Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

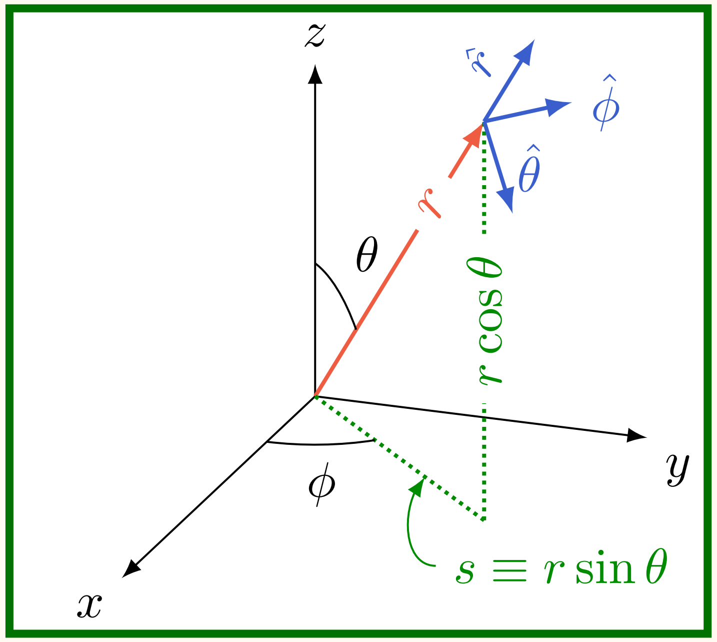
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

Aula de 26 de maio

Métodos especiais

Coordenadas esféricas



$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

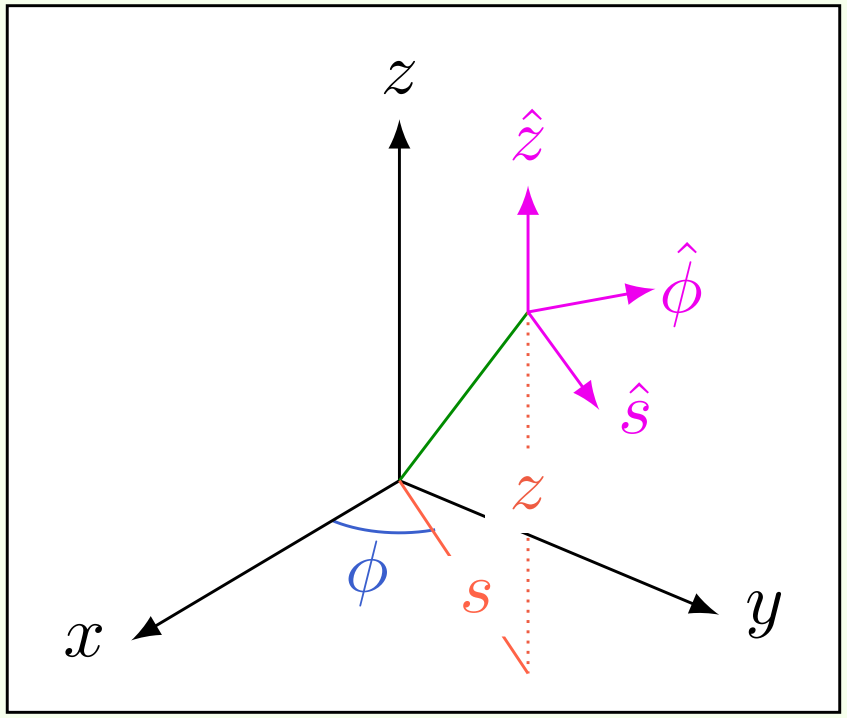
$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Coordenadas cilíndricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}s\,\hat{s} + s\,\mathrm{d}\phi\,\hat{\phi} + \mathrm{d}z\,\hat{z}$$



$$\vec{\nabla} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s}\frac{\partial (s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla} \times \vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Equação de Laplace

