface to be sufficiently far away that the stress-energy tensor is zero, then we get conservation of total momentum.

- So, in some sense the components T^{ij} tell us how much of the **momentum component** P^i is ***flowing through a surface*** element oriented in the direction S_j .

Another viewpoint on the vector potential

- These results all point to an interpretation of the Poynting vector as the EM momentum density. Now we will show how we can reduce this to what is basically the vector potential.
- We found that the EM momentum in a given volume is determined by:

$$\vec{P}_{EM} = \int d^3x \frac{1}{c^2} \vec{S} = \int d^3x \; \epsilon_0 \, \vec{E} \times \vec{B}$$

or, in components,

$$P^i_{EM} = \epsilon_0 \int d^3x \; \epsilon^{ijk} \, E_j \, B_k = \epsilon_0 \int d^3x \; \epsilon^{ijk} \, E_j \, \left(\epsilon^{klm} \partial_l A_m \right)$$

- A bit of algebra leads us to:

$$\begin{aligned} P^i_{EM} &= \epsilon_0 \int d^3x \; \left(E_j \, \partial_i A_j - E_j \, \partial_j A_i \right) \\ &= \epsilon_0 \int d^3x \; \left[\partial_i \left(E_j \, A_j \right) - \left(\partial_i E_j \right) \, A_j - \partial_j \left(E_j \, A_i \right) + \left(\partial_j E_j \right) \, A_i \right] \end{aligned}$$

- Integrating by parts the total derivatives we get surface terms, which we neglect (assuming the volume large enough), resulting in:

$$P^i_{EM} = \epsilon_0 \int d^3x \; \left(A_i \, \vec{\nabla} \cdot \vec{E} - A_j \, \partial_i E_j \right)$$

- Now, the first term just gives us $\vec{\nabla} \cdot \vec{E} = \rho / \epsilon_0$, while the second only appears if there is a time-varying magnetic field. To see that, notice that if $\vec{\nabla} \times \vec{E} = \partial \vec{B} / \partial t = 0$, then $\partial_i E_j = \partial_j E_i$, and then:

$$\begin{aligned} P^i_{EM} &= \int d^3x \; \left(\rho \, A_i - \epsilon_0 A_j \, \partial_j E_i \right) = \int d^3x \; \left[\rho \, A_i - \epsilon_0 \partial_j (A_j E_i) + \epsilon_0 (\partial_j A_j) E_i \right] \\ &= \int d^3x \; \rho \, A_i \end{aligned}$$

- So, the vector potential is a kind of ***momentum density*** of the EM field! Maxwell in fact referred to the vector potential as the “electro-kinetic momentum”!

The angular momentum of radiation

- We saw in an earlier class that stress-energy conservation also implies conservation of angular momentum. To be more specific, what we showed was:

$$\partial_\nu T^{\mu\nu} = 0 \quad \Rightarrow \quad \partial_\nu \left(x^\alpha \, T^{\mu\nu} - x^\mu \, T^{\alpha\nu} \right) = 0 \qquad (*) \quad ,$$

  where the components with $\alpha = 0$ or $\mu = 0$ are satisfied automatically with the Maxwell equations.

- This expression shows us the way to find the expression for the angular momentum of the EM field in terms of the stress-energy tensor. Consider the following object, which expresses the flux of angular momentum:

$$M_{ij} = T_{ik} \, \epsilon_{jkl} \, r_l$$

- The 3-divergence of this spatial tensor gives us exactly the spatial part of Eq. (*) above:

$$\partial_i M_{ij} = \partial_i \left(T_{ik} \, \epsilon_{jkl} \, r_l \right) = \left(\partial_i T_{ik} \right) \, \epsilon_{jkl} \, r_l + T_{ik} \, \epsilon_{jkl} \, \partial_i r_l$$

$$= f_k \, \epsilon_{jkl} \, r_l + T_{ik} \, \epsilon_{jkl} \, \delta_{il} = \epsilon_{jkl} \, f_k \, r_l = \tau_j \quad ,$$ which is exactly the torque!

