Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

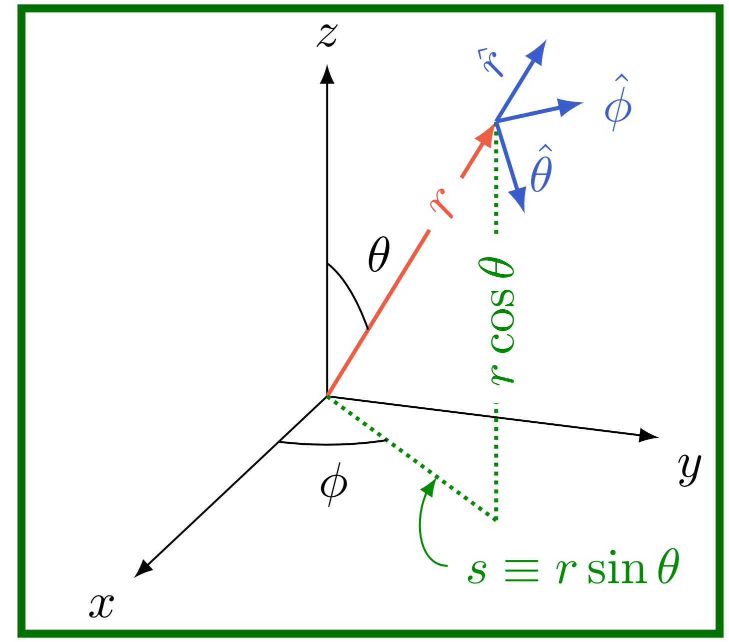
$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

Aula de 10 de maio

Eletrostática

Coordenadas esféricas

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$



$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

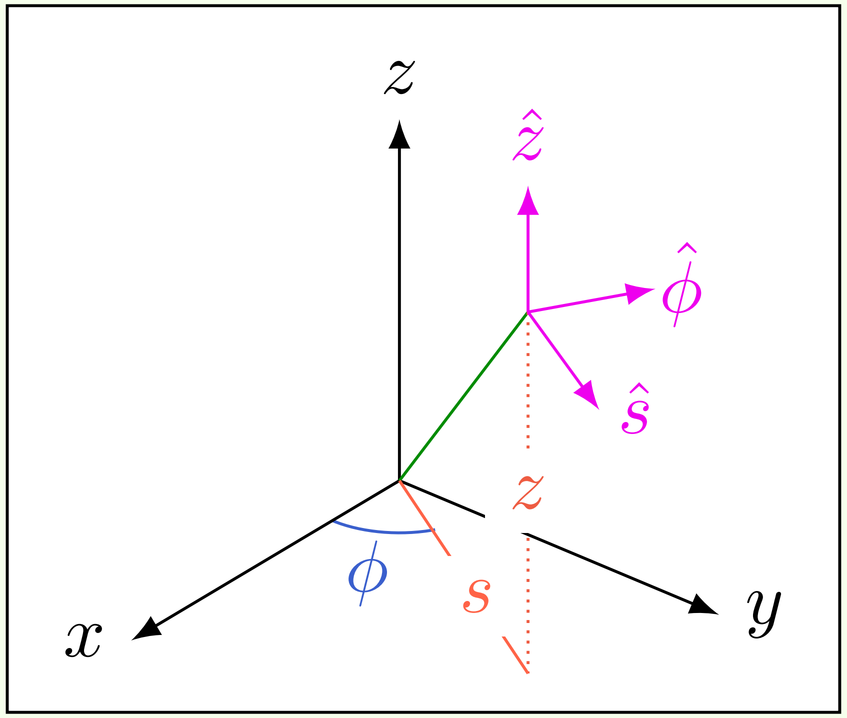
$$\vec{\nabla}\times\vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$

$$\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Coordenadas cilíndricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}s\,\hat{s} + s\,\mathrm{d}\phi\,\hat{\phi} + \mathrm{d}z\,\hat{z}$$



$$\vec{\boldsymbol{\nabla}} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{v} = \frac{1}{s}\frac{\partial (s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\boldsymbol{\nabla}} \times \vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

