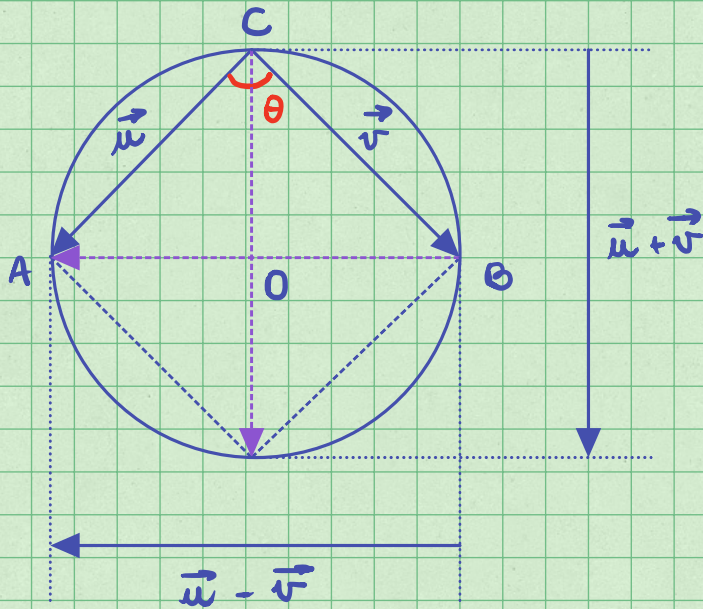
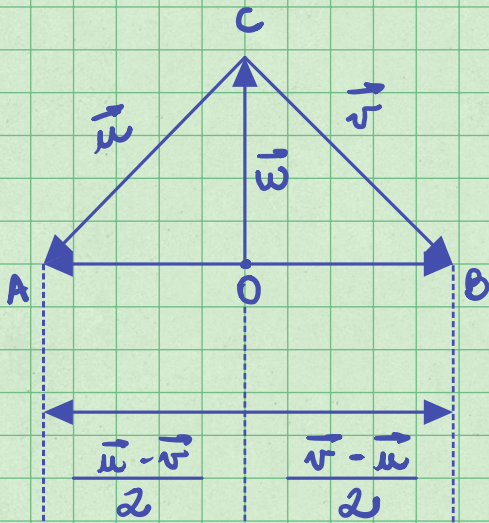


16. c)



$\theta = ?$
 $\theta = \angle u, v$



\vec{OA} e \vec{OB} ... opostos

$$\vec{OB} = -\vec{OA} = -\left(\frac{1}{2}(\vec{u} - \vec{v})\right)$$

$$\vec{OB} = \frac{1}{2}(\vec{v} - \vec{u})$$

$$\Delta COB: \vec{v} = \left(\frac{\vec{v} - \vec{u}}{2}\right) - \vec{w} \quad (1)$$

$$\Delta COA: \vec{u} = \left(\frac{\vec{u} - \vec{v}}{2}\right) - \vec{w} \quad (2)$$

$$\therefore \vec{u} \cdot \vec{v} = \left[\left(\frac{\vec{u} - \vec{v}}{2}\right) - \vec{w}\right] \cdot \left[\left(\frac{\vec{v} - \vec{u}}{2}\right) - \vec{w}\right]$$

$$= \left[\frac{1}{2}(\vec{u} - \vec{v}) - \vec{w}\right] \cdot \left[\frac{1}{2}(\vec{v} - \vec{u}) - \vec{w}\right]$$

$$\left[\frac{1}{2}(\vec{u} - \vec{v}) - \vec{w}\right] \cdot \vec{w}$$

$$\frac{1}{2}\vec{w} \cdot (-1)(\vec{u} - \vec{v})$$

$$= \frac{1}{4} (\vec{u} - \vec{v}) \cdot (\vec{v} - \vec{u}) - \frac{1}{2} \vec{w} \cdot (\vec{v} - \vec{u}) -$$

$$\frac{1}{2} (\vec{u} - \vec{v}) \cdot \vec{w} + \vec{w} \cdot \vec{w}$$

$$= \frac{1}{4} (\vec{u} - \vec{v}) \cdot (-1) (\vec{u} - \vec{v}) + \frac{1}{2} \vec{w} \cdot (\vec{u} - \vec{v}) -$$

$$\frac{1}{2} \vec{w} \cdot (\vec{u} - \vec{v}) + \vec{w} \cdot \vec{w}$$

$$\vec{u} \cdot \vec{v} = |\vec{w}|^2 - \frac{1}{4} |\vec{u} - \vec{v}|^2 \quad (3)$$

De (1) ou (2):

$$\vec{w} = -\frac{1}{2} (\vec{u} + \vec{v}) \quad \therefore |\vec{w}| = \frac{1}{2} |\vec{u} + \vec{v}|$$

$$\longrightarrow |\vec{w}|^2 = \frac{1}{4} |\vec{u} + \vec{v}|^2$$

Substituindo $|\vec{w}|^2$ em (3):

$$\vec{u} \cdot \vec{v} = \frac{1}{4} |\vec{u} + \vec{v}|^2 - \frac{1}{4} |\vec{u} - \vec{v}|^2$$

Da circunferência:

$$|\vec{u} + \vec{v}| = |\vec{u} - \vec{v}| = d, \quad d \dots \text{diâmetro}$$

Portanto:

$$\vec{u} \cdot \vec{v} = 0$$

Como $\vec{u}, \vec{v} \neq \vec{0}$ e $\vec{u} \cdot \vec{v} = 0$, $\theta = \pi/2$

Súvidas:

1 (RNP); 2 (PV)