# Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

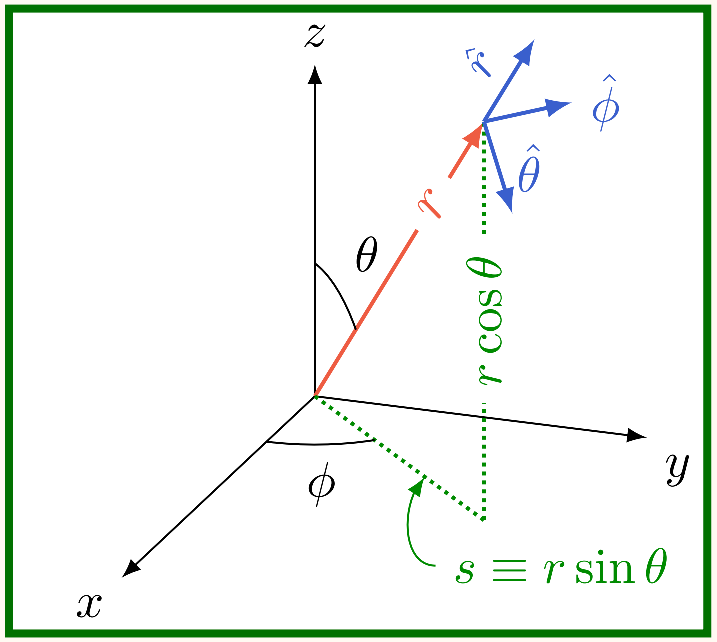
$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

Aula de 12 de maio

Eletrostática

# Coordenadas esféricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}r\,\hat{r} + r\,\mathrm{d}\theta\,\hat{\theta} + r\sin\theta\,\mathrm{d}\phi\,\hat{\phi}$$




$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

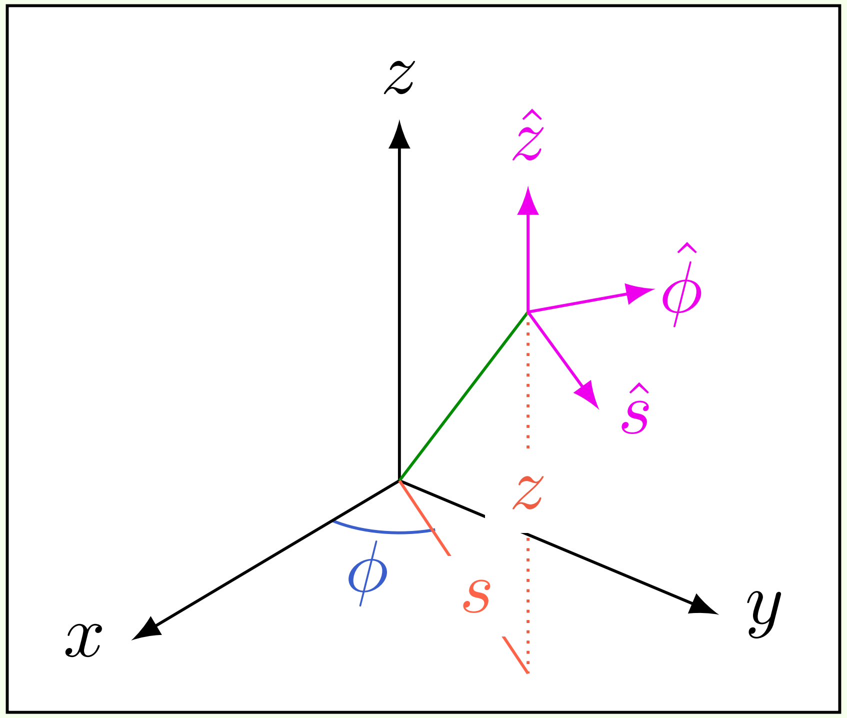
$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\, v_\phi\right) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$

$$+ \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r v_\phi\right)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\, v_\theta\right) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

# Coordenadas cilíndricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}s\,\hat{s} + s\,\mathrm{d}\phi\,\hat{\phi} + \mathrm{d}z\,\hat{z}$$




$$\vec{\boldsymbol{\nabla}} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{v} = \frac{1}{s}\frac{\partial (s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\boldsymbol{\nabla}} \times \vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

*Primeira prova*

*5 de maio de 2021*

1. A figura 1 mostra um cilindro de raio $R$ e altura $h$. Define-se um sistema de coordenadas cilíndricas centrado no ponto onde o eixo do cilindro intercepta a base do cilindro. Nesse sistema, considere o campo vetorial

$$\vec{v} = \frac{\hat{s}}{s}.$$

   (a) Calcule o divergente de $\vec{v}$ para um ponto qualquer, com coordenadas $(s, \phi, z)$, no interior do cilindro;

   (b) Calcule o fluxo total do vetor $\vec{v}$ através das três paredes do cilindro;

   (c) Determine a integral volumétrica do divergente, a partir do teorema fundamental para o divergente (de Gauss), e compare com o resultado do item (a);

   (d) Escreva uma expressão que descreva $\vec{\nabla} \cdot \vec{v}$ para qualquer ponto no interior do cilindro.

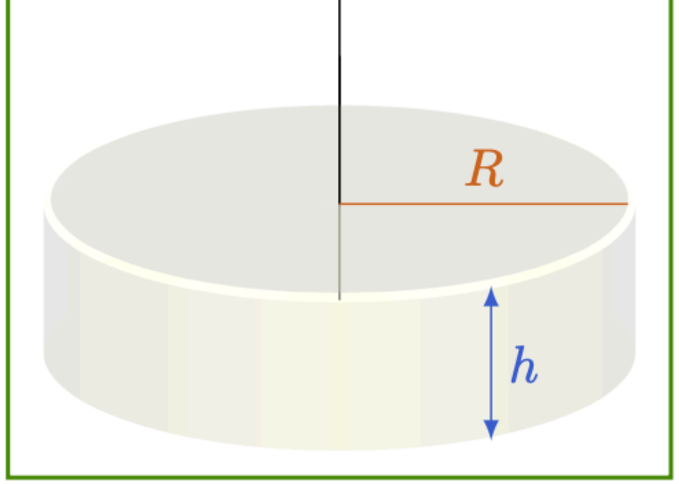


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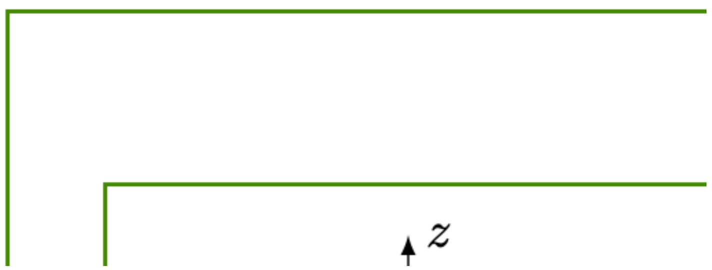
Figura 1: Questão 1



# Questão 1.d da P1

$$\overrightarrow{v} = \frac{\hat{s}}{s}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{v} = 0 \qquad (s \neq 0)$$

