

Exercício 4

$$0 = \nabla_\mu g_{\alpha\beta} = \partial_\mu g_{\alpha\beta} - \cancel{\Gamma_{\mu\alpha}^\lambda g_{\lambda\beta}} - \cancel{\Gamma_{\mu\beta}^\lambda g_{\alpha\lambda}}$$

$$0 = \nabla_\alpha g_{\beta\mu} = \partial_\alpha g_{\beta\mu} - \Gamma_{\alpha\beta}^\lambda g_{\lambda\mu} - \cancel{\Gamma_{\alpha\mu}^\lambda g_{\beta\lambda}} \quad \textcircled{-}$$

$$0 = \nabla_\beta g_{\mu\alpha} = \partial_\beta g_{\mu\alpha} - \cancel{\Gamma_{\beta\mu}^\lambda g_{\lambda\alpha}} - \cancel{\Gamma_{\beta\alpha}^\lambda g_{\mu\lambda}} \quad \textcircled{-}$$

$$0 = \partial_\mu g_{\alpha\beta} - \partial_\alpha g_{\beta\mu} - \partial_\beta g_{\mu\alpha} + 2\Gamma_{\alpha\beta}^\lambda g_{\lambda\mu} \iff g_{\lambda\mu} \Gamma_{\alpha\beta}^\lambda = \frac{1}{2} [\partial_\alpha g_{\beta\mu} + \partial_\beta g_{\mu\alpha} - \partial_\mu g_{\alpha\beta}] \iff g^{\mu\sigma}$$

$$\iff \boxed{\Gamma_{\alpha\beta}^\sigma = \frac{g^{\sigma\mu}}{2} (\partial_\alpha g_{\beta\mu} + \partial_\beta g_{\mu\alpha} - \partial_\mu g_{\alpha\beta})}$$