

Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

Aula de 17 de maio
Eletrostática

Coordenadas esféricas

$$d\vec{r} = dr \hat{r} + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}$$

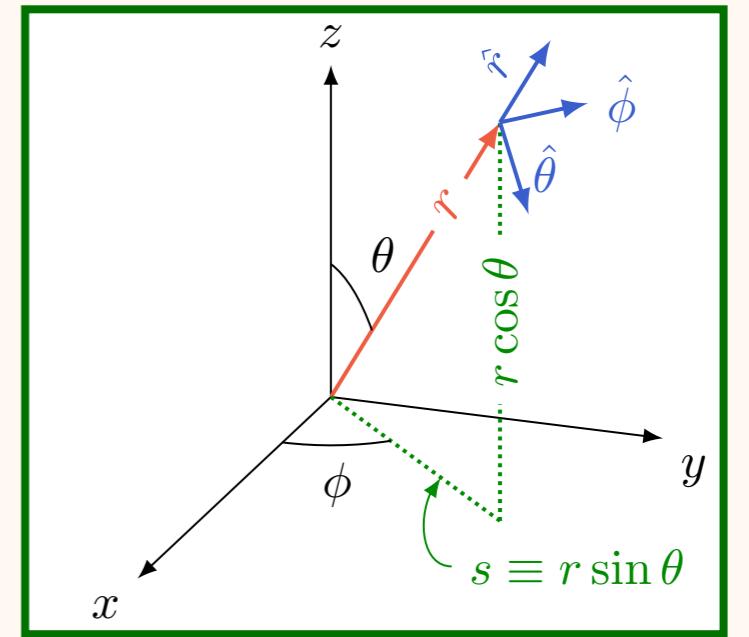
$$\vec{\nabla} t = \frac{\partial t}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial t}{\partial \theta} \hat{\theta} + \frac{1}{r \sin \theta} \frac{\partial t}{\partial \phi} \hat{\phi}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2} \frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial(\sin \theta v_\theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi}$$

$$\vec{\nabla} \times \vec{v} = \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta v_\phi) - \frac{\partial v_\theta}{\partial \phi} \right] \hat{r}$$

$$+ \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r} (r v_\phi) \right] \hat{\theta} + \frac{1}{r} \left[\frac{\partial}{\partial r} (r v_\theta) - \frac{\partial v_r}{\partial \theta} \right] \hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}$$



Coordenadas cilíndricas

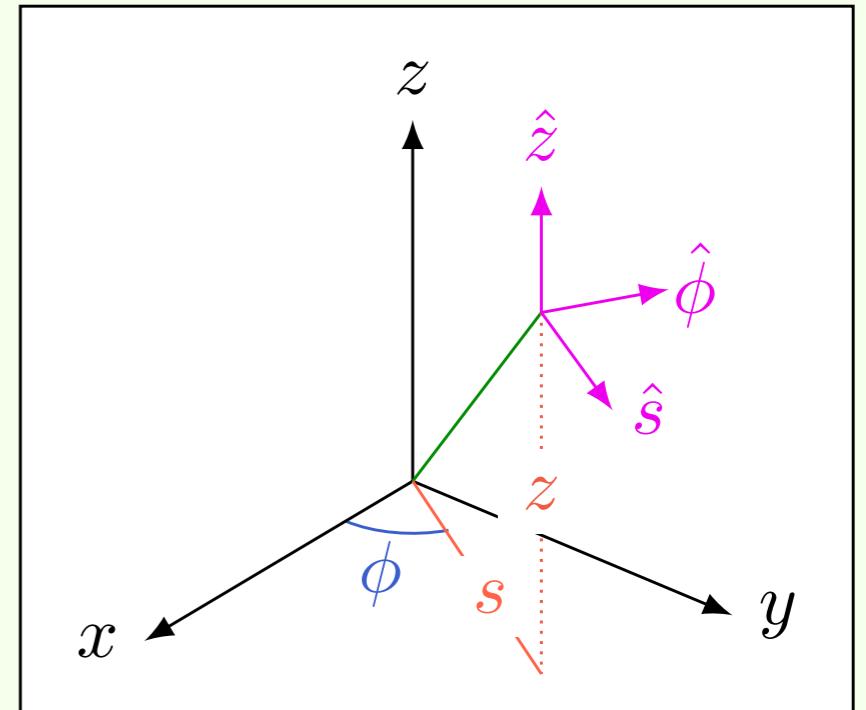
$$d\vec{\ell} = ds \hat{s} + s d\phi \hat{\phi} + dz \hat{z}$$

$$\vec{\nabla} T = \frac{\partial T}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial T}{\partial \phi} \hat{\phi} + \frac{\partial T}{\partial z} \hat{z}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s} \frac{\partial (sv_s)}{\partial s} + \frac{1}{s} \frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla} \times \vec{v} = \left(\frac{1}{s} \frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z} \right) \hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s} \right) \hat{\phi} + \frac{1}{s} \left(\frac{\partial (sv_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi} \right) \hat{z}$$

$$\nabla^2 T = \frac{1}{s} \frac{\partial}{\partial s} \left(s \frac{\partial T}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$



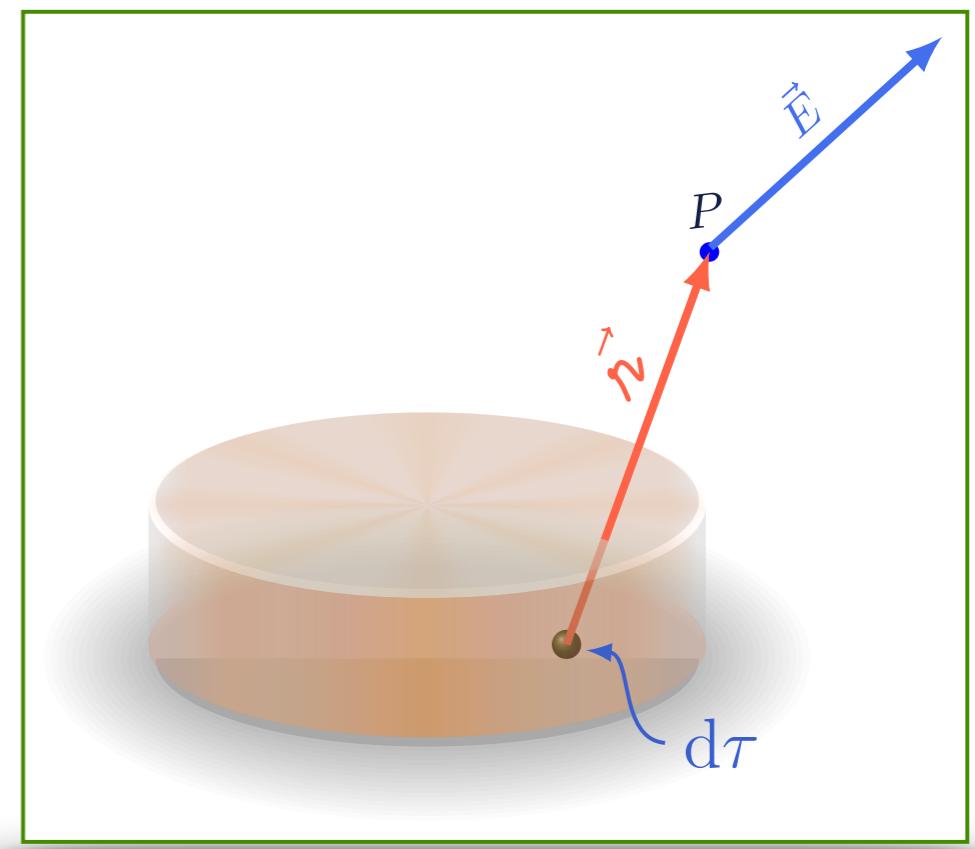
Eletrostática

Campo de distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{1}{r^2} \hat{r} dq$$

$$\vec{\nabla} \times \vec{E} = 0$$

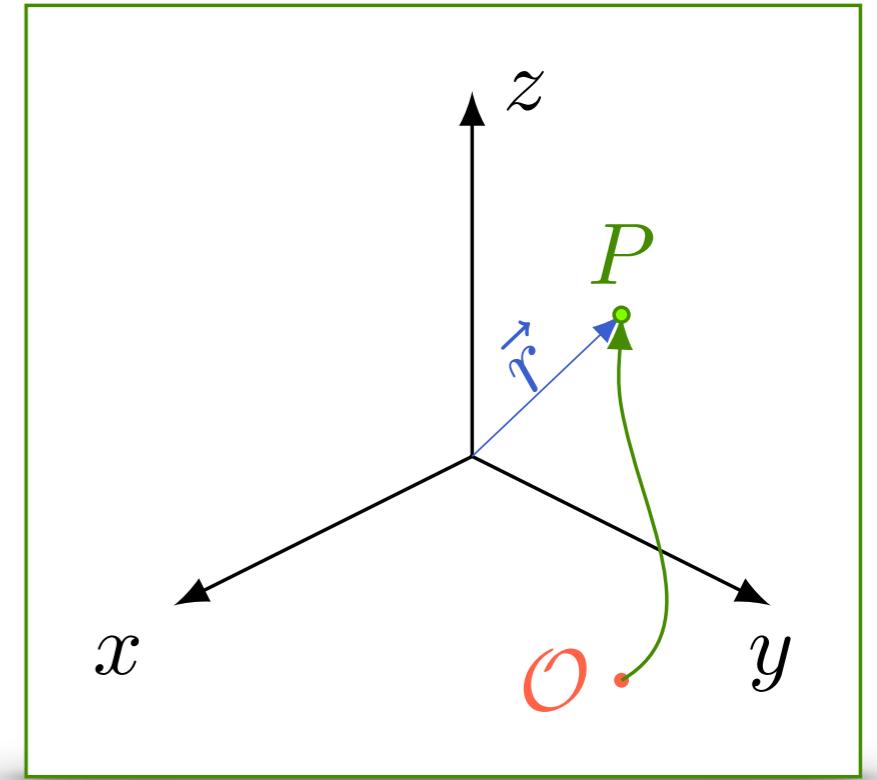
$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$



Potencial elétrico

$$\vec{\nabla} \times \vec{E} = 0$$

$$V(P) = - \int_{\mathcal{O}}^{\vec{r}} \vec{E} \cdot d\vec{\ell}$$

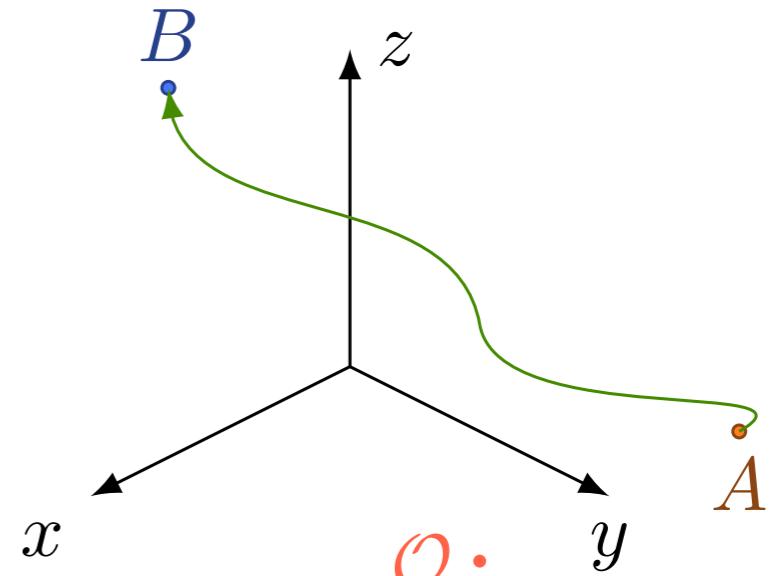


Diferença de potencial

$$\vec{\nabla} \times \vec{E} = 0$$

$$\Delta V = - \int_{\vec{r}_a}^{\vec{r}_b} \vec{E} \cdot d\vec{\ell}$$

$$\vec{E} = -\vec{\nabla}V$$



Potencial de uma carga

$$V(P) = \int_P^{\mathcal{O}} \vec{E} \cdot d\vec{\ell}$$

Referência no infinito

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

Rotacional do campo

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$
$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot d\vec{\ell} = \frac{1}{4\pi\epsilon_0} \int_{\vec{a}}^{\vec{b}} \frac{q}{r^2} \hat{r} \cdot d\vec{\ell}$$
$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot d\vec{\ell} = \frac{1}{4\pi\epsilon_0} \int_{r_a}^{r_b} \frac{q}{r^2} dr$$
$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot d\vec{\ell} = \frac{1}{4\pi\epsilon_0} \left(\frac{q}{r_a} - \frac{q}{r_b} \right)$$

