$H_0: C\underset{\sim}{\beta} = \underset{\sim}{m}$

$H_a: C\underset{\sim}{\beta} \neq \underset{\sim}{m}$

$$L(\underset{\sim}{\beta}, \sigma^2 | \underset{\sim}{y}, X) = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \exp\left[-\frac{1}{2\sigma^2}(\underset{\sim}{y} - X\underset{\sim}{\beta})'(\underset{\sim}{y} - X\underset{\sim}{\beta})\right]$$

$$\Theta = \{(\underset{\sim}{\beta}, \sigma^2),\ -\infty < \beta_j < \infty,\ j = 0, 1, 2, \dots K,\ \sigma^2 > 0\}$$

$$\Theta_0 = \{(\underset{\sim}{\beta}, \sigma^2) \in \Theta \mid C\underset{\sim}{\beta} = \underset{\sim}{m}\}$$

$$= \frac{\sup\limits_{(\underset{\sim}{\beta}, \sigma^2) \in \Theta_0} L(\underset{\sim}{\beta}, \sigma^2 | \underset{\sim}{y}, X)}{\sup\limits_{(\underset{\sim}{\beta}, \sigma^2) \in \Theta} L(\underset{\sim}{\beta}, \sigma^2 | \underset{\sim}{y}, X)}$$

$$\sup_{(\underset{\sim}{\beta}, \sigma^2) \in \Theta} L(\underset{\sim}{\beta}, \sigma^2 | \underset{\sim}{y}, X) = L(\hat{\underset{\sim}{\beta}}, \hat{\sigma}^2_{mv} | \underset{\sim}{y}, X) =$$

$$\hat{\underset{\sim}{\beta}} = (X'X)^{-1}X'\underset{\sim}{Y} \qquad \hat{\sigma}^2_{mv} = \frac{(\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}})'(\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}})}{n}$$

$$= (2\pi)^{-n/2}(\hat{\sigma}^2_{mv})^{-n/2} e^{-n/2}$$

Cálculo de $\sup\limits_{(\underset{\sim}{\beta}, \sigma^2) \in \Theta_0} L(\underset{\sim}{\beta}, \sigma^2 | \underset{\sim}{y}, X)$

$H_0: C\underset{\sim}{\beta} = \underset{\sim}{m}$

$H_a: C\underset{\sim}{\beta} \neq \underset{\sim}{m}$

$$L(\underset{\sim}{\beta}, \sigma^2 \mid \underset{\sim}{y}, X) = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \exp\left[-\frac{1}{2\sigma^2}(\underset{\sim}{y} - X\underset{\sim}{\beta})'(\underset{\sim}{y} - X\underset{\sim}{\beta})\right]$$

$$\Theta = \{(\underset{\sim}{\beta}, \sigma^2),\ -\infty < \beta_j < \infty,\ j = 0, 1, 2 \ldots K,\ \sigma^2 > 0\}$$

$$\Theta_0 = \{(\underset{\sim}{\beta}, \sigma^2) \in \Theta \mid C\underset{\sim}{\beta} = \underset{\sim}{m}\}$$

$$= \frac{\sup_{(\underset{\sim}{\beta}, \sigma^2) \in \Theta_0} L(\underset{\sim}{\beta}, \sigma^2 \mid \underset{\sim}{y}, X)}{\sup_{(\underset{\sim}{\beta}, \sigma^2) \in \Theta} L(\underset{\sim}{\beta}, \sigma^2 \mid \underset{\sim}{y}, X)}$$

$$\sup_{(\underset{\sim}{\beta}, \sigma^2) \in \Theta} L(\underset{\sim}{\beta}, \sigma^2 \mid \underset{\sim}{y}, X) = L(\hat{\underset{\sim}{\beta}}, \hat{\sigma}^2_{mv} \mid \underset{\sim}{y}, X) =$$

$$\hat{\underset{\sim}{\beta}} = (X'X)^{-1}X'\underset{\sim}{Y} \qquad \hat{\sigma}^2_{mv} = \frac{(\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}})'(\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}})}{n}$$

$$= (2\pi)^{-n/2}(\hat{\sigma}^2_{mv})^{-n/2} e^{-n/2}$$

Cálculo de $\sup_{(\underset{\sim}{\beta}, \sigma^2) \in \Theta_0} L(\underset{\sim}{\beta}, \sigma^2 \mid \underset{\sim}{y}, X)$

