Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

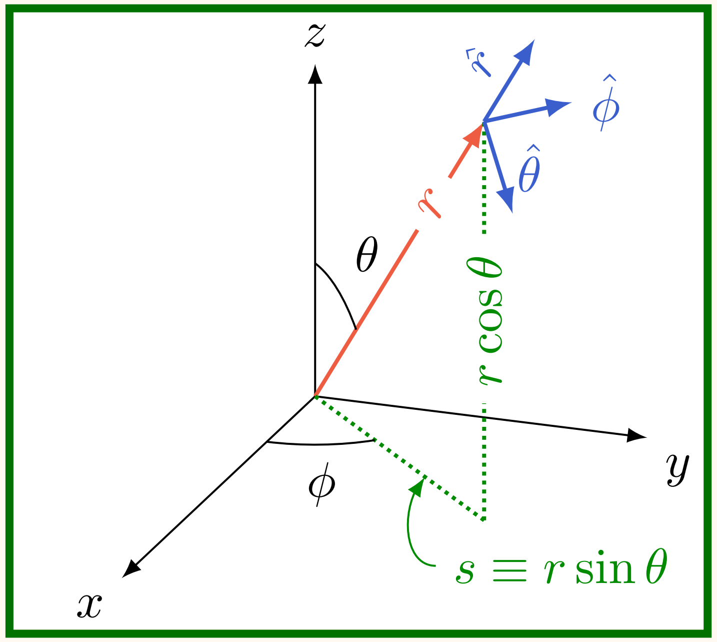
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

Aula de 2 de junho
Métodos especiais

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

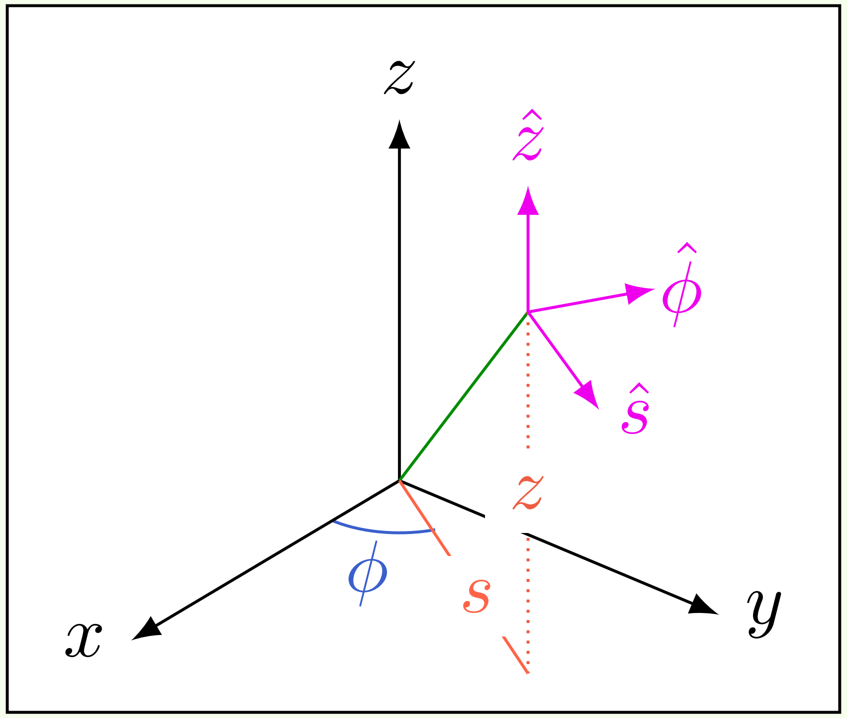
$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$

$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\, v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Coordenadas cilíndricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}s\,\hat{s} + s\,\mathrm{d}\phi\,\hat{\phi} + \mathrm{d}z\,\hat{z}$$



$$\vec{\nabla} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{s}\frac{\partial(s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla}\times\vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial(s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

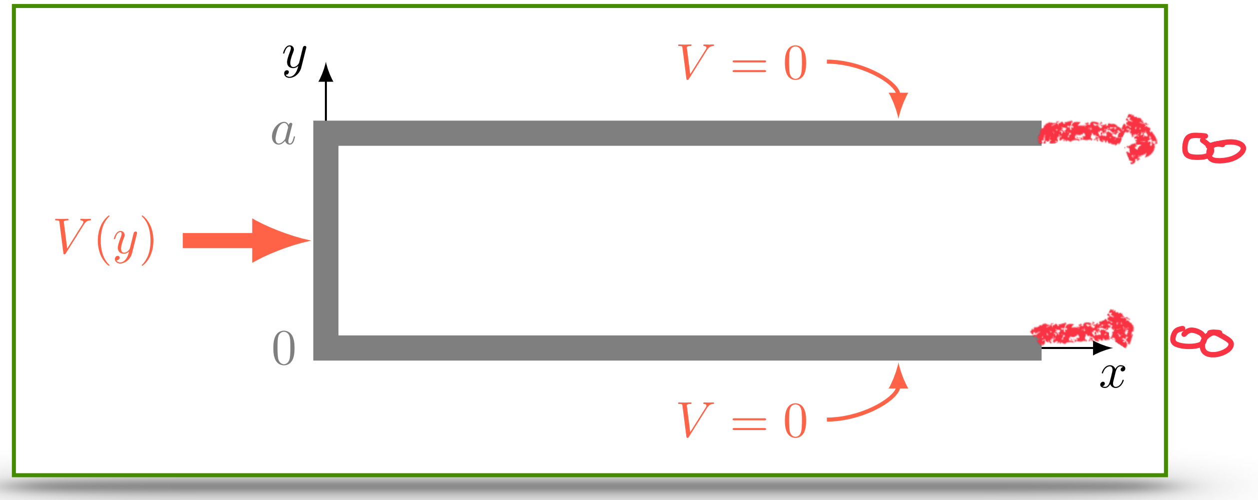
$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

Duas dimensões, por exemplo

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

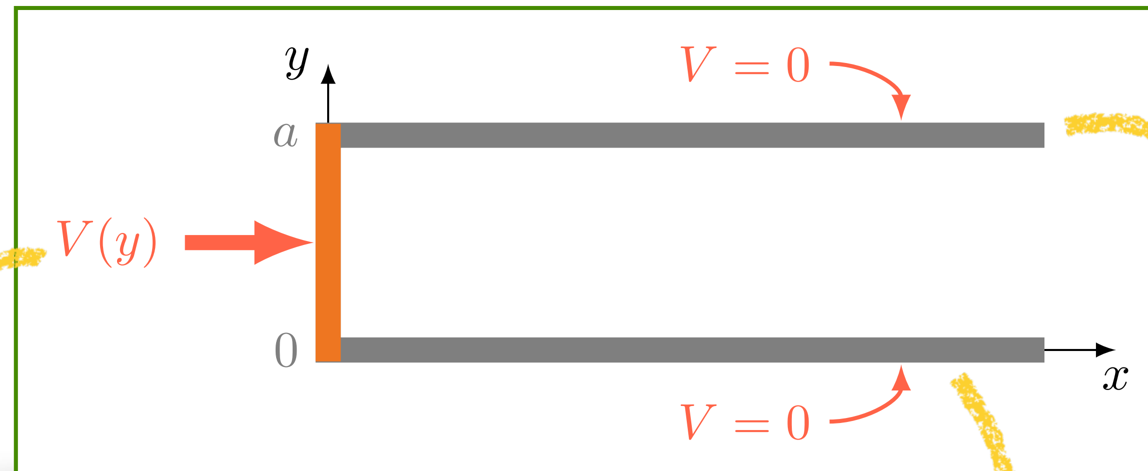


Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

Duas dimensões, por exemplo

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$



$$V(x,y) = X(x)Y(y)$$

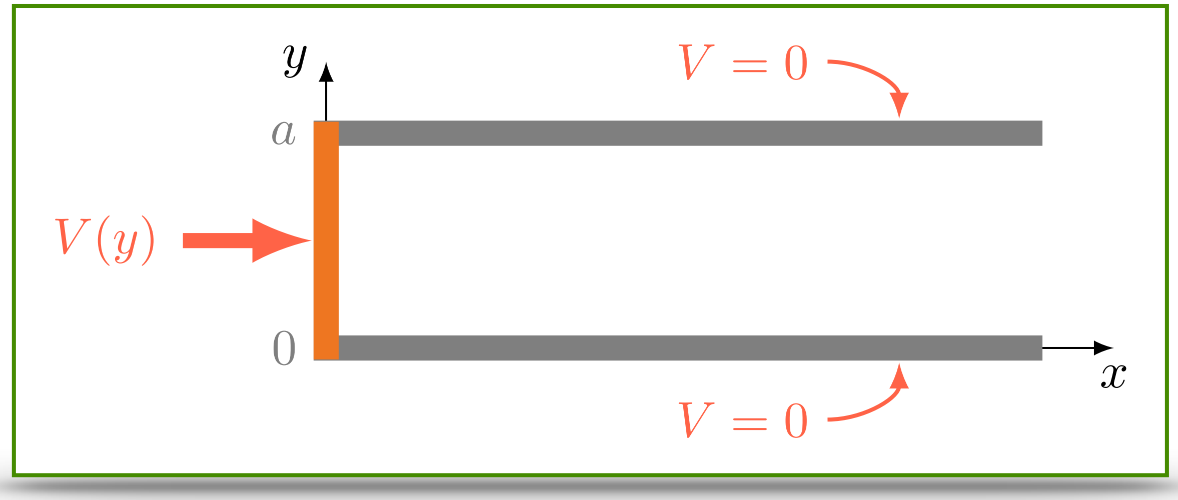
FÁCIL DESCREVER CONDIÇÕES DE CONTORNO.

- $y=0$ => CONDIÇÃO DE CONTORNO SE REDUZ A $Y(0)=0$
- $y=a$ => CONDIÇÃO DE CONTORNO SE REDUZ A $Y(a)=0$
- $x=0$ => CONDIÇÃO DE CONTORNO SE REDUZ A $X(0)Y(y)=V(y)$
- $x\to\infty$ => CONDIÇÃO DE CONTORNO SE REDUZ A $X(\infty)=0$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