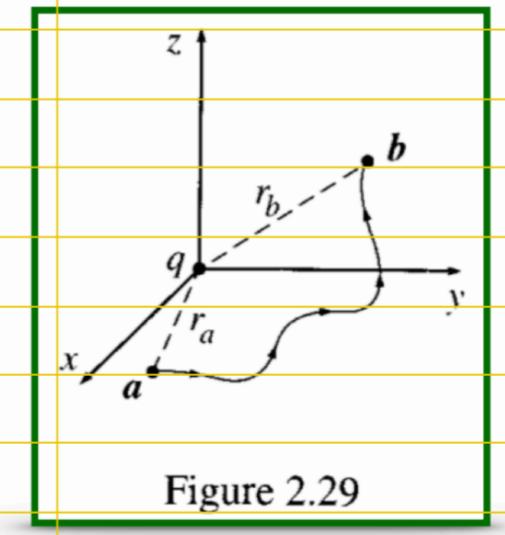
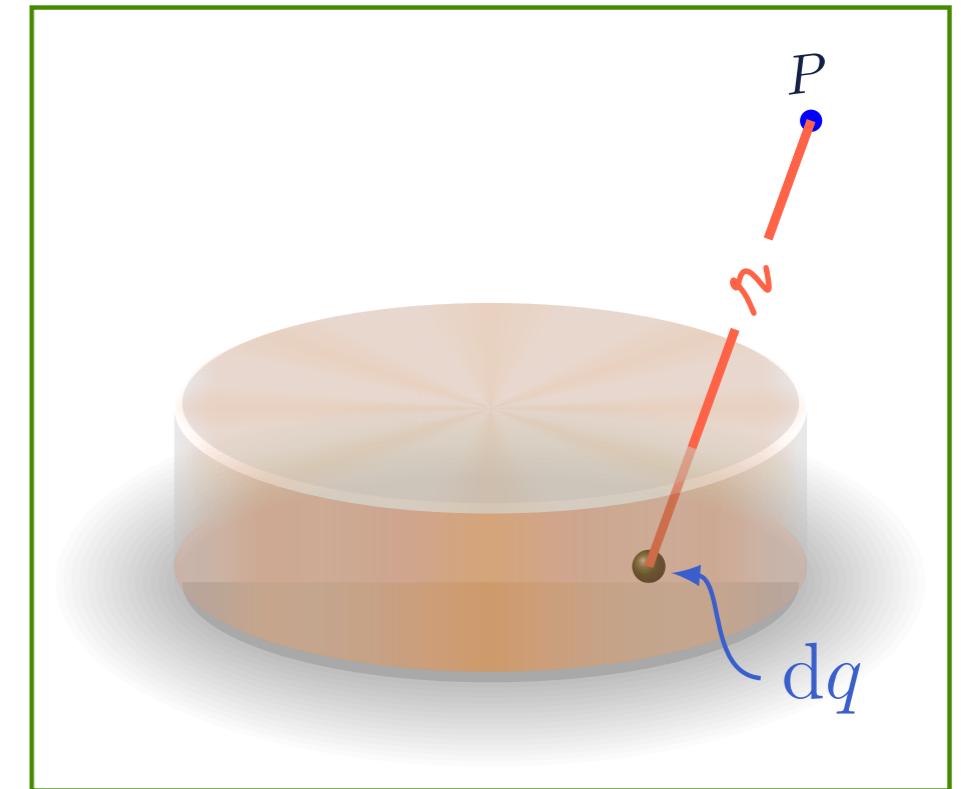


Figure 2.29

Potencial de distribuição de cargas

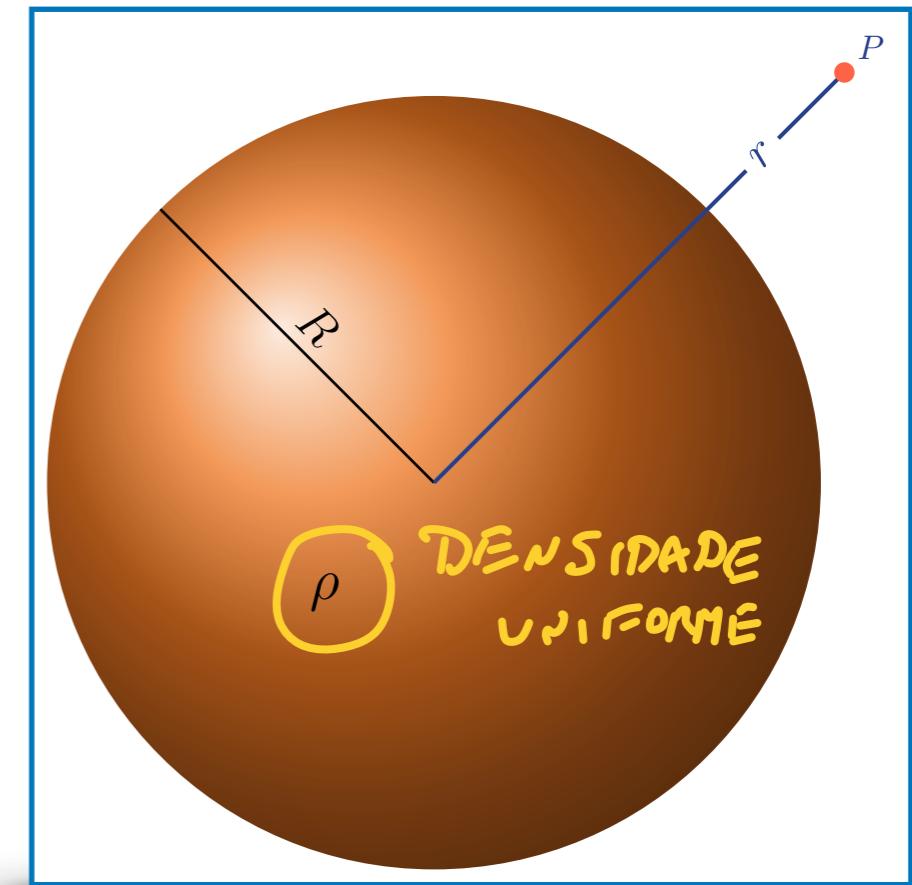
$$V(P) = \frac{1}{4\pi\epsilon_0} \int_V \frac{1}{r} dq$$



$$\Delta V = \int_{\vec{r}_b}^{\vec{r}_a} \vec{E} \cdot d\vec{l}$$

Pratique o que aprendeu

EXERCÍCIO 9 DA SEGUNDA LISTA



Pratique o que aprendeu

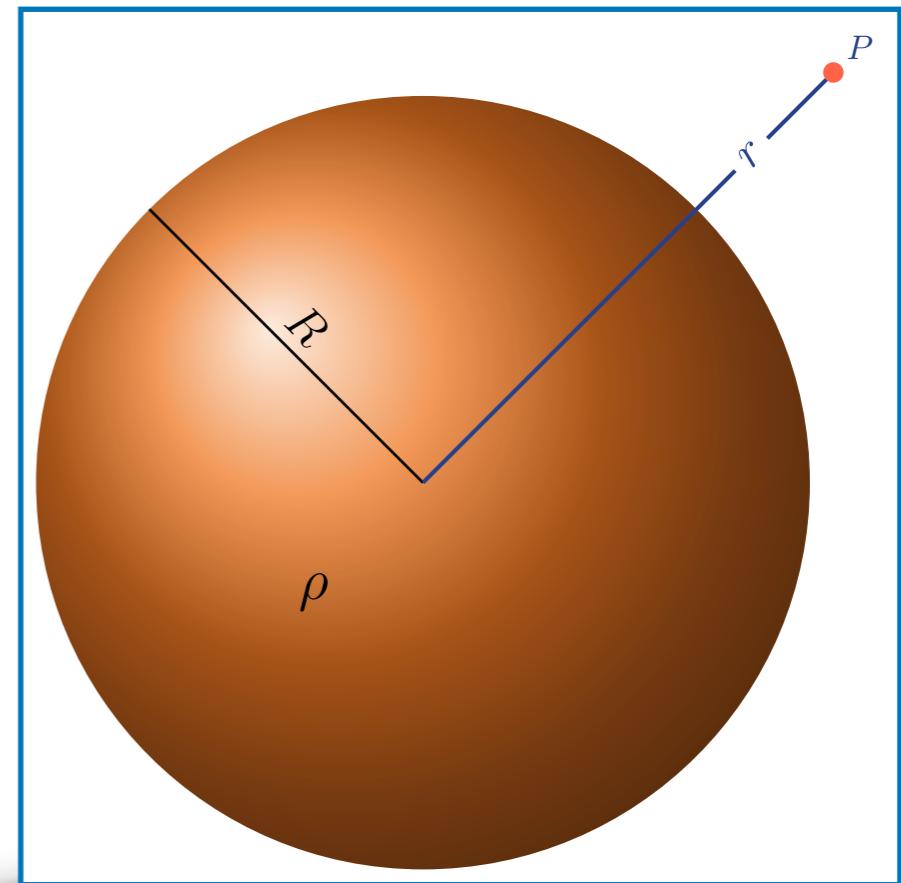
$$\Delta V = \int_{\vec{r}_b}^{\vec{r}_a} \vec{E} \cdot d\vec{l}$$

$$r > R$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

→ CAMPO ELÉTRICO FORA DA
ESFERA EQUIVALE AO DE
UMA CARGA PONTUAL q
NO CENTRO DA ESFERA

- POTENCIAL TAMBÉM EQUIVALE
AO DE UMA CARGA PONTUAL



Pratique o que aprendeu

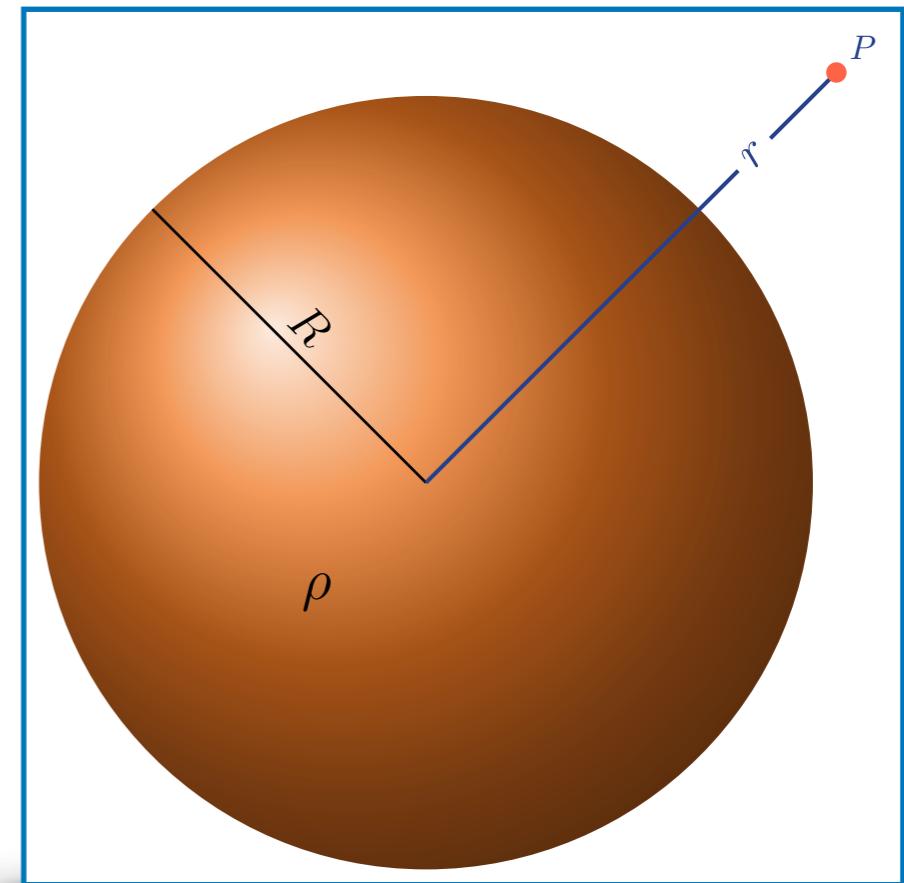
$$\Delta V = \int_{\vec{r}_b}^{\vec{r}_a} \vec{E} \cdot d\vec{\ell}$$

$$r > R$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$q = \frac{4\pi R^3}{3} \rho$$

