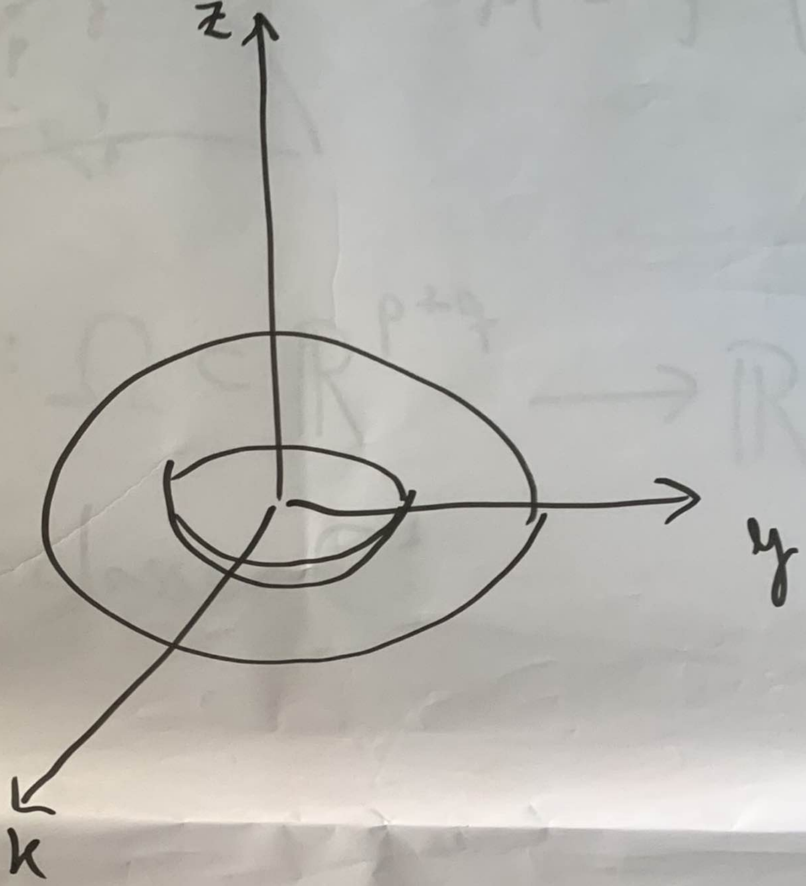
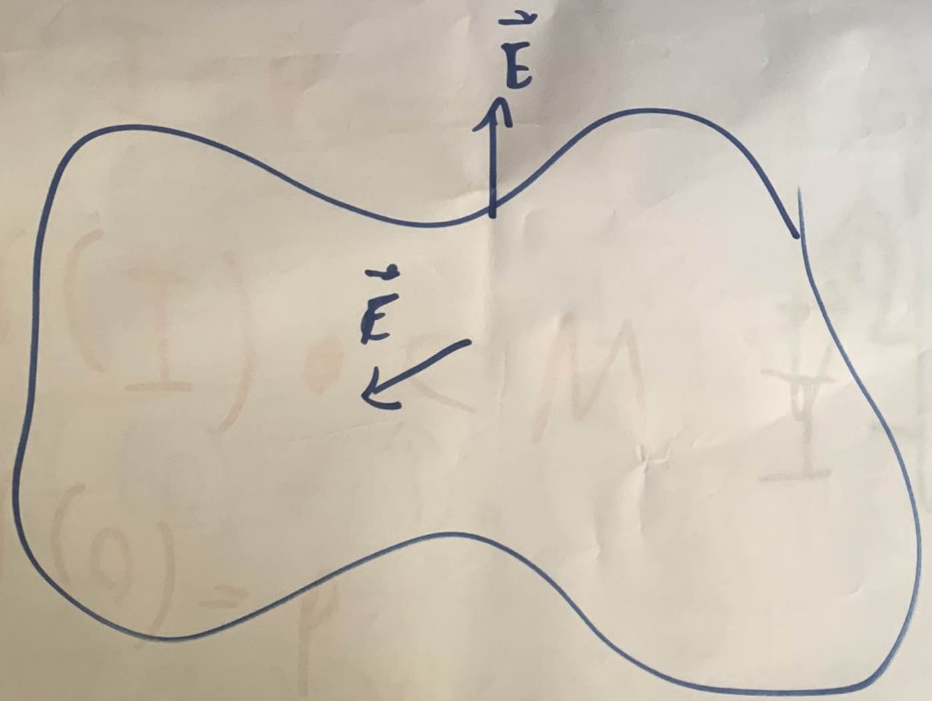
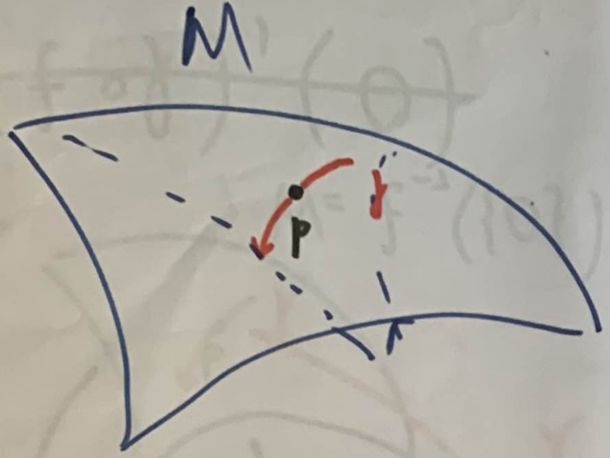
MONITORIA MAT 4 27/05







