Exercicio

$$\rho = \frac{k}{r^2}$$

1. **2.23** Para a configuração do Problema 2.15 (casca esférica carregada com densidade $\rho = k/r^2$), encontre o potencial no centro, tomando o infinito como ponto de referência.

Problem 2.23

$$V(0) = -\int_\infty^0 \mathbf{E}\cdot d\mathbf{l} = -\int_\infty^b \left(\frac{k}{\epsilon_0}\frac{(b-a)}{r^2}\right)dr - \int_b^a \left(\frac{k}{\epsilon_0}\frac{(r-a)}{r^2}\right)dr - \int_a^0 (0)dr = \frac{k}{\epsilon_0}\frac{(b-a)}{b} - \frac{k}{\epsilon_0}\left(\ln\left(\frac{a}{b}\right) + a\left(\frac{1}{a} - \frac{1}{b}\right)\right)$$

$$= \frac{k}{\epsilon_0}\left\{1 - \frac{a}{b} - \ln\left(\frac{a}{b}\right) - 1 + \frac{a}{b}\right\} = \boxed{\frac{k}{\epsilon_0}\ln\left(\frac{b}{a}\right).}$$

$r < a$

$$\oiint \vec{E}\cdot d\vec{S} = \frac{Q_{int}}{\varepsilon_0}$$

$Q_{int} = 0 \Rightarrow \vec{E} = \vec{0}$

$a < r < b$

$$\oiint \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}\int \rho\, d\tau = \frac{1}{\varepsilon_0}\int \rho\, r'^2 dr' \,\mathrm{sen}\,\theta\, d\theta\, d\phi$$

$$E \times 4\pi r^2 = \frac{1}{\varepsilon_0}\int \frac{k}{\cancel{r'^2}}\cancel{r'^2}\, dr' \,\mathrm{sen}\,\theta\, d\theta\, d\phi$$

$$= \frac{k}{\varepsilon_0}\int_a^r dr \int_0^\pi \mathrm{sen}\,\theta\, d\theta \int_0^{2\pi} d\phi$$

$$\boxed{\frac{k}{\varepsilon_0}\ln\frac{b}{a}}$$

$$E \times 4\pi r^2 = \frac{1}{\varepsilon_0} k \left[r' \right]_a^r \times \left[-\cos\theta \right]_0^\pi \left[\phi \right]_0^{2\pi}$$

$$= \frac{1}{\varepsilon_0} (r-a)(+1+1) \times 2\pi$$

$$E = \frac{\cancel{4\pi}(r-a)k}{\varepsilon_0 \times \cancel{4\pi} r^2} \Rightarrow \vec{E} = \frac{k(r-a)}{\varepsilon_0 r^2} \vec{e_r}$$

$r > b$

$$\int \vec{E} \cdot \vec{ds} = \frac{Q_{int}}{\varepsilon} = \frac{1}{\varepsilon_0} \int_\tau \rho \, d\tau = \int_\tau \frac{k}{\cancel{r'^2}} \cancel{r'^2} \operatorname{sen}\theta \, d\theta \, d\phi \, dr'$$

$$E \times 4\pi r^2 = \frac{2\pi \times 2}{\varepsilon_0} \int_a^b k \, dr' \Rightarrow \vec{E} = \frac{\cancel{4\pi} k \left[r \right]_a^b}{\cancel{4\pi} \varepsilon_0 r^2} \vec{e_r}$$

$$\boxed{\vec{E} = \frac{k(b-a)}{\varepsilon_0 r^2} \vec{e_r}}$$

Potencial $\quad V_\infty = 0$

$$V(0) - \underset{\overset{\parallel}{0}}{V_\infty} = -\int_\infty^0 \vec{E}\cdot\vec{dl} = -\int_\infty^b \vec{E}\cdot\vec{dl} - \int_b^a \vec{E}\cdot\vec{dl} - \int_a^0 \vec{E}\cdot\vec{dl}$$

$$= -\int_\infty^b \frac{k(b-a)}{\varepsilon_0 \, r^2}\cdot dr - \int_b^a \frac{k(r-a)}{\varepsilon_0 \, r^2}\, dr + 0$$

$$V(0) = \frac{k(b-a)}{\varepsilon_0}\int_\infty^b \frac{dr}{r^2} - \frac{k}{\varepsilon_0}\int_b^a \frac{r-a}{\varepsilon_0 r^2}\, dr$$

$$V(0) = \frac{k}{\varepsilon_0}(b-a)\left[-\frac{1}{r}\right]_\infty^b - \frac{k}{\varepsilon_0}\left(\int_b^a \frac{\cancel{r}\,dr}{\varepsilon_0 r^{\cancel{2}}} - \int_b^a \frac{a\,dr}{r^2}\right)$$

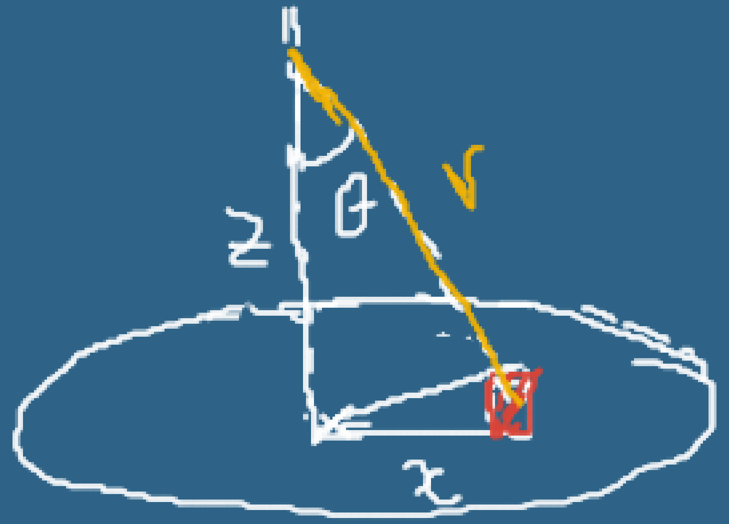
$$V(0) = \frac{k(b-a)}{\varepsilon_0}\left(-\frac{1}{b} + \frac{1}{\infty}\right) - \frac{k}{\varepsilon_0}\left[\left(\ln r\right)_b^a + a\left(-\frac{1}{r}\right)_b^a\right]$$