Duas dimensões, por exemplo



$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

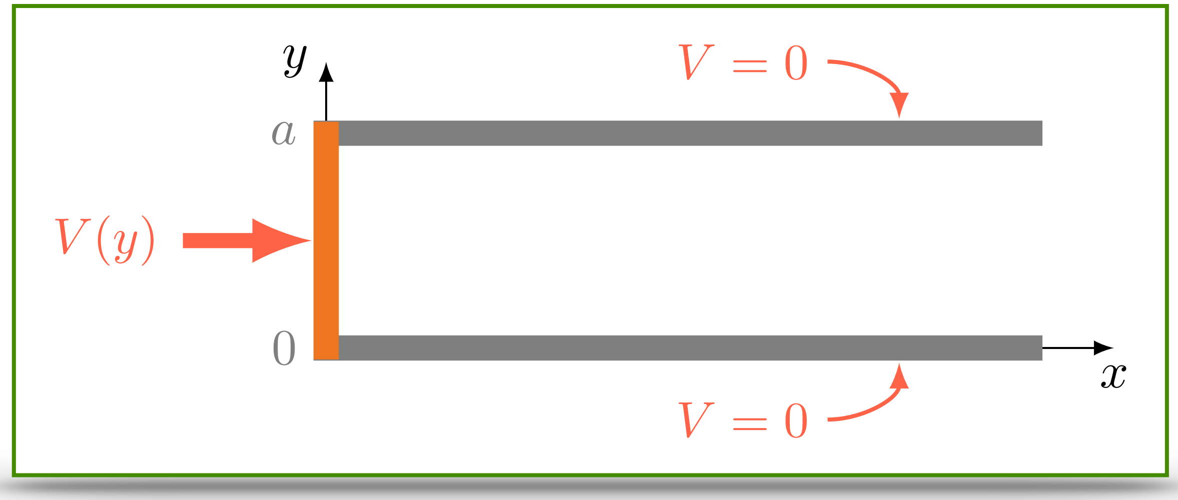
$$V(x, y) = X(x)Y(y)$$

$$\frac{Y \frac{\mathrm{d}^2 X}{\mathrm{d}x^2}}{XY} + \frac{X \frac{\mathrm{d}^2 Y}{\mathrm{d}y^2}}{XY} = 0$$

$$\nabla^2 V = 0$$

Equação de Laplace
Separação de variáveis

Duas dimensões, por exemplo



$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

$$V(x, y) = X(x) Y(y)$$

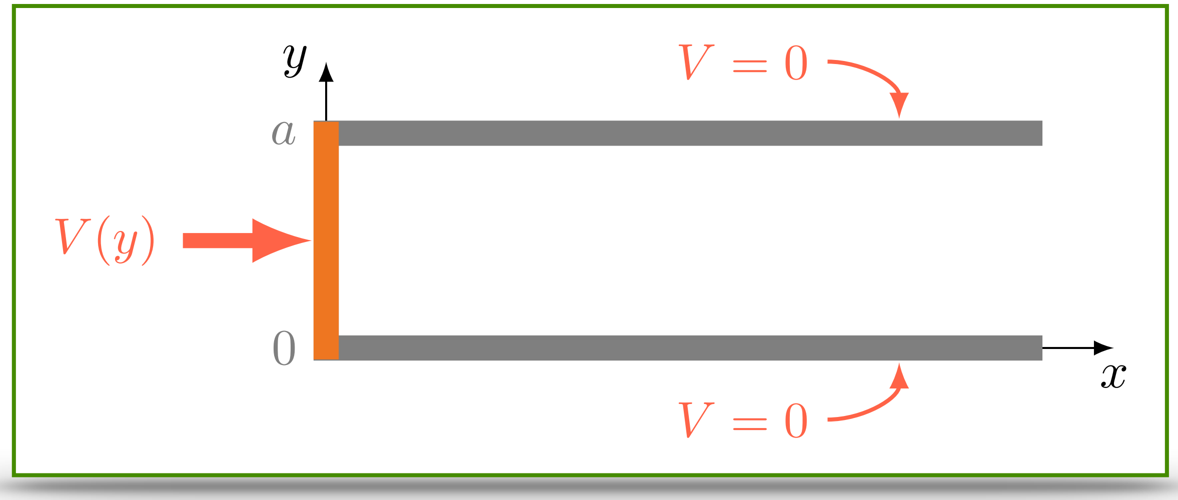
$$Y \frac{\mathrm{d}^2 X}{\mathrm{d}x^2} + X \frac{\mathrm{d}^2 Y}{\mathrm{d}y^2} = 0$$

$$\frac{1}{X} \frac{\mathrm{d}^2 X}{\mathrm{d}x^2} + \frac{1}{Y} \frac{\mathrm{d}^2 Y}{\mathrm{d}y^2} = 0 \quad \left\{ \begin{array}{l} \dfrac{1}{X} \dfrac{\mathrm{d}^2 X}{\mathrm{d}x^2} = k^2 \\ \\ \dfrac{1}{Y} \dfrac{\mathrm{d}^2 Y}{\mathrm{d}y^2} = -k^2 \end{array} \right.$$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

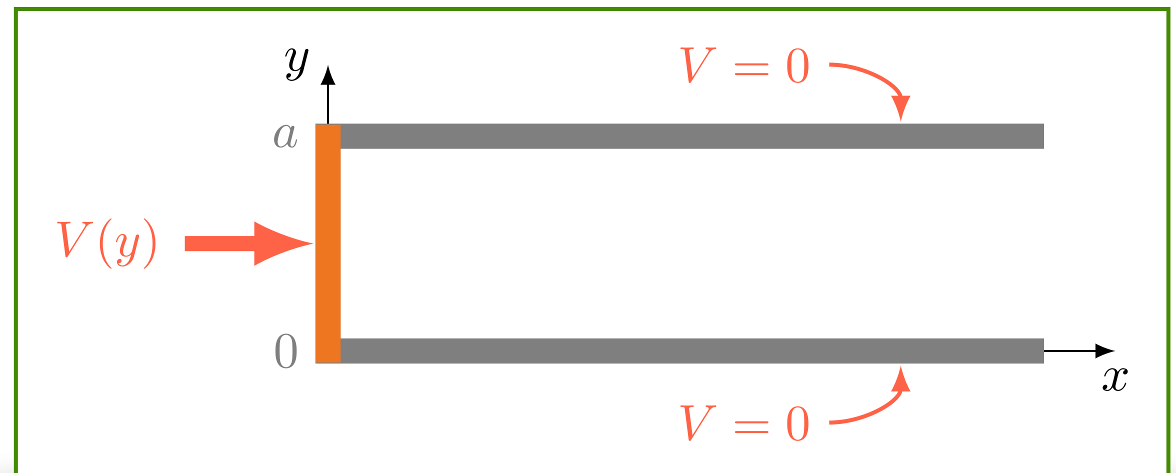
$$\frac{1}{X}\frac{\mathrm{d}^2X}{\mathrm{d}x^2} + \frac{1}{Y}\frac{\mathrm{d}^2Y}{\mathrm{d}y^2} = 0 \quad \begin{cases} \dfrac{1}{X}\dfrac{\mathrm{d}^2X}{\mathrm{d}x^2} = k^2 \\ \dfrac{1}{Y}\dfrac{\mathrm{d}^2Y}{\mathrm{d}y^2} = -k^2 \end{cases}$$



Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$\frac{1}{X}\frac{\mathrm{d}^2 X}{\mathrm{d}x^2} + \frac{1}{Y}\frac{\mathrm{d}^2 Y}{\mathrm{d}y^2} = 0 \quad \begin{cases} \dfrac{1}{X}\dfrac{\mathrm{d}^2 X}{\mathrm{d}x^2} = k^2 \\ \\ \dfrac{1}{Y}\dfrac{\mathrm{d}^2 Y}{\mathrm{d}y^2} = -k^2 \end{cases}$$



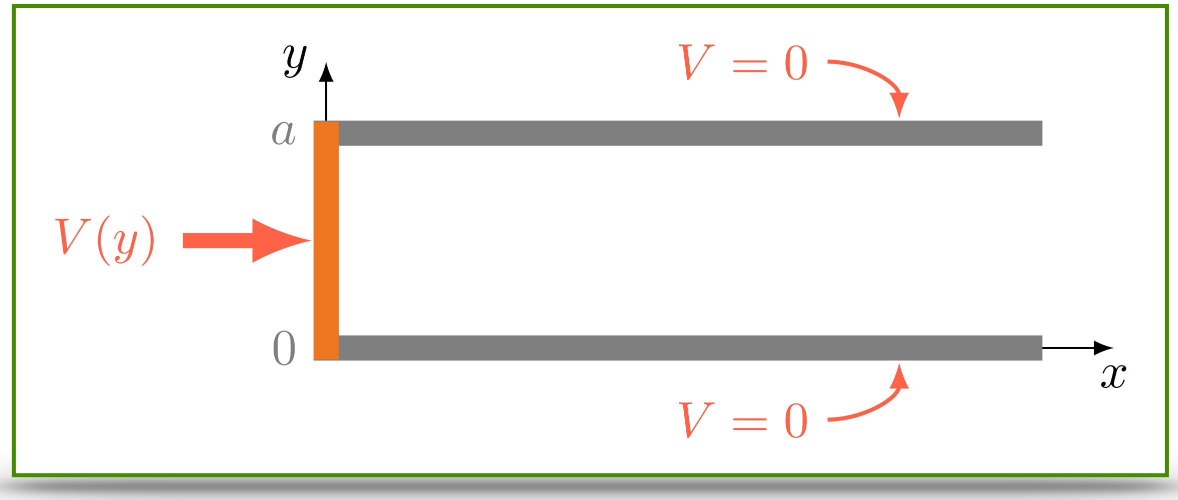
$$\frac{\mathrm{d}^2 Y}{\mathrm{d}y^2} + k^2 Y = 0 \quad \Rightarrow \quad Y(y) = C\sin(ky) + D\cos(ky)$$

PERMITE IMPOR $Y(0) = Y(a) = 0$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$\frac{1}{X}\frac{\mathrm{d}^2 X}{\mathrm{d}x^2} + \frac{1}{Y}\frac{\mathrm{d}^2 Y}{\mathrm{d}y^2} = 0 \quad \begin{cases} \dfrac{1}{X}\dfrac{\mathrm{d}^2 X}{\mathrm{d}x^2} = k^2 \\[2em] \dfrac{1}{Y}\dfrac{\mathrm{d}^2 Y}{\mathrm{d}y^2} = -k^2 \end{cases}$$



$$\frac{\mathrm{d}^2 Y}{\mathrm{d}y^2} + k^2 Y = 0 \quad \Rightarrow \quad Y(y) = C\sin(ky) + D\cos(ky)$$

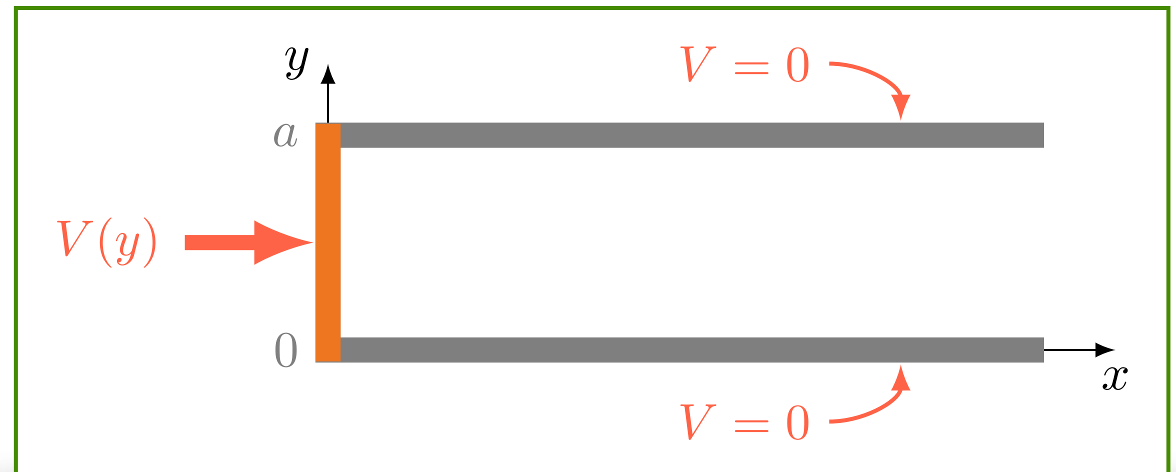
$$\frac{\mathrm{d}^2 X}{\mathrm{d}x^2} - k^2 X = 0 \quad \Rightarrow \quad X(x) = \underbrace{A\exp(kx) + B\exp(-kx)}$$