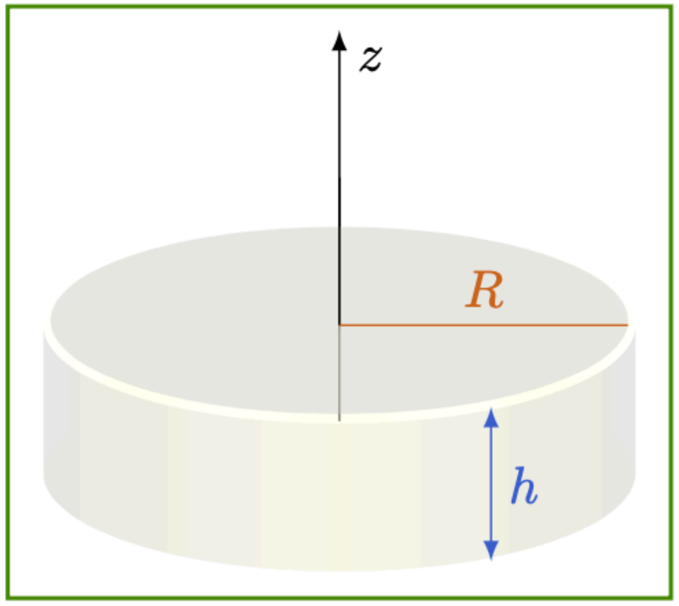



Figura 1: Questão 1

# Questão 1.d da P1

$$\overrightarrow{v} = \frac{\hat{s}}{s}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{v} = 0 \qquad (s \neq 0, \forall z)$$

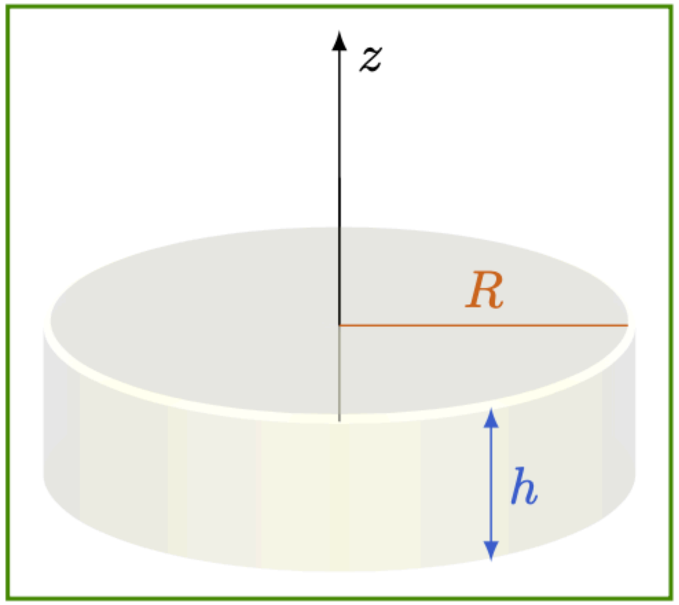



Figura 1: Questão 1

# Questão 1.d da P1

$$\vec{v} = \frac{\hat{s}}{s}$$

$$\vec{\nabla} \cdot \vec{v} = 0 \qquad (s \neq 0, \forall z)$$

$$\vec{\nabla} \cdot \vec{v} = \alpha \delta(x) \delta(y)$$

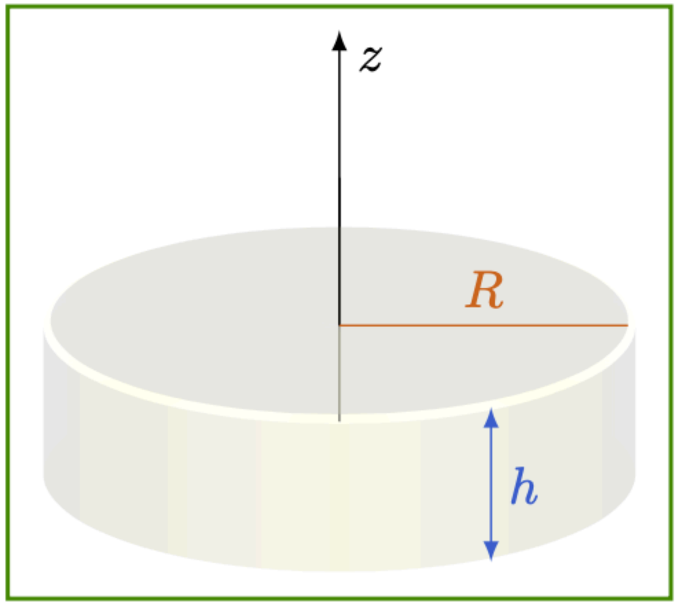



Figura 1: Questão 1

# Questão 1.d da P1

$$\overrightarrow{v} = \frac{\hat{s}}{s}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{v} = 0 \qquad (s \neq 0, \forall z)$$

↳ DA QUESTÃO 1a

$$\overrightarrow{\nabla} \cdot \overrightarrow{v} = \alpha \delta(x) \delta(y)$$

$$\int_{\mathscr{V}} \overrightarrow{\nabla} \cdot \overrightarrow{v} \, \mathrm{d}\tau = \alpha \int_{\mathscr{V}} \delta(x) \delta(y) \, \mathrm{d}\tau$$

→ VOLUME DO CILINDRO

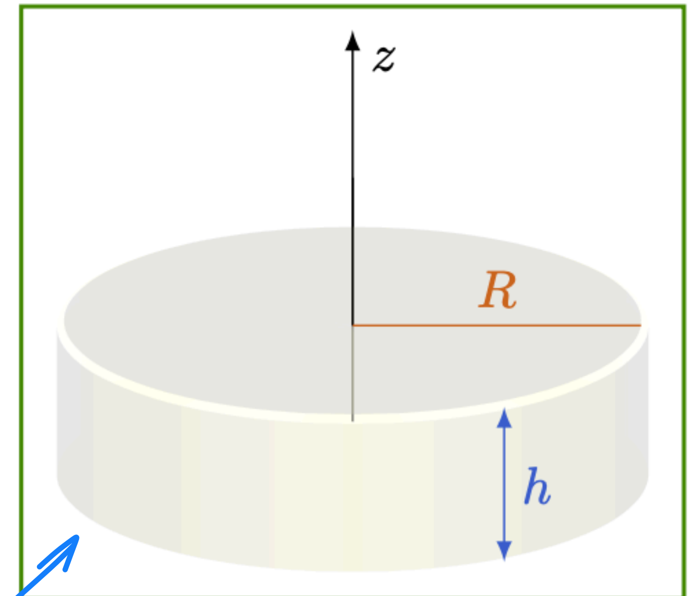



Figura 1: Questão 1

# Questão 1.d da P1

$$\vec{v} = \frac{\hat{s}}{s}$$

$$\vec{\nabla} \cdot \vec{v} = 0 \qquad (s \neq 0, \forall z)$$

$$\vec{\nabla} \cdot \vec{v} = \alpha \delta(x)\delta(y)$$

$$\int_{\mathscr{V}} \vec{\nabla} \cdot \vec{v} \, \mathrm{d}\tau = \alpha \int_{\mathscr{V}} \delta(x)\delta(y) \, \mathrm{d}\tau$$

$$2\pi h = \alpha \int_{\mathscr{V}} \delta(x)\delta(y) \, \mathrm{d}x\mathrm{d}y\mathrm{d}z$$