VOLUME DA ESFERA



Pratique o que aprendeu

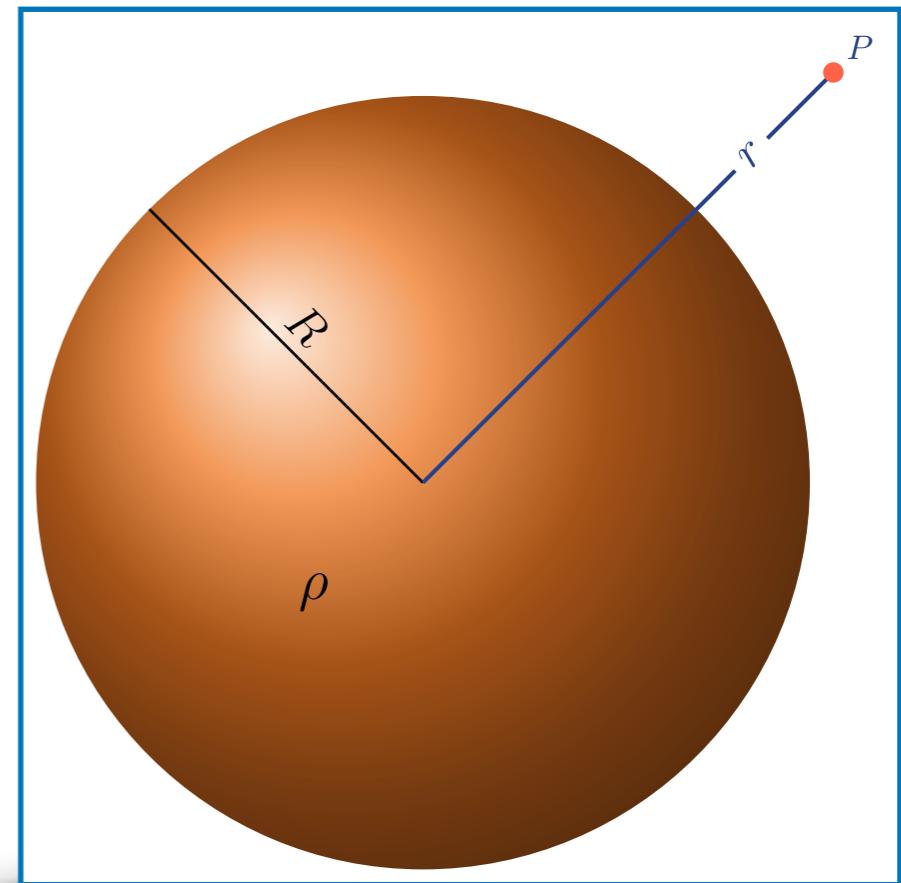
$$\Delta V = \int_{\vec{r}_b}^{\vec{r}_a} \vec{E} \cdot d\vec{\ell}$$

$$r > R$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$q = \frac{4\pi R^3}{3} \rho$$

$$V(r) = \frac{R^3}{3\epsilon_0} \frac{\rho}{r}$$



$$\Delta V = \int_{\vec{r}_b}^{\vec{r}_a} \vec{E} \cdot d\vec{\ell}$$

Pratique o que aprendeu

$$r = R$$

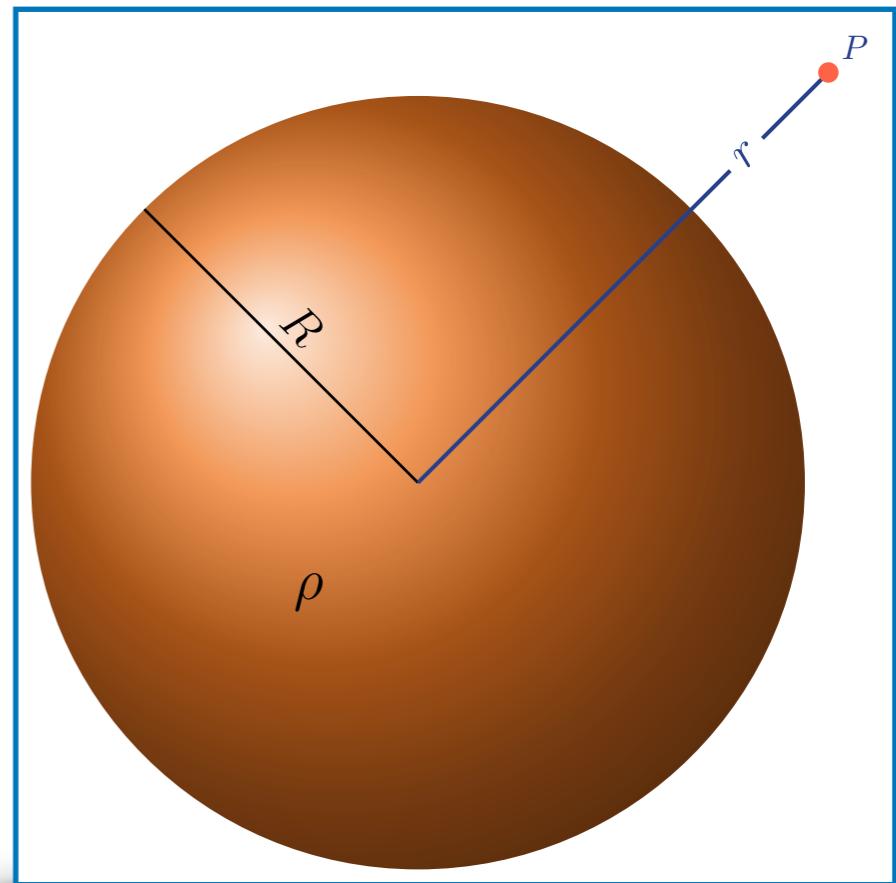
$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

$$q = \frac{4\pi R^3}{3} \rho$$

$$V(r) = \frac{R^3}{3\epsilon_0} \frac{\rho}{r}$$

$V(r = R) = \frac{R^2}{3\epsilon_0} \rho$

 JÁ TIL PARA CALCULAR $V(r < R)$



$$\Delta V = \int_{\vec{r}_b}^{\vec{r}_a} \vec{E} \cdot d\vec{l}$$

Pratique o que aprendeu

$$r < R$$

$$V(r = R) = \frac{R^2}{3\epsilon_0} \rho$$

SUPERFÍCIE ($r=R$)

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

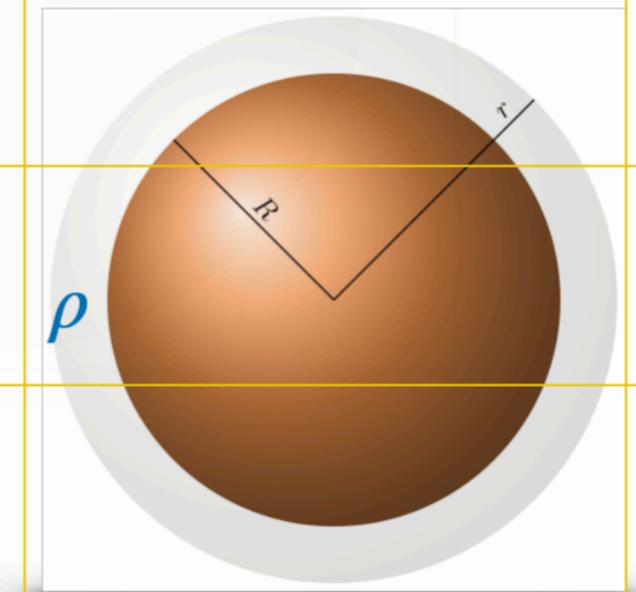
Pratique o que aprendeu

$$\frac{1}{r^2} \frac{d}{dr} (r^2 E(r)) = \frac{\rho}{\epsilon_0}$$

$$\frac{d}{dr} (r^2 E(r)) = \frac{\rho}{\epsilon_0} r^2$$

$$E(r) - E(0) = \frac{\rho}{\epsilon_0 r^2} \int_0^r r'^2 dr'$$

$$\rightarrow E(r) = \frac{\rho r}{\epsilon_0 3}$$



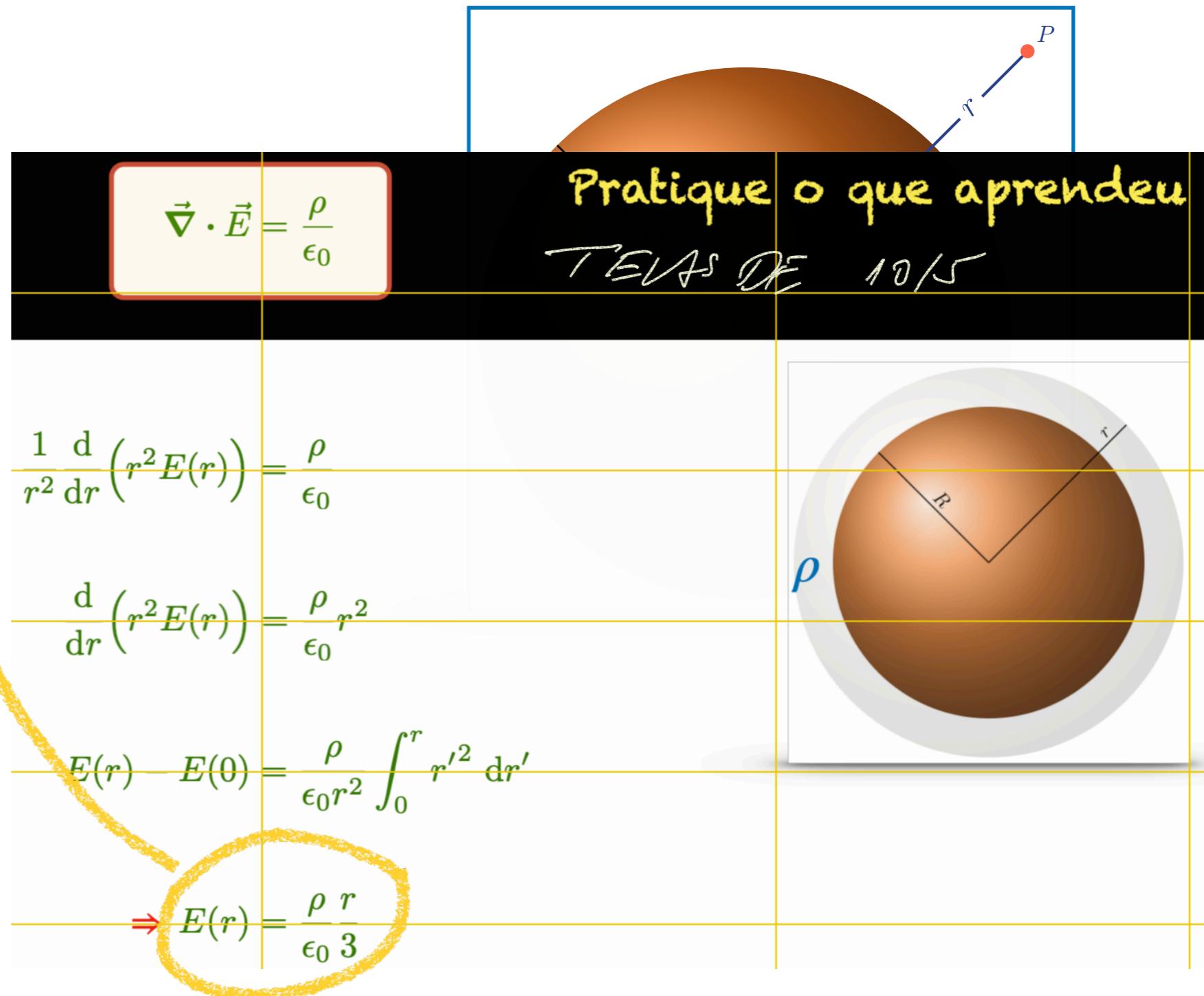
$$\Delta V = \int_{\vec{r}_b}^{\vec{r}_a} \vec{E} \cdot d\vec{\ell}$$