$$\log L(\underset{\sim}{\beta}, \sigma^2 \mid \underset{\sim}{y}, X) =$$

$$= -\frac{n}{2}\left[\log(2\pi) + \log \sigma^2\right] - \frac{1}{2\sigma^2}(\underset{\sim}{y} - X\underset{\sim}{\beta})'(\underset{\sim}{y} - X\underset{\sim}{\beta})$$

$$l = \log L - \underset{\sim}{\lambda}'(C\underset{\sim}{\beta} - \underset{\sim}{m})$$

$$\frac{\partial l}{\partial \underset{\sim}{\beta}} = -\frac{1}{2\sigma^2}(2X'X\underset{\sim}{\beta} - 2X'\underset{\sim}{Y}) - C'\underset{\sim}{\lambda} = \underset{\sim}{0}$$

$$\frac{\partial l}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \frac{1}{2\sigma^4}(\underset{\sim}{y} - X\underset{\sim}{\beta})'(\underset{\sim}{y} - X\underset{\sim}{\beta}) = 0$$

$$\frac{\partial l}{\partial \underset{\sim}{\lambda}} = -(C\underset{\sim}{\beta} - \underset{\sim}{m}) = \underset{\sim}{0}$$

$$\frac{-n\sigma^2 + (\underset{\sim}{y} - X\underset{\sim}{\beta})'(\underset{\sim}{y} - X\underset{\sim}{\beta})}{2\sigma^4} = 0$$

$$\hat{\sigma}^2_0 = \frac{(\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}}_0)'(\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}}_0)}{n}$$ estimador de máxima verossimilhança de σ^2 sob H_0 (ou em Θ_0)

$\hat{\underset{\sim}{\beta}}_0$ estimador de máxima verossimilhança de $\underset{\sim}{\beta}$ sob H_0 (ou em Θ_0).

$$\frac{\partial \ell}{\partial \underset{\sim}{\beta}} = \underset{\sim}{0} \Rightarrow \frac{1}{2\sigma_0^2}\left(2X'X\underset{\sim}{\beta} - 2X'\underset{\sim}{y}\right) + C'\underset{\sim}{\lambda} = \underset{\sim}{0}$$

$$X'X\hat{\underset{\sim}{\beta}}_0 = X'\underset{\sim}{y} - C'\underset{\sim}{\lambda}\hat{\sigma}_0^2$$

$$\hat{\underset{\sim}{\beta}}_0 = (X'X)^{-1}X'\underset{\sim}{y} - (X'X)^{-1}C'\underset{\sim}{\lambda}\hat{\sigma}_0^2$$

$$= \hat{\underset{\sim}{\beta}} - (X'X)^{-1}C'\underset{\sim}{\lambda}\hat{\sigma}_0^2$$

$$\frac{\partial \ell}{\partial \underset{\sim}{\lambda}} = \underset{\sim}{0} \Rightarrow C\hat{\underset{\sim}{\beta}}_0 = \underset{\sim}{m}$$

$\times C$ $$C\hat{\underset{\sim}{\beta}} - C(X'X)^{-1}C'\underset{\sim}{\lambda}\hat{\sigma}_0^2 = \underset{\sim}{m}$$

$$\underset{\sim}{\lambda} = \frac{\left[C(X'X)^{-1}C'\right]^{-1}\left(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m}\right)}{\hat{\sigma}_0^2}$$

$$\therefore \hat{\underset{\sim}{\beta}}_0 = \hat{\underset{\sim}{\beta}} - (X'X)^{-1}C'\left[C(X'X)^{-1}C'\right]^{-1}\left(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m}\right)$$

Obs:

$\hat{\sigma}_0^2$ e $\hat{\underset{\sim}{\beta}}_0$ são os estimadores de máxima verossimilhança de σ^2 e $\underset{\sim}{\beta}$ sob $H_0: C\underset{\sim}{\beta} = \underset{\sim}{m}$.

Coincidem com os estimadores de máxima verossimilhança no modelo reduzido sob H_0.

$$\sup_{\theta \in \Theta_0} L(\underset{\sim}{\beta}, \sigma^2 \mid \underset{\sim}{y}, X) = L(\hat{\underset{\sim}{\beta}}_0, \hat{\sigma}^2_0 \mid \underset{\sim}{y}, X) =$$

$$= (2\pi)^{-n/2} (\hat{\sigma}^2_0)^{-n/2} e^{-n/2}$$

$$\Lambda = \left(\frac{\hat{\sigma}^2_{mv}}{\hat{\sigma}^2_0} \right)^{n/2}$$

RC: $\Lambda = \left(\frac{\hat{\sigma}^2_{mv}}{\hat{\sigma}^2_0} \right)^{n/2} \leq \lambda_0$