DA QUESTÃO 1.C

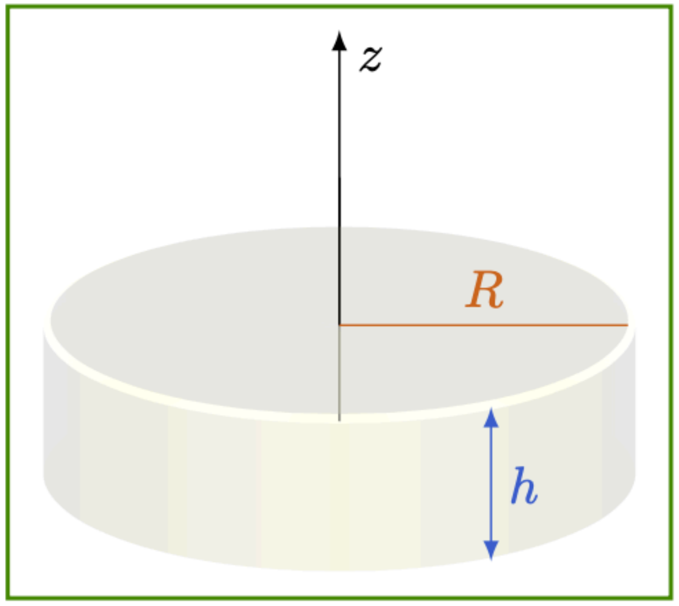



Figura 1: Questão 1

# Questão 1.d da P1

$$\vec{v} = \frac{\hat{s}}{s}$$

$$\vec{\nabla} \cdot \vec{v} = 0 \qquad (s \neq 0, \forall z)$$

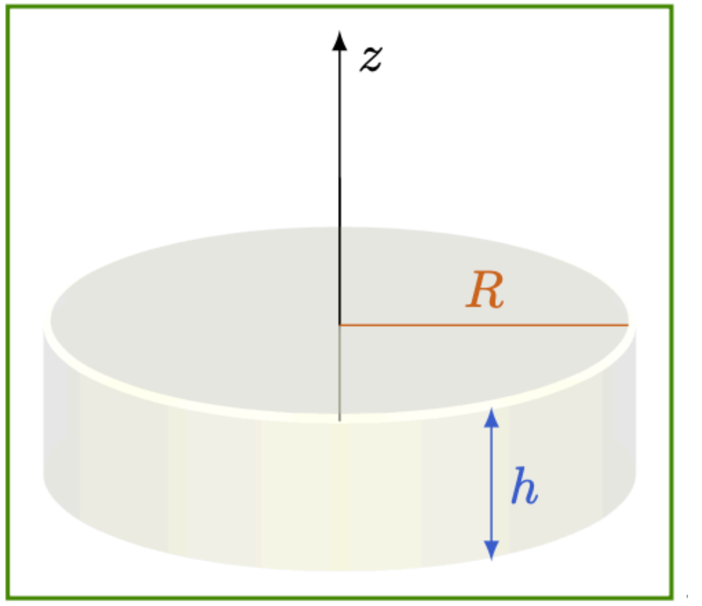



Figura 1: Questão 1

$$\vec{\nabla} \cdot \vec{v} = \alpha\delta(x)\delta(y)$$

$$\int_{\mathscr{V}} \vec{\nabla} \cdot \vec{v}\, \mathrm{d}\tau = \alpha \int_{\mathscr{V}} \delta(x)\delta(y)\, \mathrm{d}\tau$$

$$2\pi h = \alpha \int_{\mathscr{V}} \delta(x)\delta(y)\, \mathrm{d}x\mathrm{d}y\mathrm{d}z \qquad 2\pi h = \alpha h$$

$$\left.\begin{array}{l}\int \delta(x)\delta(y)\, dx\, dy = 1 \\ \int dz = h\end{array}\right\}$$

# Questão 1.d da P1

$$\vec{v} = \frac{\hat{s}}{s}$$

$$\vec{\nabla} \cdot \vec{v} = 0 \qquad (s \neq 0, \forall z)$$

$$\vec{\nabla} \cdot \vec{v} = \alpha\delta(x)\delta(y)$$

$$\int_{\mathscr{V}} \vec{\nabla} \cdot \vec{v}\, \mathrm{d}\tau = \alpha \int_{\mathscr{V}} \delta(x)\delta(y)\, \mathrm{d}\tau$$

$$2\pi h = \alpha \int_{\mathscr{V}} \delta(x)\delta(y)\, \mathrm{d}x\mathrm{d}y\mathrm{d}z \qquad 2\pi h = \alpha h$$

$$\vec{\nabla} \cdot \vec{v} = 2\pi\delta(x)\delta(y)$$

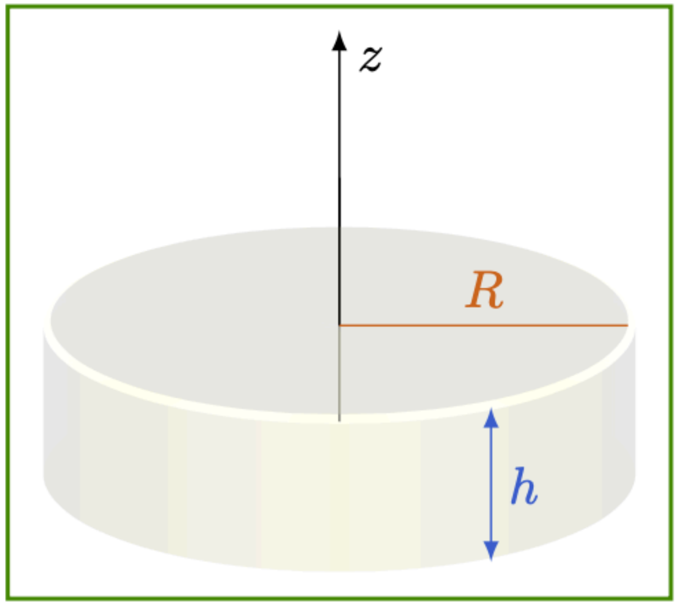



Figura 1: Questão 1

# Questão 1.d da P1

$$\vec{v} = \frac{\hat{s}}{s}$$

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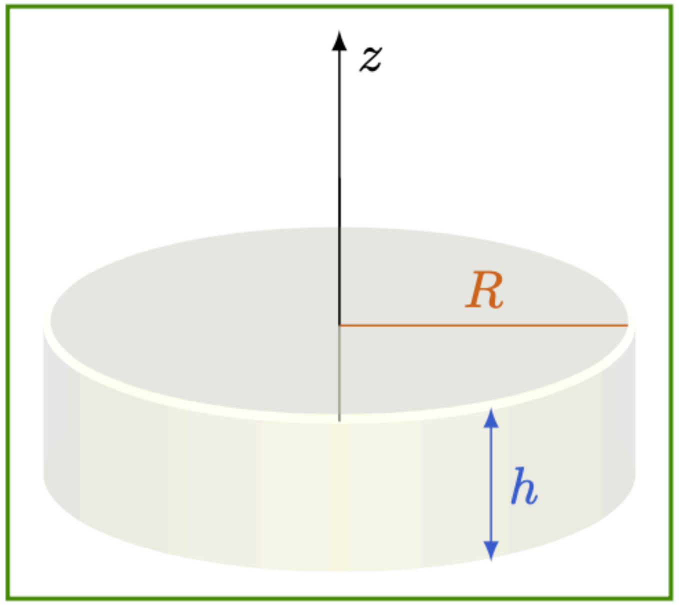