$$V(0) = -\frac{k\,(b-a)}{\varepsilon_0\, b} - \frac{k}{\varepsilon_0}\left(\ln\frac{a}{b} + a\left(-\frac{1}{a} + \frac{1}{b}\right)\right)$$

$$V(0) = -\frac{\cancel{k b}}{\cancel{\varepsilon_0 b}} + \frac{k\cancel{a}}{\cancel{b\,\varepsilon_0}} - \frac{k}{\varepsilon_0}\ln\frac{a}{b} + \frac{\cancel{k a}}{\cancel{a\,\varepsilon_0}} - \frac{\cancel{k a}}{\cancel{b\,\varepsilon_0}}$$

$$\boxed{V(0) = \frac{k}{\varepsilon_0}\ln\frac{a}{b}}$$

3. **2.25** Encontre o potencial a uma distância z acima do centro da distribuição de cargas em forma de disco na figura 1. Calcule $\vec{E} = -\vec{\nabla} V$ e compare com o campo elétrico calculado diretamente a partir da distribuição de cargas.



$$V = \int \frac{dq}{4\pi\varepsilon_0 r}$$

$$dq = \sigma\, ds$$

$$r^2 = x^2 + z^2 \Rightarrow r = (x^2+z^2)^{1/2}$$

$$ds = x\, d\phi\, dx$$

$$V = \int_0^R \int_0^{2\pi} \frac{\sigma\, x\, dx\, d\phi}{4\pi\varepsilon_0 (x^2+z^2)}$$

$$V = \frac{2\pi\sigma}{4\pi\varepsilon_0} \int_0^R \frac{x\, dx}{(x^2+z^2)^{1/2}}$$

$$V = \frac{2\pi\sigma}{4\pi\varepsilon_0} \left[\sqrt{x^2+z^2} \right]_0^R$$

$$V = \frac{2\pi\sigma}{4\pi\varepsilon_0} \left(\sqrt{R^2+z^2} - |z| \right)$$

$$\vec{E} = -\vec{\nabla} V = -\frac{2\pi\sigma}{4\pi\varepsilon_0} \left(\frac{1}{\cancel{2}} \times \cancel{2} z \left(R^2+z^2\right)^{-1/2} - 1 \right)$$

$$\vec{E} = -\frac{\cancel{2\pi}\sigma}{2\cancel{4}\pi\varepsilon_0} \left(\frac{z}{\sqrt{R^2+z^2}} - 1 \right) \vec{e_z}$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0} \left(1 - \frac{z}{\sqrt{R^2+z^2}} \right) \vec{e_z}$$

$$d\vec{E} = \frac{dq \cos\theta}{4\pi\varepsilon_0 r^2} \vec{e_z} \; ; \; \cos\theta = \frac{z}{r}$$

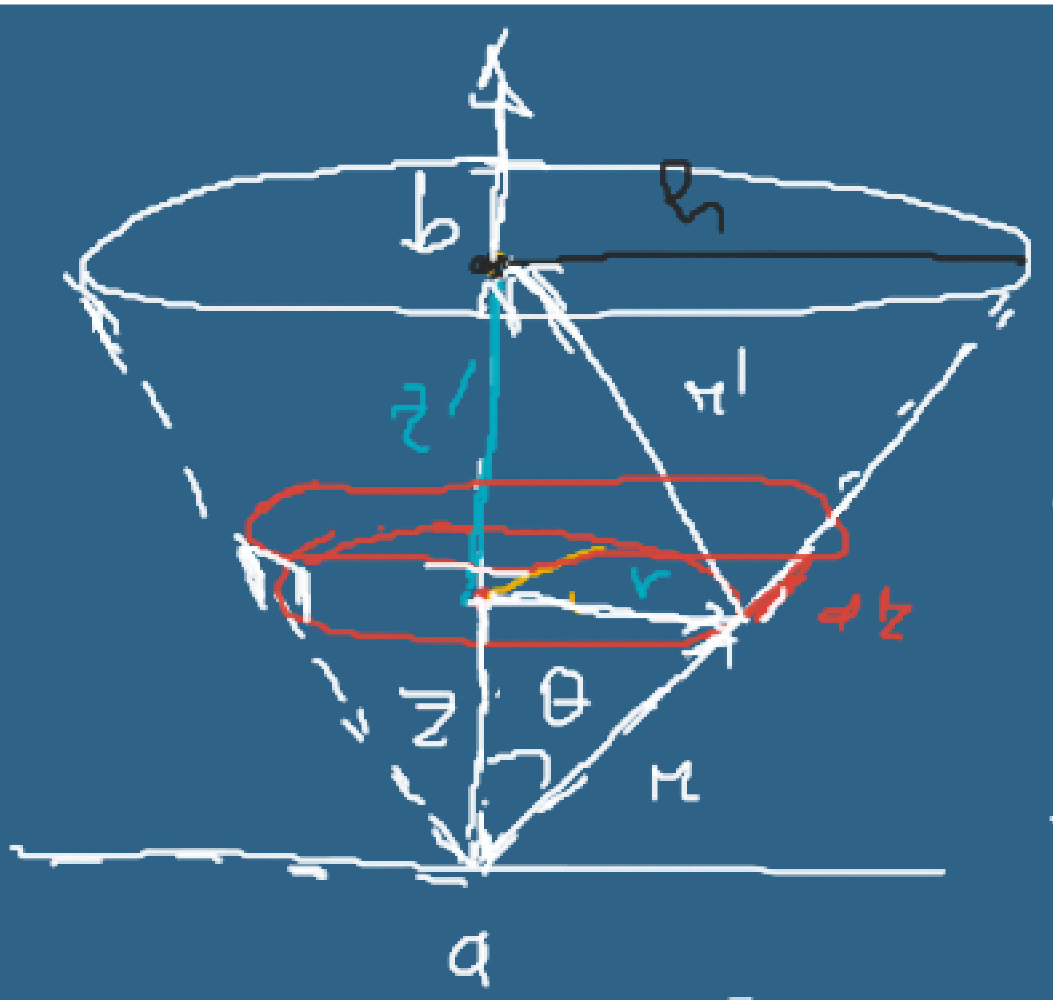
$$d\vec{E} = \frac{dq\, z}{4\pi\varepsilon_0 r^3} \vec{e_z}$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{x\,dx}{(x^2+z^2)^{3/2}} = \int_0^R \frac{x\,dx}{(x^2+z^2)^{3/2}} = \left[\frac{1}{\sqrt{x^2+z^2}}\right]_0^R$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{\sqrt{R^2+z^2}} - \frac{1}{\sqrt{z^2}}\right)\vec{e_r}$$