$$\nabla^2 V = 0$$

Equação de Laplace

$$\nabla^2 V = 0$$

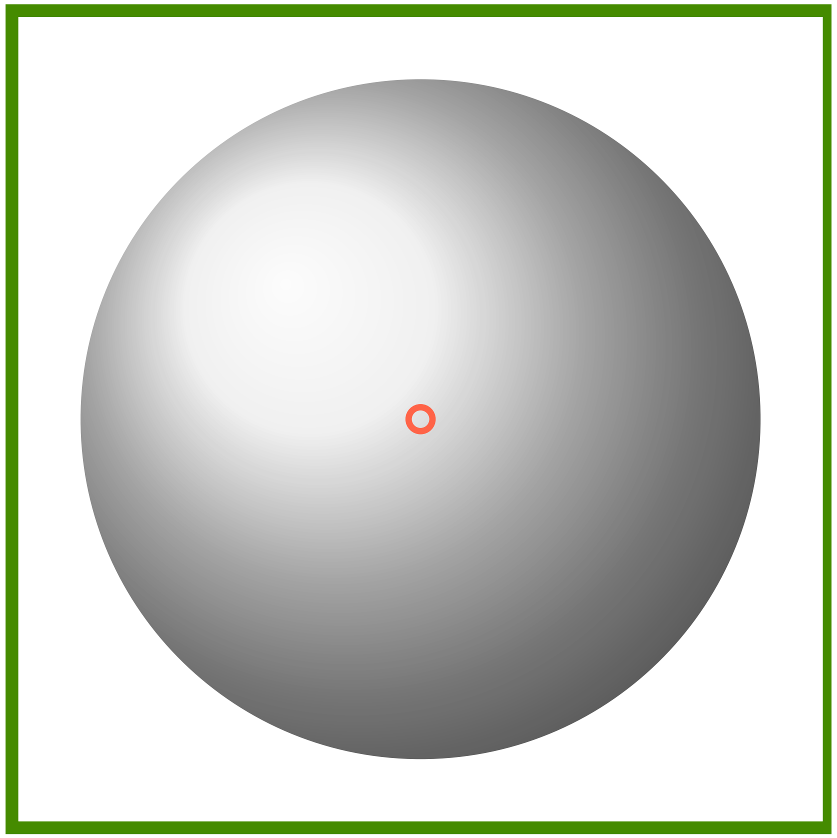
Propriedades

Equação de Laplace

$$\nabla^2 V = 0$$

Propriedades

1. $\displaystyle\int_A V(\vec{r})\,\mathrm{d}A = 4\pi R^2 V(0)$



Equação de Laplace

$$\nabla^2 V = 0$$

Propriedades

1. $\int_A V(\vec{r})\,\mathrm{d}A = 4\pi R^2 V(0)$