Figura 1: Questão 1

$$\int_{\mathscr{V}} \vec{\nabla} \cdot \vec{v} \, \mathrm{d}\tau = \alpha \int_{\mathscr{V}} \delta(x)\delta(y) \, \mathrm{d}\tau$$

$$2\pi h = \alpha \int_{\mathscr{V}} \delta(x)\delta(y) \, \mathrm{d}x\mathrm{d}y\mathrm{d}z \qquad 2\pi h = \alpha h$$

$$\vec{\nabla} \cdot \vec{v} = 2\pi\delta(x)\delta(y) = 2\pi\delta^2(\vec{s})$$

→ DEFINE $\delta^2(\vec{s})$

# Questão 1.d da P1

ALTERNATIVA

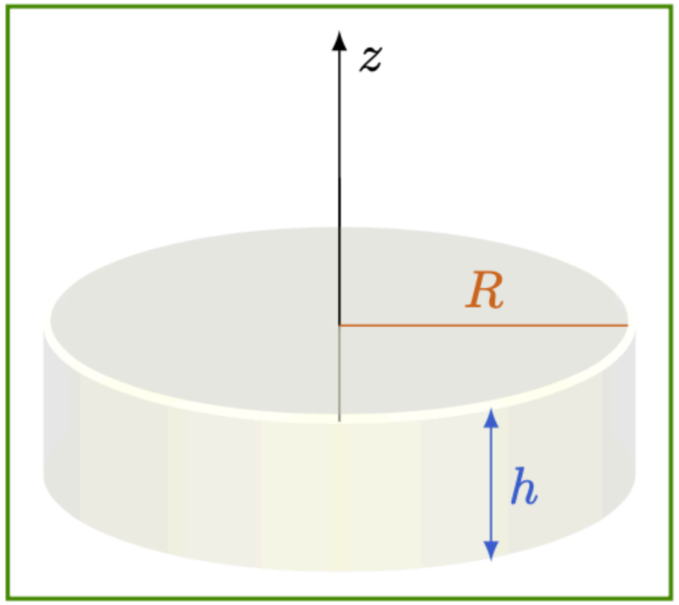


Figura 1: Questão 1

$$\vec{v} = \frac{\hat{s}}{s}$$

$$\vec{\nabla} \cdot \vec{v} = 0 \qquad (s \neq 0, \forall z)$$

$$\int_{\mathscr{V}} \vec{\nabla} \cdot \vec{v} \, \mathrm{d}\tau = 2\pi h$$

$$\mathrm{d}\tau = \mathrm{d}z \, s \, \mathrm{d}\phi \, \mathrm{d}s$$

PARA CANCELAR

$$\vec{\nabla} \cdot \vec{v} = \alpha \frac{1}{s} \delta(s)$$

$$\int_{\mathscr{V}} \vec{\nabla} \cdot \vec{v} \, \mathrm{d}\tau = 2\pi h \alpha \int \delta(s) \mathrm{d}s$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s} \delta(s)$$

# Cavitação

# Cavitação

## MECÂNICA DE FLUIDOS

EQ. BERNOULLI ⇒ $P + \frac{\rho v^2}{2} + \rho g h = constante$

- ONDE VELOCIDADE É ALTA, PRESSÃO É BAIXA
- PRESSÃO BAIXA PERMITE QUE BOLHAS SE FORMEM

EXS:
- ESTEIRA DE HÉLICE (TELA ANTERIOR)
- ÁGUA MINERAL C/ GÁS, REFRIGERANTE OU CERVEJA EM GARRAFA QUE FOI AGITADA
-  TETO DE RESIDÊNCIA QUE VOA EM FURACÃO

- AO PASSAR POR REGIÃO ONDE PRESSÃO É NORMAL, BOLHAS ENTRAM EM COLAPSO ⇐ <u>CAVITAÇÃO</u>

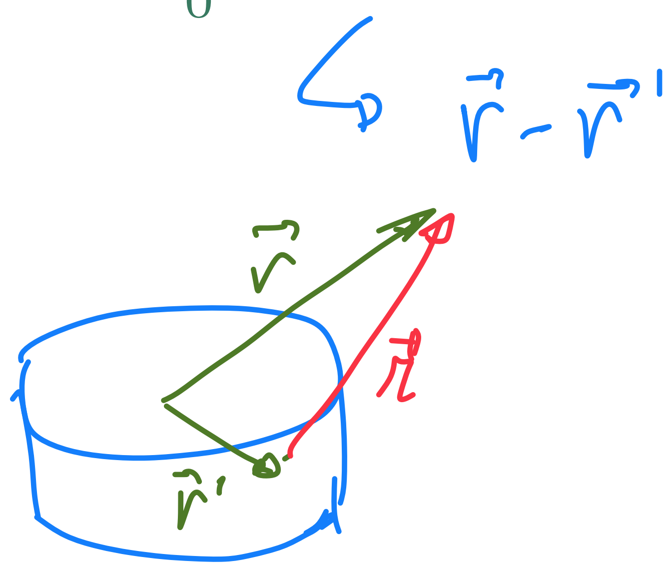
EXS:
- TORNEIRA QUE CANTA
- OLEODUTO ENFERRUJADO
- TUBARÃO QUE MATA SARDINHAS COM RABANADA NA ÁGUA (VER VÍDEO INDICADO NO MOODLE)

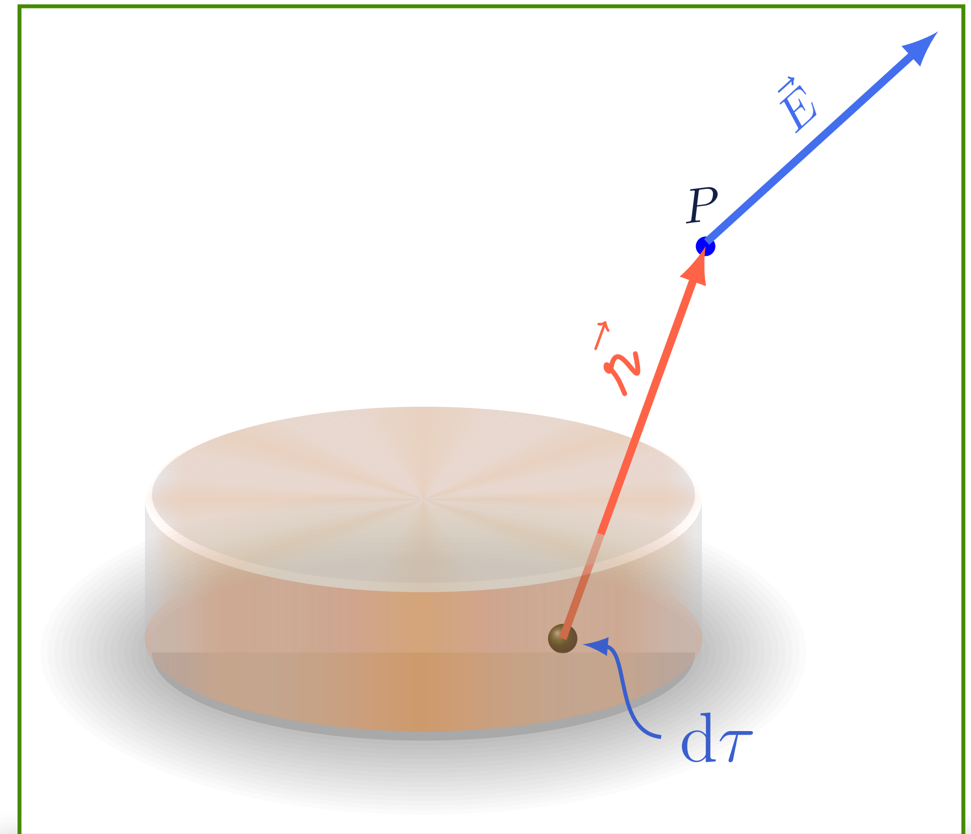
- CAVITAÇÃO TAMBÉM OCORRE POR INÉRCIA (VER VÍDEO)