$$\boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{\sqrt{R^2+z^2}} - \frac{1}{|z|}\right)\vec{e_r}}$$

4. **2.26** Uma superfície cônica (formato de uma casquinha de sorvete, vazia) está carregada com densidade superficial uniforme σ. A altura da casquinha é h, igual ao raio do topo. Encontre a diferença de potencial entre os pontos $\vec{a}$ (vértice) e $\vec{b}$ (o centro do topo).



$$V(a) = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{\mathcal{R}}$$

$$dq = \sigma\, ds = \sigma\, r\, d\theta\, d\mathcal{R}$$

$$\mathcal{R}^2 = z^2 + r^2 = 2r^2$$

$$V(a) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\, 2\pi r\, d\mathcal{R}}{\mathcal{R}}$$

$$V(a) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\, 2\pi r\, dr}{\mathcal{R}} \quad ; \quad \frac{r}{h} = \frac{z}{h} \Rightarrow r = z$$

$$V(a) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\, 2\pi r\, d\mathcal{R}}{\mathcal{R}}$$

$$\mathcal{R}^2 = r^2 + r^2 = 2r^2$$

$$\mathcal{R} = \sqrt{2}\, r$$

$$V(a) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\, 2\pi \cancel{r}}{\sqrt{2}\,\cancel{r}}\, d\mathcal{R} = \frac{1}{4\pi\varepsilon_0}\int \frac{2\pi\sigma}{\sqrt{2}}\, d\mathcal{R}$$

$$V(r) = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^{\sqrt{2}h} \frac{d\mathcal{R}}{\sqrt{2}}$$

$$V(r) = \frac{2\pi\sigma}{4\pi\varepsilon_0\sqrt{2}}\Big[\mathcal{R}\Big]_0^{\sqrt{2}h}$$

$$V = \frac{2\pi\sigma}{2\, 4\pi\varepsilon_0 \cancel{\sqrt{2}}}\left(\cancel{\sqrt{2}}\, h\right)$$

$$\boxed{V = \frac{\sigma h}{2\varepsilon_0}}$$

$$\vec{z}\,' = \vec{\mathcal{R}}\,' + \vec{r}$$

$$\vec{\mathcal{R}}\,' = \vec{h} - \vec{r}$$

$$= h^2 + r^2 - 2h$$

$$V(b) = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{\mathcal{R}'}$$

$$V(b) = \frac{1}{4\pi\varepsilon_0}\int_0 \frac{\sigma\, ds}{} = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma\, r\, d\theta\, dz}{\mathcal{R}'}$$

$$V(b) = \frac{1}{4\pi\varepsilon_0}\int \frac{\delta\, r\, d\theta\, dr}{r'} = \int \frac{\delta\, r\, d\theta\, dr}{r'} \quad ; \quad r'^2 = z'^2 + r^2$$

