Lista 1

Exercícios: ~~13~~ - ~~21~~ - ~~23~~ - ~~16~~ - ~~28~~ - ~~20~~ - ~~12~~ - ~~24~~ - ~~8~~ - ~~15~~

13 $\{\vec{u}, \vec{v}, \vec{w}\}$ é LI

$\vec{t} = a\vec{u} + b\vec{v} + c\vec{w}$

$\{\vec{u}+\vec{t}, \vec{v}+\vec{t}, \vec{w}+\vec{t}\}$ é LI $\iff$ $a+b+c \neq -1$

$\alpha(\vec{u}+\vec{t}) + \beta(\vec{v}+\vec{t}) + \gamma(\vec{w}+\vec{t}) = \vec{0}$, $\alpha = \beta = \gamma = 0$

$\alpha(\vec{u} + a\vec{u} + b\vec{v} + c\vec{w}) +$

$\beta(\vec{v} + a\vec{u} + b\vec{v} + c\vec{w}) +$

$\gamma(\vec{w} + a\vec{u} + b\vec{v} + c\vec{w}) = \vec{0}$

$(\alpha + \alpha a + \beta a + \gamma a)\vec{u} + (\alpha b + \beta + \beta b + \gamma b)\vec{v} +$

$(\alpha c + \beta c + \gamma + \gamma c)\vec{w} = \vec{0}$

$$\begin{cases} (1+a)\alpha + a\beta + a\gamma = 0 \\ b\alpha + (1+b)\beta + b\gamma = 0 \\ c\alpha + c\beta + (1+c)\gamma = 0 \end{cases}$$

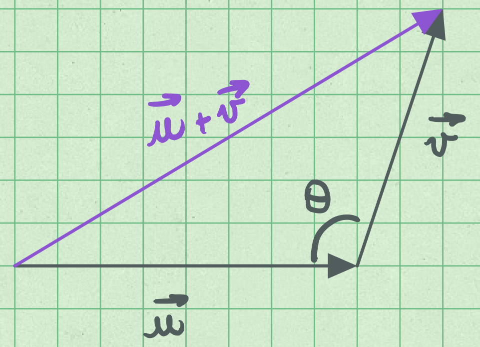
De acordo com o SLH acima, a solução obtida será somente a trivial se:

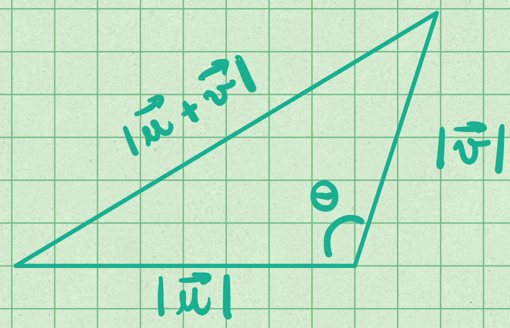
$$\begin{vmatrix} (1+a) & a & a \\ b & (1+b) & b \\ c & c & (1+c) \end{vmatrix} \neq 0$$

⋮

$a+b+c \neq -1$ //

21





$\theta > \pi/2 \therefore \cos\theta < 0$

Lei dos Cossenos

$$|\vec{u}+\vec{v}|^2 = |\vec{u}|^2 + 2\,\vec{u}\cdot\vec{v} + |\vec{v}|^2$$

$$\left.\begin{aligned} |\vec{u}| &= 1 \\ |\vec{v}| &= 2 \\ \theta &= 2\pi/3 \end{aligned}\right\} \qquad |\underbrace{2\vec{u}}_{=\vec{a}} + \underbrace{4\vec{v}}_{=\vec{b}}|^2 = ?$$

$$\begin{aligned} \therefore |\vec{a}+\vec{b}|^2 &= |\vec{a}|^2 + 2\,\vec{a}\cdot\vec{b} + |\vec{b}|^2 \\ &= |2\vec{u}|^2 + 2(2\vec{u})\cdot(4\vec{v}) + |4\vec{v}|^2 \quad \vec{u},\vec{v} \\ &= (|2|\,|\vec{u}|)^2 + 16(\vec{u}\cdot\vec{v}) + (|4|\,|\vec{v}|)^2 \\ &= 4\,|\vec{u}|^2 + 16(|\vec{u}|\,|\vec{v}|\cos\theta) + 16\,|\vec{v}|^2 \\ &\;\;\vdots \end{aligned}$$