2. Condição de contorno: V na superfície

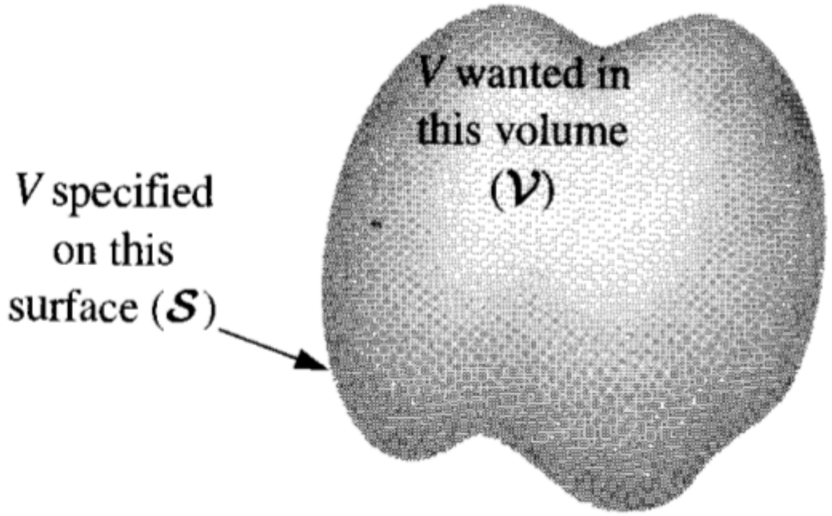


Figure 3.5

Equação de Laplace

$$\nabla^2 V = 0$$

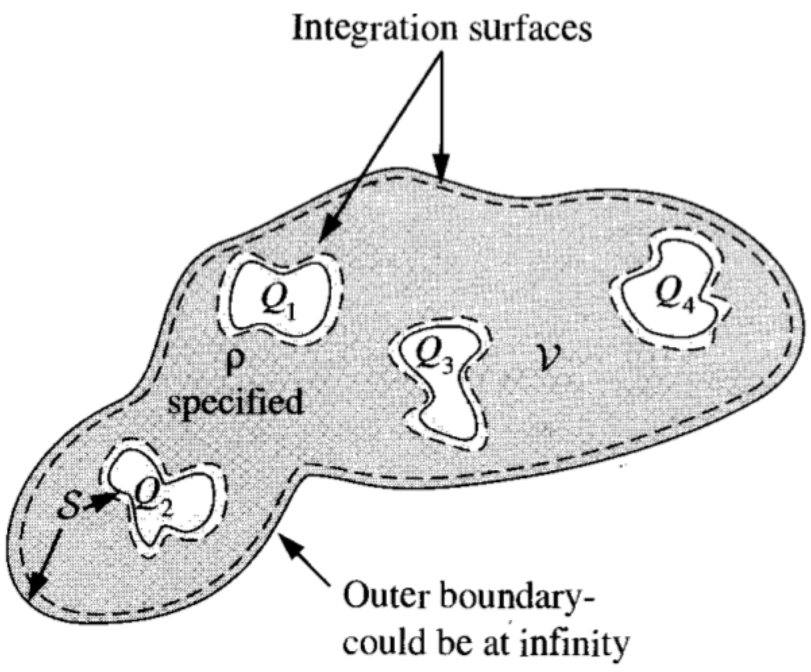


Figure 3.6

Propriedades

1. $\displaystyle\int_A V(\vec{r})\,\mathrm{d}A = 4\pi R^2 V(0)$

→ DEMONSTRAÇÃO A SEGUIR

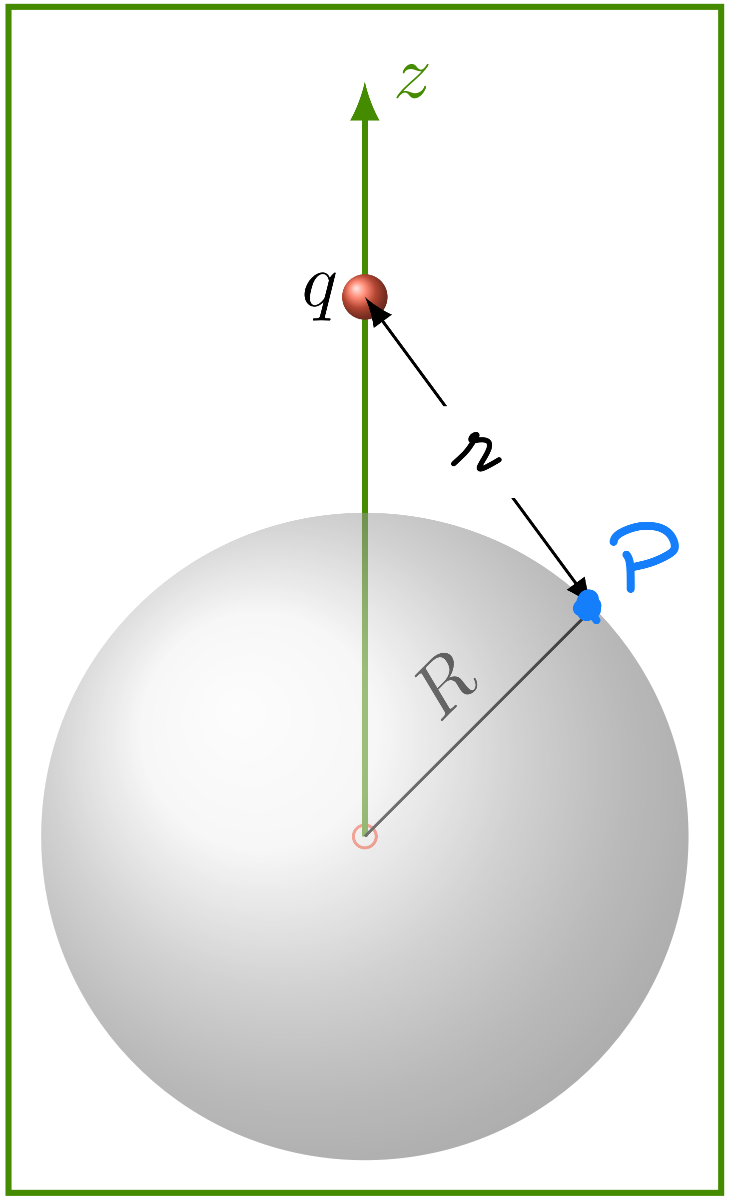
2. Condição de contorno: V na superfície