$$\frac{z'}{h} = \frac{r}{h} \Rightarrow r'^2 = 2r^2$$

$$\vec{r}' + \vec{r} = \vec{h} \Rightarrow r'^2 = (\vec{h} - \vec{r})^2$$

$$r'^2 = h^2 + r^2 - \sqrt{2}\, h\, r$$

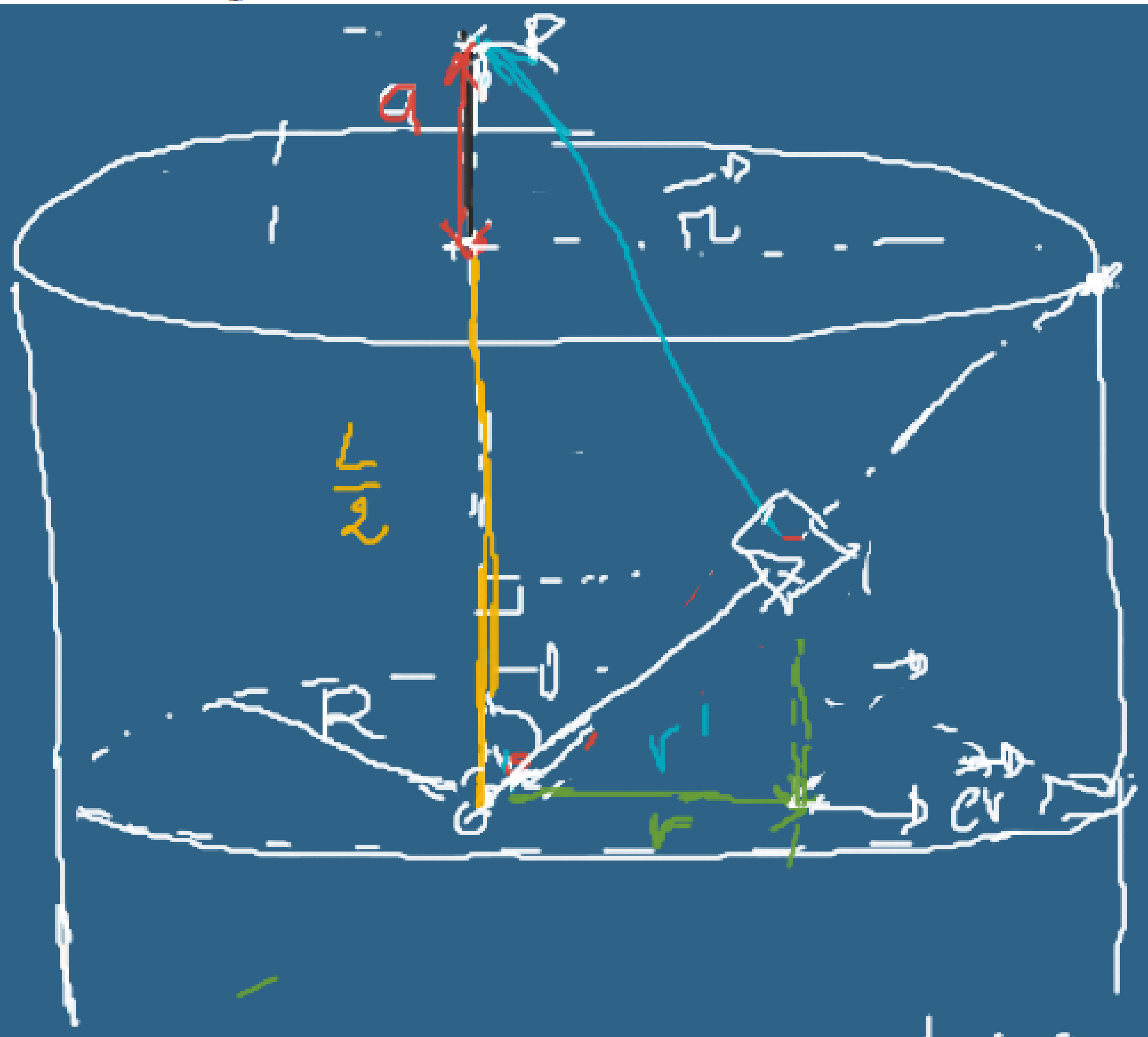
$$\frac{r}{\cancel{h}} = \frac{z'}{\cancel{h}} \qquad V(b) = \frac{1}{4\pi\varepsilon_0}\int_0^{\sqrt{2}h} \frac{\delta\, r\, d\theta\, dr}{\sqrt{r^2 + h^2 - \sqrt{2}\, r h}} = \frac{\delta \times 2\pi}{4\pi\varepsilon_0}\int_0^{\sqrt{2}h} \frac{r\, dr}{\sqrt{r^2 + h^2 - \sqrt{2}\, r h}}$$

$$V(b) = \frac{2\pi\delta}{4\pi\varepsilon_0}\int_0^{\sqrt{2}h} \frac{r\, dr}{\sqrt{r^2 + h^2 - \sqrt{2}\, r h}} \quad ; \quad V(b) = \frac{\delta h}{2\varepsilon_0}\ln(1 + \sqrt{2})$$

$$V(a) = \frac{\delta h}{2\varepsilon_0}$$

$$\boxed{V(a) - V(b) = \frac{\delta h}{2\varepsilon_0}\left[1 - \ln(1 + \sqrt{2})\right]}$$

5. **2.27** Um cilindro tem comprimento L, raio R e densidade uniforme de carga ρ Encontre o potencial no eixo de um cilindro sólido uniformemente carregado, a uma distância $z > L/2$ do centro.. Aproveite o resultado para calcular o campo elétrico no mesmo ponto.



$$V = \int \frac{\rho \, d\tau}{4\pi\varepsilon_0 \eta}$$

$$A = a + \frac{L}{2}$$

$$\vec{r'} = r\vec{er} + z'\vec{e_z}$$

$$\vec{A} = \vec{\eta} + \vec{r'} \Rightarrow \vec{\eta} = \vec{A} - \vec{r'}$$

$$\vec{\eta} = \vec{A} - r\vec{er} - z'\vec{e_z}$$

$$\vec{\eta} = (A - z')\vec{e_z} - r\vec{er}$$

$$\vec{er} = \vec{es}$$

$$S = r$$

$$d\tau = r\,d\phi\,dr\,dz', \quad \eta = \sqrt{(A-z') + r^2}$$

$$V = \int_{\tau} \frac{\rho\, d\tau}{4\pi\varepsilon_0 \eta} = \int_{\tau} \frac{\rho\, r\, d\phi\, dr\, dz'}{4\pi\varepsilon_0\sqrt{(z-z') + r^2}}$$

$$V = \int_0^{2\pi} \rho\, d\phi \int_{-L/2+a}^{\frac{L}{2}+a} dz' \int_0^R \frac{r\,dr}{4\pi\varepsilon_0\sqrt{(A-z') + r^2}} = \frac{\rho \times 2\pi}{4\pi\varepsilon_0} \int_{-L/2}^{\frac{L}{2}+a} \frac{1}{2}\left[\sqrt{r^2 + (A-z')^2}\right]_0 dz'$$

$$V = \frac{2\pi \rho}{4\pi \varepsilon_0} \int \frac{1}{2} \left[\sqrt{r^2 + (A - z')^2} \right]_0^R dz'$$