$\vec{p} = k\vec{v}$, $k \in \mathbb{R}$

$\hookrightarrow |\vec{p}| = |k|\,|\vec{v}|$

23

$$\left.\begin{aligned} \vec{v} &= (-1,1,1) \\ \vec{u} &= (-2,1,2) \end{aligned}\right\} \qquad \text{proj}_{\vec{u}}\vec{v} = \left(\frac{\vec{v}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\right)\vec{u}$$

$\vdots$

16 OUTRA FOLHA (a, b)

25

$\vec{AB} = (2, \sqrt{3}, 1)$

$\vec{AC} = (-1, \sqrt{3}, 1)$

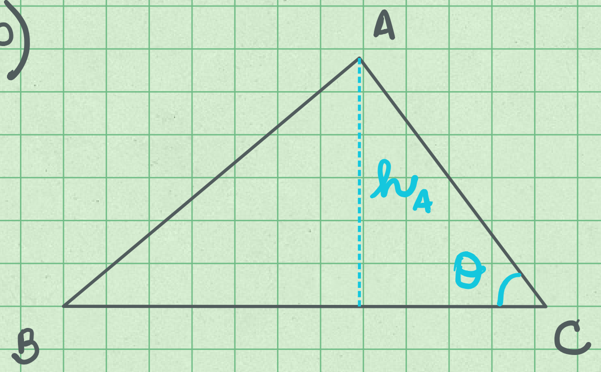
a) Verificar que são vértices de um triângulo

b) h_A, Área

a) $\vec{AB}$ e $\vec{AC}$ são vértices de um triângulo $\Leftrightarrow$

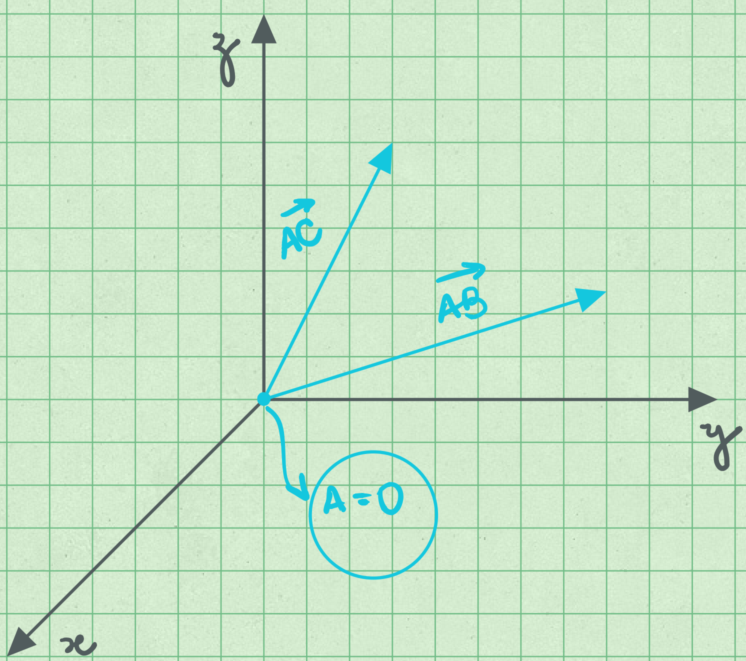
$\nexists\, k \in \mathbb{R} \,/\, \vec{AB} = k\,\vec{AC}$.

b)