$M = f^{-1}(0)$

$f : \Omega \subset \mathbb{R}^{p+q} \longrightarrow \mathbb{R}^q$

é de classe C^1

$g : \Omega \longrightarrow \mathbb{R}$ a maximizar

$\gamma : I \subset \mathbb{R} \longrightarrow \Omega$

$\gamma(I) \subset M$

$\gamma(0) = p$

$f\circ\gamma : I \to \mathbb{R}$

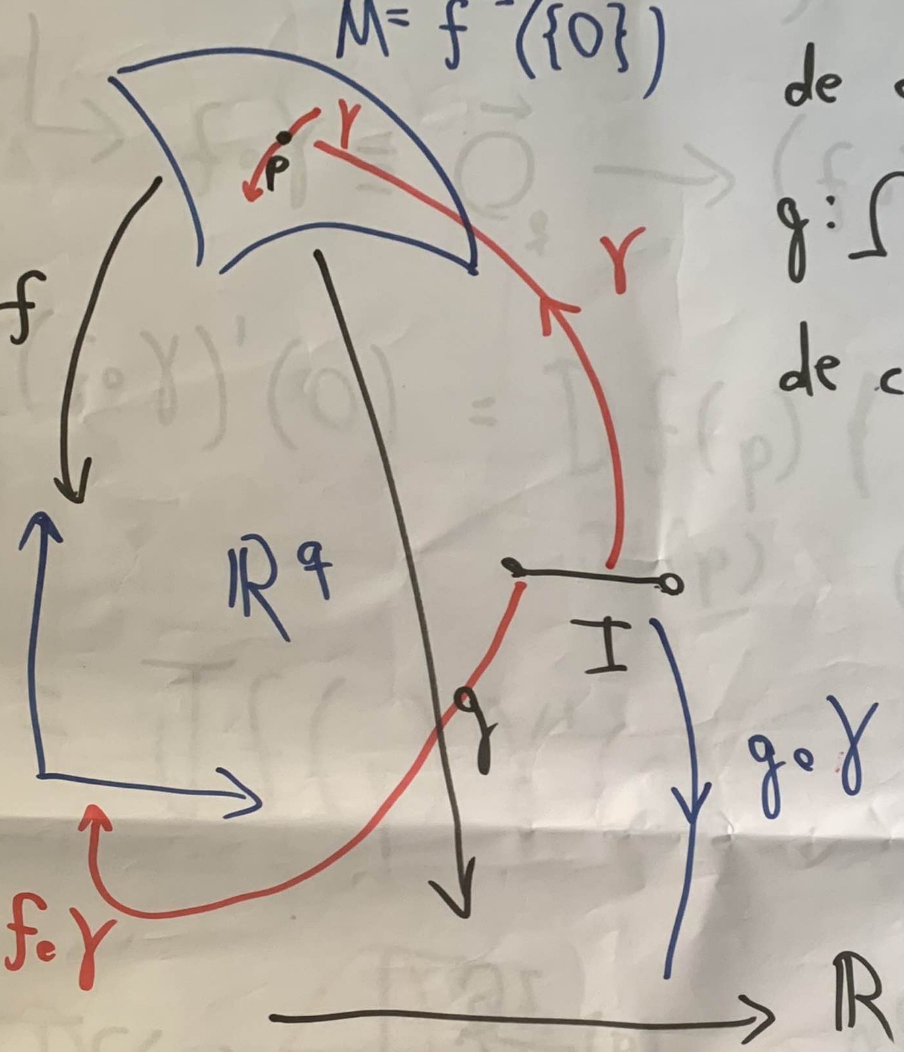
$\cancel{(f\circ\gamma)'(0)}$

$f : \Omega \subset \mathbb{R}^{p+q} \longrightarrow \mathbb{R}^q$

de classe C^1

$g : \Omega \to \mathbb{R}$

de classe C^1



$$\cancel{(f\circ\gamma}\ \boxed{(g\circ\gamma)'(0)} = Dg(\gamma(0))\circ D\gamma(0) = Dg(p)(\gamma'(0)) =$$

$$= \boxed{\langle \nabla g(p), \gamma'(0)\rangle}$$

$$\gamma(I) \subset M,\ f(M) = \{0\}$$

$$\hookrightarrow f \circ \gamma \equiv \vec{0}_q \longrightarrow (f \circ \gamma)'(0) = \vec{0}_q$$

$$(f \circ \gamma)'(0) = Df(p)\left(\gamma'(0)\right) =$$

$$Jf(p) \qquad (\ \ldots\ ) \to \begin{bmatrix} \vdots \end{bmatrix}$$

$$= Jf(p) \cdot \gamma'(0)$$

$$Jf(p) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(p) & \cdots & \frac{\partial f}{\partial x} \\ \vdots & & \end{bmatrix}$$

$$Jf(p) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(p) & \cdots & \frac{\partial f_1}{\partial x_{p+q}}(p) \\ \vdots & & \\ \frac{\partial f_q}{\partial x_1}(p) & \cdots & \frac{\partial f_q}{\partial x_{p+q}}(p) \end{bmatrix}$$

$$\gamma = (\gamma_1, \dots, \gamma_{p+q})$$

$$Jf(p)\cdot\gamma'(0) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_{p+q}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_q}{\partial x_{p+q}} & \cdots & \frac{\partial f_q}{\partial x_{p+q}} \end{bmatrix} \begin{bmatrix} \gamma_1'(0) \\ \vdots \\ \gamma_{p+q}'(0) \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^{p+q} \frac{\partial f_1}{\partial x_i}(p)\cdot\gamma_i'(0) \\ \vdots \\ \sum_{i=1}^{p+q} \frac{\partial f_q}{\partial x_i}(p)\cdot\gamma_i'(0) \end{bmatrix} =$$