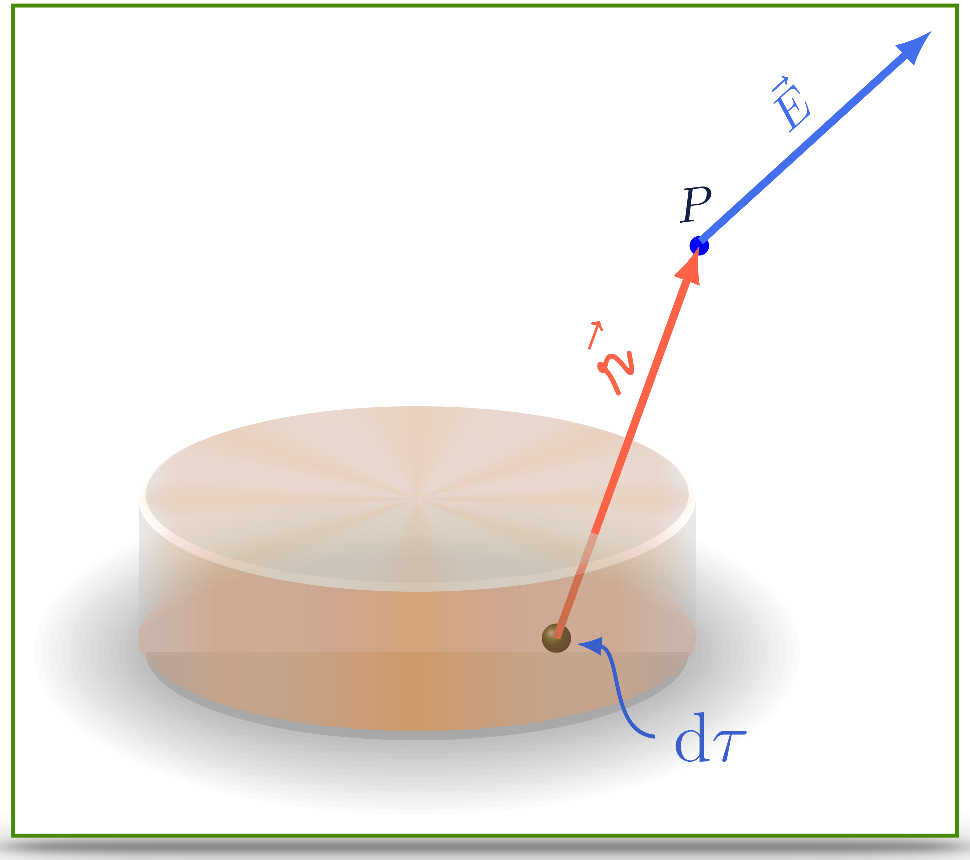
$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Eletrostática

Campo de distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{1}{\mathscr{r}^2} \hat{\mathscr{r}} \, \mathrm{d}q$$

$$\mathrm{d}q = \begin{cases} \lambda \, \mathrm{d}\ell & \text{(linear)} \\ \sigma \, \mathrm{d}A & \text{(superficial)} \\ \rho \, \mathrm{d}\tau & \text{(volumétrica)} \end{cases}$$

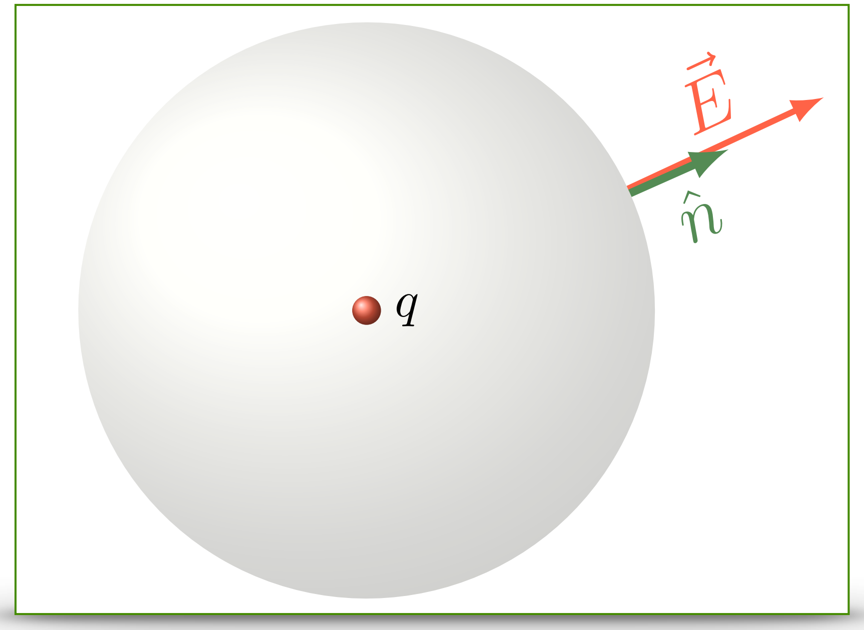


Eletrostática

Lei de Gauss

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$$

$$\boxed{\int \vec{E}\cdot\hat{n}\ \mathrm{d}A = \frac{q}{\epsilon_0}}$$



- Superfície não precisa ser esférica.
- Carga pode estar distribuída dentro da superfície.