Pratique o que aprendeu

$$r < R$$

$$V(r = R) = \frac{R^2}{3\epsilon_0} \rho$$

$$E(r) = \frac{\rho}{3\epsilon_0} r$$



Pratique o que aprendeu

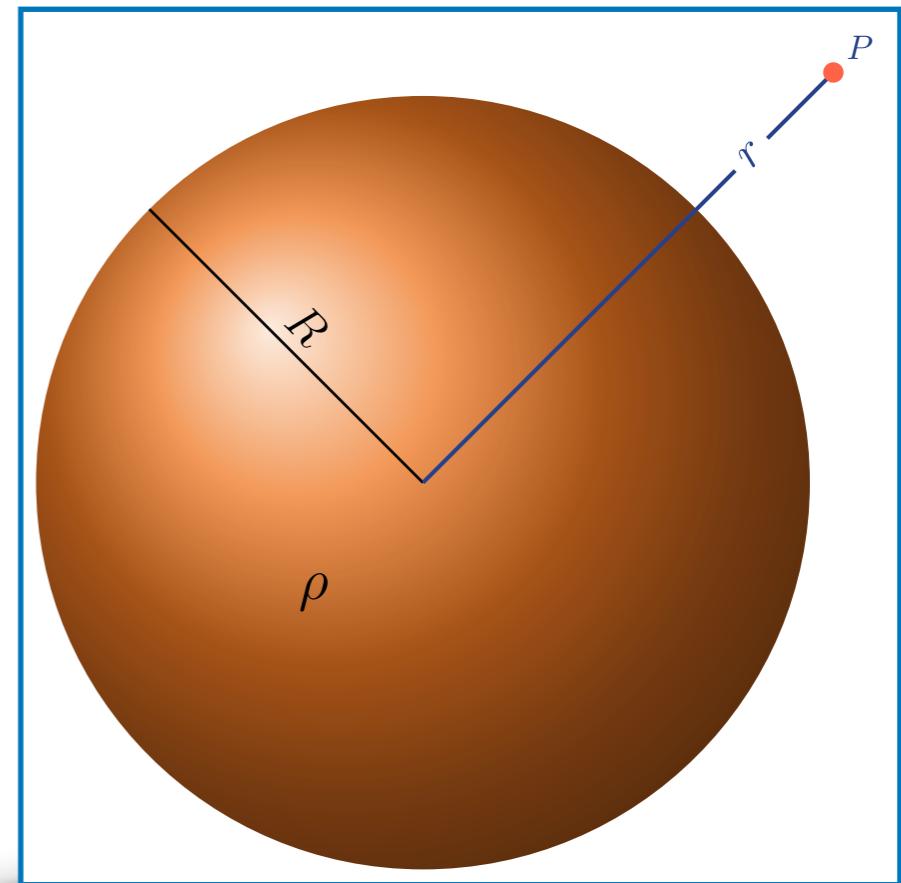
$$\Delta V = \int_{\vec{r}_b}^{\vec{r}_a} \vec{E} \cdot d\vec{\ell}$$

$$r < R$$

$$V(r = R) = \frac{R^2}{3\epsilon_0} \rho$$

$$E(r) = \frac{\rho}{3\epsilon_0} r$$

$$V(r) - V(R) = \int_r^R E(r') dr'$$



Pratique o que aprendeu

$$\Delta V = \int_{\vec{r}_b}^{\vec{r}_a} \vec{E} \cdot d\vec{\ell}$$

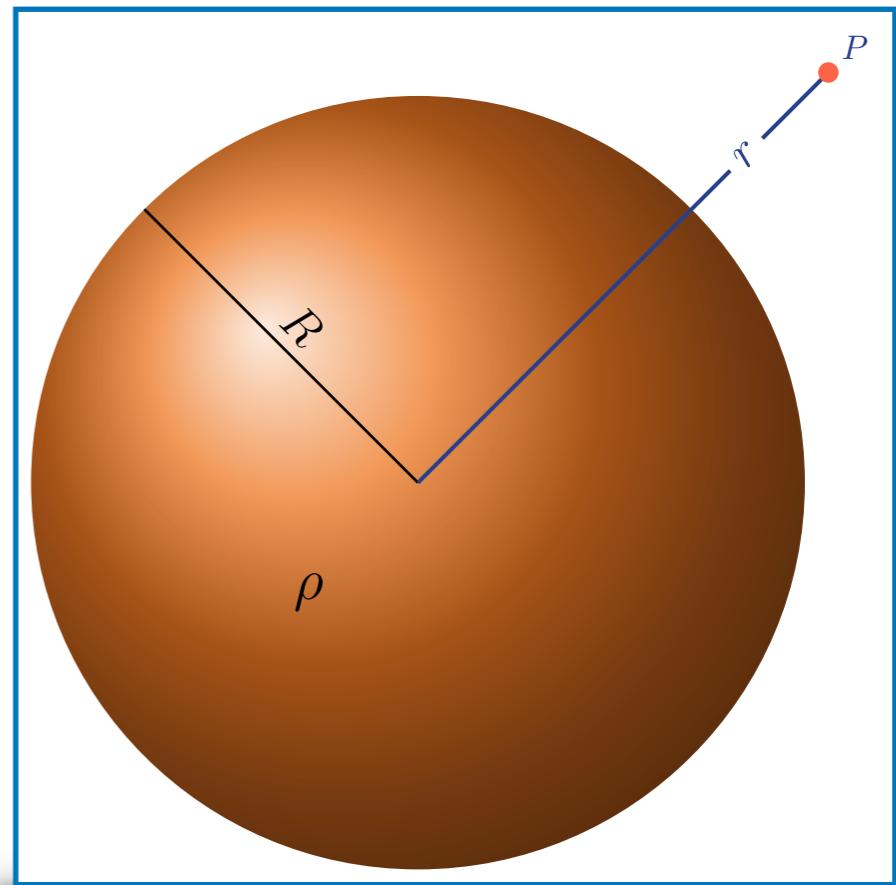
$$r < R$$

$$V(r = R) = \frac{R^2}{3\epsilon_0} \rho$$

$$E(r) = \frac{\rho}{3\epsilon_0} r$$

$$V(r) - V(R) = \int_r^R E(r') dr'$$

$$V(r) - V(R) = \frac{\rho}{6\epsilon_0} (R^2 - r^2)$$



Pratique o que aprendeu

$$\Delta V = \int_{\vec{r}_b}^{\vec{r}_a} \vec{E} \cdot d\vec{\ell}$$

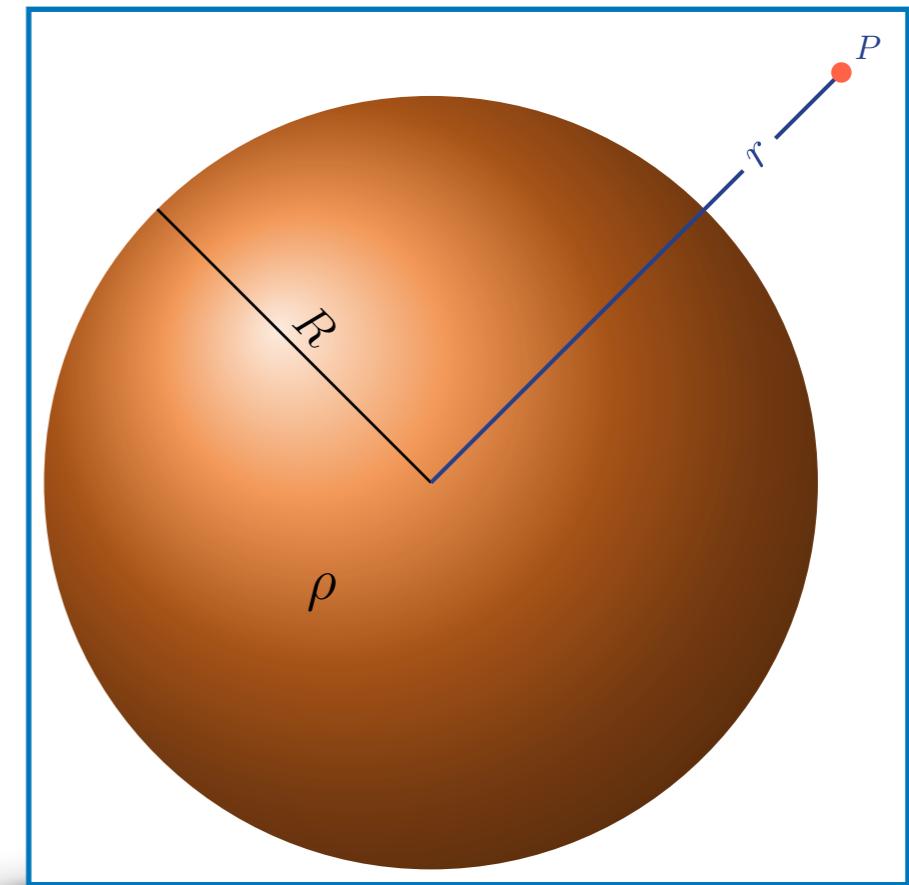
$$r < R$$

$$V(r = R) = \frac{R^2}{3\epsilon_0} \rho$$

$$E(r) = \frac{\rho}{3\epsilon_0} r$$

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$$V(r) - V(R) = \frac{\rho}{6\epsilon_0} (R^2 - r^2)$$

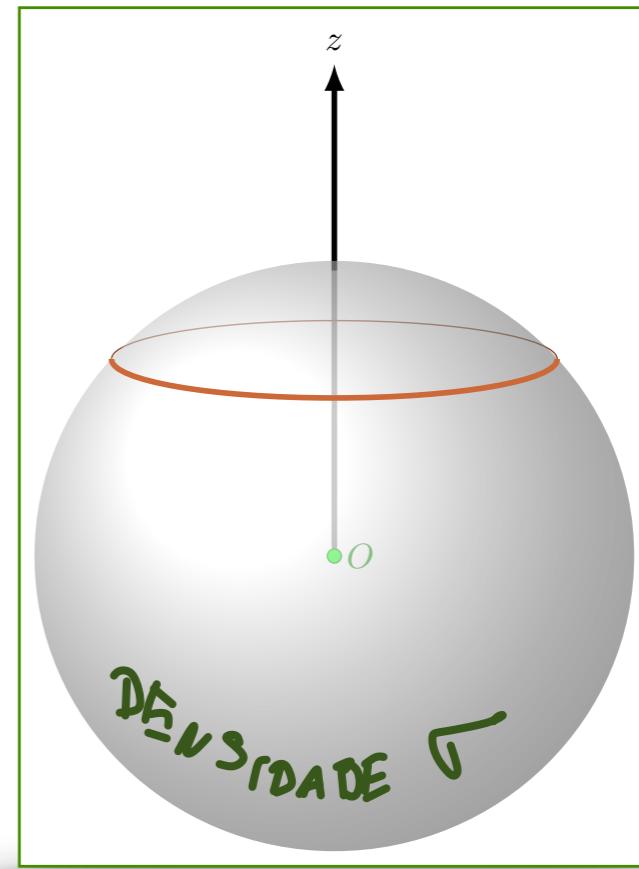


$$\Rightarrow V(r) = \frac{\rho}{2\epsilon_0} \left(R^2 - \frac{r^2}{3} \right)$$