$h_A = |\vec{AC}| \operatorname{sen}\theta$

$\theta = ?$



$\vec{AC} = C - A = C - (0,0,0)$

$\downarrow$

$C(-1, \sqrt{3}, 1)$

$\vec{AB} = B - A = B - (0,0,0)$

$\downarrow$

$B(2, \sqrt{3}, 1)$

$\vec{CB} = B - C = (3,0,0)$

$\theta = \measuredangle\, \vec{CA}$ e $\vec{CB} \Rightarrow \cos\theta = \dfrac{\vec{CA} \cdot \vec{CB}}{|\vec{CB}|\,|\vec{CB}|}$

oposto de $\vec{AC}$

$\cos\theta$

$\downarrow$

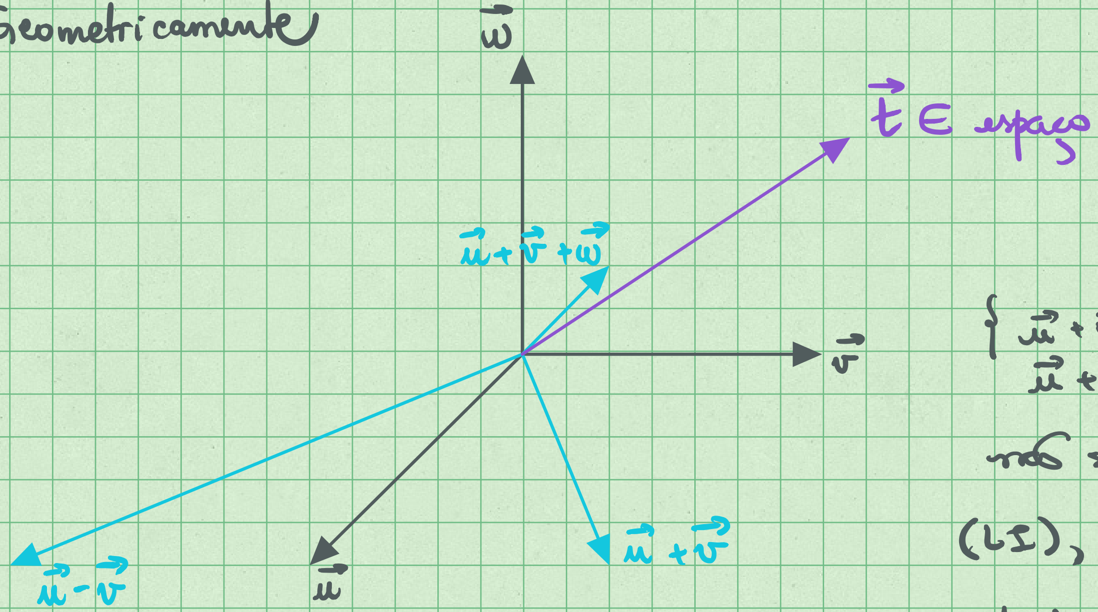
$\operatorname{sen}\theta$

$\downarrow$

h_A

20 AULA PASSADA (ATENDIMENTO)

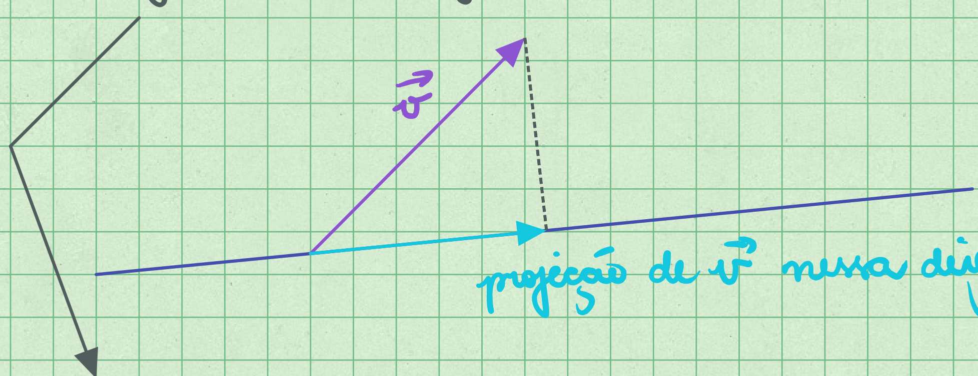
12 Geometricamente



$\{\vec{u}+\vec{v}, \vec{u}-\vec{v}, \vec{u}+\vec{v}+\vec{w}\}$ não são coplanares (LI), garante-se que todo $\vec{t} \in$ espaço pode ser escrito como CL dos 3 vetores. Logo, eles formam base.

O mesmo vale para $\{\vec{u}, \vec{v}, \vec{w}\}$.

24 $\text{proj}_{k\vec{u}}\vec{v} = \text{proj}_{\vec{u}}\vec{v}$, $k \neq 0$, $\vec{u} \neq \vec{0}$



projeção de $\vec{v}$ nessa direção depende
{ do comp. $\vec{v}$
{ da direção em si

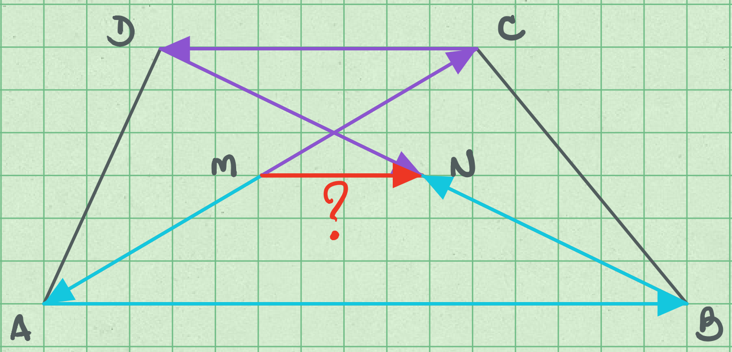