- So, the force and torque laws are written as:

$$\partial_i T_{ik} = f_k = \frac{dP_k}{dt} \qquad , \qquad \partial_i M_{ij} = \tau_j = \frac{dL_j}{dt}$$

- Angular momentum conservation for the total system (matter + EM fields) implies that the

$$\int d^3x \left\{ \frac{d\, l_i}{dt} + \partial_j M_{ij} \right\} = \int d^3x \left\{ \frac{d}{dt} \left[\vec{x} \times \left(\vec{p} + \vec{p}_{EM} \right) \right]_i + \partial_j M_{ij} \right\} = 0$$

- Therefore, we get that the total angular momentum of the system obeys the conservation law:

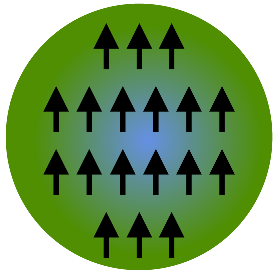
$$\frac{d}{dt} \left(\vec{L} + \vec{L}_{EM} \right) = \int d^3x \, \partial_j M_{ij} = \oint d^2 S_j \, M_{ij}$$

The angular momentum of radiation

- As an example, consider a sphere of radius R with **constant magnetization** $\vec{M} = M\,\hat{z}$ and **charge** Q equally distributed on its surface, so $\sigma = Q/4\pi R^2$. The magnetic and electric fields are:

$$\vec{E} = \frac{\sigma R^2}{\epsilon_0 r^2}\hat{r} \quad \text{outside the sphere, and} \quad \vec{E} = 0 \quad \text{inside the sphere ,}$$

$$\vec{B} = \frac{\mu_0 M R^3}{3\,r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right) \quad \text{outside the sphere, and} \quad \vec{B} = \frac{2}{3}\mu_0\,M\,\hat{z} \quad \text{inside the sphere}$$

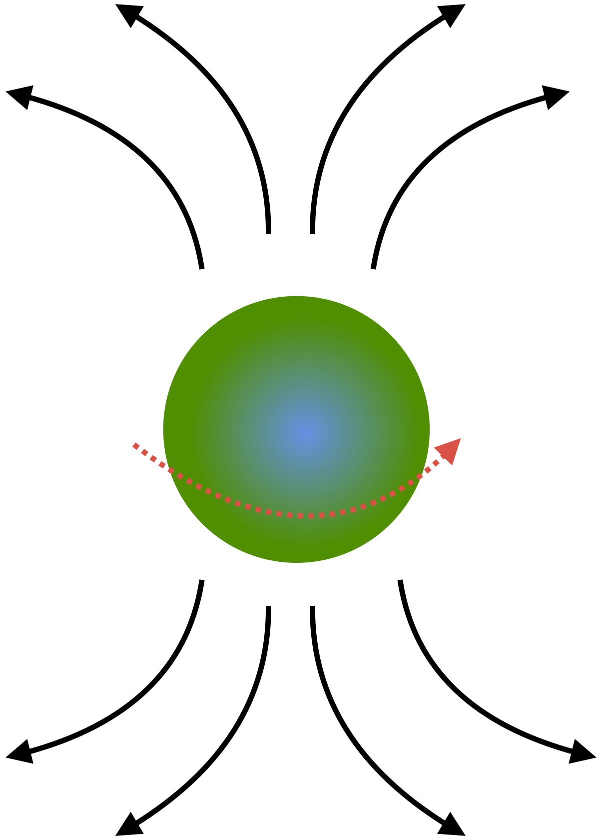


- Now, let's say that this magnetization of the sphere is lost: e.g., the sphere is heated up above the critical temperature (the Curie temperature). The sphere will then "eject" the magnetic field, generating an electromotive force on the charges, which will rotate the sphere. The angular momentum of the sphere must then be compensated by the angular momentum in the field that "flies away".

- So, let's compute that angular momentum which is exchanged. The angular momentum in the EM field is:

$$\vec{L}_{EM} = \epsilon_0 \int d^3x\,\vec{r} \times (\vec{E} \times \vec{B})$$

$$= \epsilon_0 \int_R^\infty dr\,r^2 \int_{-1}^1 d(\cos\theta) \int_0^{2\pi} d\varphi\,\vec{r} \times \left[\left(\frac{\sigma R^2}{\epsilon_0 r^2}\hat{r}\right) \times \left(\frac{\mu_0 M R^3}{3\,r^3}\sin\theta\,\hat{\theta}\right)\right]$$



- Since $\hat{r} \times \hat{\theta} = \hat{\varphi}$ and $\hat{r} \times \hat{\varphi} = -\hat{\theta} = \sin\theta\,\hat{z} - \cos\theta(\cos\varphi\,\hat{x} + \sin\varphi\,\hat{y})$, and $\int_0^{2\pi} d\varphi\,\sin\varphi = \int_0^{2\pi} d\varphi\,\cos\varphi = 0$, we get:

$$\vec{L}_{EM} = \mu_0 \frac{2\pi\sigma M R^5}{3}\,\hat{z} \int_R^\infty dr\,r^2 \int_{-1}^1 d(\cos\theta)\,r^{-4}\sin^2\theta$$

$$= \mu_0 \frac{2\pi\sigma M R^5}{3}\,\hat{z} \int_R^\infty dr\,r^{-2} \int_{-1}^1 d\mu\,\left(1 - \mu^2\right)$$