Pratique o que aprendeu

$$V(P) = \frac{1}{4\pi\epsilon_0} \int_V \frac{1}{r} dq$$

Potencial de uma casca esférica

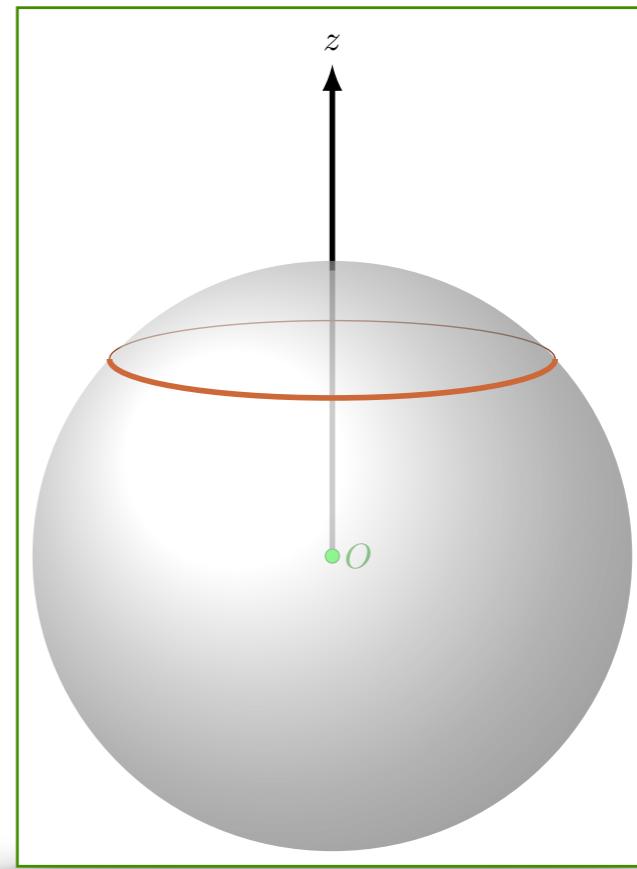


Pratique o que aprendeu

$$V(P) = \frac{1}{4\pi\epsilon_0} \int_V \frac{1}{r} dq$$

Potencial de uma casca esférica

$$V(P) = \frac{1}{4\pi\epsilon_0} \int_A \frac{\sigma}{d} dA$$



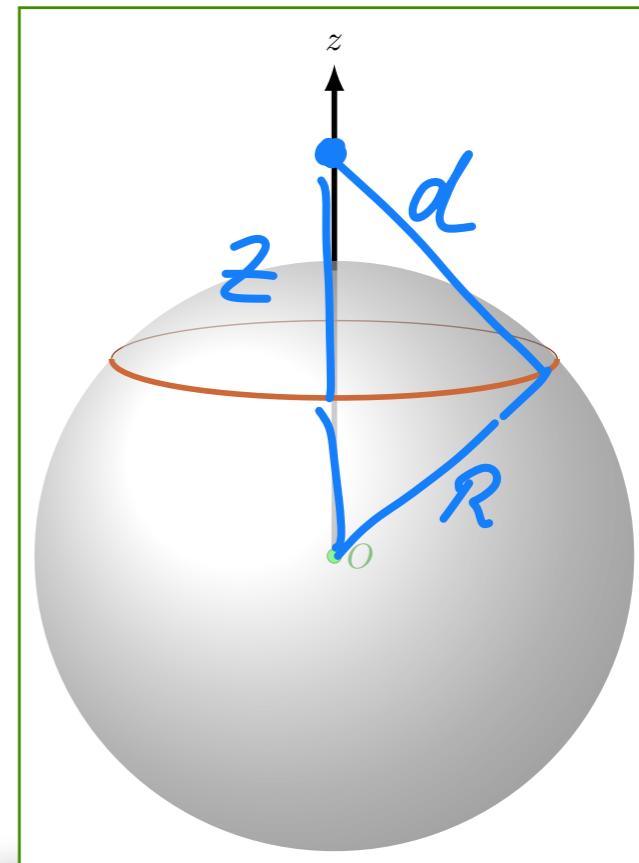
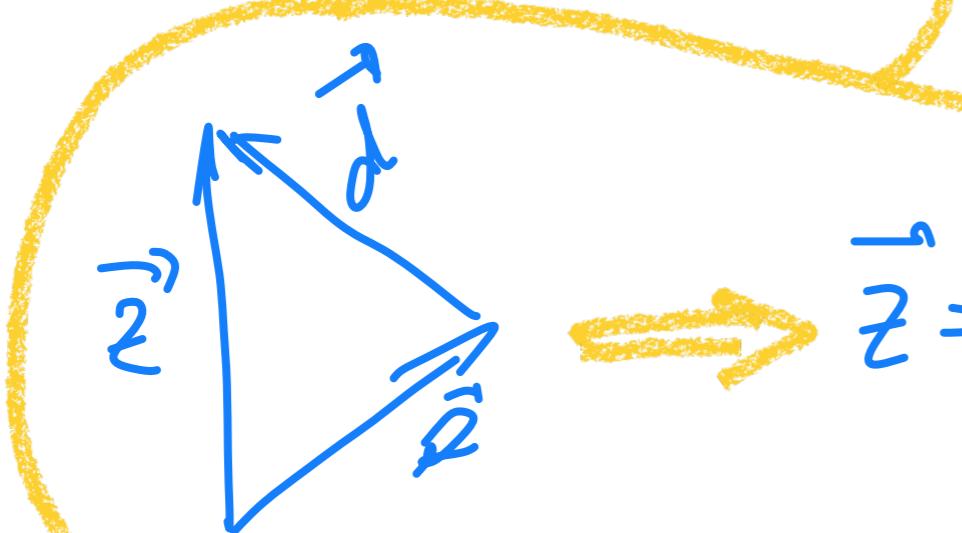
Pratique o que aprendeu

$$V(P) = \frac{1}{4\pi\epsilon_0} \int_V \frac{1}{r} dq$$

Potencial de uma casca esférica

$$V(P) = \frac{1}{4\pi\epsilon_0} \int_A \frac{\sigma}{d} dA$$

$$d^2 = R^2 + z^2 - 2\vec{z} \cdot \vec{R}$$



$$\begin{aligned} \vec{z} &= \vec{R} + \vec{d} \Rightarrow \vec{d} = \vec{z} - \vec{R} \\ \Rightarrow d^2 &= (\vec{z} - \vec{R}) \cdot (\vec{z} - \vec{R}) \\ &= z^2 + R^2 - 2\vec{z} \cdot \vec{R} \end{aligned}$$

Pratique o que aprendeu

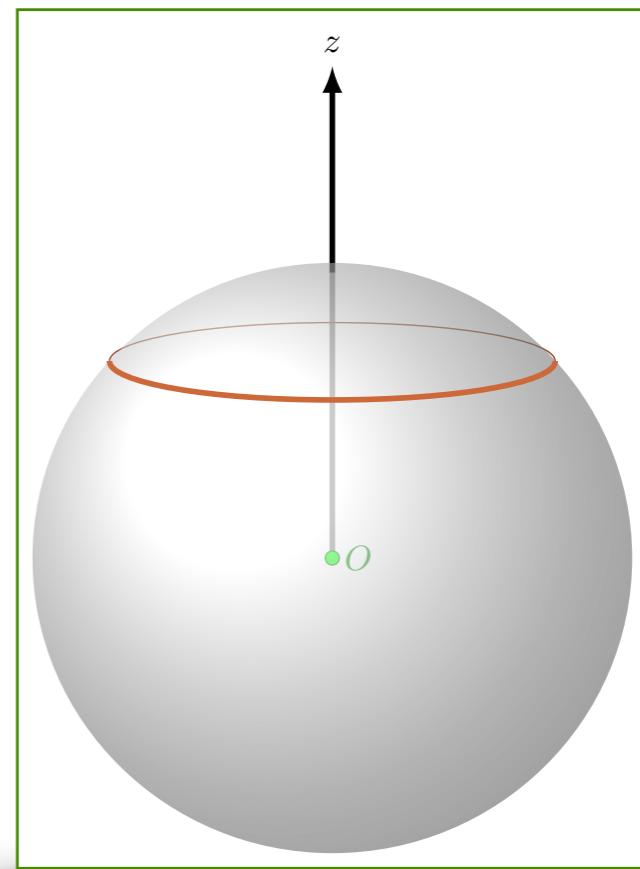
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Potencial de uma casca esférica

$$V(P) = \frac{1}{4\pi\epsilon_0} \int_A \frac{\sigma}{d} dA$$

$$d^2 = R^2 + z^2 - 2\vec{z} \cdot \vec{R}$$

$$d = \sqrt{R^2 + z^2 - 2zR \cos \theta}$$



Pratique o que aprendeu

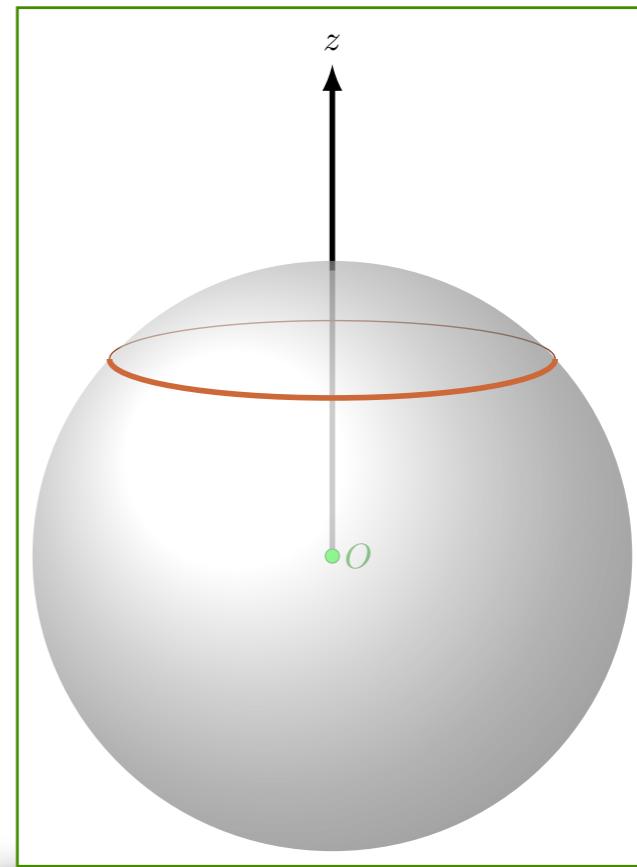
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Potencial de uma casca esférica

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$$d^2 = R^2 + z^2 - 2\vec{z} \cdot \vec{R}$$

$$d = \sqrt{R^2 + z^2 - 2zR \cos \theta}$$



$$V = \frac{\sigma}{4\pi\epsilon_0} \int_0^{2\pi} \int_{-1}^1 \frac{R^2}{\sqrt{R^2 + z^2 - 2zRu}} du d\phi$$

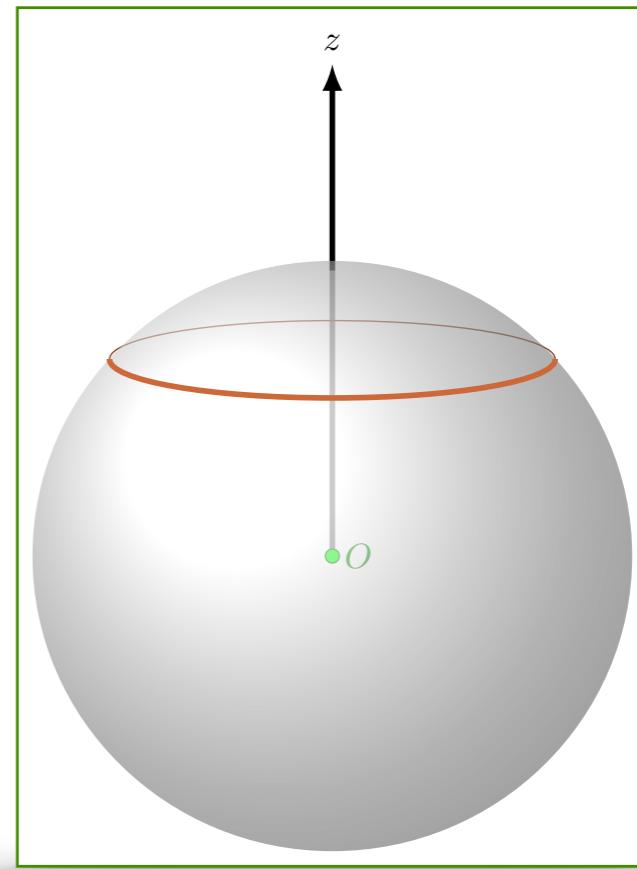
$$u = \cos \theta \Rightarrow \sin \theta d\theta = -du$$

$$\begin{aligned} d\vec{r} &= R^2 \sin \theta d\theta d\phi d\vec{r} \\ &= dA d\vec{r} \\ \Rightarrow dA &= R^2 \sin \theta d\theta d\phi \end{aligned}$$

Pratique o que aprendeu

$$V(P) = \frac{1}{4\pi\epsilon_0} \int_V \frac{1}{r} dq$$

Potencial de uma casca esférica



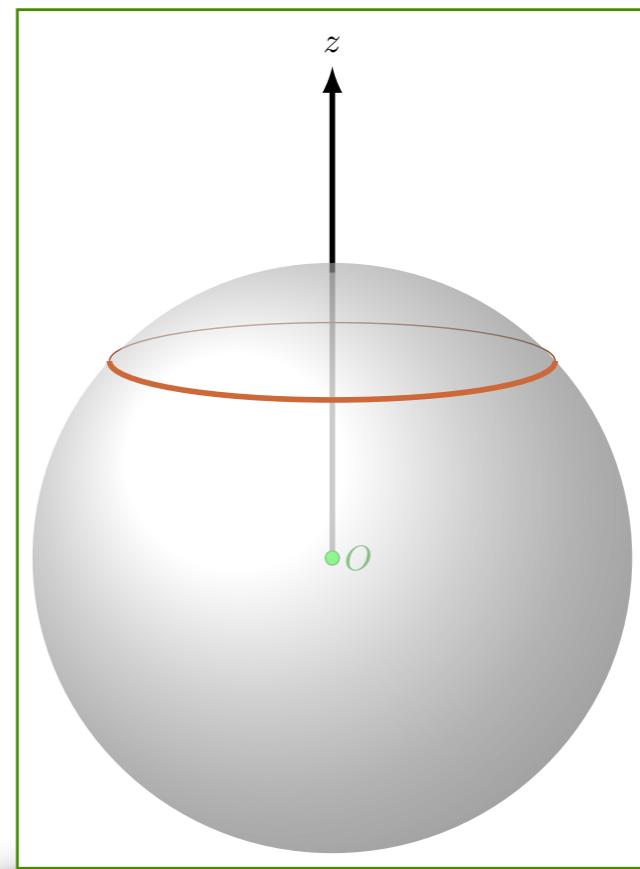
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Potencial de uma casca esférica