$$\left(\frac{\vec{v}\cdot k\vec{u}}{k\vec{u}\cdot k\vec{u}}\right)(k\vec{u}) = \left(\frac{\vec{v}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\right)\vec{u}$$

$$\left(\frac{\cancel{k}\;\vec{v}\cdot\vec{u}}{k^2\;\vec{u}\cdot\vec{u}}\right)k(\vec{u}) =$$

$$\cancel{k}\left(\frac{\vec{v}\cdot\vec{u}}{\cancel{k}\,\vec{u}\cdot\vec{u}}\right)\vec{u} =$$

$$\left(\frac{\vec{v}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\right)\vec{u}$$

8



$$\vec{MN} = \vec{MC} + \vec{CD} + \vec{DN}$$
$$\vec{MN} = \vec{MA} + \vec{AB} + \vec{BN}$$

$(+)$

$$2\,\vec{MN} = \underbrace{\vec{MC} + \vec{MA}}_{=\vec{0}} + \vec{AB} + \vec{CD} + \underbrace{\vec{DN} + \vec{BN}}_{=\vec{0}}$$

$$2\,\vec{MN} = \vec{AB} + \vec{CD}$$
$$2\,\vec{MN} = \vec{AB} - \vec{DC} \longrightarrow \vec{MN} = \frac{1}{2}\left(\vec{AB} - \vec{DC}\right)$$

Def. Trapézio: $\vec{AB} \parallel \vec{DC} \quad \therefore \quad \vec{AB} - \vec{DC} \parallel \vec{AB}$ e $\vec{DC}$

Com isso: $\vec{MN} = \frac{1}{2}\left(\vec{AB} - \vec{DC}\right) \parallel \vec{AB}$ e $\vec{DC}$