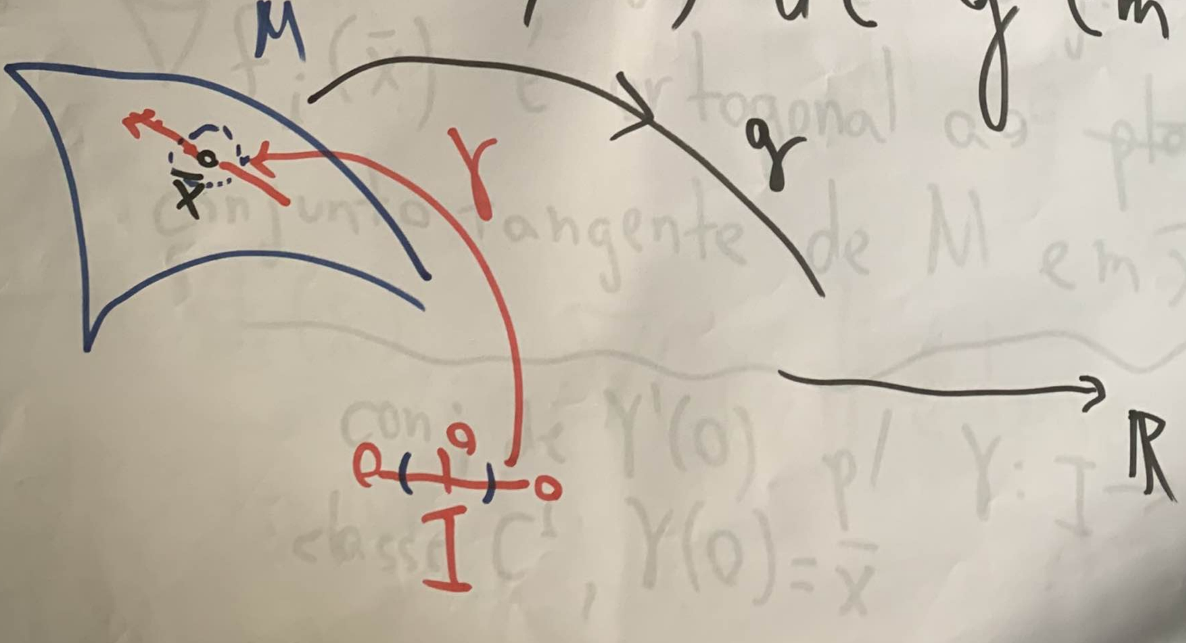
$$= \begin{bmatrix} \langle \nabla f_1(p), \gamma'(0) \rangle \\ \vdots \\ \langle \nabla f_q(p), \gamma'(0) \rangle \end{bmatrix}$$

Conclusão: ($(f_i \circ \gamma)'(0) = \vec{0}$)

$$\langle \nabla f_i(p), \gamma'(0) \rangle = 0$$

↳ ~~Os vetores~~ Os campos de vetores $\nabla f_1, \nabla f_2, \dots, \nabla f_q$ são normais a M

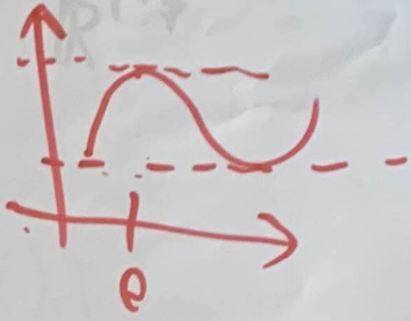
Suponha que $\bar{x}$ é ponto de extremo local (máx., mín.) de g em M



0 é um ponto de extremo local de $g \circ \gamma : I \to \mathbb{R}$

mat. I

$\hookrightarrow (g \circ \gamma)'(0) = 0$



$\hookrightarrow \langle \nabla g(\bar{x}), \gamma'(0) \rangle = 0$

Juntando tudo: Se $\bar{x}$ é um pto. de extremo local de g em M, então:

- $\nabla g(\bar{x})$ é ortogonal ao conj. tangente
- $\nabla f_i(\bar{x})$ é ortogonal ao ~~plan espaço~~ conjunto tangente de M em $\bar{x}$, $T_{\bar{x}}M$

conj de $\gamma'(0)$ p/ $\gamma : I \to M$ de classe C^1, $\gamma(0) = \bar{x}$

Hip.: $Df(p)$ é sobrej. $\forall\, p \in M$

TFImp

- $T_{\bar{x}} M$ é ~~s~~ esp. vet. de $\mathbb{R}^{p+q}$
- $\dim T_{\bar{x}} M = p$
- $\{\nabla f_1(p), \dots, \nabla f_q(p)\}$ é l.i.