PERMITE IMPOR $X(x\to\infty) = 0$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$\frac{1}{X}\frac{\mathrm{d}^2X}{\mathrm{d}x^2} + \frac{1}{Y}\frac{\mathrm{d}^2Y}{\mathrm{d}y^2} = 0 \begin{cases} \frac{1}{X}\frac{\mathrm{d}^2X}{\mathrm{d}x^2} = k^2 \\ \frac{1}{Y}\frac{\mathrm{d}^2Y}{\mathrm{d}y^2} = -k^2 \end{cases}$$



$$\frac{\mathrm{d}^2Y}{\mathrm{d}y^2} + k^2Y = 0 \quad \Rightarrow \quad Y(y) = C\sin(ky) + D\cos(ky)$$

$$D = 0 \quad \leftarrow \quad Y(0) = 0$$

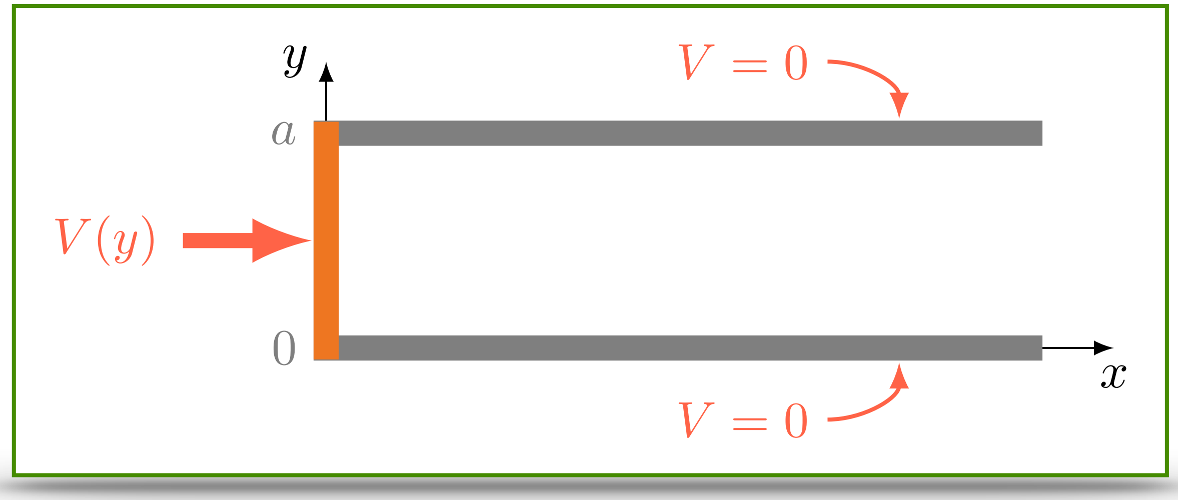
$$ka = n\pi \quad \Rightarrow \quad Y(y) = C\sin\left(\frac{n\pi y}{a}\right)$$

$$\uparrow \; Y(a) = 0$$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$\frac{1}{X}\frac{\mathrm{d}^2 X}{\mathrm{d}x^2} + \frac{1}{Y}\frac{\mathrm{d}^2 Y}{\mathrm{d}y^2} = 0 \quad \begin{cases} \dfrac{1}{X}\dfrac{\mathrm{d}^2 X}{\mathrm{d}x^2} = k^2 \\ \dfrac{1}{Y}\dfrac{\mathrm{d}^2 Y}{\mathrm{d}y^2} = -k^2 \end{cases}$$



$$\frac{\mathrm{d}^2 X}{\mathrm{d}x^2} - k^2 X = 0 \quad \Rightarrow \quad X(x) = A\exp(kx) + B\exp(-kx)$$

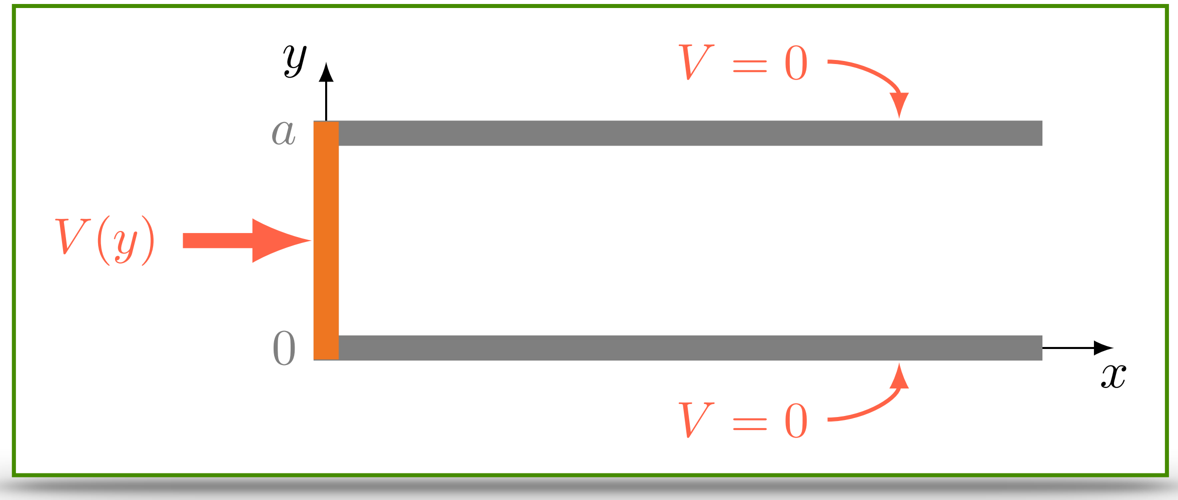
$A = 0$ ← $X(x \to \infty) = 0$

$$X_n(x) = B\exp\left(-\frac{n\pi x}{a}\right)$$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} = 0 \begin{cases} \frac{1}{X}\frac{d^2X}{dx^2} = k^2 \\ \frac{1}{Y}\frac{d^2Y}{dy^2} = -k^2 \end{cases}$$



$$\frac{d^2X}{dx^2} - k^2 X = 0 \quad \Rightarrow \quad X(x) = A\exp(kx) + B\exp(-kx)$$

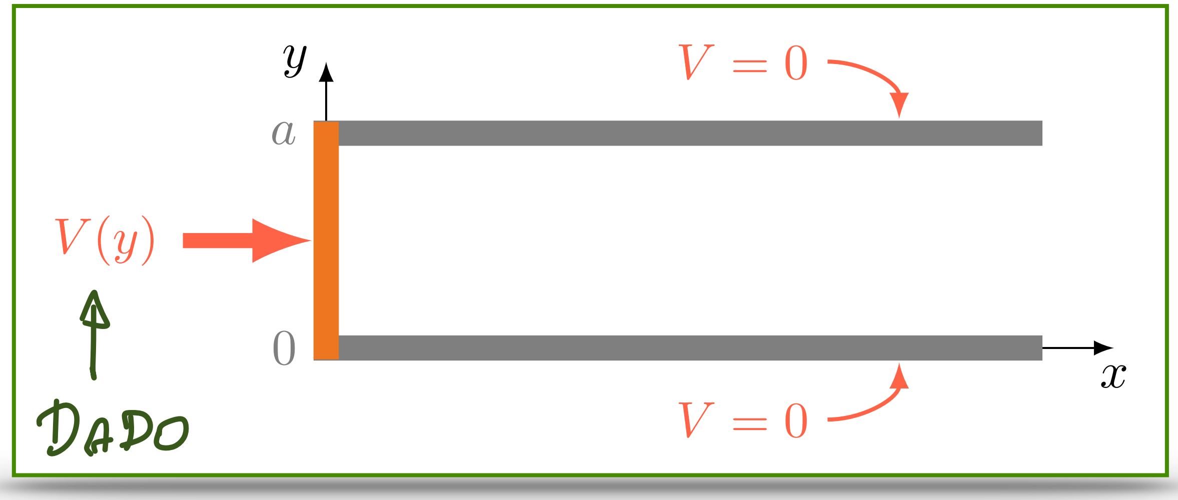
$$A = 0$$

$$X_n(x) = B\exp\left(-\frac{n\pi x}{a}\right) \quad \Rightarrow \quad V(x,y) = V_n \exp\left(-\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$V(x,y) = V_n \exp\left(-\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$



EM GERAL, NÃO SATISFAZ $V(0,y) = V(y)$

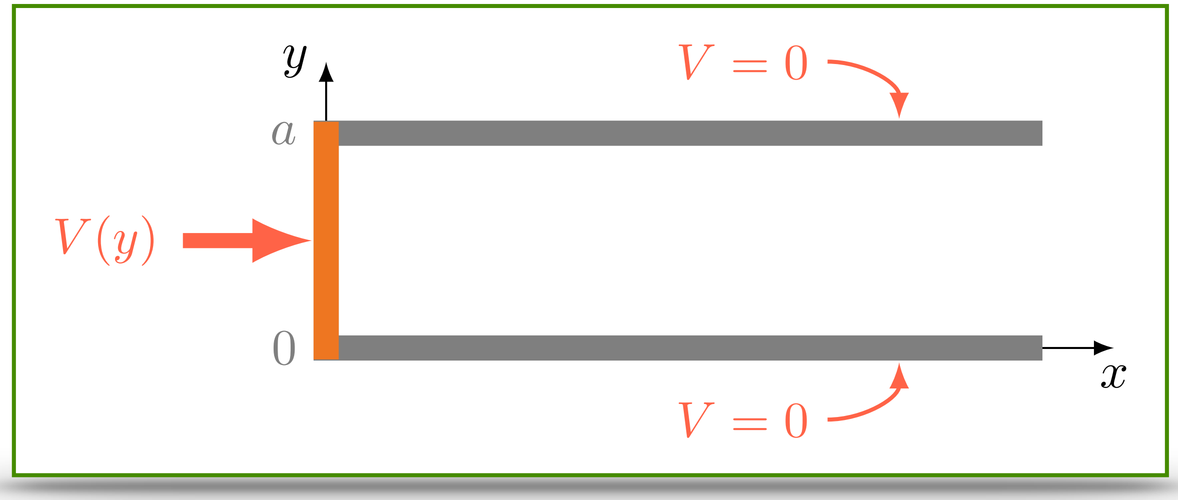
$$V(x,y) = \sum_{n=1}^{\infty} V_n \exp\left(-\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$

DEVEM SER ESCOLHIDOS PARA GARANTIR QUE $V(0,y) = V(y)$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$V(x,y) = V_n \exp\left(-\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$



$$V(x,y) = \sum_{n=1}^{\infty} V_n \exp\left(-\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$

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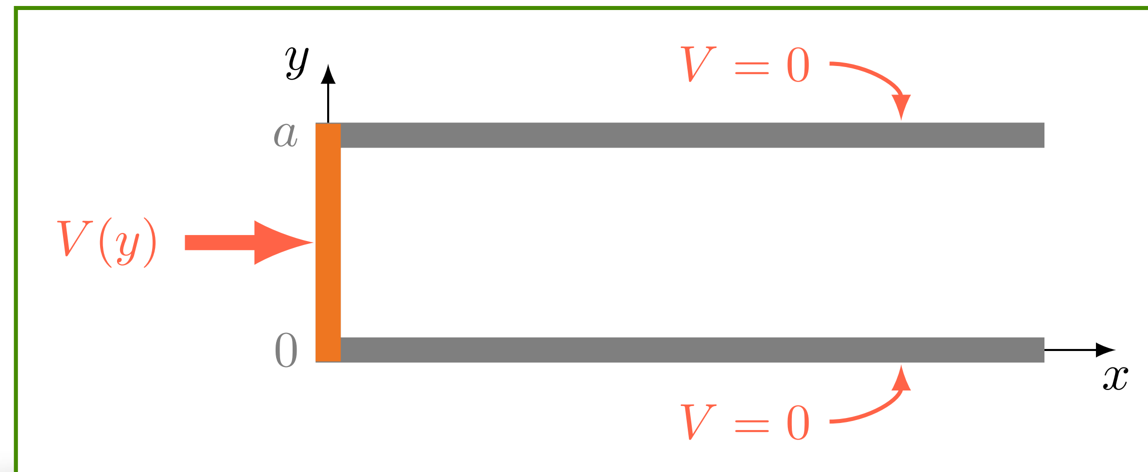
Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$V(x,y) = V_n \exp\left(-\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$



$$V(x,y) = \sum_{n=1}^{\infty} V_n \exp\left(-\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$