3. Condição de contorno para condutores:
Q em cada condutor

Equação de Laplace

$$\nabla^2 V = 0$$

1. $\int_A V(\vec{r})\ \mathrm{d}A = 4\pi R^2 V(0)$

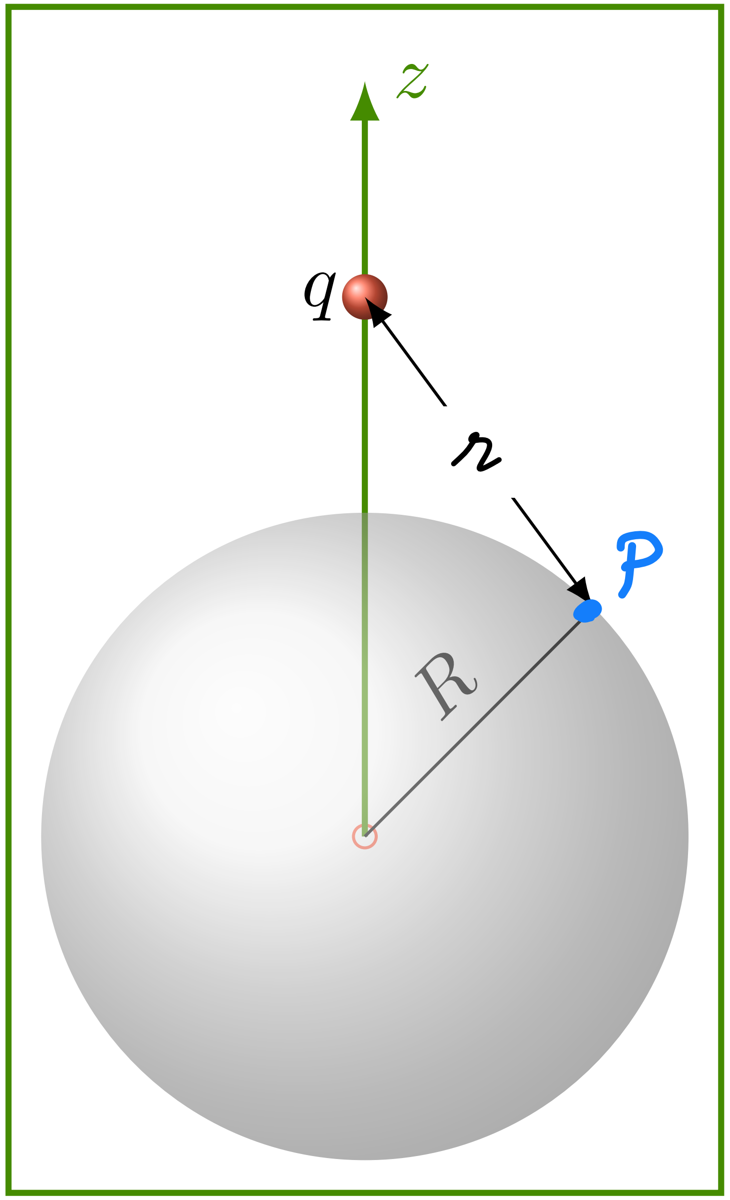


Equação de Laplace

$$\nabla^2 V = 0$$

1. $$\int_A V(\vec{r})\, \mathrm{d}A = 4\pi R^2 V(0)$$

$$V(P) = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r}}$$