# Eletrostática

## Campo de distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}\,\mathrm{d}q$$



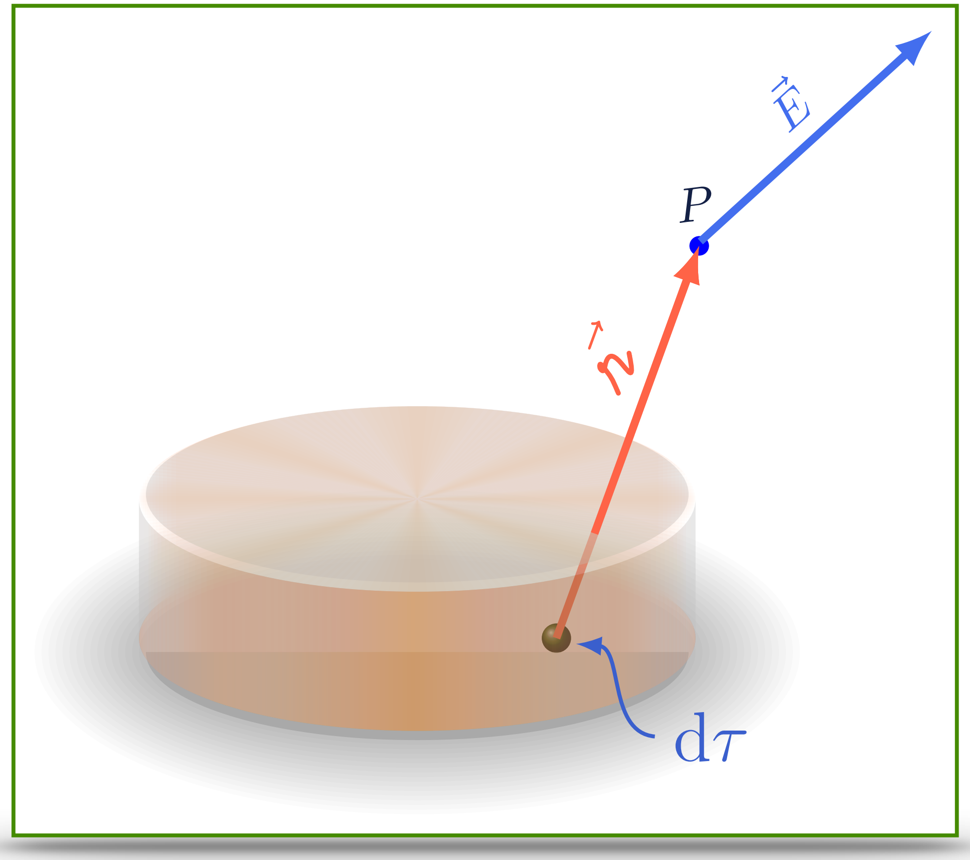


# Eletrostática

Campo de distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{1}{\mathscr{r}^2} \hat{\mathscr{r}} \, \mathrm{d}q$$

$$\vec{\nabla} \times \vec{E} = \frac{1}{4\pi\epsilon_0} \int \vec{\nabla} \times \left( \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \right) \mathrm{d}q$$



# Eletrostática

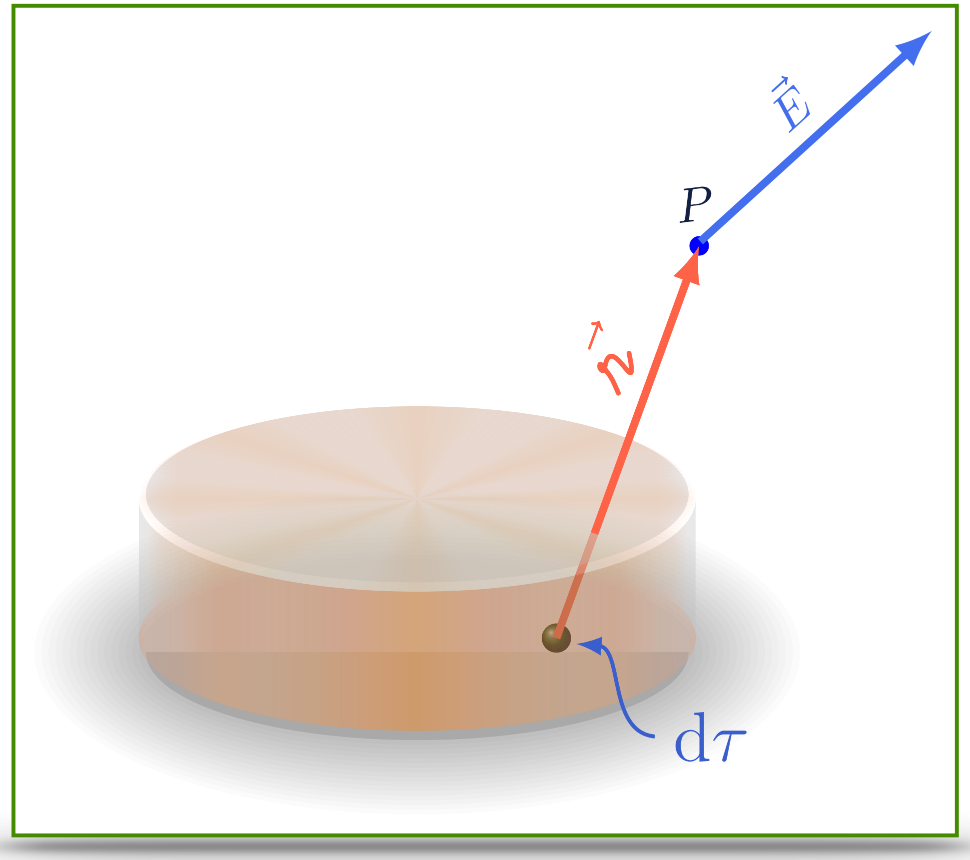
Campo de distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathcal{r}^2}\hat{\mathcal{r}}\,\mathrm{d}q$$

$$\vec{\nabla}\times\vec{E} = \frac{1}{4\pi\epsilon_0}\int \vec{\nabla}\times\left(\frac{\hat{\mathcal{r}}}{\mathcal{r}^2}\right)\mathrm{d}q$$