$$V(0,y) = \sum_{n=1}^{\infty} V_n \sin\left(\frac{n\pi y}{a}\right)$$

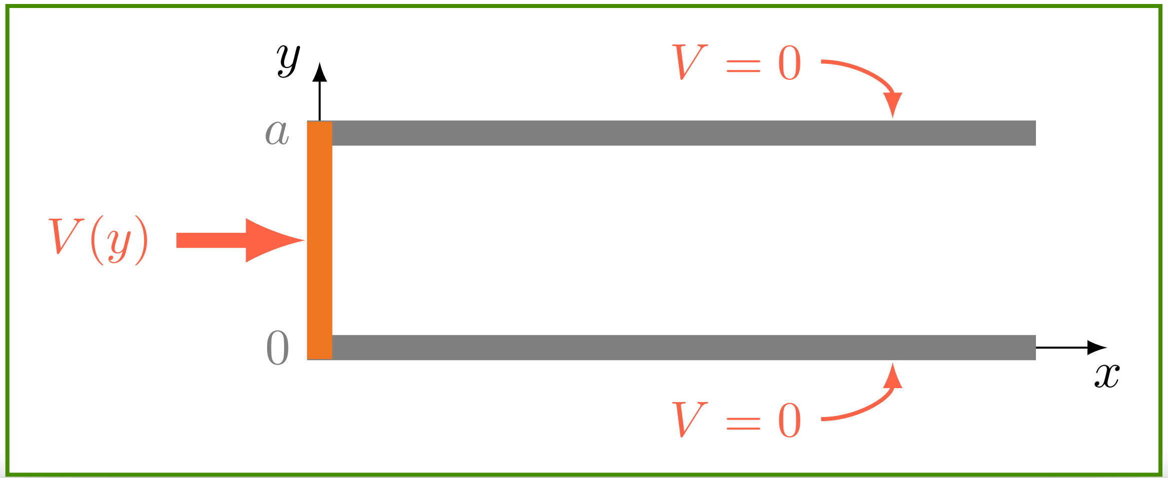
$$\int_0^a V(0,y)\sin\left(\frac{m\pi y}{a}\right)\,\mathrm{d}y = \sum_{n=1}^{\infty} V_n \underbrace{\int_0^a \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{m\pi y}{a}\right)\,\mathrm{d}y}_{\frac{a}{2}\delta_{m,n}}$$

→ FUNÇÕES ORTOGONAIS, PARA $m \neq n$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$V(x,y) = V_n \exp\left(-\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$



$$V(x,y) = \sum_{n=1}^{\infty} V_n \exp\left(-\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$

$$V_m = \frac{2}{a}\int_0^a V(0,y)\sin\left(\frac{m\pi y}{a}\right)\,\mathrm{d}y$$

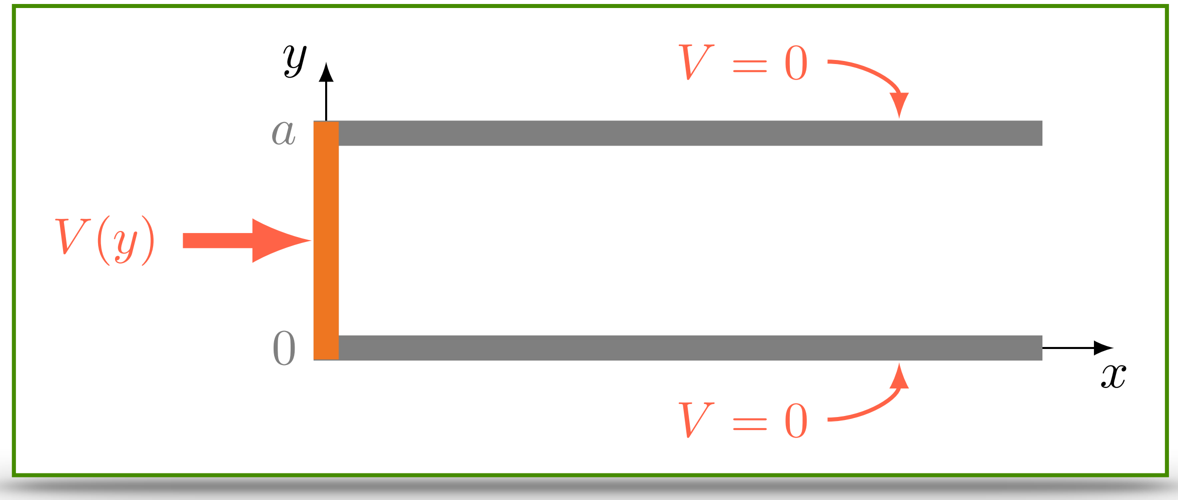
$$V(0,y) = \sum_{n=1}^{\infty} V_n \sin\left(\frac{n\pi y}{a}\right)$$

$$\int_0^a V(0,y)\sin\left(\frac{m\pi y}{a}\right)\,\mathrm{d}y = \sum_{n=1}^{\infty} V_n \int_0^a \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{m\pi y}{a}\right)\,\mathrm{d}y$$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$V(x,y) = \sum_{n=1}^{\infty} V_n \exp\left(-\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$

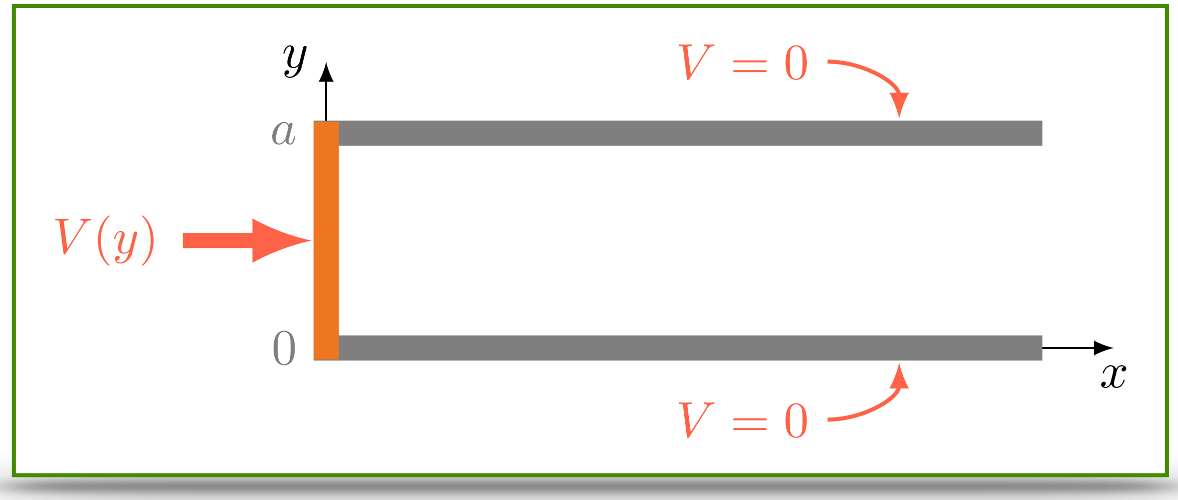


$$V_m = \frac{2}{a}\int_0^a V(0,y)\sin\left(\frac{m\pi y}{a}\right)\,\mathrm{d}y$$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

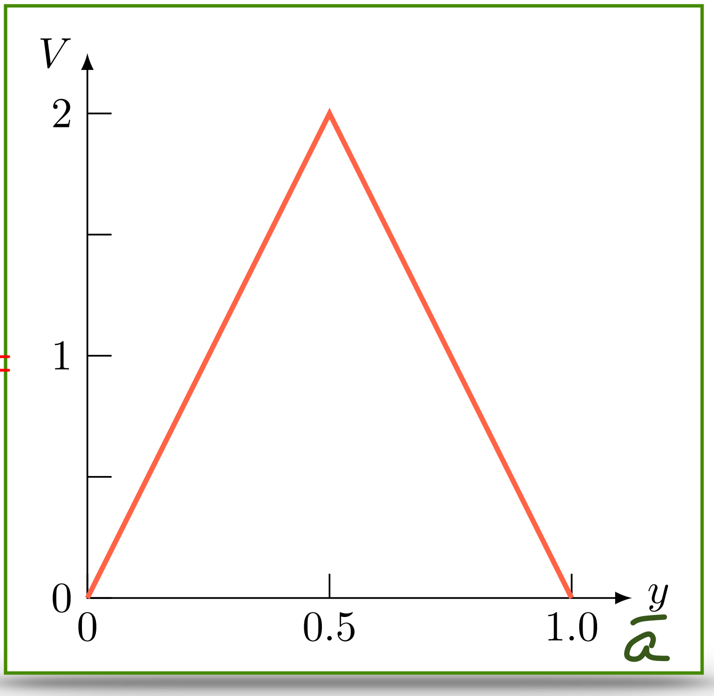
$$V(x,y) = \sum_{n=1}^{\infty} V_n \exp\left(-\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$



$$V_m = \frac{2}{a}\int_0^a V(0,y)\sin\left(\frac{m\pi y}{a}\right)\,dy$$

EXEMPLO

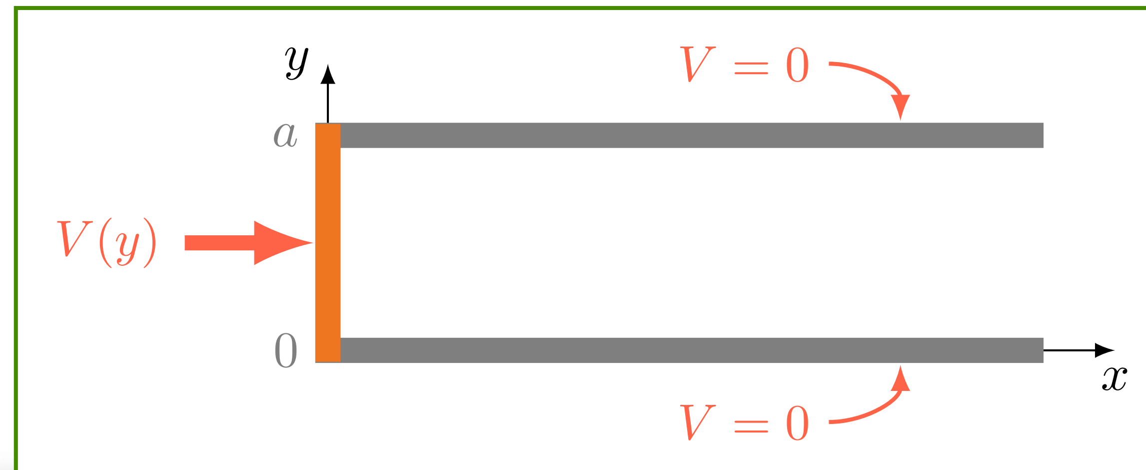
$$V_m = \frac{2}{a}\frac{1}{(m\pi)^2} \qquad (m \text{ ímpar})$$



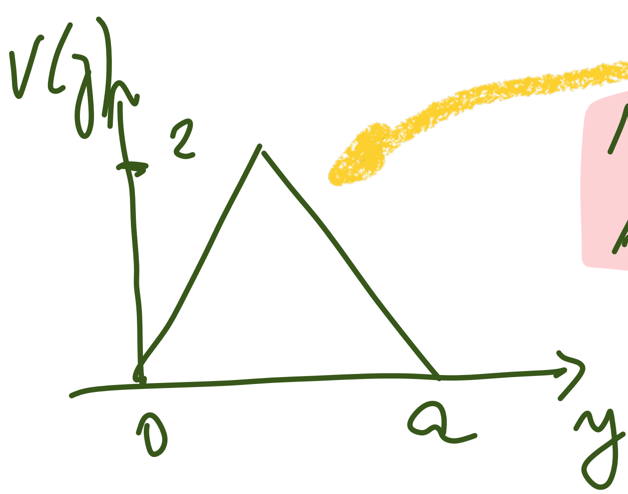
Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$V(x,y) = \sum_{n=1}^{\infty} V_n \exp\left(-\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$$