Equação de Laplace

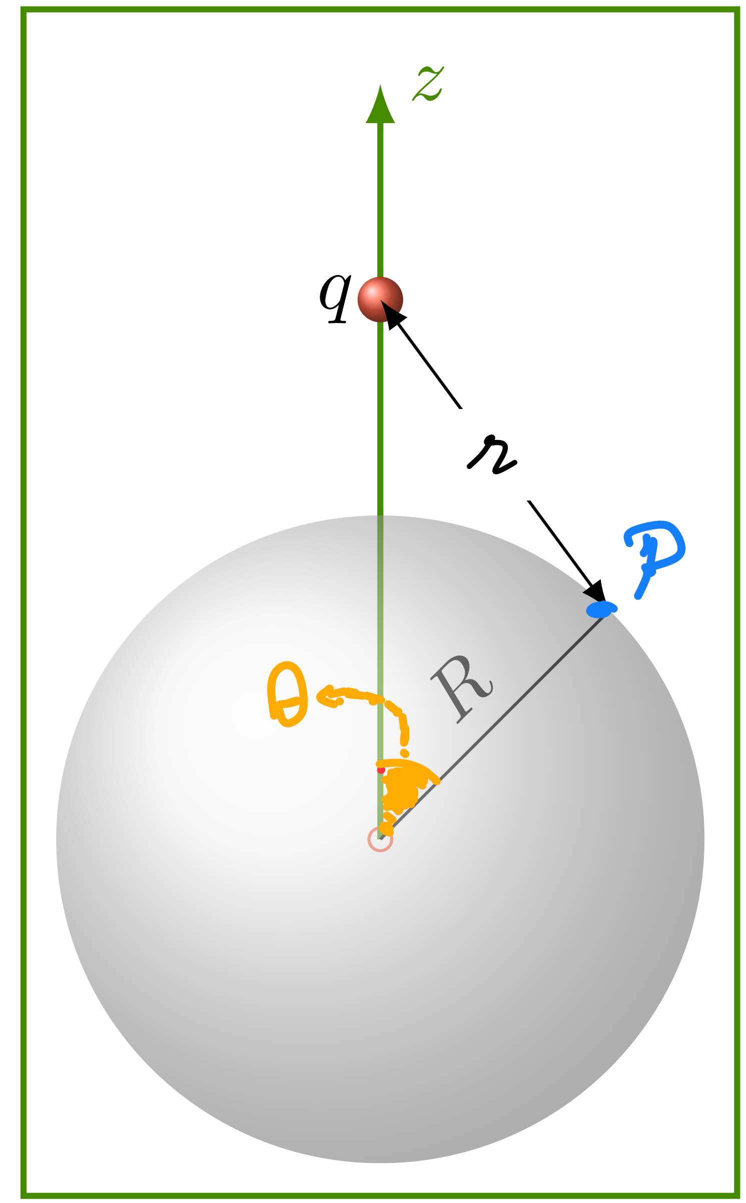
$$\nabla^2 V = 0$$

1. $$\int_A V(\vec{r})\, \mathrm{d}A = 4\pi R^2 V(0)$$

$$V(P) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

$$r = \sqrt{z^2 + R^2 - 2Rz\cos(\theta)}$$

→ LEI DOS COSSENOS



Equação de Laplace

$$\nabla^2 V = 0$$

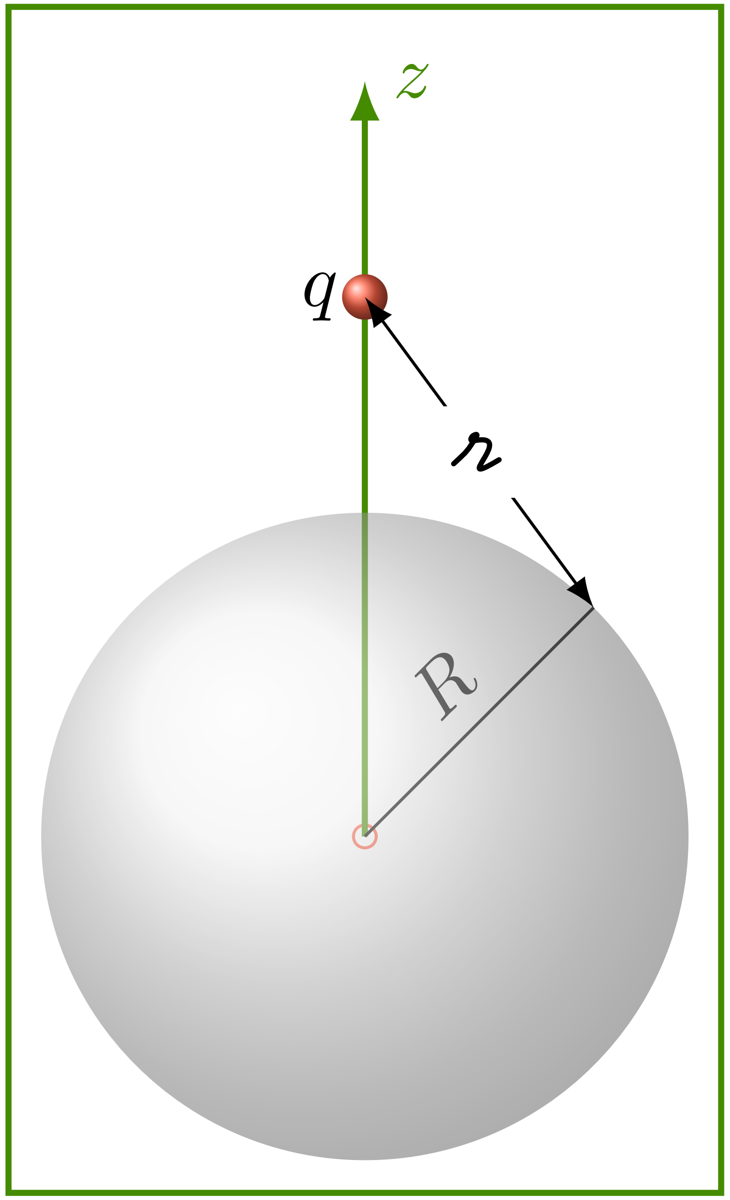
1. $$\int_A V(\vec{r})\,\mathrm{d}A = 4\pi R^2 V(0)$$

$$V(P) = \frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r}}$$

$$\mathscr{r} = \sqrt{z^2 + R^2 - 2Rz\cos(\theta)}$$

$$V_{\text{med}} = \frac{1}{4\pi R^2}\frac{q}{4\pi\epsilon_0}\int_0^{2\pi}\int_{-1}^{1}\frac{1}{\sqrt{z^2+R^2-2Rzu}}R^2\,\mathrm{d}u\,\mathrm{d}\phi$$