Anteriormente, vimos que $\nabla g(\bar{x})$, $\nabla f_1(\bar{x}), \dots, \nabla f_q(\bar{x}) \in T_{\bar{x}} M^{\perp}$

Fato de AlgeLin: $\dim T_{\bar{x}} M^{\perp} =$
$= \dim \mathbb{R}^{p+q} - \dim T_{\bar{x}} M = q$

$\{\nabla f_1(\bar{x}), \dots, \nabla f_q(\bar{x})\}$ é base de $T_{\bar{x}} M^{\perp}$

Conclusão: $\exists \lambda_1, \dots, \lambda_q \in \mathbb{R}$ t.q.

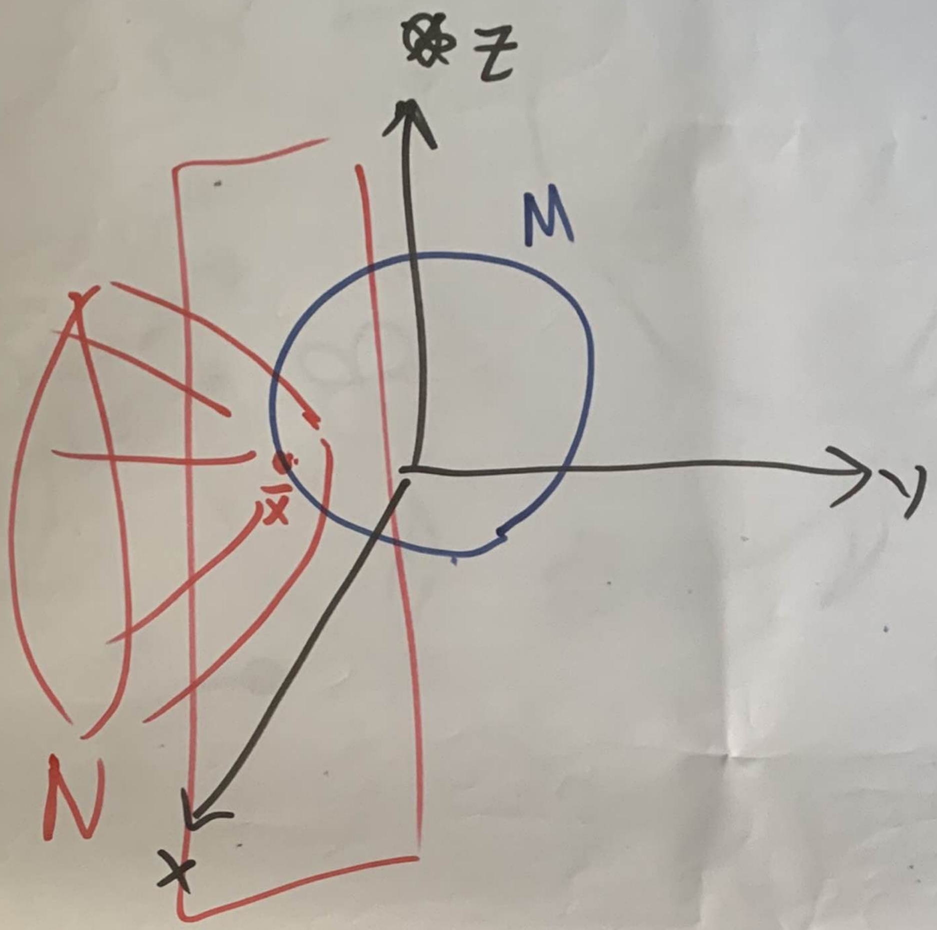
$$\nabla g(\bar{x}) = \lambda_1 \nabla f_1(\bar{x}) + \cdots + \lambda_q \nabla f_q(\bar{x})$$

~~Sabemos~~ $N = g^{-1}(\{g(\bar{x})\})$

$T_{\bar{x}} N$ é subesp. de $\mathbb{R}^{p+q}$ de dim $p+q-1$

e $T_{\bar{x}} N^{\perp} = \{\lambda \nabla g(\bar{x}); \lambda \in \mathbb{R}\}$

$\hookrightarrow T_{\bar{x}} M$ é subesp. de $T_{\bar{x}} N$


Z
M
Y
N
X