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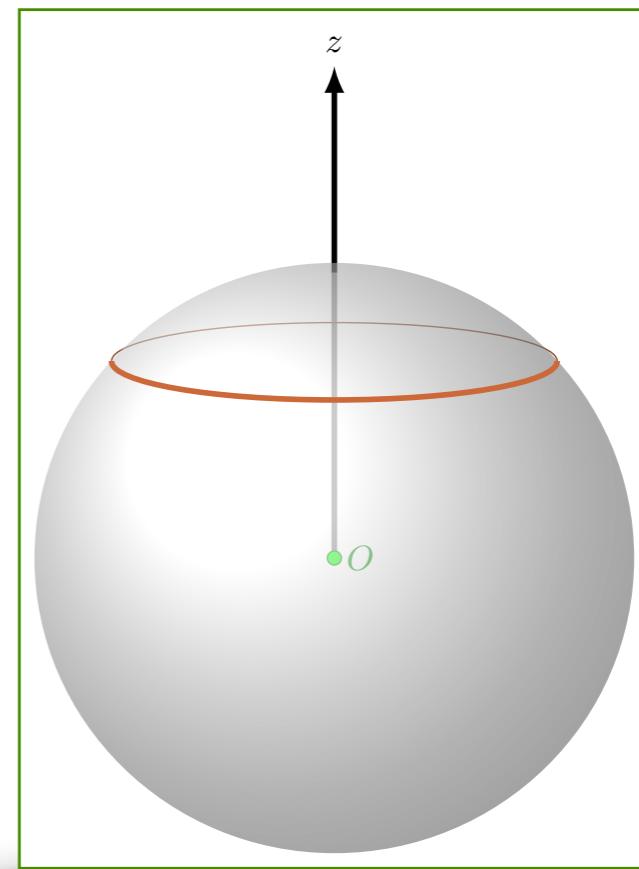
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Potencial de uma casca esférica

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$$w = R^2 + z^2 - 2zRu$$



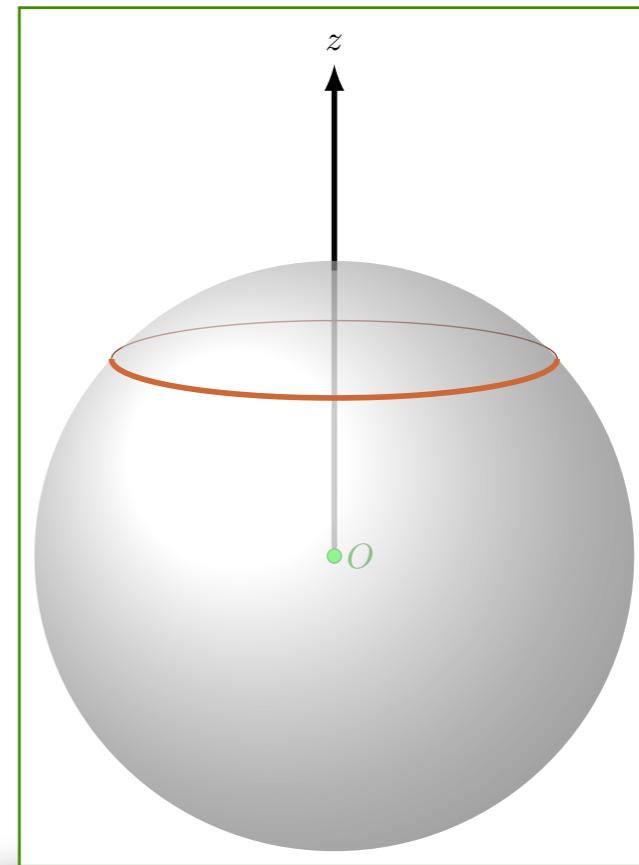
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Potencial de uma casca esférica

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$$w = R^2 + z^2 - 2zRu$$



$$dw = -2zR du$$

$$w = \begin{cases} (z - R)^2 & (u = 1) \\ (z + R)^2 & (u = -1) \end{cases}$$

Pratique o que aprendeu

$$V(P) = \frac{1}{4\pi\epsilon_0} \int_V \frac{1}{r} dq$$

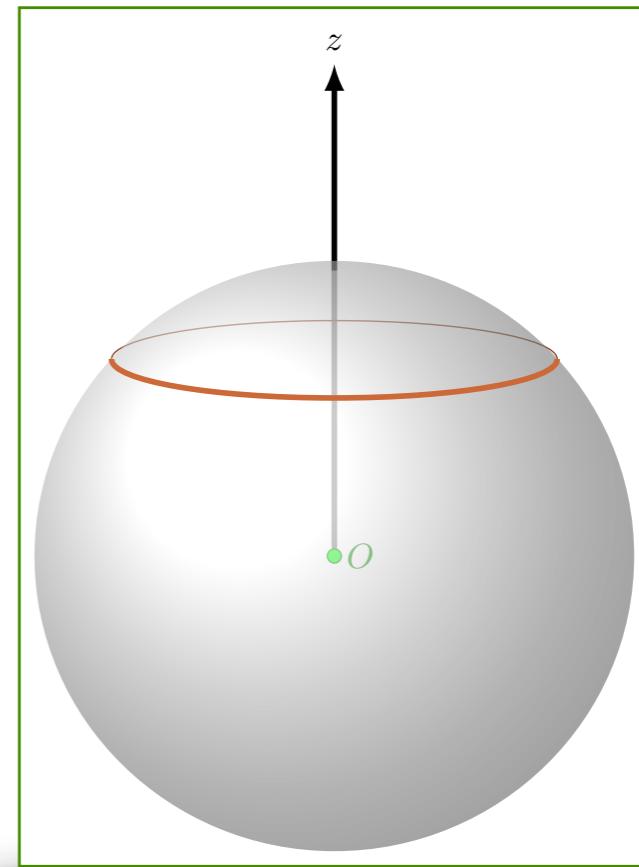
Potencial de uma casca esférica

$$V = \frac{\sigma}{4\pi\epsilon_0} \int_0^{2\pi} \int_{-1}^1 \frac{R^2}{\sqrt{R^2 + z^2 - 2zRu}} du d\phi$$

$$w = R^2 + z^2 - 2zRu$$

$$V = \frac{2\pi\sigma}{4\pi\epsilon_0} \frac{(-R)}{2z} \int_{(z+R)^2}^{(z-R)^2} \frac{dw}{\sqrt{w}}$$

$$\rightarrow \frac{R^2}{-2zR}$$



$$dw = -2zR du$$

$$w = \begin{cases} (z - R)^2 & (u = 1) \\ (z + R)^2 & (u = -1) \end{cases}$$

Pratique o que aprendeu

$$V(P) = \frac{1}{4\pi\epsilon_0} \int_V \frac{1}{r} dq$$

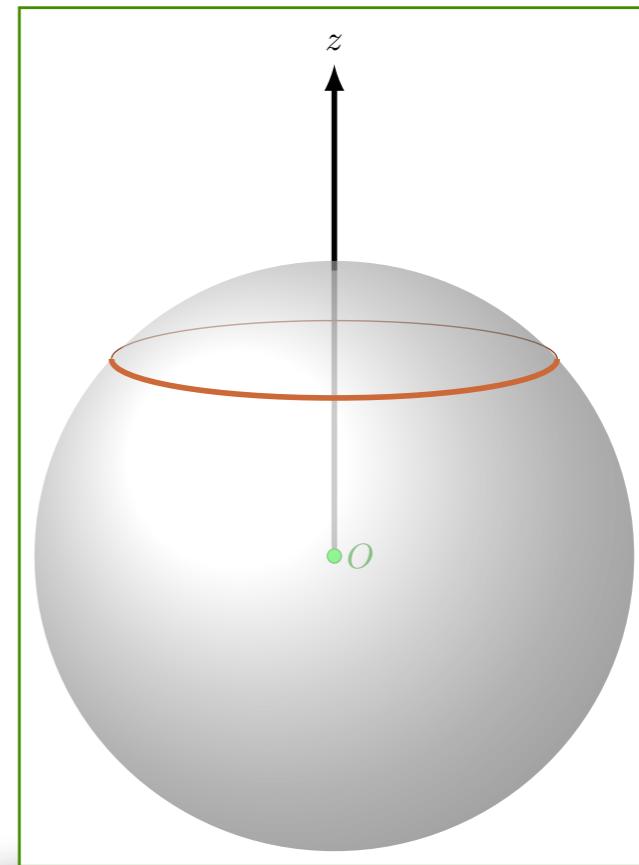
Potencial de uma casca esférica

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$$V = \frac{2\pi\sigma}{4\pi\epsilon_0} \frac{(-R)}{2z} \int_{(z+R)^2}^{(z-R)^2} \frac{dw}{\sqrt{w}}$$

$$V = \frac{\sigma}{4\epsilon_0} \frac{(-R)}{z} \left(2\sqrt{w}\right)_{(z+R)^2}^{(z-R)^2}$$



$$dw = -2zR du$$

$$w = \begin{cases} (z - R)^2 & (u = 1) \\ (z + R)^2 & (u = -1) \end{cases}$$

Pratique o que aprendeu

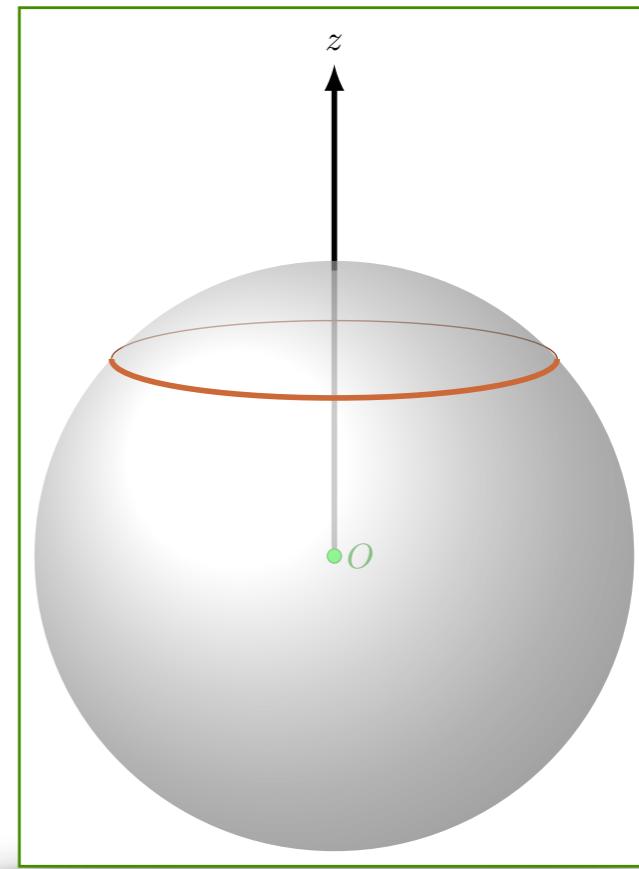
$$V(P) = \frac{1}{4\pi\epsilon_0} \int_V \frac{1}{r} dq$$

Potencial de uma casca esférica

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$$w = R^2 + z^2 - 2zRu$$

$$V = \frac{2\pi\sigma}{4\pi\epsilon_0} \frac{(-R)}{2z} \int_{(z+R)^2}^{(z-R)^2} \frac{dw}{\sqrt{w}}$$



$$V = \frac{\sigma}{4\epsilon_0} \frac{(-R)}{z} \left(2\sqrt{w}\right)_{(z+R)^2}^{(z-R)^2} = \frac{\sigma}{\epsilon_0} \frac{R}{2z} (z+R - |z-R|)$$

$z > R \Rightarrow V = \frac{\sigma}{\epsilon_0} \frac{R}{2z} (z + R - (z - R)) = \frac{\sigma}{\epsilon_0} \frac{R^2}{z}$

$\left. \begin{array}{l} z - R \\ R - z \end{array} \right\} \begin{array}{l} (z > R) \\ (z < R) \end{array}$