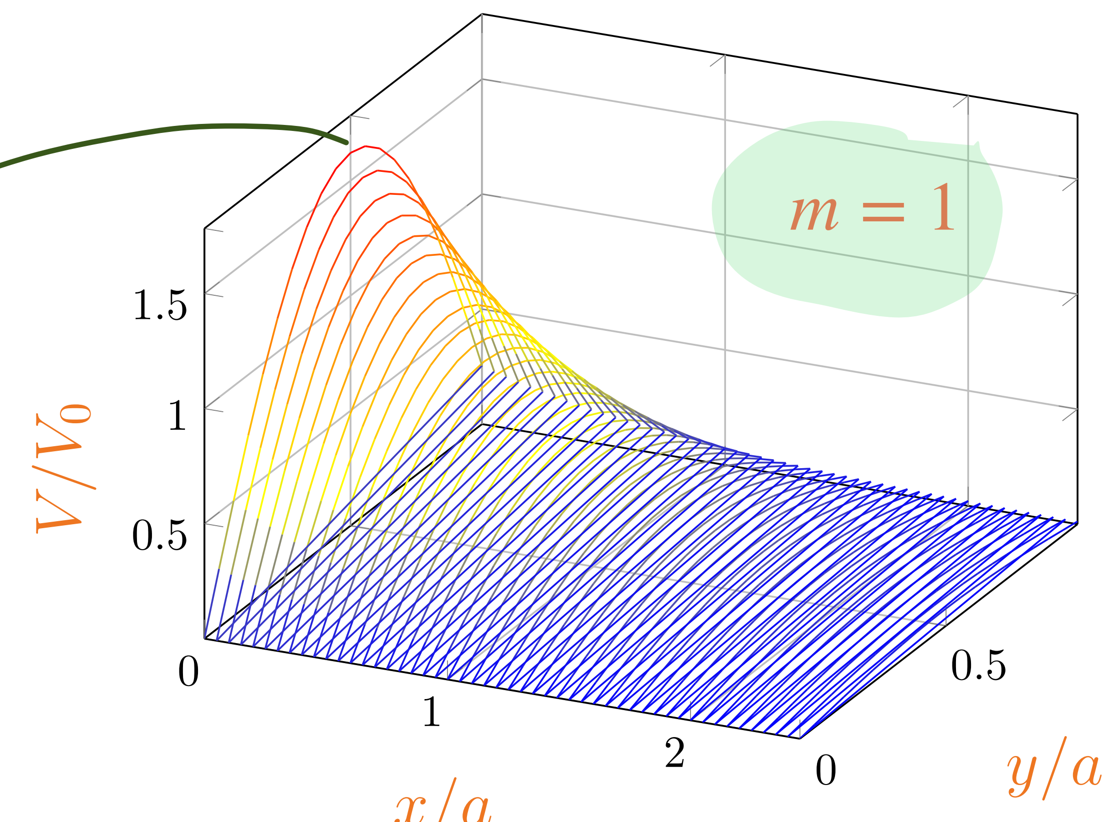


$$V_m = \frac{2}{a}\int_0^a V(0,y)\sin\left(\frac{m\pi y}{a}\right)\,dy$$



COMPARE COM

NÃO CHEGA A 2, ARREDONDADO

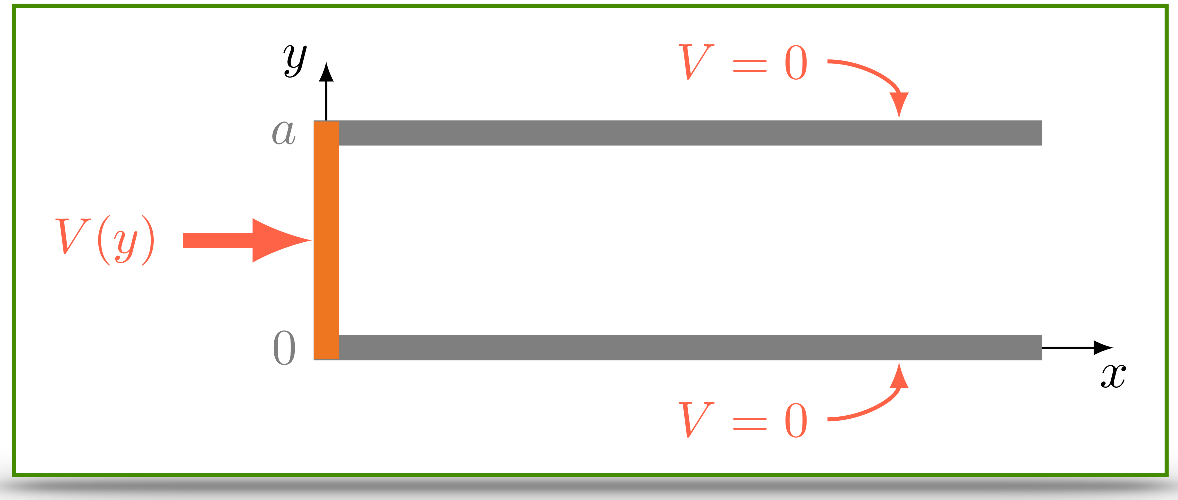


Equação de Laplace
Separação de variáveis

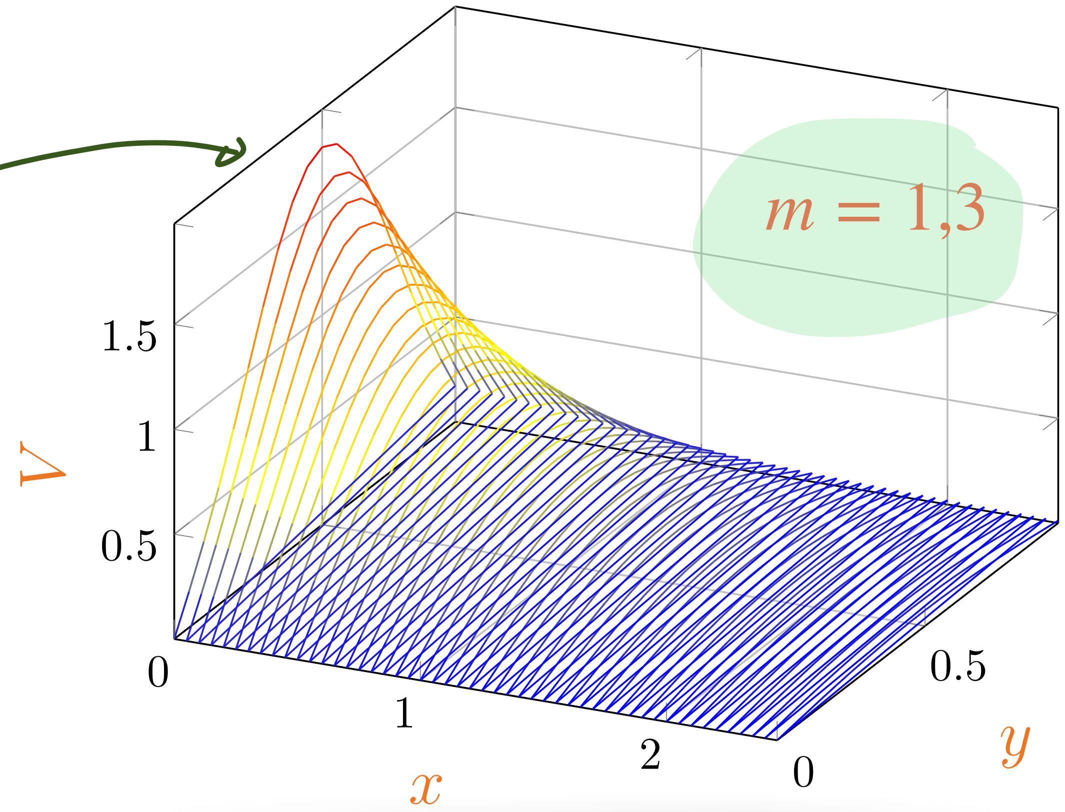
$$\nabla^2 V = 0$$

$$V(x,y) = \sum_{n=1}^{\infty} V_n \exp\left(-\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$

$$V_m = \frac{2}{a}\int_0^a V(0,y)\sin\left(\frac{m\pi y}{a}\right)\,\mathrm{d}y$$



MELHOROU, MAS AINDA MENOR QUE 2



Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$V(x,y) = \sum_{n=1}^{\infty} V_n \exp\left(-\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$

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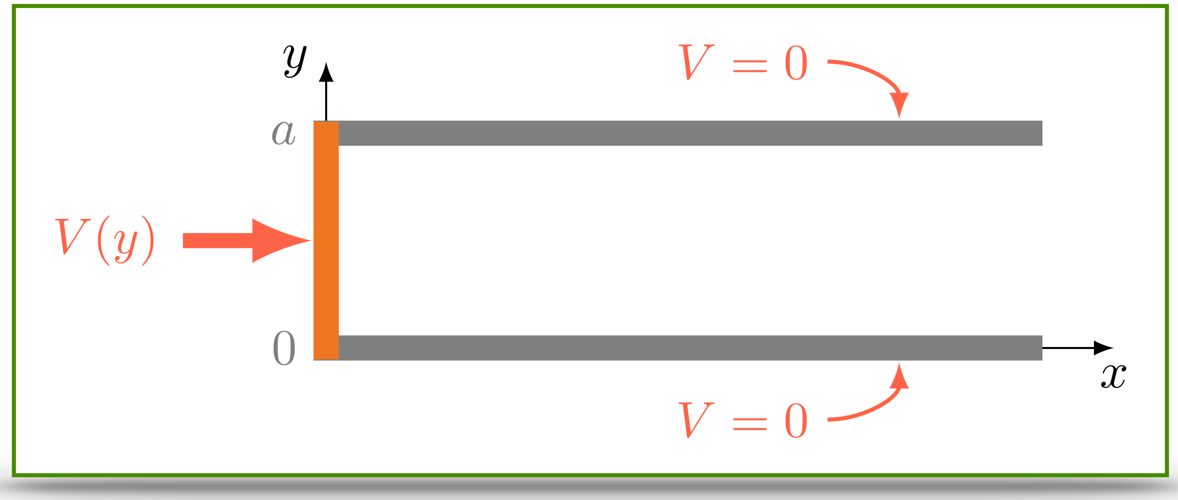
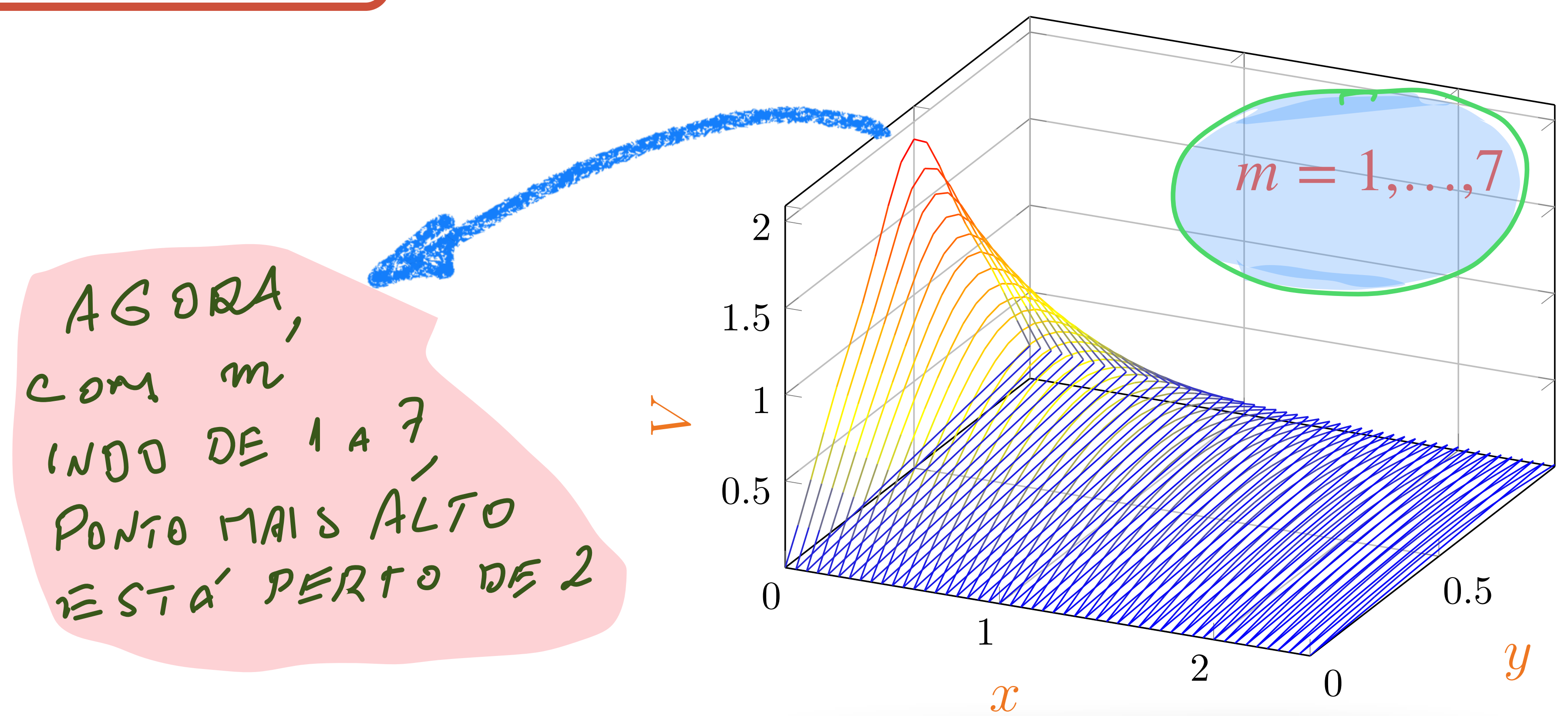