ATUA SOBRE $\vec{r}$
EM $\vec{\mathcal{r}} = \vec{r} - \vec{r}'$

$$\vec{\nabla}\times\vec{E} = 0$$



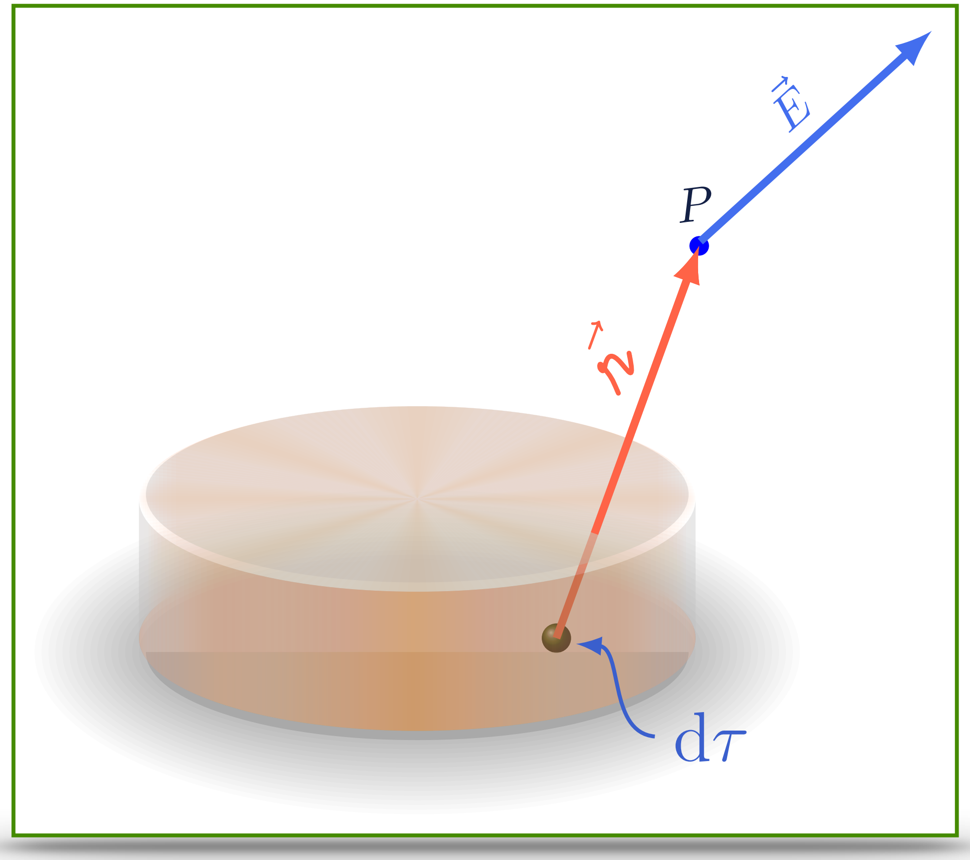
# Eletrostática

Campo de distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathcal{r}^2}\hat{\mathcal{r}}\,\mathrm{d}q$$

$$\vec{\nabla}\times\vec{E} = \frac{1}{4\pi\epsilon_0}\int \vec{\nabla}\times\left(\frac{\hat{\mathcal{r}}}{\mathcal{r}^2}\right)\mathrm{d}q$$

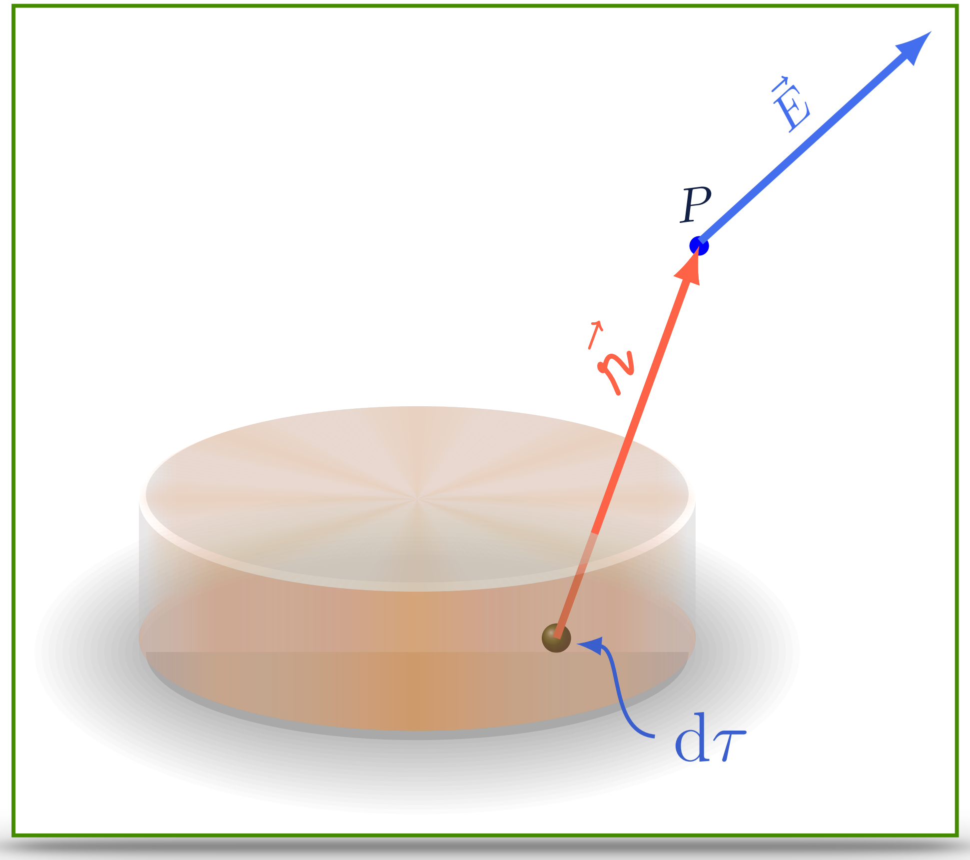
$$\vec{\nabla}\times\vec{E}=0$$



# Eletrostática

Campo de distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{1}{r^2}\hat{r}\,\mathrm{d}q$$