Pratique o que aprendeu

$$V(P) = \frac{1}{4\pi\epsilon_0} \int_V \frac{1}{r} dq$$

Potencial de uma casca esférica

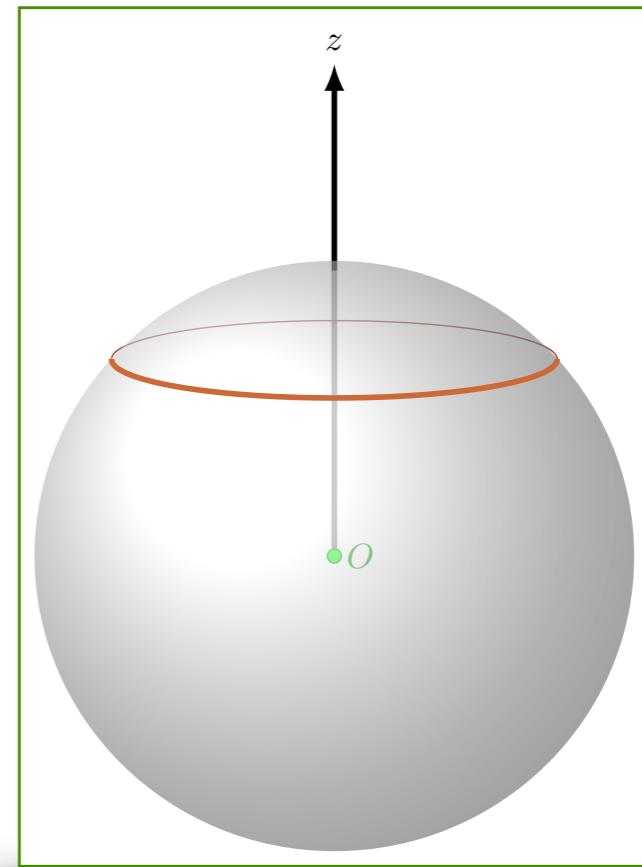
$$V = \frac{\sigma}{4\pi\epsilon_0} \int_0^{2\pi} \int_{-1}^1 \frac{R^2}{\sqrt{R^2 + z^2 - 2zRu}} du d\phi$$

$$w = R^2 + z^2 - 2zRu$$

$$V = \frac{2\pi\sigma}{4\pi\epsilon_0} \frac{(-R)}{2z} \int_{(z+R)^2}^{(z-R)^2} \frac{dw}{\sqrt{w}}$$

$$V = \frac{\sigma}{4\epsilon_0} \frac{(-R)}{z} \left(2\sqrt{w}\right)_{(z+R)^2}^{(z-R)^2}$$

$z < R \Rightarrow V = \frac{\sigma}{\epsilon_0} \frac{R}{2z} (z + R - (R - z)) = \frac{\sigma}{\epsilon_0} R$



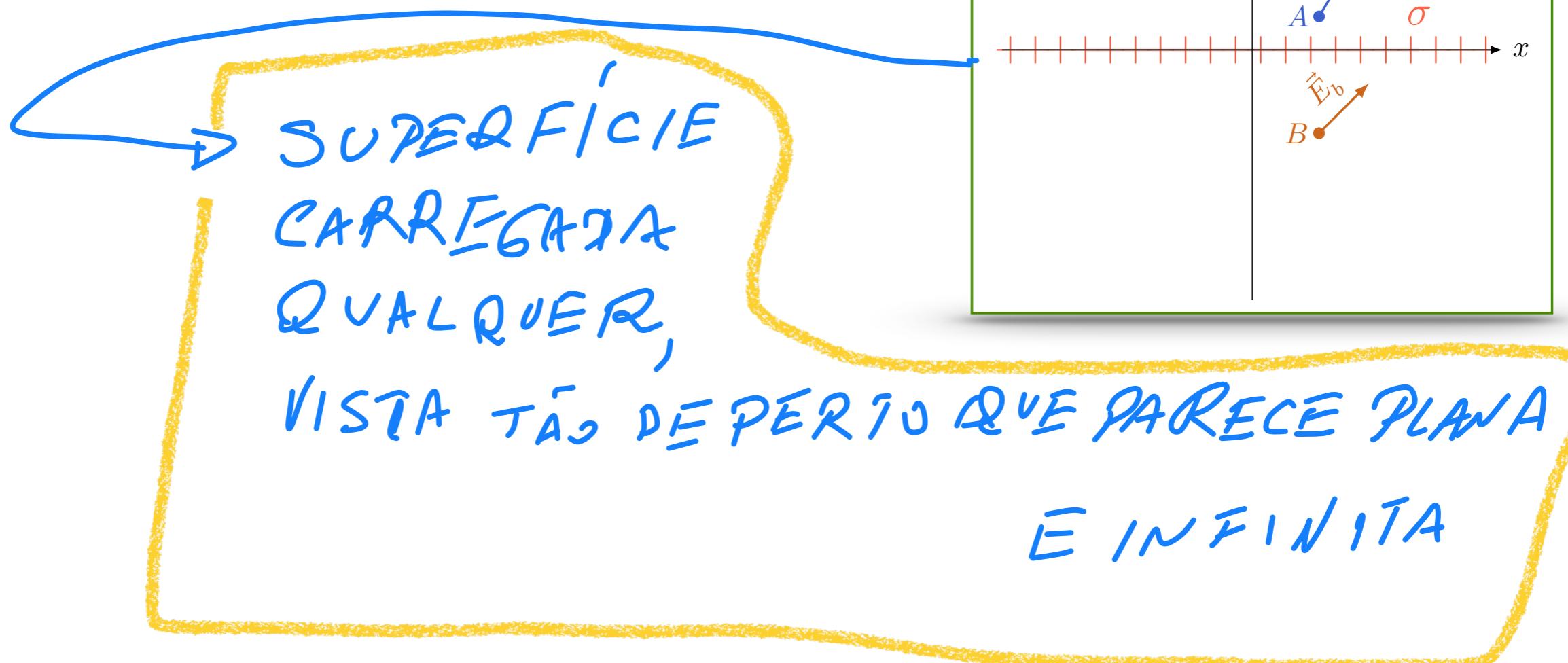
$$|z - R| = R - z \quad (z \leq R)$$

\Rightarrow CONSIANSE $\Rightarrow \vec{E} = 0$

$$\vec{E} = 0$$

Condições de contorno

Descontinuidade do campo elétrico



Condições de contorno

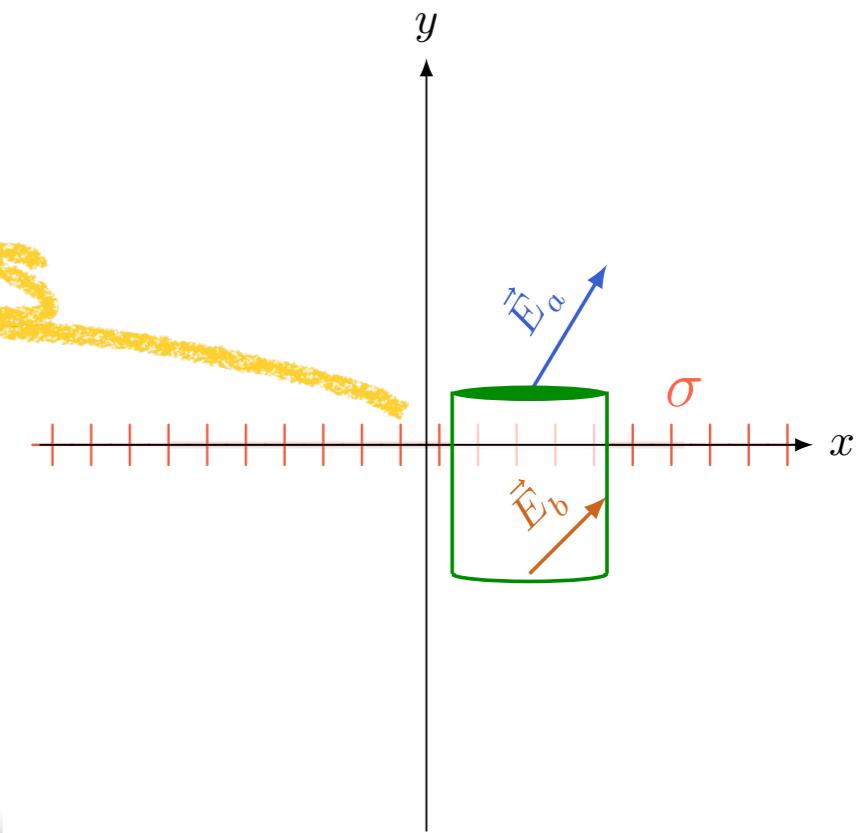
Descontinuidade do campo elétrico

$$E_{y,a}A - E_{y,b}A = \frac{\sigma A}{\epsilon_0}$$

FLUXO
ATRAVÉS
DAS TAMPAS

CARGA NO
INTERIOR
DO CILINDRO

GAUSS



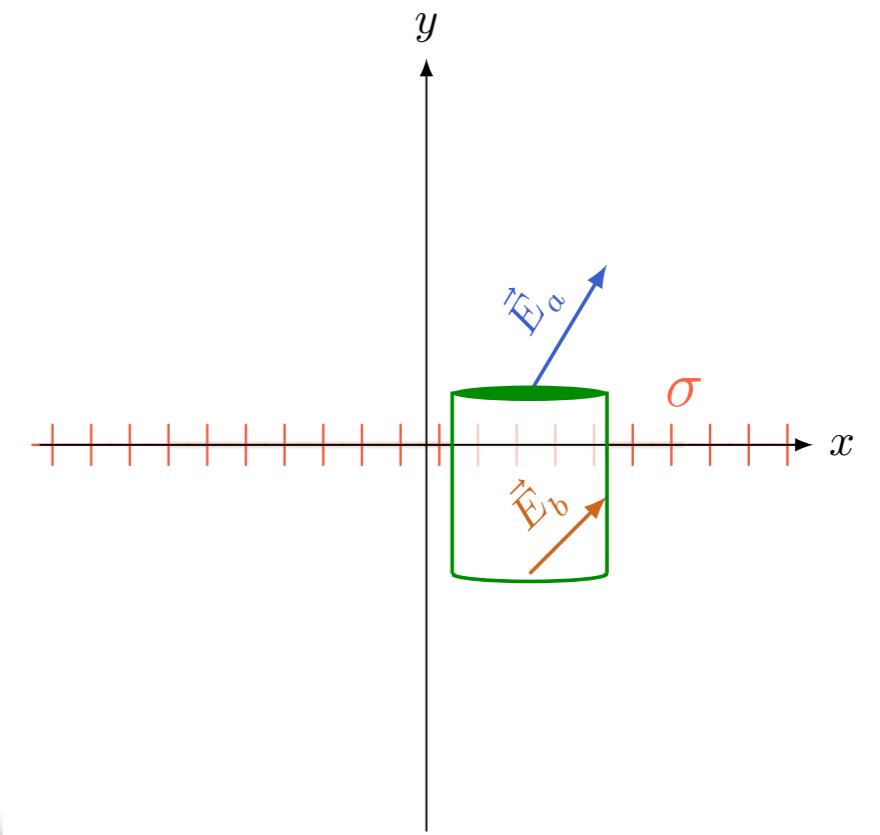
FLUXO ATRAVÉS
DA PAREDE
LATERAL r'
DUITO PEQUENO,
PQ ALTURA Δz
WE IPOTESINAL

Condições de contorno

Descontinuidade do campo elétrico

$$E_{y,a}A - E_{y,b}A = \frac{\sigma A}{\epsilon_0}$$

$$\Delta E_y = \frac{\sigma}{\epsilon_0}$$



Condições de contorno

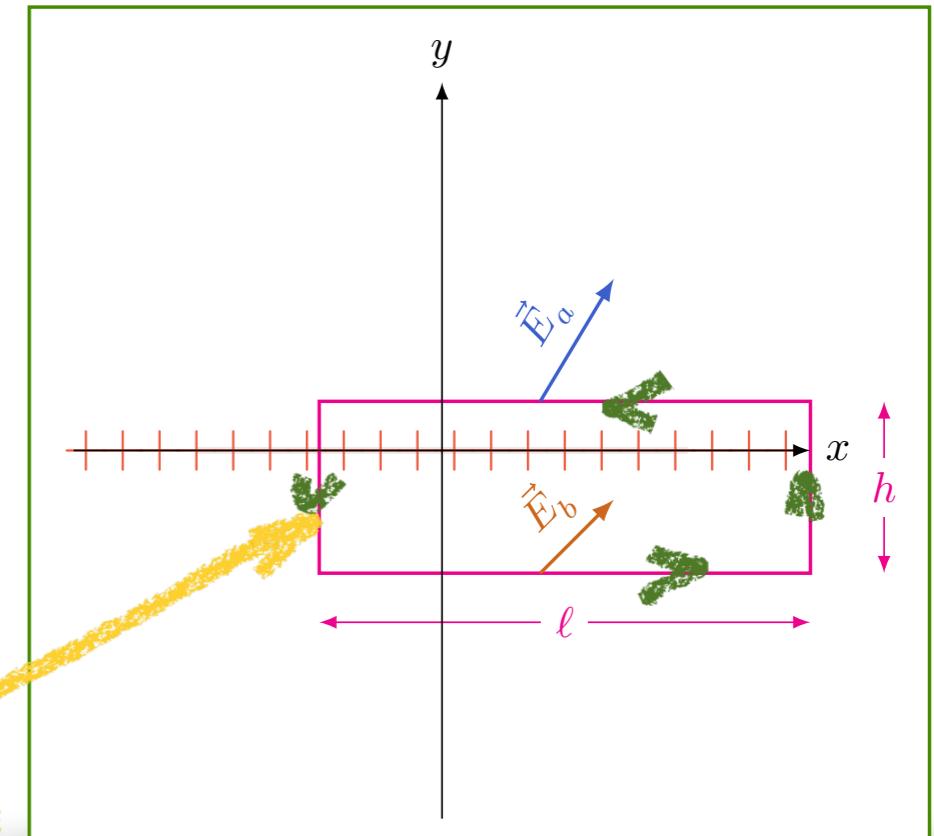
Descontinuidade do campo elétrico

$$E_{y,a}A - E_{y,b}A = \frac{\sigma A}{\epsilon_0}$$

$$\Delta E_y = \frac{\sigma}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0$$