15

$\vec{u} = 3\vec{e_1} + 4\vec{e_2}$

$\vec{v} = \vec{e_1} + a\vec{e_2}$, $E = \{\vec{e_1}, \vec{e_2}\}$ é ortonormal

$a = ?$

a) $\vec{u} \perp \vec{v}$

b) $\theta = \frac{\pi}{4}$

$\vec{u} = (3,4)_E$

$\vec{v} = (1,a)_E$

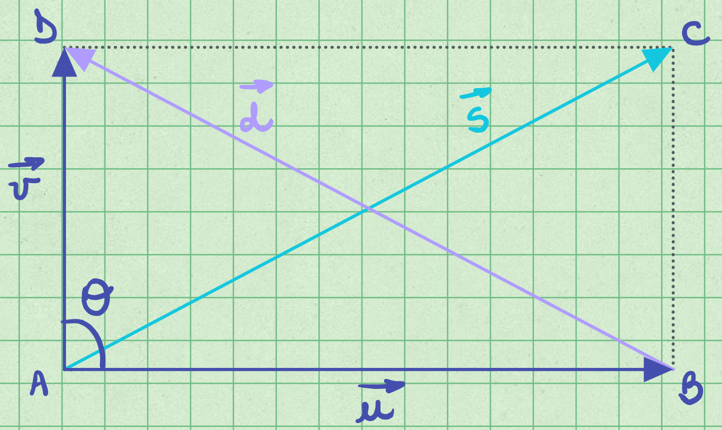
Como a base é ortonormal,

a) $\vec{u} \perp \vec{v} \iff \vec{u} \cdot \vec{v} = 0$

b) $\theta = \frac{\pi}{4} \Rightarrow \cos\theta = \frac{\vec{u} \cdot \vec{v}}{|\vec{u}|\,|\vec{v}|}$

⋮

16 a) As diagonais de um retângulo têm o mesmo comprimento:



$\vec{s} = \vec{u} + \vec{v}$

$\vec{d} = \vec{u} - \vec{v}$

$\theta = \frac{\pi}{2}$

Mostrar que: $|\vec{d}| = |\vec{s}|$

Triângulo ABC:

$|\vec{s}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2\, \vec{u} \cdot \vec{v}\, \underbrace{\cancel{\cos\theta}}_{=0}$

$|\vec{s}|^2 = |\vec{u}|^2 + |\vec{v}|^2 \quad (i)$

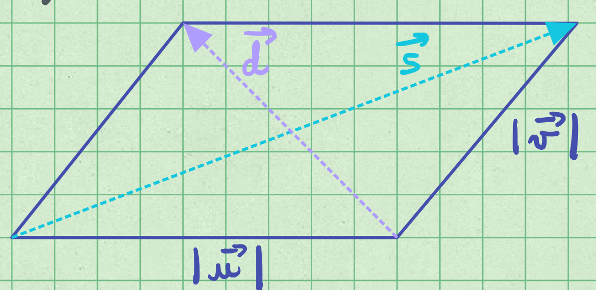
Triângulo DAB:

$|\vec{d}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2\, \vec{u} \cdot \vec{v}\, \underbrace{\cancel{\cos\theta}}_{=0}$

$|\vec{d}|^2 = |\vec{u}|^2 + |\vec{v}|^2 \quad (ii)$

Comparando (i) e (ii) conclui-se que: $|\vec{d}| = |\vec{s}|$.
Portanto, as diagonais de um retângulo têm o mesmo comprimento (módulo).

b)



$|\vec{u}|^2 + |\vec{v}|^2 + |\vec{u}|^2 + |\vec{v}|^2 = |\vec{s}|^2 + |\vec{d}|^2$

$\vec{s} = \vec{u} + \vec{v} \quad \therefore \quad |\vec{s}| = |\vec{u} + \vec{v}|$

$\vec{d} = \vec{u} - \vec{v} \quad \therefore \quad |\vec{d}| = |\vec{u} - \vec{v}|$

$\vdots$ Ex. 21 : $|\vec{u} + \vec{v}|^2$