Usar a estatística $W = (\Lambda^{-2/n} - 1) \frac{n-k-1}{p}$ facilidade de obter a distribuição

— W é uma função monótona de Λ

— $\Lambda \leq \lambda_0 \iff \Lambda^{-1} \geq \lambda_0^{-1} \iff \Lambda^{-2/n} \geq \lambda_0^{-2/n}$

$$(\Lambda^{-2/n} - 1) \frac{n-k-1}{p} \geq (\lambda_0^{-2/n} - 1) \frac{n-k-1}{p} \iff$$

$\iff W \geq w$ forma da região crítica

$$W = \left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}_{mv}^2} - 1\right)\frac{n-K-1}{p} = \frac{\hat{\sigma}_0^2 - \hat{\sigma}_{mv}^2}{\hat{\sigma}_{mv}^2}\,\frac{n-K-1}{p} \quad *$$

$$\hat{\sigma}_0^2 = \frac{(\underset{\sim}{y} - X\underset{\sim}{\hat{\beta}}_0)'(\underset{\sim}{y} - X\underset{\sim}{\hat{\beta}}_0)}{n}$$

$$\underset{\sim}{\hat{\beta}}_0 = \underset{\sim}{\hat{\beta}} - \underbrace{(X'X)^{-1}C'[C(X'X)^{-1}C']^{-1}(C\underset{\sim}{\hat{\beta}} - \underset{\sim}{m})}_{\underset{\sim}{H}_{(K+1)\times 1}}$$

$$= \underset{\sim}{\hat{\beta}} - \underset{\sim}{H}$$

$$\hat{\sigma}_0^2 = \frac{1}{n}\left[(\underset{\sim}{y} - X\underset{\sim}{\hat{\beta}} + X\underset{\sim}{H})'(\underset{\sim}{y} - X\underset{\sim}{\hat{\beta}} + X\underset{\sim}{H})\right] =$$

$$= \frac{1}{n}\left[(\underset{\sim}{y} - X\underset{\sim}{\hat{\beta}})'(\underset{\sim}{y} - X\underset{\sim}{\hat{\beta}}) + (\underset{\sim}{y} - X\underset{\sim}{\hat{\beta}})'XH + \underset{\sim}{H}'X'(\underset{\sim}{y} - X\underset{\sim}{\hat{\beta}}) + \underset{\sim}{H}'X'X\underset{\sim}{H}\right]$$

$$\underset{\sim}{H}'X'(\underset{\sim}{y} - X\underset{\sim}{\hat{\beta}}) = \underset{\sim}{H}'(X'\underset{\sim}{y} - X'X\underset{\sim}{\hat{\beta}}) = \underset{\sim}{H}'(X'\underset{\sim}{y} - X'\underset{\sim}{y}) = 0$$

$$(\underset{\sim}{y} - X\underset{\sim}{\hat{\beta}})'X\underset{\sim}{H} = [\underset{\sim}{H}'X'(\underset{\sim}{y} - X\underset{\sim}{\hat{\beta}})]' = 0$$

$$= \hat{\sigma}^2 + \frac{\underset{\sim}{H}'X'X\underset{\sim}{H}}{n} = \hat{\sigma}^2 + \frac{1}{n}(C\underset{\sim}{\hat{\beta}} - \underset{\sim}{m})'[C(X'X)^{-1}C']^{-1}C(X'X)^{-1}(X'X)$$

$$(X'X)^{-1}C'\left[C(X'X)^{-1}C'\right]^{-1}(C\hat{\underset{\sim}{\beta}}-\underset{\sim}{m}) =$$

$$= \hat{\sigma}^2_{mv} + \frac{1}{n}(C\hat{\underset{\sim}{\beta}}-\underset{\sim}{m})'\left[C(X'X)^{-1}C'\right]^{-1}(C\hat{\underset{\sim}{\beta}}-\underset{\sim}{m})$$