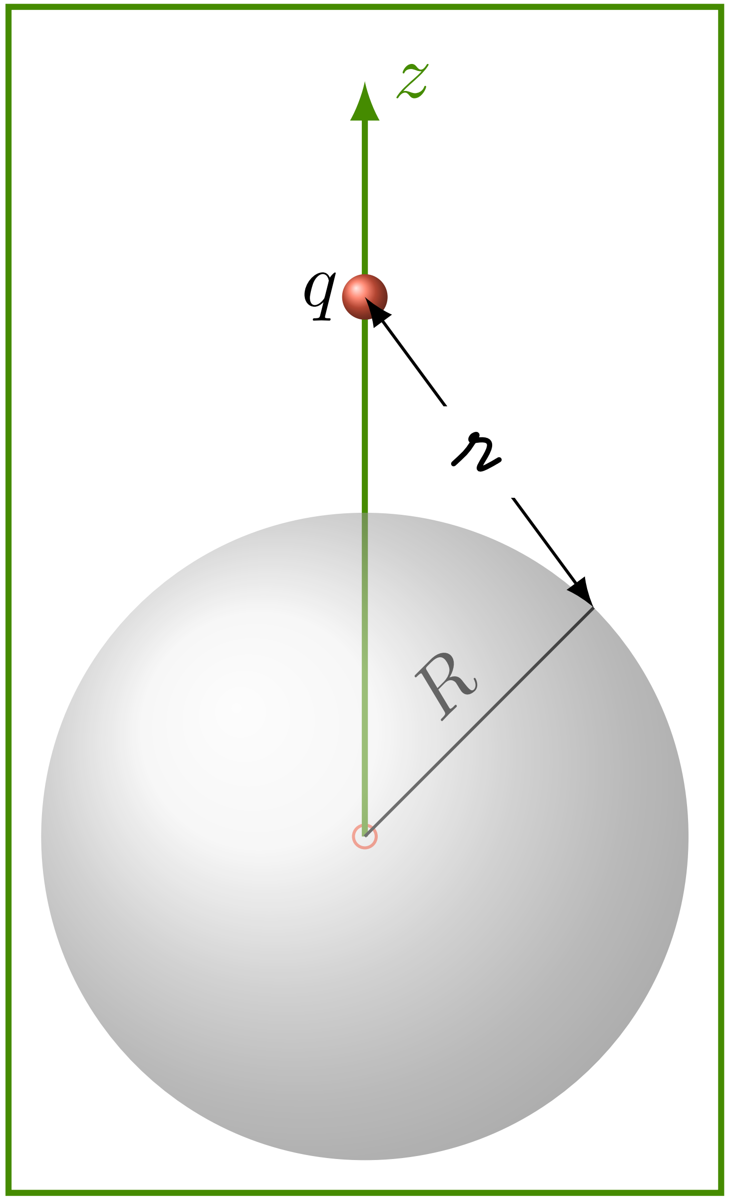
$$\hookrightarrow \frac{\int V(\vec{r})\,dA}{\int dA} = \frac{\int V(\vec{r})\,dA}{4\pi R^2}$$



Equação de Laplace

$$\nabla^2 V = 0$$

1. $$\int_A V(\vec{r})\, \mathrm{d}A = 4\pi R^2 V(0)$$



$$V(P) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

$$r = \sqrt{z^2 + R^2 - 2Rz\cos(\theta)}$$

$$V_{\text{med}} = \frac{1}{4\pi R^2}\frac{q}{4\pi\epsilon_0}\int_0^{2\pi}\int_{-1}^{1}\frac{1}{\sqrt{\underbrace{z^2 + R^2 - 2Rzu}_{w}}}R^2\, \mathrm{d}u\, \mathrm{d}\phi$$

$$\left\{ \begin{array}{l} dw = -2Rz\, du \\ w = \left\{ \begin{array}{ll} z^2+R^2-2Rz = (z-R)^2 & (u=1) \\ z^2+R^2+2Rz = (z+R)^2 & (u=-1) \end{array} \right. \end{array} \right.$$

$$V_{\text{med}} = \frac{q}{4\pi\epsilon_0 z}$$

$\hookrightarrow V_{med} = V(\hat{r}=0)$