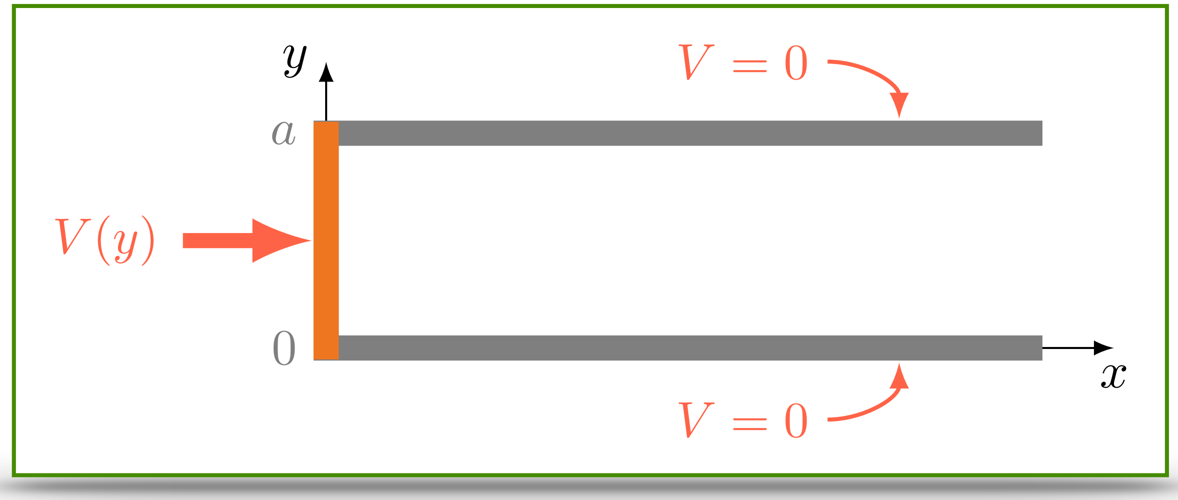


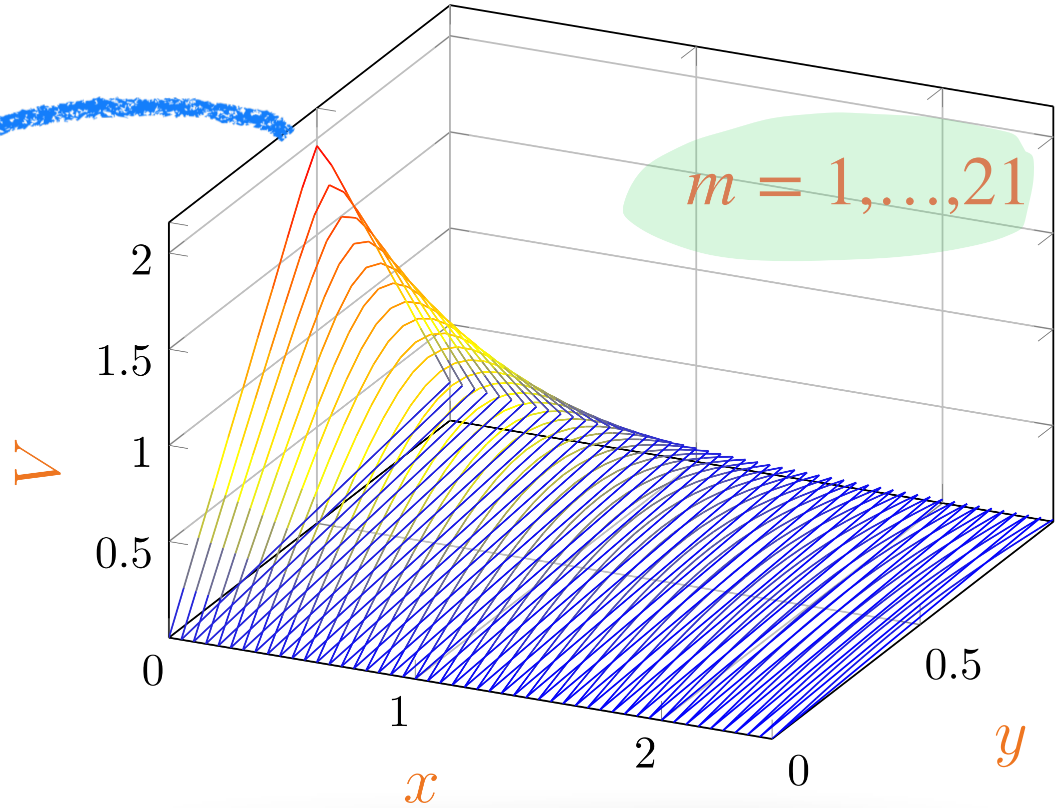
Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

$$V(x,y) = \sum_{n=1}^{\infty} V_n \exp\left(-\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi y}{a}\right)$$

$$V_m = \frac{2}{a} \int_0^a V(0,y) \sin\left(\frac{m\pi y}{a}\right)\, \mathrm{d}y$$