Substituindo em *

$$W = \frac{(C\hat{\underset{\sim}{\beta}}-\underset{\sim}{m})'\left[C(X'X)^{-1}C'\right]^{-1}(C\hat{\underset{\sim}{\beta}}-\underset{\sim}{m})}{n\,\hat{\sigma}^2_{mv}} \; \frac{n-k-1}{p}$$

$$= \frac{(C\hat{\underset{\sim}{\beta}}-\underset{\sim}{m})'\left[C(X'X)^{-1}C'\right]^{-1}(C\hat{\underset{\sim}{\beta}}-\underset{\sim}{m})}{p\;\dfrac{(\underset{\sim}{Y}-X\hat{\underset{\sim}{\beta}})'(\underset{\sim}{Y}-X\hat{\underset{\sim}{\beta}})}{n-k-1}} =$$

$$= \frac{(C\hat{\underset{\sim}{\beta}}-\underset{\sim}{m})'\left[C(X'X)^{-1}C'\right]^{-1}(C\hat{\underset{\sim}{\beta}}-\underset{\sim}{m})}{p\;MSE}$$ estatística de teste

↳ do Modelo Completo $\underset{\sim}{Y} = X\underset{\sim}{\beta} + \underset{\sim}{\epsilon}$

Distribuição de W

$$\frac{(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m})'[C(X'X)^{-1}C']^{-1}(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m})}{\sigma^2}$$

$\hat{\underset{\sim}{\beta}} \sim N_{k+1}(\underset{\sim}{\beta}, (X'X)^{-1}\sigma^2)$

$C\hat{\underset{\sim}{\beta}} \sim N_p(C\underset{\sim}{\beta}, C(X'X)^{-1}C'\sigma^2)$

$(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m}) \sim N_p(C\underset{\sim}{\beta} - \underset{\sim}{m}, C(X'X)^{-1}C'\sigma^2)$

$A\Sigma = \frac{[C(X'X)^{-1}C']^{-1}}{\sigma^2} C(X'X)^{-1}C'\sigma^2 = I$ ($A\Sigma$ é idempotente)

$\lambda = \frac{1}{2}(C\underset{\sim}{\beta} - \underset{\sim}{m})' \frac{[C(X'X)^{-1}C']^{-1}}{\sigma^2}(C\underset{\sim}{\beta} - \underset{\sim}{m})$

$$\therefore \frac{(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m})'[C(X'X)^{-1}C']^{-1}(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m})}{\sigma^2} \sim \chi^2_{p,\lambda}$$

$\hat{\underset{\sim}{\beta}}$ e SSE são independentes $\Rightarrow$

$W \sim F_{p, n-k-1, \lambda}$

Sob H_0 $C\underset{\sim}{\beta} = \underset{\sim}{m}$ $\lambda = 0$ e $W \sim F_{p, n-k-1}$ central

O teste da razão de verossimilhança generalizada rejeita H_0 para $W \geq w$.

Tomar $w \mid P(F_{p,n-k-1} \geq w) = \alpha$.

Obs:

1) $$W = \frac{\hat{\sigma}_0^2 - \hat{\sigma}_{mv}^2}{\hat{\sigma}_{mv}^2} \frac{n-k-1}{p}$$

$\hat{\sigma}_{mv}^2$ - estimador de max. veross. de σ^2 no modelo original

$\hat{\sigma}_0^2$ - estimador de max. veross. de σ^2 no modelo reduzido sob H_0 (ambos têm denominador n)

$\therefore$ W pode ser escrita como

$$W = \frac{SSE_r - SSE_c}{SSE_c} \frac{n-k-1}{p}$$

onde

SSE_c - soma de quadrados do resíduo do modelo completo (modelo original) $= (\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}})'(\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}})$

SSE_r - soma de quadrados do resíduo do modelo reduzido sob $H_0 = (\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}}_0)'(\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}}_0) =$

$$= (\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}})'(\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}}) + (C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m})'[C(X'X)^{-1}C']^{-1}(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m})$$

$$SSE_r - SSE_c = (C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m})'[C(X'X)^{-1}C']^{-1}(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m}) > 0$$

pois $C(X'X)^{-1}C'$ é positiva definida

Resultados

i) $X_{n \times p}$ $r(X) = p \Rightarrow X'X$ é positiva definida

ii) Se A é positiva definida então $r(CAC') = r(C)$

$\Rightarrow [C(X'X)^{-1}C']$ é inversível

iii) Se A é p.d. $n \times n$ e C é $p \times n$ com $r(C) = p$ então CAC' é positiva definida $\Rightarrow C(X'X)^{-1}C'$ é positiva definida Seber pag 386

(Não escrever $SSE_r - SSE_c = SSR_c - SSR_r$, pois isto pode não valer.

$$SST = SSR_c + SSE_c$$

$$SST = SSR_r + SSE_r$$

$$\Rightarrow SSE_r - SSE_c = SSR_c - SSR_r$$

Em alguns casos (raros), a redução sob H_0 altera a variável resposta e as somas de quadrados totais não são as mesmas)