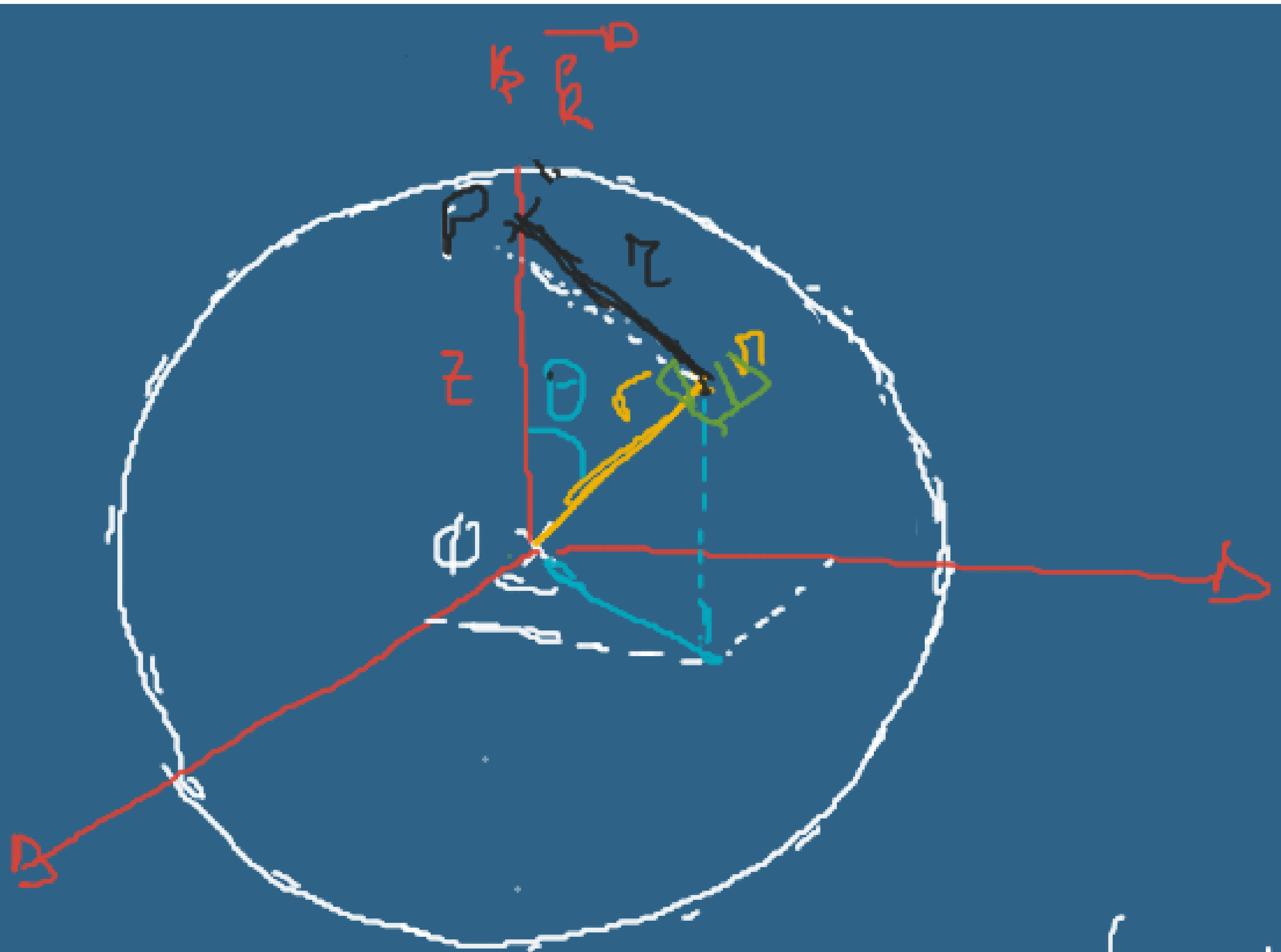
$$A = \frac{L}{2} + a$$

$$V = \frac{2\pi \rho}{4\pi \varepsilon_0} \times \frac{1}{2} \int \left(\sqrt{R^2 + (A - z')^2} - \sqrt{(z - z')^2} \right) dz'$$

$$V = \frac{2\pi \rho}{2 \times 4\pi \varepsilon_0} \left[\int_{-\frac{L}{2}+a}^{\frac{L}{2}+a} \sqrt{R^2 + (A - z')^2}\, dz + \int_{-\frac{L}{2}+a}^{\frac{L}{2}+a} (A - z')\, dz' \right]$$

$$V = \frac{2\pi \rho}{2 \times 4\pi \varepsilon_0} \int_{-L/2}^{\frac{L}{2}+a} \sqrt{R^2 + (A - z')^2}\, dz + \frac{2\pi \rho}{2 \times 4\pi \varepsilon_0} \left[-\frac{1}{2} (A - z')^2 \right]_{-L/2+a}^{\frac{L}{2}+a}$$

6. **2.28** Use a Eq. 2.29 para calcular o potencial dentro de uma esfera sólida uniformemente carregada com raio R e carga total q. Compare o resultado com o da questão 9 da segunda lista.