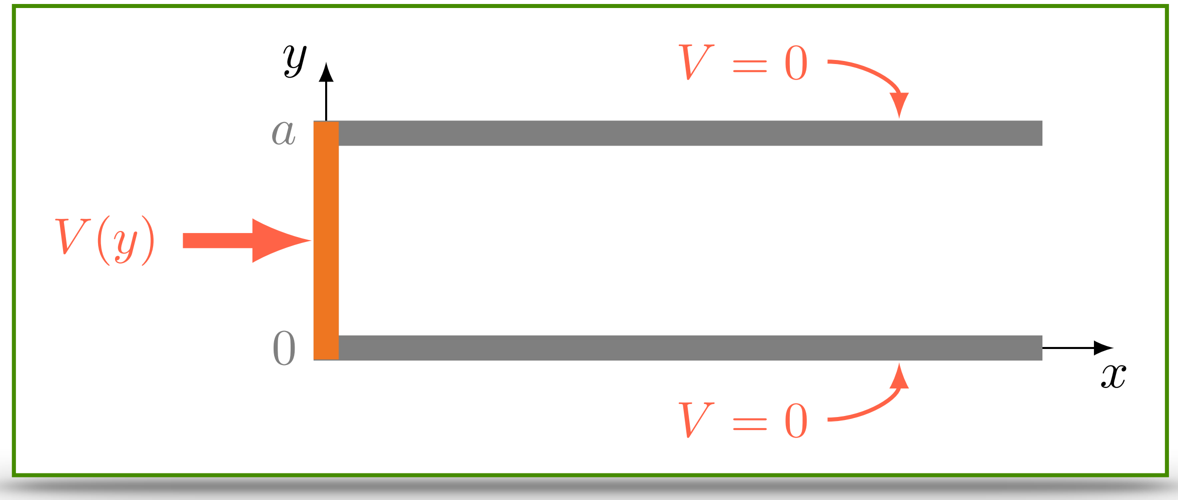


BEM PERTO DE 2, E A MUDANÇA SÚBITA NA INCLINAÇÃO COMEÇA A APARECER

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

Em resumo



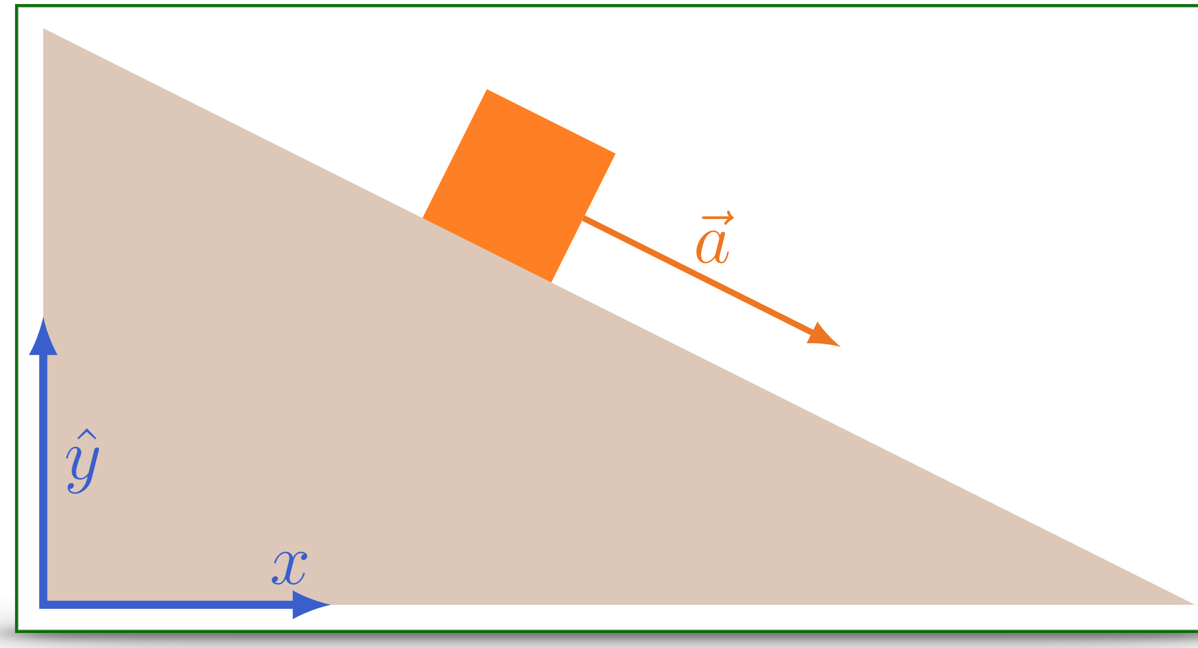
1. $V(x, y) = X(x)Y(y)$

2. $\nabla^2 V = 0 \Rightarrow \begin{cases} X_n(x) \\ Y_n(y) \end{cases}$

3. $V(x, y) = \sum_n V_n X_n(x) Y_n(y)$

4. Cond. contorno $\Rightarrow V_n$

$$\nabla^2 V = 0$$



Em resumo

1. $V(x,y) = X(x)Y(y)$

2. $\nabla^2 V = 0 \Rightarrow \begin{cases} X_n(x) \\ Y_n(y) \end{cases}$

$X_n Y_n$ ORTOGONAL A $X_m Y_m$, PARA $m \neq n$

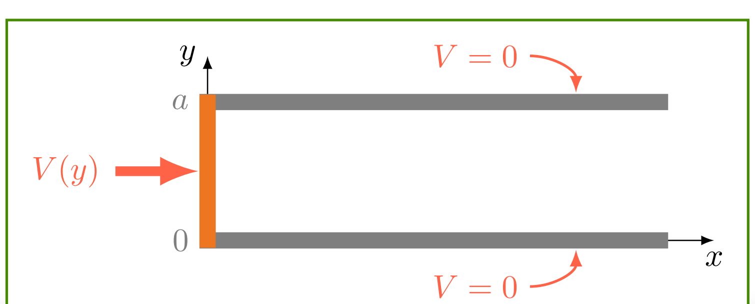
3. $V(x,y) = \sum_n V_n X_n(x) Y_n(y)$

COEFICIENTES A DETERMINAR

4. Cond. contorno $\Rightarrow V_n$

$$\left[V_n = \frac{2}{a} \int_0^a V(0,y)\, Y_n(y)\, \mathrm{d}y \right]$$

INTEGRAL PODE SER VISTA COMO PRODUTO ESCALAR



PROCEDIMENTOS ANÁLOGOS

1. Usar vetores

2. Base: $\{\hat{x},\ \hat{y}\}$

ORTOGONAIS

3. $\vec{a} = \alpha\hat{x} + \beta\hat{y}$

4. $\begin{cases} \alpha = \vec{a}\cdot\hat{x} \\ \beta = \vec{a}\cdot\hat{y} \end{cases}$

Equação de Laplace
Separação de variáveis

$$\nabla^2 V = 0$$

Simetria esférica

$$V(r, \theta, \phi) = R(r)\Theta(\theta)$$

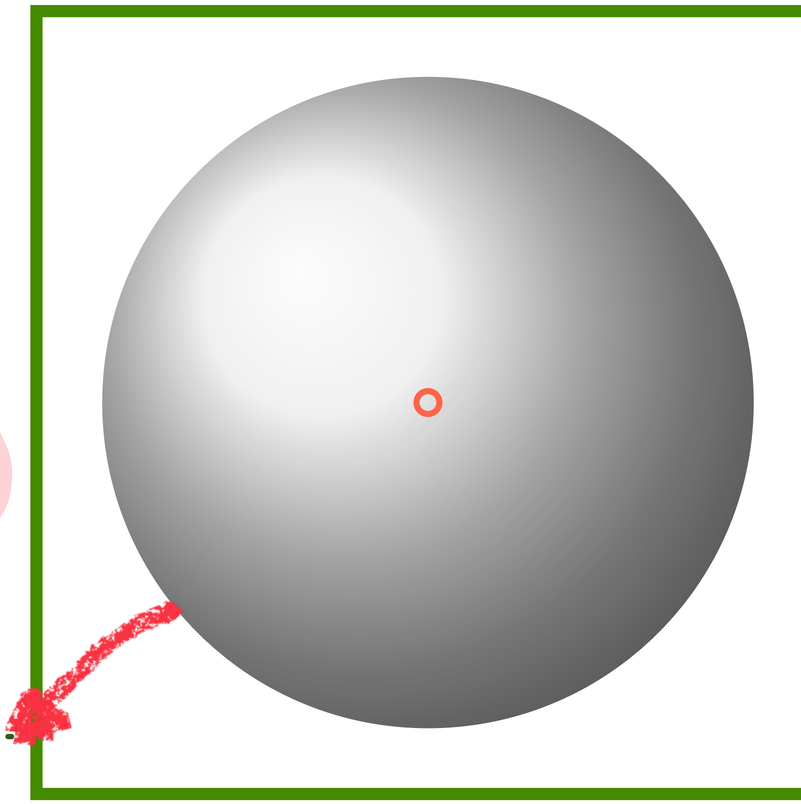
↳ PERMITE ESPECIFICAR CONDIÇÕES DE CONTORNO

POR EXEMPLO,
SE POTENCIAL FOR $V(\theta)$
NA SUPERFÍCIE,

↳ DADO

$$\Rightarrow V(\theta) = R(R)\,\Theta(\theta)$$

↳ CONSTANTE



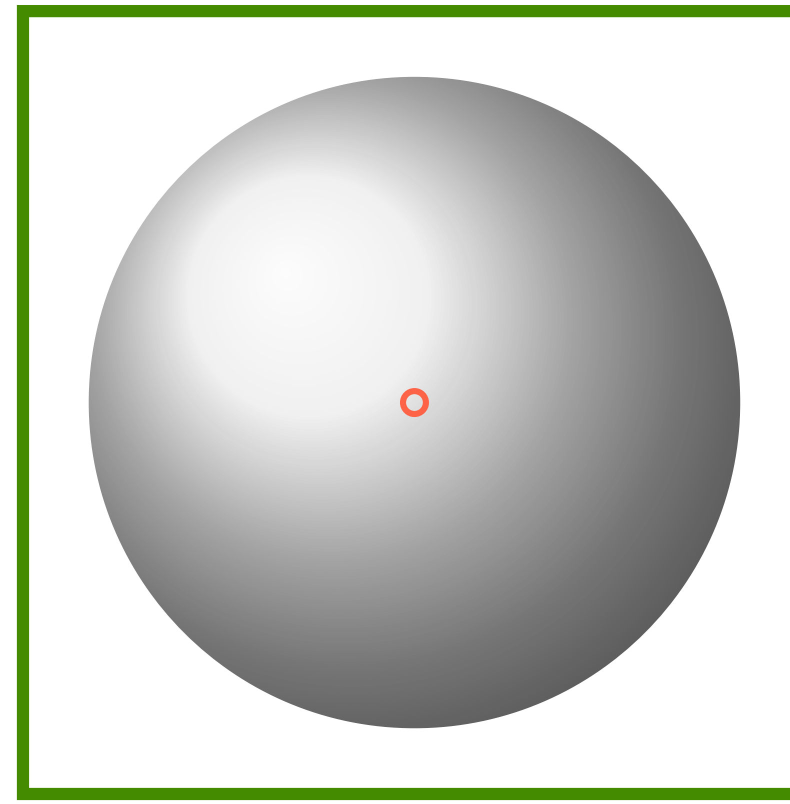
$$\nabla^2 V = 0$$

Equação de Laplace
Separação de variáveis

Simetria esférica E AZIMUTAL

$$V(r,\theta,\phi) = R(r)\Theta(\theta)$$

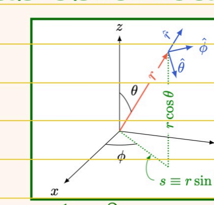
CONDIÇÕES DE CONTORNO INDEPENDENTES DE ϕ



Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$



$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\, v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\, v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Equação de Laplace
Separação de variáveis

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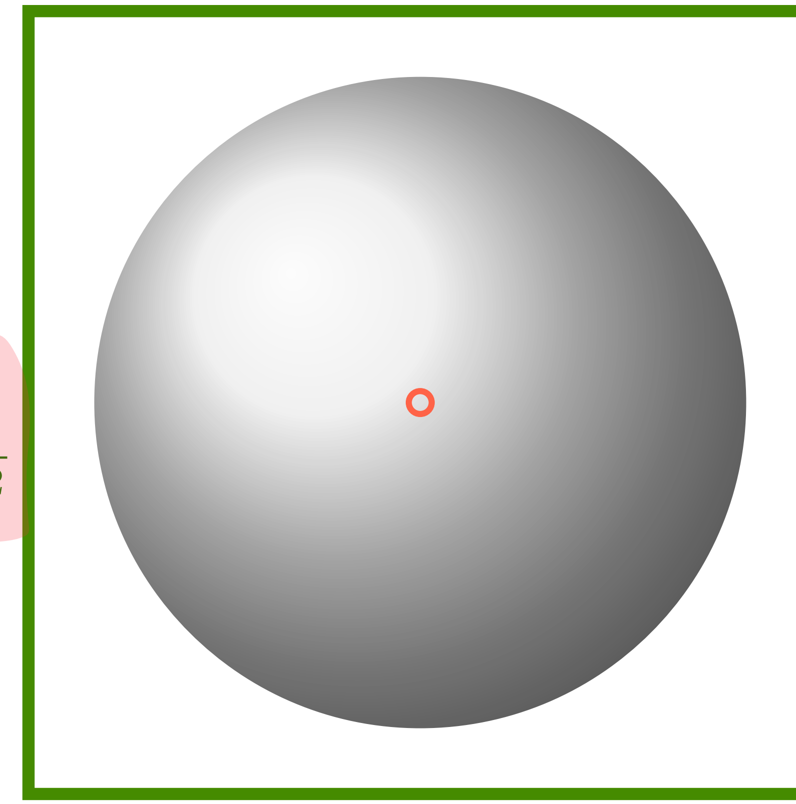
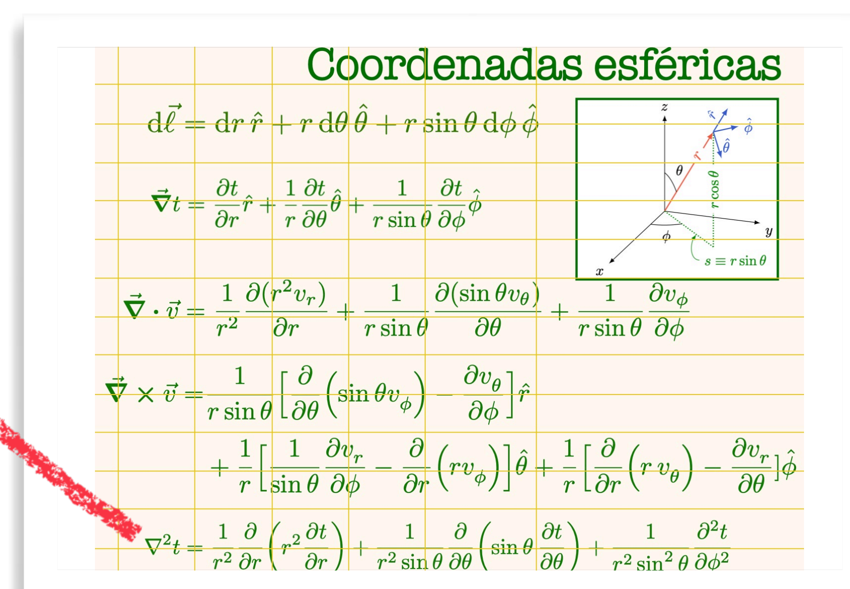
Simetria esférica

$$V(r,\theta,\phi) = R(r)\Theta(\theta)$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial t}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$





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$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$

$$+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\, v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

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Equação de Laplace
Separação de variáveis