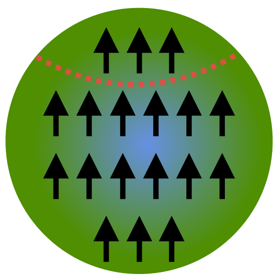
$$= \mu_0 \frac{2\pi\sigma M R^5}{3}\,\hat{z}\,\frac{1}{R}\left(2 - \frac{2}{3}\right) = \mu_0 \frac{2\,Q\,M\,R^2}{9}\,\hat{z}$$

The angular momentum of radiation

- Now let's check that this angular momentum is transferred to the sphere. First, let's compute the electromotive force on charges along a circular path on the surface of the sphere, aligned with the z axis , so its radius is $\rho = r \sin\theta$. The electric field on that circular path is:

$$2\pi (r \sin\theta)\, E_\varphi = -\frac{d\Phi_B}{dt} = -\left(\pi r^2 \sin^2\theta\right)\frac{dB_z}{dt} \quad \Rightarrow \quad E_\varphi = -\frac{1}{2} r \sin\theta \frac{dB_z}{dt} \quad ,$$

where B_z is the z component of the field ***inside the sphere***



- The torque on the sphere is therefore:

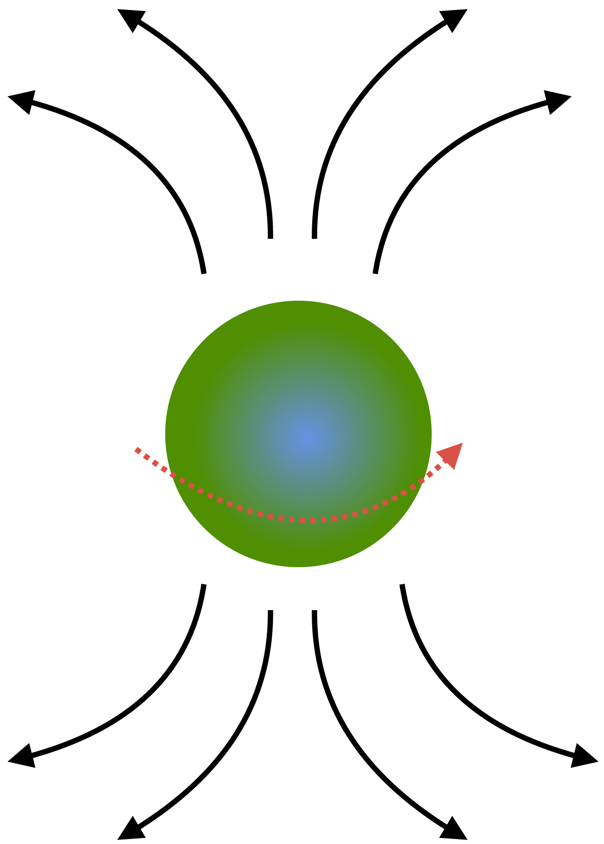
$$\vec{\tau} = \int d^3x\, \vec{r} \times (\rho \vec{E}) = \sigma R \int d^2S\, \hat{r} \times (E_\varphi \hat{\varphi})$$

$$= -\frac{1}{2}\sigma R^4 \frac{dB_z}{dt} \int_{-1}^{1} d(\cos\theta) \int_0^{2\pi} d\varphi \sin\theta\, (-\hat{\theta})$$

- Again, the components along the x and y axis are zero by symmetry, and we are left with the only component:

$$\vec{\tau} = -\frac{1}{2}\sigma R^4 \frac{dB_z}{dt}\, \hat{z} \int_{-1}^{1} d(\cos\theta) \int_0^{2\pi} d\varphi \sin^2\theta$$

$$= -\frac{4\pi}{3}\sigma R^4 \frac{dB_z}{dt}\, \hat{z} = -\frac{1}{3} Q R^2 \frac{dB_z}{dt}\, \hat{z}$$



- If the magnetization decays, all the magnetic field inside the sphere decays as well, and the total variation is:

$$\Delta \vec{L} = -\frac{1}{3} Q R^2 \Delta B_z\, \hat{z} = -\frac{1}{3} Q R^2 \times \left(0 - \frac{2}{3}\mu_0 M \hat{z}\right)$$

$$\vec{L} = \frac{2}{9}\mu_0 Q R^2 M \hat{z}$$

which is exactly what we obtained before! So, the angular momentum which was "stored" in the field gets transferred to the angular momentum of the sphere! Angular momentum is conserved!!

Next class:

- Plasma phenomena
- R. Fitzpatrick, “Classical Electromagnetism”, Ch. 9
- PDF: http://farside.ph.utexas.edu/teaching/jk1/jk1.html