$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, d\tau}{\mathscr{r}}$$

$$\vec{r} + \vec{\mathscr{r}} = \vec{z} \Rightarrow \vec{\mathscr{r}} = |\vec{z} - \vec{r}|$$

$$\mathscr{r}^2 = |\vec{z} - \vec{r}| = z^2 + r^2 - 2rz\cos\theta$$

$$\mathscr{r}^2 = z^2 + r^2 - 2rz\cos\theta$$

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho\, r^2 \operatorname{sen}\theta\, d\theta\, d\phi\, dr}{\sqrt{z^2 + r^2 - 2rz\cos\theta}}$$

$$V = \frac{\rho \times 2\pi}{4\pi\varepsilon_0}\int \frac{r^2\, dr\, \operatorname{sen}\theta\, d\theta}{\sqrt{z^2 + r^2 - 2rz\cos\theta}}$$

$$\begin{cases} u = \cos\theta \\ du = \operatorname{sen}\theta\, d\theta \end{cases}$$

$$V = \frac{2\pi\rho}{4\pi\varepsilon_0}\int_0^R\int_{-1}^{1} \frac{r^2\, dr\, du}{(z^2 + r^2 - 2rzu)^{1/2}}$$

$$\int_{-1}^{1} \frac{du}{(z^2 + r^2 - 2rz} = ?$$

$$I = \int_{-1}^{1} \frac{dy}{\sqrt{z^2+r^2-2zru}} = \frac{1}{rz}\left[\sqrt{z^2+r^2-2zru}\right]_{-1}^{1}$$

$$Q = \rho V$$

$$\frac{3Q}{R^3} = \rho \frac{4\pi \cancel{R^3}}{\cancel{3}}$$

$$I = \frac{1}{rz}\left(\sqrt{z^2+r^2+2zr} - \sqrt{z^2+r^2-2rz}\right)$$

Se $r<z \Rightarrow r-z<0$

$$I = \frac{z+r-|-r+z|}{rz} = \frac{2\cancel{r}}{\cancel{r}z} = \frac{2}{z}$$

$$I = \frac{1}{rz}\left(|z+r| - |r-z|\right)$$

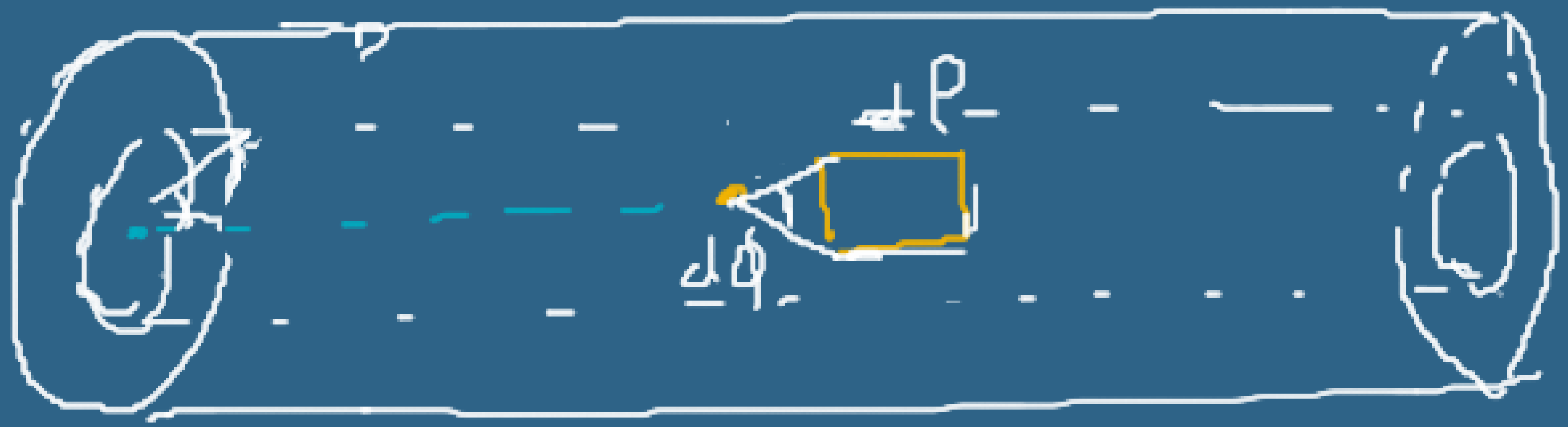
se $r>z \Rightarrow I = \frac{2}{r}$

$$I = \begin{cases} \frac{2}{z} & \text{se } r<z \\ \frac{2}{r} & \text{se } r>z \end{cases} \Rightarrow V = \frac{2\pi\rho}{4\pi\varepsilon_0}\left(\int_0^z \frac{2}{z} r^2\,dr + \int_z^R \frac{2}{\cancel{r}} r^{\cancel{2}}\,dr\right)$$

$$V = \frac{2\pi\rho}{4\pi\varepsilon_0}\left(\left[\frac{2}{z}\times\frac{r^3}{3}\right]_0^z + \left[\frac{2r^2}{2}\right]_z^R\right) = \frac{2\pi\rho}{4\pi\varepsilon_0}\left(\frac{2z^{\cancel{3}2}}{3\cancel{z}} + \frac{2R^2}{2} - \frac{2z^2}{2}\right)$$

$$V = \frac{4\pi\rho}{4\pi\varepsilon_0}\left(\frac{z^2}{3} + \frac{R^2-z^2}{2}\right) = \frac{3Q}{4\pi\varepsilon_0 R^3}\left(\frac{2z^2-3z^2}{6}+\frac{R^2}{2}\right) = \frac{3Q}{4\pi\varepsilon_0 R^3}\left(\frac{R^2}{2}-\frac{z^2}{6}\right)$$

7. **2.30(b)** Use a lei de Gauss para encontrar o campo elétrico dentro e fora de um casca cilíndrica muito comprida, carregada uniformemente com densidade superficial σ. Verifique que o resultado é consistente com a Eq. 2.33.