$$\vec{\nabla} \times \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

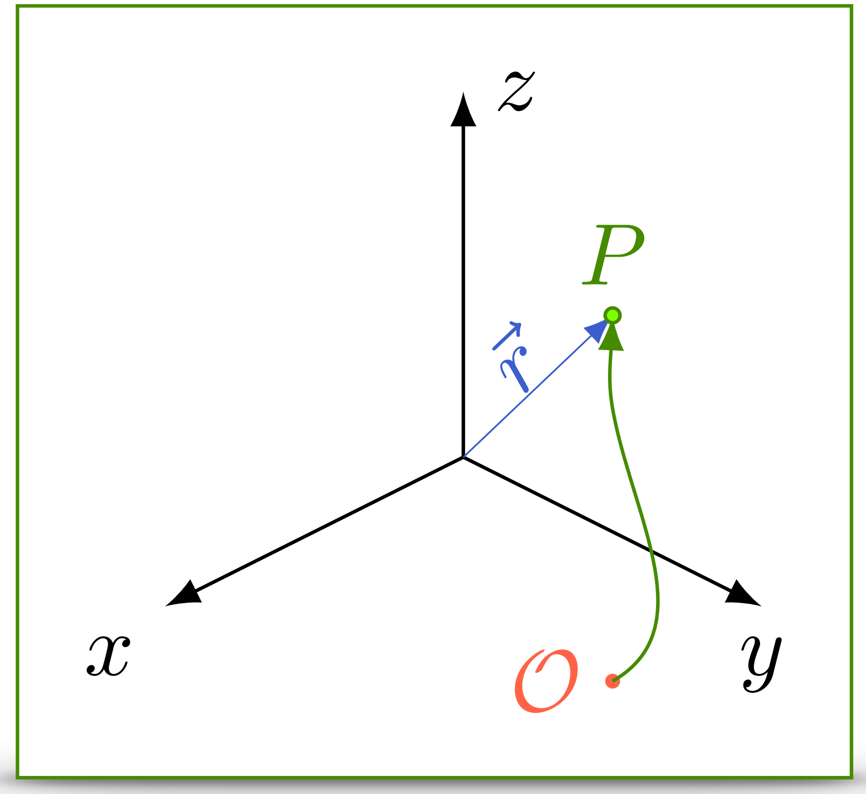
ELETROSTÁTICA

# Potencial elétrico

$$\vec{\nabla} \times \vec{E} = 0$$

$$V(P) = -\int_{\mathcal{O}}^{\vec{r}} \vec{E} \cdot d\vec{\ell}$$

INDEPENDE DO CAMINHO DE INTEGRAÇÃO

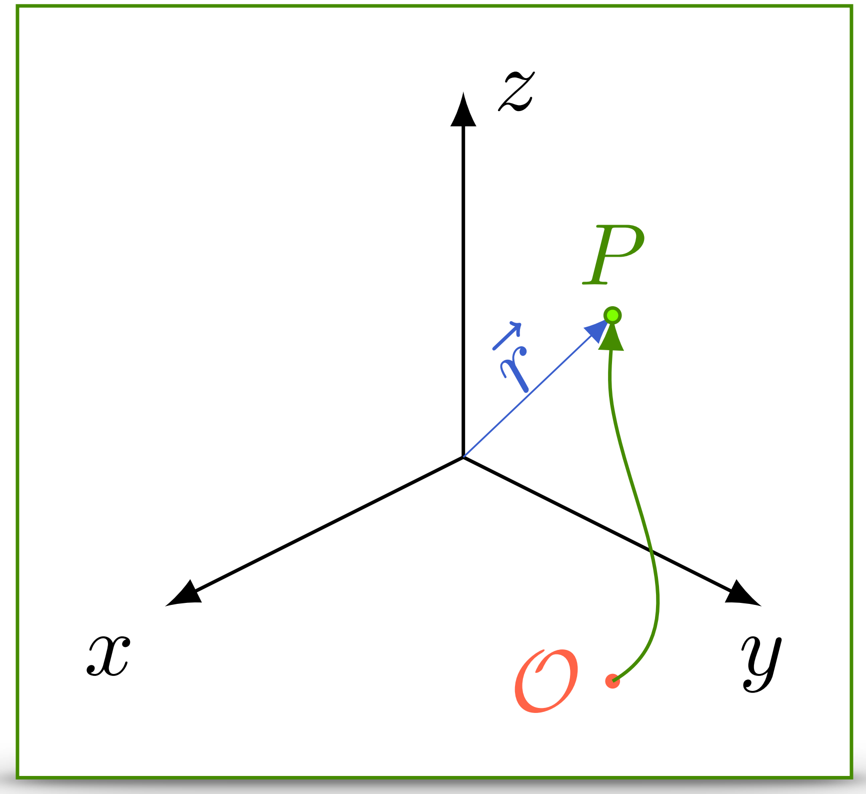


# Potencial elétrico

$$\vec{\nabla} \times \vec{E} = 0$$

$$V(P) = -\int_{\mathcal{O}}^{\vec{r}} \vec{E} \cdot \mathrm{d}\vec{\ell}$$

Unidade = $V$OLT

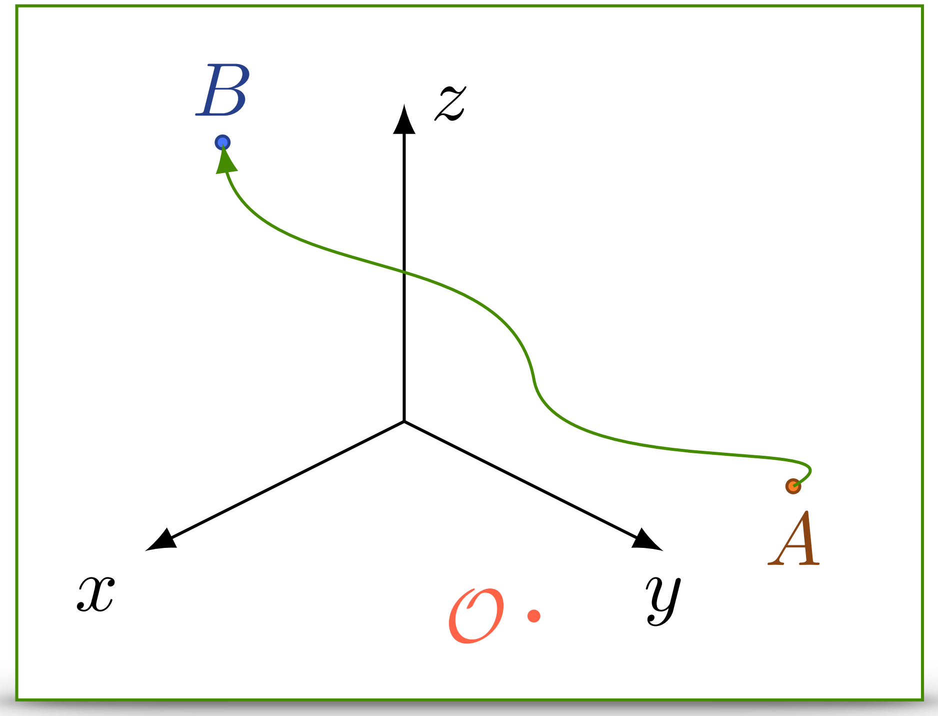


# Diferença de potencial

$$\vec{\nabla} \times \vec{E} = 0$$

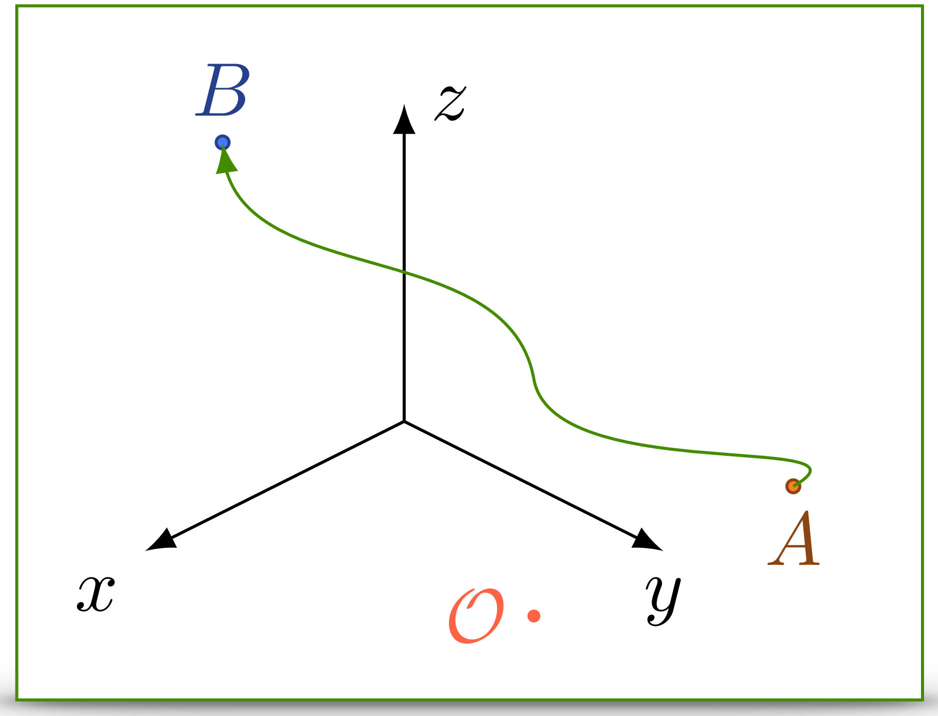
$$\Delta V = -\int_{\vec{r}_a}^{\vec{r}_b} \vec{E} \cdot \mathrm{d}\vec{\ell}$$