Ex: $Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$

$H_0: \beta_1 = \beta_2 + 4$

Sob H_0 $\quad Y = \beta_0 + (\beta_2 + 4) x_1 + \beta_2 x_2 + \epsilon$

$$Y - 4x_1 = \beta_0 + \beta_2 (x_1 + x_2) + \epsilon$$

2) $Var(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m}) = Var(C\hat{\underset{\sim}{\beta}}) = C(X'X)^{-1}C'\sigma^2$

$$\widehat{Var}(C\hat{\underset{\sim}{\beta}}) = C(X'X)^{-1}C' \, MSE$$

3) Para cada valor de λ (parâmetro de não centralidade da distribuição de W), o poder do teste é

$$\Pi(\lambda) = \int_{w_\alpha}^{\infty} F(p, n-k-1, \lambda)\, dw$$

onde $F(p, n-k-1, \lambda)$ - densidade da $F_{p, n-k-1, \lambda}$

e w_α tal que $P(F_{p, n-k-1} \geq w_\alpha) = \alpha$,

α - nível de significância do teste.

4) $\hat{\underset{\sim}{\beta}}_0 = \hat{\underset{\sim}{\beta}} - (X'X)^{-1}C'[C(X'X)^{-1}C']^{-1}(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m})$

est. max. veross. de $\underset{\sim}{\beta}$ sob $H_0: C\underset{\sim}{\beta} = \underset{\sim}{m}$

é também o estimador de mínimos quadrados de $\underset{\sim}{\beta}$ sob H_0. [Minimiza $(\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}}_0)'(\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}}_0)$ sujeito à condição $C\hat{\underset{\sim}{\beta}}_0 = \underset{\sim}{m}$]

5) A estatística W para testar $H_0: C\underset{\sim}{\beta} = \underset{\sim}{m}$ contra $H_a: C\underset{\sim}{\beta} \neq \underset{\sim}{m}$ fica inalterada se testarmos $H_0: QC\underset{\sim}{\beta} = Q\underset{\sim}{m}$ contra $H_a: QC\underset{\sim}{\beta} \neq Q\underset{\sim}{m}$, onde Q é uma matriz $p \times p$ não singular.

$C - p \times (k+1)$

$QC - p \times (k+1)$

$C\underset{\sim}{\beta} = \underset{\sim}{m} \iff$

$QC\underset{\sim}{\beta} = Q\underset{\sim}{m}$

$$W_1 = \frac{(QC\hat{\underset{\sim}{\beta}} - Q\underset{\sim}{m})'[QC(X'X)^{-1}C'Q']^{-1}(QC\hat{\underset{\sim}{\beta}} - Q\underset{\sim}{m})}{p\,MSE}$$

$$= \frac{(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m})'Q'Q'^{-1}[C(X'X)^{-1}C']^{-1}Q^{-1}Q(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m})}{p\,MSE} =$$

$$= \frac{(C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m})' \left[C (X'X)^{-1} C' \right]^{-1} (C\hat{\underset{\sim}{\beta}} - \underset{\sim}{m})}{p\,MSE}$$

No exemplo 4, $Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \epsilon$

$H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 \iff C\underset{\sim}{\beta} = \underset{\sim}{m}$

$$C = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix}_{3\times 5} \qquad \underset{\sim}{m} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{array}{l} \beta_1 - \beta_2 = 0 \\ \beta_1 - \beta_3 = 0 \\ \beta_1 - \beta_4 = 0 \end{array}$$

H_0 pode ser escrita alternativamente como

$H_0: C_1 \underset{\sim}{\beta} = \underset{\sim}{m}_1$

$$C_1 = \begin{bmatrix} 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \qquad \underset{\sim}{m}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{array}{l} \beta_1 - \beta_2 = 0 \\ \beta_2 - \beta_3 = 0 \\ \beta_3 - \beta_4 = 0 \end{array}$$

Temos que $C_1 = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}_{3\times 3}}_{Q} C \qquad C_1 = QC$

$$|Q| = \begin{vmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix} = 1 \qquad Q_{3\times 3} \text{ não singular } \therefore$$

$QC\underset{\sim}{\beta} = Q\underset{\sim}{m}$

W_1 para testar $H_0: C_1 \underset{\sim}{\beta} = \underset{\sim}{m}_1$ coincide com W que testa $H_0: C\underset{\sim}{\beta} = \underset{\sim}{m}$. Os graus de liberdade são os mesmos $\Rightarrow$ as conclusões são as mesmas.