lei de Gauss

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{int}}{\varepsilon_0}$$

se $r < R \Rightarrow Q_{int} = 0$

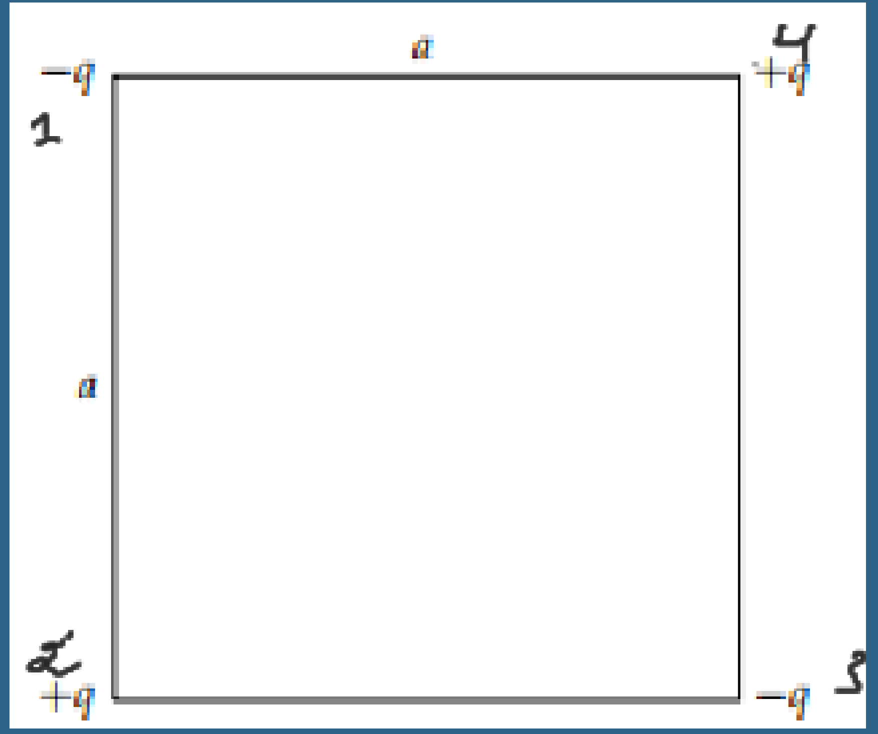
se $r > R \Rightarrow Q_{int} = \int_S \sigma \, dS$

$$\oint \vec{E} \cdot d\vec{S} = \int \sigma \, d\vec{S} \Rightarrow \int_0^{2\pi}\int_0^{L} E \cdot s \, d\phi \, d\ell = \int_0^{2\pi}\int_0^{L} \frac{\sigma R \, d\phi \, d\ell}{\varepsilon_0}$$

$$\cancel{2\pi} s E \cancel{L} = \frac{\sigma}{\varepsilon_0} R \times \cancel{2\pi} \cancel{L}$$

$$\boxed{E = \frac{\sigma R}{\varepsilon_0 s}}$$

8. **2.31(b)** Qual o trabalho necessário para montar a configuração de quatro cargas no retângulo da figura 2.



$$W = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\sum_{j>i}\frac{q_i q_j}{r_{ij}}$$

$$W_{total} = W_{12} + W_{13} + W_{14} + W_{23} + W_{24} + W_{34}$$

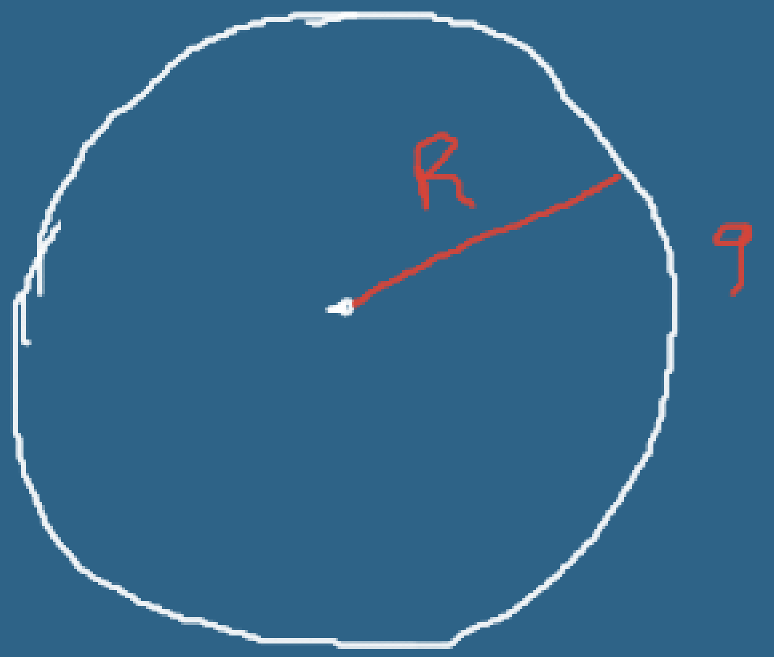
$$W_{12} = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} = \frac{-q^2}{4\pi\varepsilon_0 a}, \quad W_{13} = \frac{q_1 q_3}{4\pi\varepsilon_0 r_1 r_3} = \frac{q^2}{4\pi\varepsilon_0\sqrt{2}a}; \quad W_{14} = \frac{-q^2}{4\pi\varepsilon_0 a}$$

$$W_{23} = \frac{-q^2}{4\pi\varepsilon_0 a}; \quad W_{24} = \frac{q^2}{4\pi\varepsilon_0\sqrt{2}a}; \quad W_{34} = \frac{-q^2}{4\pi\varepsilon_0 a}$$

$$W_{total} = \frac{-4q^2}{4\pi\varepsilon_0 a} + \frac{2q^2}{4\pi\varepsilon_0\sqrt{2}a} = \frac{q^2}{4\pi\varepsilon_0 a}\left(-4 + \frac{2}{\sqrt{2}}\right) = \frac{2q^2}{4\pi\varepsilon_0 a}\left(-2 + \frac{1}{\sqrt{2}}\right)$$

$$W_{total} = \frac{2q^2}{4\pi\varepsilon_0 a}\left(-2 + \frac{1}{\sqrt{2}}\right)$$

