10- Derivação de Funções Trigonométricas

$\frac{d}{du}(\text{sen}\, u) = \cos u \cdot du$

$\frac{d}{du}(\cos u) = -\text{sen}\, u \cdot du$

$\frac{d}{du}(\text{tg}\, u) = \sec^2 u \cdot du$

$\frac{d}{du}(\text{cossec}\, u) = -\text{cossec}\, u \cdot \text{cotg}\, u \cdot du$

$\frac{d}{du}(\sec u) = \sec u \cdot \text{tg}\, u$

$\frac{d}{du}(\text{cotg}\, u) = -\text{cossec}^2 u\, du$

a) $f(x) = 3x^2 - 2\cos x$

$f'(x) = 6x + 2\,\text{sen}\, x$

$f'(x) = 2(3x + \text{sen}\, x)$

$\frac{dy}{dx}$ ou $\frac{df(x)}{dx}$

b) $f(x) = \underbrace{\text{tg}\, x}_{u} \cdot \underbrace{\sec x}_{v}$ → Regra do Produto $d(u \cdot v) = u\,dv + v\,du$

$u = \text{tg}\, x$ $\quad v = \sec x$

$\underbrace{\frac{du}{dx}}_{u'} = \sec^2 x$ $\quad \underbrace{\frac{dv}{dx}}_{v'} = \sec x \cdot \text{tg}\, x$

$f'(x) = \text{tg}\, x \cdot \sec x \cdot \text{tg}\, x + \sec x \cdot \sec^2 x$

$f'(x) = \text{tg}^2 x \cdot \sec x + \sec^3 x$

$\boxed{1 + \text{tg}^2 x = \sec^2 x}$

$f'(x) = (\sec^2 - 1) \cdot \sec x + \sec^3 x$

$f'(x) = \sec^3 x - \sec x + \sec^3 x$

$f'(x) = 2\sec^3 x - \sec x$

$f'(x) = \sec x\,(2\sec^2 x - 1)$

c) $y = \dfrac{\operatorname{sen} x \; u}{1 - \cos x \; v}$

Regra do Quociente

$$d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}$$

$u = \operatorname{sen} x \qquad v = 1 - \cos x$

$\underbrace{u'}_{\frac{du}{dx}} = \cos x \qquad \underbrace{v'}_{\frac{dv}{dx}} = \operatorname{sen} x$

$$y' = \frac{(1-\cos x).\cos x - \operatorname{sen} x . \operatorname{sen} x}{(1-\cos x)^2}$$

$$y' = \frac{\cos x - \cos^2 x - \operatorname{sen}^2 x}{(1-\cos x)^2}$$

$$y' = \frac{\cos x - \overbrace{(\operatorname{sen}^2 x + \cos^2 x)}^{=1}}{(1-\cos x)^2}$$

$$y' = \frac{\cos x - 1}{(1-\cos x)^2}$$

$$y' = \frac{\cancel{-(1-\cos x)}}{(1-\cos x)^{\cancel{2}}}$$

$$y = \frac{-1}{1-\cos x}$$

11- Derivação da Função Logarítmica

$$\frac{d}{du}(\ln u) = \frac{1}{u}\,du$$

Obs. Ler Stewart (7ª ed.) Pag. 196

a) $f(x) = \ln\left(\frac{1+x}{1-x}\right)$ $\quad u = 1+x \quad v = 1-x$

$\underbrace{\frac{du}{dx}}_{u'} = 1 \quad \underbrace{\frac{dv}{dx}}_{v'} = -1$

$$\boxed{d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}}$$

$$f'(x) = \frac{1}{\frac{1+x}{1-x}} \cdot \frac{(1-x).1 - (1+x).(-1)}{(1-x)^2}$$

$$f'(x) = \frac{\cancel{1-x}}{1+x} \cdot \frac{1-\cancel{x}+1+\cancel{x}}{(1-x)^{\cancel{2}}}$$

$$f'(x) = \frac{2}{1-x^2}$$

b) $f(x) = \ln(\underbrace{\operatorname{sen}^2 x}_{u})$ $\quad u = \operatorname{sen}^2 x$

$\frac{du}{dx} = 2\operatorname{sen}x \cdot \cos x$

$$f'(x) = \frac{1}{\operatorname{sen}^2 x} \cdot 2\operatorname{sen}x \cdot \cos x$$

$$f'(x) = \frac{2\cancel{\operatorname{sen}x}\cos x}{\operatorname{sen}^{\cancel{2}} x} = 2\operatorname{cotg}x$$

$$d(u^n) = n\,u^{n-1} \cdot du$$

c) $y = \left[\ln(x^2+1)\right]^3$

$$\underbrace{y'}_{\frac{dy}{dx}} = 3\left[\ln(x^2+1)\right]^2 \cdot \frac{1}{x^2+1} \cdot 2x$$

$$y' = \frac{6x}{x^2+1}\left[\ln(x^2+1)\right]^2$$

$$\frac{d}{du}(a^u) = a^u \log_e a \, du$$

$$\frac{d}{du}\left(\log_a u\right) = \frac{1}{u} \log_a e \, du$$

d) $y = \log_3 \operatorname{sen}^9 7x$ $\Rightarrow \log_a x^b = b \log_a x$

$y = 9 \log_3 \operatorname{sen} 7x$

$\underbrace{y'}_{\frac{dy}{dx}} = 9 \cdot \frac{1}{\operatorname{sen} 7x} \cdot \log_3 e \cdot \cos 7x \cdot 7$

$y' = 63 \frac{\cos 7x}{\operatorname{sen} 7x} \log_3 e$

$y' = 63 \operatorname{cotg} 7x \log_3 e$

e) $y = \log_e \operatorname{sen}^2 5x \cdot \operatorname{tg}^3(3x+1)$

$\Rightarrow \log_a x \cdot y = \log_a x + \log_a y$

$y = \log_e \operatorname{sen}^2 5x + \log_e \operatorname{tg}^3(3x+1)$

$y = 2 \log_e \operatorname{sen} 5x + 3 \log_e \operatorname{tg}(3x+1)$

$y' = 2 \frac{1}{\operatorname{sen} 5x} \cdot \log_e \operatorname{sen} 5x \cdot \cos 5x \cdot 5 + 3 \frac{1}{\operatorname{tg}(3x+1)} \log_e \operatorname{tg}(3x+1) \sec^2(3x+1) \cdot 3$

$y' = 10 \operatorname{cotg} 5x \log_e \operatorname{sen} 5x + 9 \cdot \frac{1}{\frac{\operatorname{sen}(3x+1)}{\cancel{\cos(3x+1)}}} \log_e \operatorname{tg}(3x+1) \cdot \frac{1}{\cos^2(3x+1)}$

$y' = 10 \operatorname{cotg} 5x \log_e \operatorname{sen} 5x + 9 \operatorname{cosec}(3x+1) \cdot \log_e \operatorname{tg}(3x+1) \cdot \sec(3x+1)$