INDEPENDE DO PONTO DE REFERÊNCIA



# Diferença de potencial

$$\vec{\nabla} \times \vec{E} = 0$$



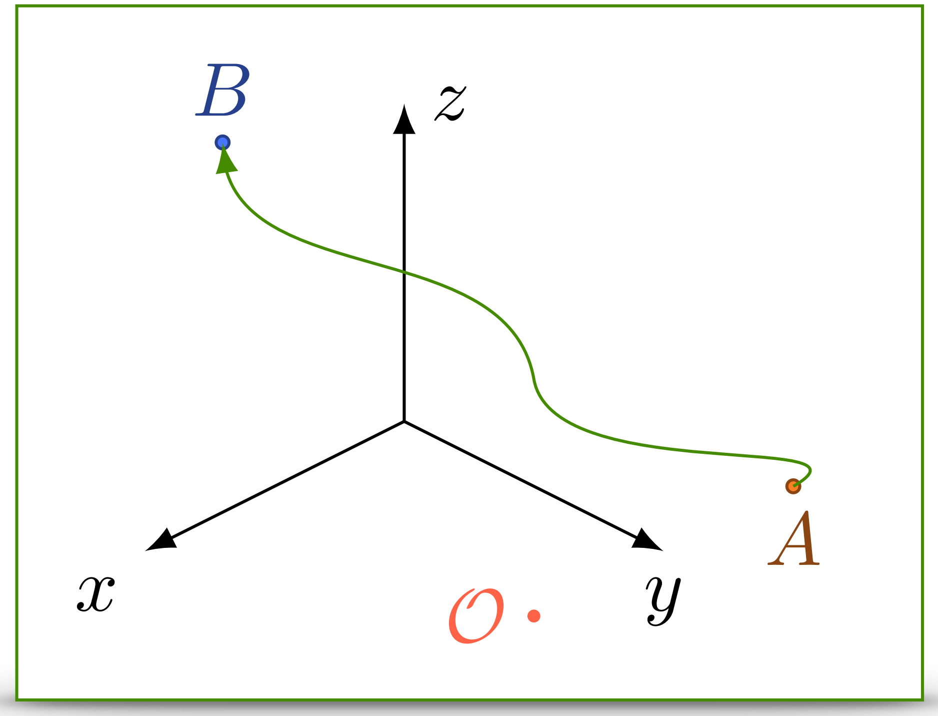

$$\Delta V = -\int_{\vec{r}_a}^{\vec{r}_b} \vec{E} \cdot \mathrm{d}\vec{\ell}$$

$$\Delta V = \int_{\vec{r}_a}^{\vec{r}_b} \vec{\nabla} V \cdot \mathrm{d}\vec{\ell}$$

TEOREMA FUNDAMENTAL DO GRADIENTE

# Diferença de potencial

$$\vec{\nabla} \times \vec{E} = 0$$




$$\Delta V = -\int_{\vec{r}_a}^{\vec{r}_b} \vec{E} \cdot d\vec{\ell}$$

$$\Delta V = \int_{\vec{r}_a}^{\vec{r}_b} \vec{\nabla} V \cdot d\vec{\ell}$$

$$\vec{E} = -\vec{\nabla} V$$

# Potencial de uma carga

$$V(P) = -\int_{\mathcal{O}}^{\vec{r}} \vec{E}\cdot \mathrm{d}\vec{\ell}$$

## Rotacional do campo

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r})\cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{\vec{a}}^{\vec{b}} \frac{q}{r^2}\hat{r}\cdot \mathrm{d}\vec{\ell}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r})\cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{q}{r^2}\,\mathrm{d}r$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r})\cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right)$$

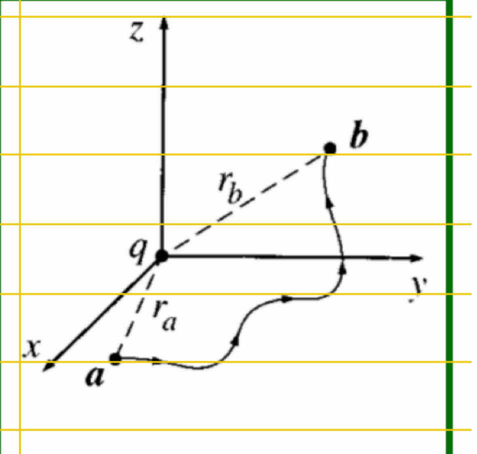



Figure 2.29

# Potencial de uma carga

$$V(P) = \int_P^{\mathcal{O}} \vec{E} \cdot \mathrm{d}\vec{\ell}$$

SINAL TROCADO PORQUE LIMITES DE INTEGRAÇÃO INVERTIDOS

## Rotacional do campo

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{\vec{a}}^{\vec{b}} \frac{q}{r^2}\hat{r} \cdot \mathrm{d}\vec{\ell}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{q}{r^2}\,\mathrm{d}r$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right)$$

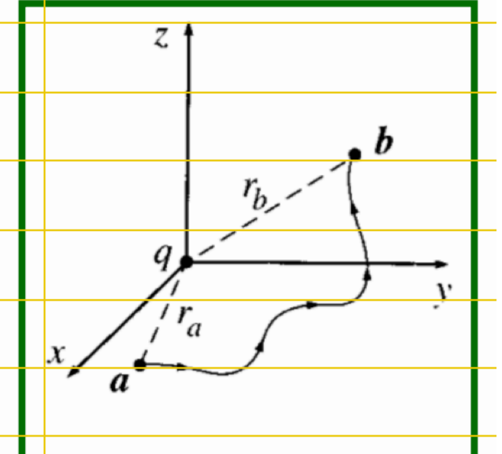



Figure 2.29

# Potencial de uma carga

$$V(P) = \int_P^{\mathcal{O}} \vec{E} \cdot \mathrm{d}\vec{\ell}$$

Referência no infinito

## Rotacional do campo

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{\vec{a}}^{\vec{b}} \frac{q}{r^2}\hat{r} \cdot \mathrm{d}\vec{\ell}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{q}{r^2}\,\mathrm{d}r$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right)$$

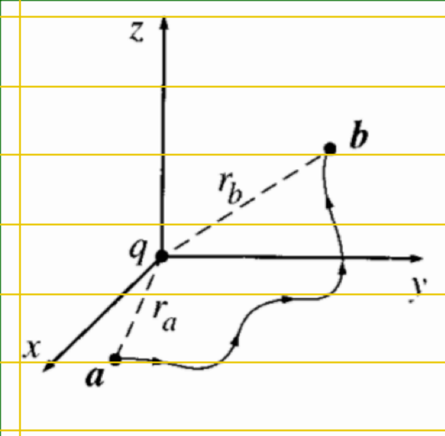



Figure 2.29

# Potencial de uma carga

$$V(P) = \int_P^{\mathcal{O}} \vec{E} \cdot d\vec{\ell}$$

TELA DE 10 DE MAIO

## Rotacional do campo

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot d\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{\vec{a}}^{\vec{b}} \frac{q}{r^2}\hat{r} \cdot d\vec{\ell}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot d\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{q}{r^2}\, dr$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot d\vec{\ell} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right)$$

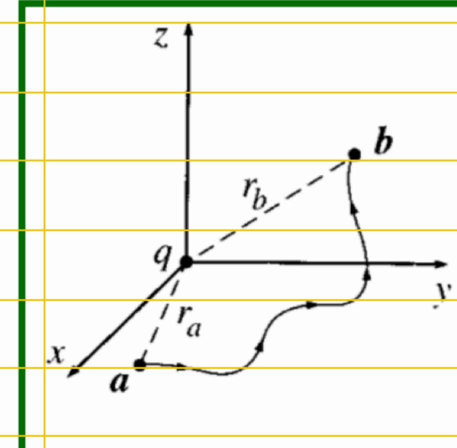



Figure 2.29

**Referência no infinito**

$$\int_{\infty}^{\mathcal{O}} \vec{E} \cdot d\vec{r} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right)$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

↳ 1 CARGA q NA ORIGEM

# Potencial de distribuição de cargas

$V(P) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$ ← 1 CARGA

$$V(P) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{1}{r}\,\mathrm{d}q$$ ← DISTRIBUIÇÃO DE CARGAS