9. **2.32** Encontre a energia armazenada em uma esfera sólida de raio R, carregada uniformemente com carga total q.



$$U = \frac{\varepsilon_0}{2}\int E^2 dV$$

$$U = \frac{\varepsilon_0}{2}\int E^2 dV$$

$$U = \frac{\varepsilon_0}{2}\int E^2 dV = \frac{\varepsilon_0}{2}\int\left(\frac{q}{4\pi\varepsilon_0 r^2}\right)^2 d\tau$$

$$d\tau = r^2 \operatorname{sen}\theta \, d\theta \, d\phi \, dr$$

$$U = \frac{\varepsilon_0}{2}\iiint \frac{q^2}{(4\pi\varepsilon_0 r^2)^2} r^2 \operatorname{sen}\theta \, d\theta \, d\phi \, dr$$

$$U = \frac{\varepsilon_0}{2}\int_0^{2\pi} d\phi \int_0^{\pi} \operatorname{sen}\theta \, d\theta \int \frac{q \cancel{r^2}}{(4\pi\varepsilon_0)^2 r^{\cancel{4}2}} dr$$

$$U = \frac{\varepsilon_0}{2} \times 2\pi \, (2) \int \frac{q}{(4\pi\varepsilon_0)^2} \frac{dr}{r^2}$$

$$U = \frac{\cancel{\varepsilon_0}}{2} \times \cancel{4\pi} \times \frac{q^2}{(4\pi\varepsilon_0)^{\cancel{2}}} \int \frac{dr}{r^2}$$

$$U = \frac{q^2}{8\pi\varepsilon_0}\left[-\frac{1}{r}\right]_R^{+\infty} = \frac{+q^2}{8\pi\varepsilon_0 R}$$

$$U = \frac{q^2}{8\pi\varepsilon_0 R}$$

10. **2.34** Considere duas cascas esféricas de raios a e b. Suponha que a interna tem carga q, e a externa, carga $-q$ (ambas uniformemente distribuídas sobre a superfície). Calcule a energia dessa configuração

(a) A partir do potencial a que está sujeita cada carga;

(b) A partir da integral de E^2.

Exo 10

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$\vec{E}_1 = \vec{E}_{\text{esfera } 1}$$

$$\vec{E}_2 = \vec{E}_{\text{esfera } 2}$$

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q_{int}}{\varepsilon_0}$$

A : $Q_{int} = 0 \Rightarrow \vec{E}_1 = 0 \; ; \; \vec{E}_2 = 0$

Zona B $\begin{cases} \vec{E}_1 = \dfrac{q}{4\pi\varepsilon_0 r^2} \\ \vec{E}_2 = 0 \end{cases} \Rightarrow \vec{E} = \dfrac{q}{4\pi\varepsilon_0 r^2}$

Zona C $\begin{cases} \vec{E}_1 = \dfrac{q}{4\pi\varepsilon_0 r^2} \\ \vec{E}_2 = \dfrac{-q}{4\pi\varepsilon_0 r^2} \end{cases}$ $\quad \vec{E} = \vec{E}_1 + \vec{E}_2 ; \quad \vec{E} = 0$

$$V_\infty = 0 \Rightarrow V = -\int_\infty^a \vec{E} \cdot d\vec{l}$$

$$\underbrace{V - V_\infty}_{\shortparallel \, 0} = -\int_\infty^0 \vec{E} \cdot d\vec{l}$$

$$V = -\underbrace{\int_\infty^b \vec{E} \cdot d\vec{l}}_{\shortparallel \, 0} - \int_b^a \vec{E} \cdot d\vec{l} - \underbrace{\int_a^0 \vec{E} \cdot d\vec{l}}_{\shortparallel \, 0}$$

$$V = -\int_b^a \vec{E} \cdot d\vec{l} = -\int_b^a \frac{q \, dr}{4\pi\varepsilon_0 r^2}$$

$$V = -\frac{q}{4\pi\varepsilon_0} \int_a^b \frac{dr}{r^2}$$

$$V = \frac{q}{4\pi\varepsilon_0} \left[-\frac{1}{r}\right]_b^a = \frac{-q}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right)$$

Zona A = $\vec{E}_1 = \vec{0}$; Zona B $\vec{E} = \dfrac{q}{4\pi\varepsilon_0 r^2}\vec{e_r}$; Zona C = $\vec{E} = \vec{0}$

$$W = \frac{\varepsilon_0}{2}\int E^2 \, d\tau \qquad \begin{cases} r < a \\ a\,r < b \\ r > b \end{cases}$$

$$W = \frac{\varepsilon_0}{2}\int_a^b \frac{q^2}{(4\pi\varepsilon_0 r^2)^2}\, d\tau = \frac{q^2}{(4\pi\varepsilon_0)^2}\int_a^b \frac{1}{r^2}\, d\tau = \frac{q^2}{(4\pi\varepsilon_0)^2}\,\frac{\varepsilon_0}{2}\int_V \frac{\cancel{r^2}\,\mathrm{sen}\,\theta\, d\theta\, d\phi\, dr}{\cancel{r^2}}$$

$$W = \frac{\varepsilon_0}{2}\frac{q^2}{(4\pi\varepsilon_0)^2}\int_0^\pi \mathrm{sen}\,\theta\, d\theta \int_0^{2\pi} d\phi \int \frac{dr}{r^2} = \frac{\varepsilon_0}{2}\,\frac{q^2 \times 4\pi}{(4\pi\varepsilon_0)^2}\left[-\frac{1}{r}\right]_a^b = \frac{\cancel{4\pi\varepsilon_0}\, q^2}{2(4\pi\varepsilon_0)^{\cancel{2}}}\left[\frac{1}{r}\right]_a^b$$

$$\boxed{W = \frac{q^2}{4\pi\varepsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)}$$