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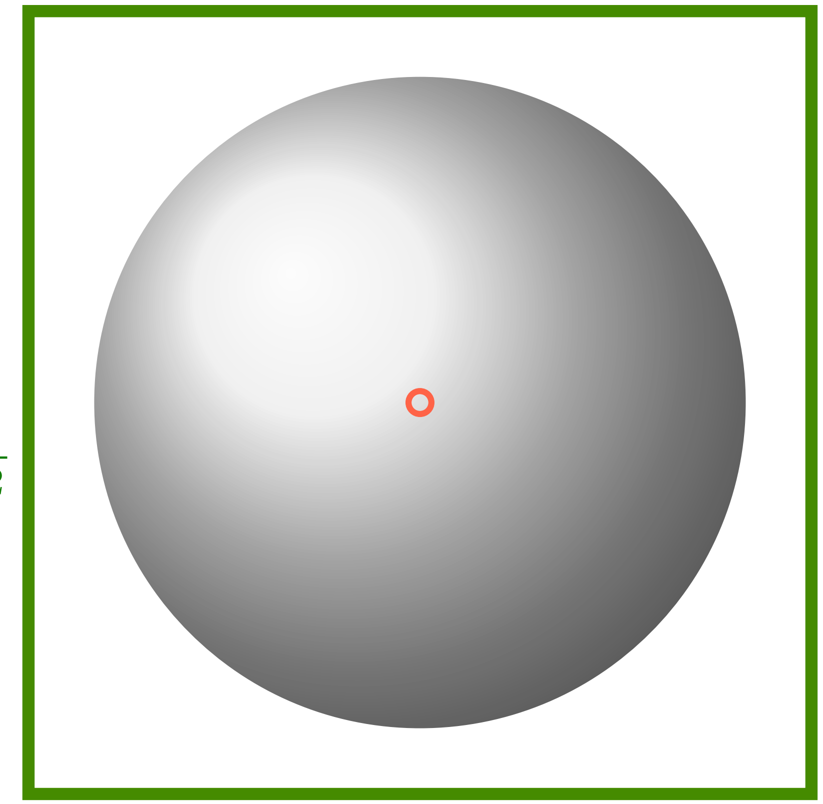
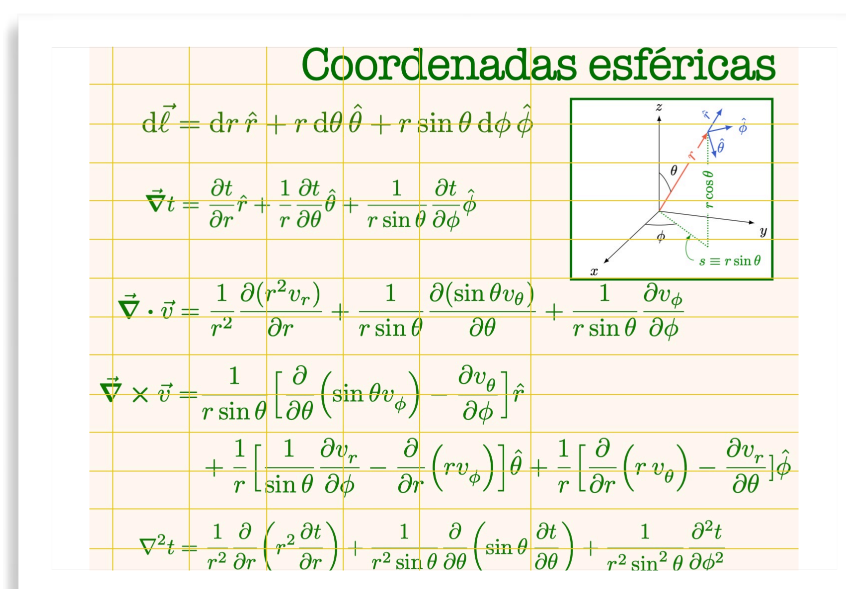


$$\Theta(\theta)\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + R(r)\frac{1}{r^2\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

$\Theta(\theta)\,R(r)$ $\qquad$ $\Theta(\theta)\,R(r)$



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$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

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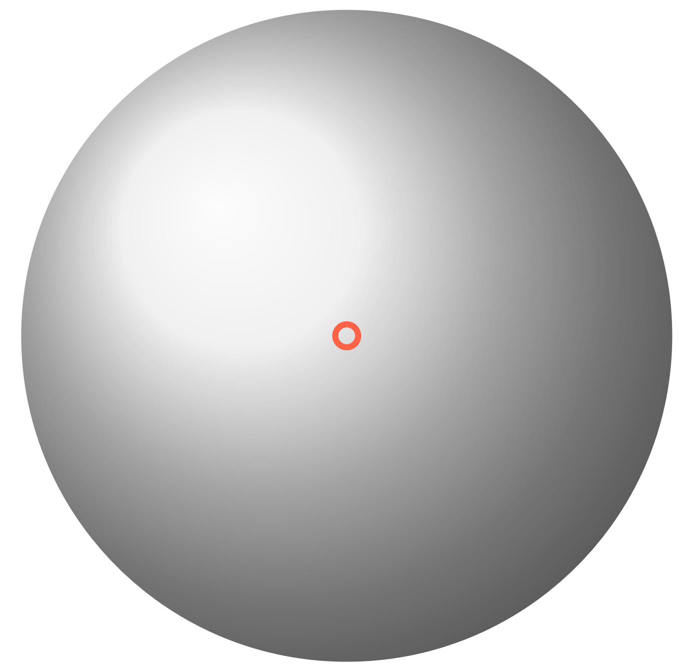


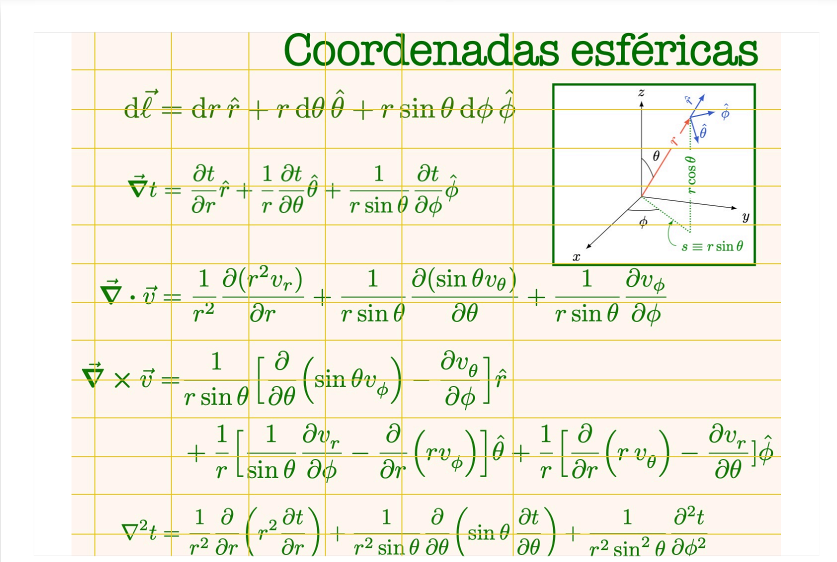
$$\Theta(\theta)\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + R(r)\frac{1}{r^2\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \frac{1}{\Theta}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

CONSTANTE — CHAMAREMOS DE $\ell(\ell+1)$

$-$CONSTANTE $= -\ell(\ell+1)$

↳ SÓ PORQUE É CONVENIENTE



Coordenadas esféricas

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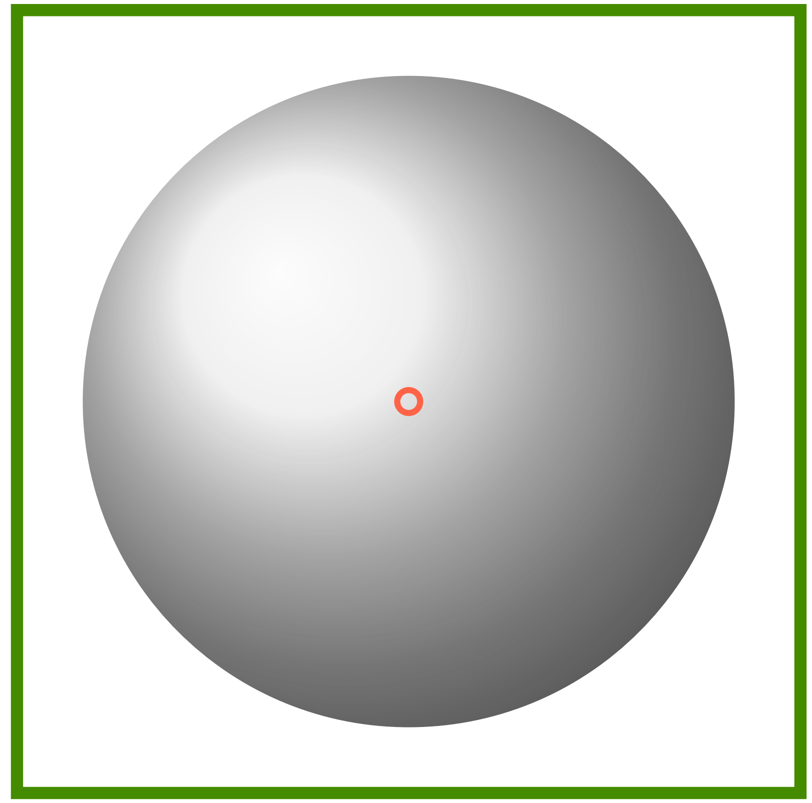
Equação de Laplace
Separação de variáveis



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$$\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) + \frac{1}{\Theta}\frac{1}{\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\sin\theta\frac{\mathrm{d}\Theta}{\mathrm{d}\theta}\right) = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) = \ell(\ell+1)R$$

PROCURAR SOLUÇÃO DA FORMA $R(r) = r^s$ → CONSTANTE A DETERMINAR

$$\frac{d}{dr}\left(r^2 s r^{s-1}\right) = \ell(\ell+1) r^s$$

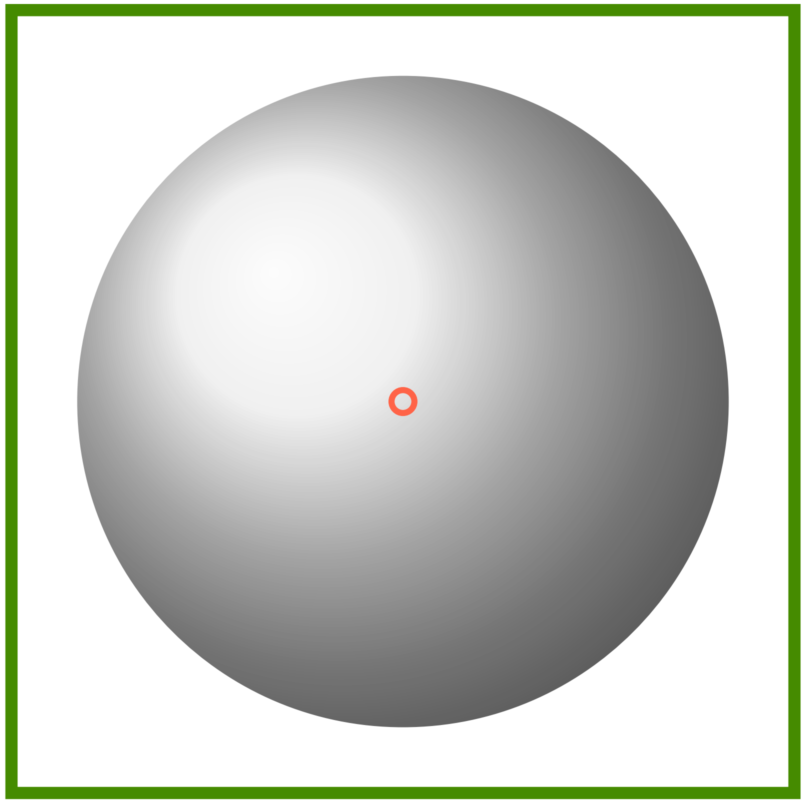
→ $s(s+1) \cancel{r^s} = \ell(\ell+1) \cancel{r^s}$. → EQ. DO SEGUNDO GRAU P/ S

DUAS SOLUÇÕES: $s = \begin{cases} \ell \\ -\ell-1 \end{cases}$ → $R(r) = A r^\ell + \dfrac{B}{r^{\ell+1}}$

$$\nabla^2 V = 0$$

Equação de Laplace
Separação de variáveis

Simetria esférica



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$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right) = \ell(\ell+1)R$$

$$R(r) = Ar^\ell + \frac{B}{r^{\ell+1}}$$