STOKES NO CIRCUITO



Condições de contorno

Descontinuidade do campo elétrico

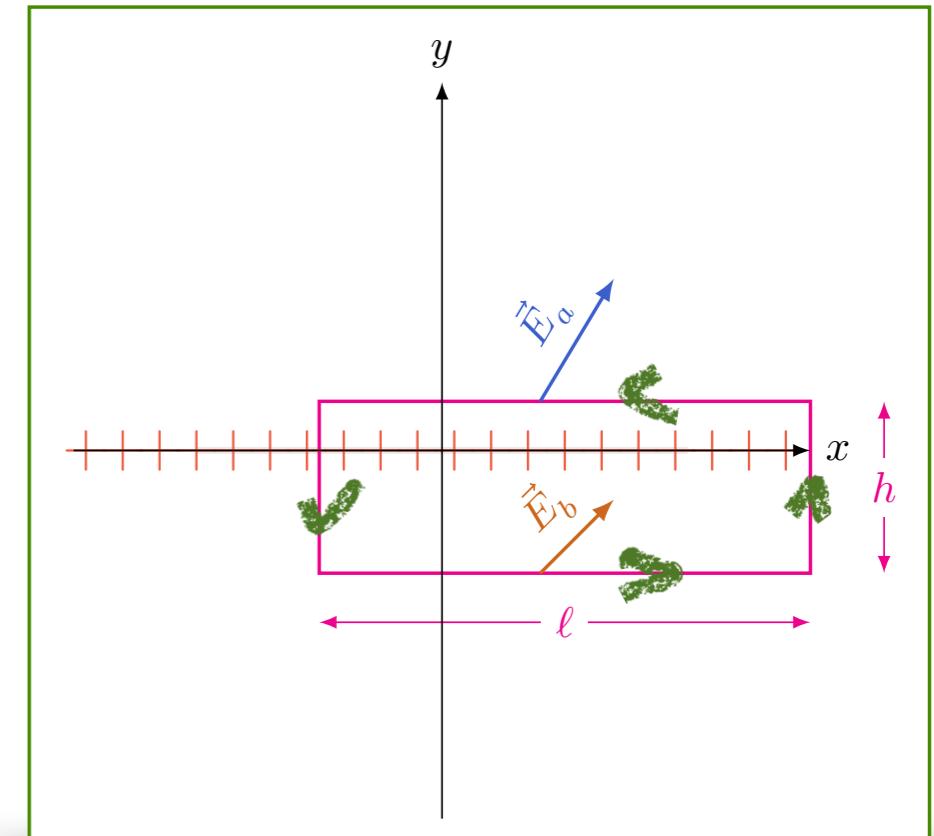
$$E_{y,a}A - E_{y,b}A = \frac{\sigma A}{\epsilon_0}$$

$$\Delta E_y = \frac{\sigma}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0$$

$$E_{x,b}\ell + E_y h - E_{x,a}\ell - E_y h = 0$$

→ TÃO PEQUENOS QUANTO QUISERMOS



Condições de contorno

Descontinuidade do campo elétrico

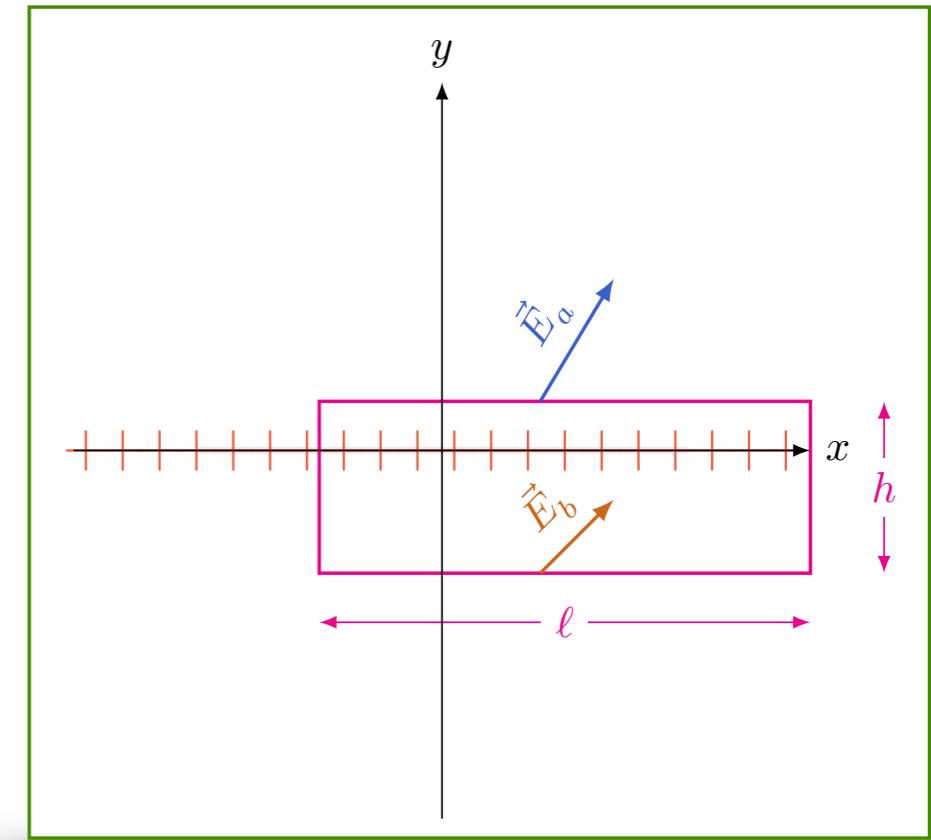
$$E_{y,a}A - E_{y,b}A = \frac{\sigma A}{\epsilon_0}$$

$$\Delta E_y = \frac{\sigma}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0$$

$$E_{x,b}\ell + E_y h - E_{x,a}\ell - E_y h = 0$$

$$\Rightarrow (E_{x,b} - E_{x,a})\ell = 0$$



Condições de contorno

Descontinuidade do campo elétrico

$$E_{y,a}A - E_{y,b}A = \frac{\sigma A}{\epsilon_0}$$

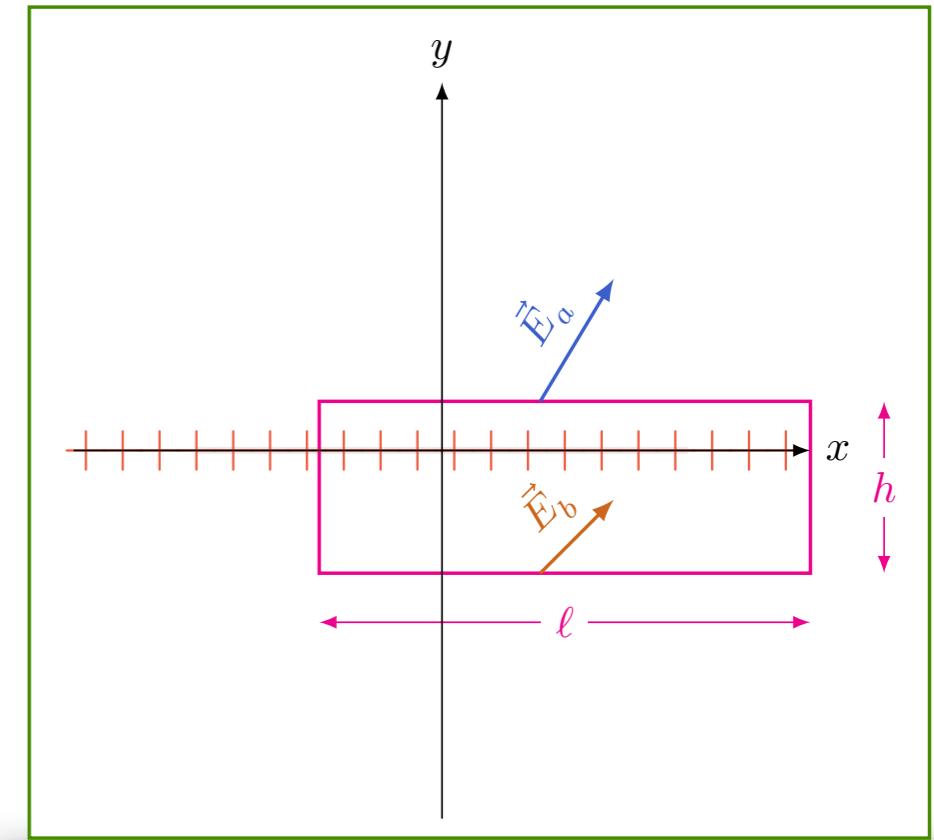
$$\Delta E_y = \frac{\sigma}{\epsilon_0}$$

$$E_{x,b}\ell + E_y h - E_{x,a}\ell - E_y h = 0$$

$$(E_{x,b} - E_{x,a})\ell = 0$$

$$\Delta E_x = 0$$

$\Delta E_z = 0$ (PERPENDICULAR À TELA)



$$\Delta \vec{E} \cdot \hat{n} = \frac{\sigma}{\epsilon_0}$$

$$\hat{n} = \hat{j}$$

Condições de contorno

Descontinuidade do campo elétrico

$$E_{y,a}A - E_{y,b}A = \frac{\sigma A}{\epsilon_0}$$

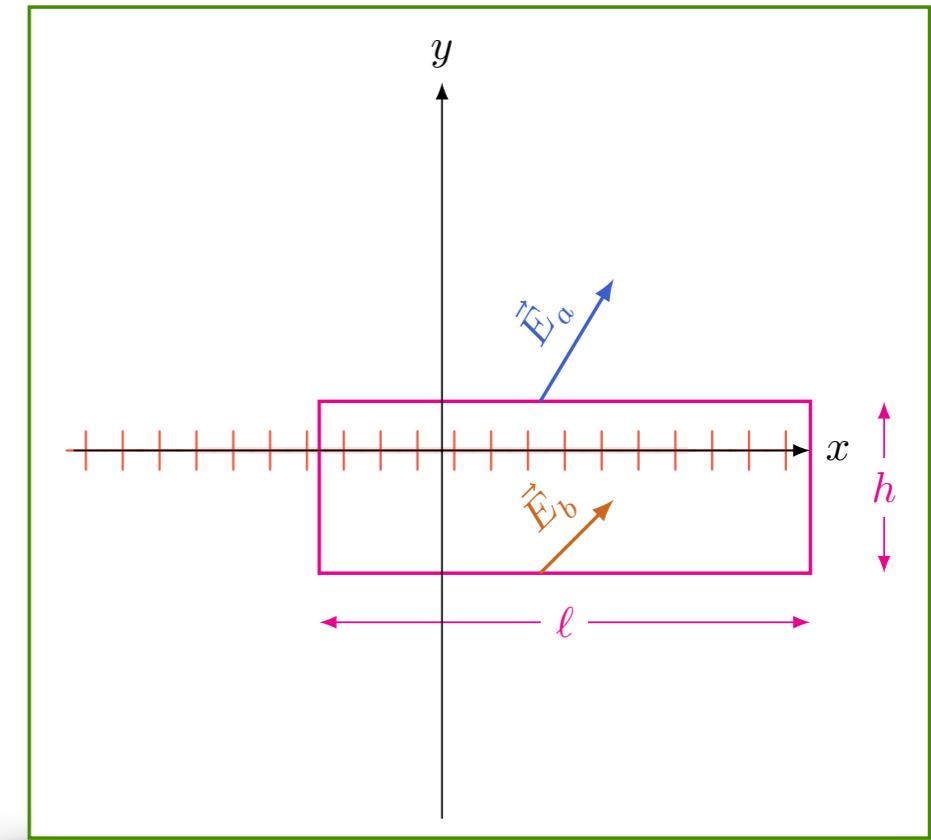
$$\Delta E_y = \frac{\sigma}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0$$

$$E_{x,b}\ell + E_y h - E_{x,a}\ell - E_y h = 0$$

$$(E_{x,b} - E_{x,a})\ell = 0$$

$$\Delta E_x = 0$$



$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\Delta \vec{E} \cdot \hat{n} = \frac{\sigma}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0$$