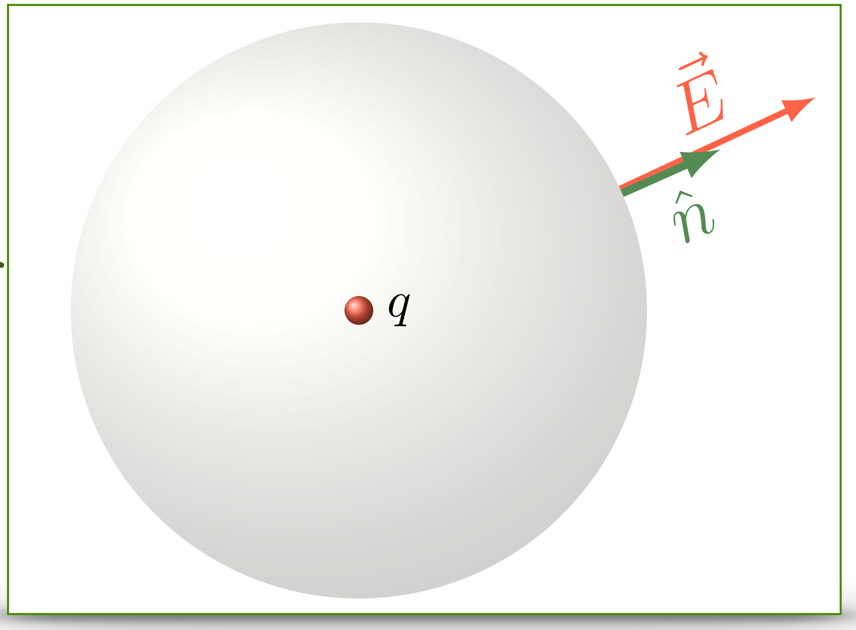
Eletrostática

Lei de Gauss

$$\boxed{\int \vec{E} \cdot \hat{n}\ \mathrm{d}A = \frac{q}{\epsilon_0}}$$

Teorema Gauss

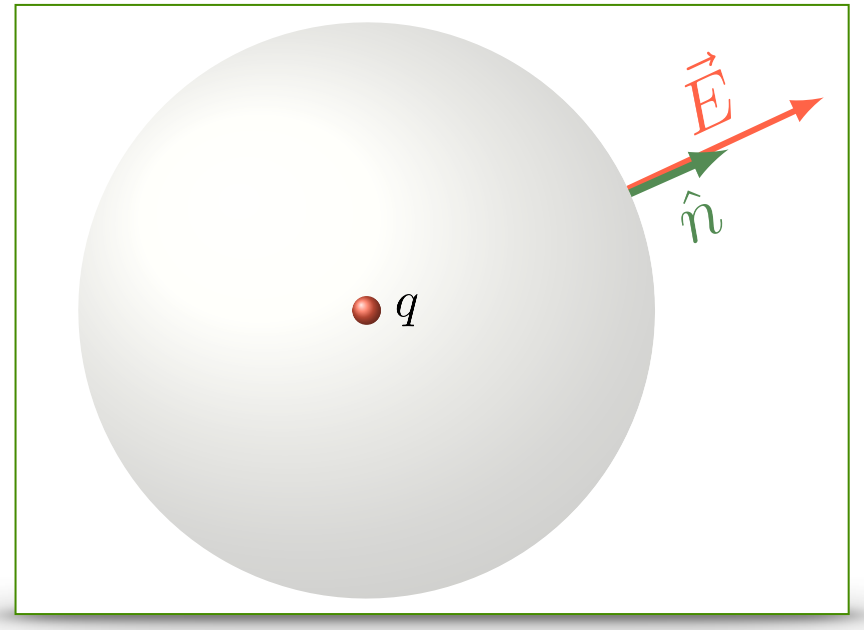
$$\int_{\mathcal{S}} \vec{E} \cdot \hat{n}\ \mathrm{d}A = \int_{\mathcal{V}} \vec{\boldsymbol{\nabla}} \cdot \vec{E}\ \mathrm{d}\tau$$



Eletrostática

Lei de Gauss

$$\boxed{\int \vec{E} \cdot \hat{n} \; \mathrm{d}A = \frac{q}{\epsilon_0}}$$



$$\int_{\mathcal{S}} \vec{E} \cdot \hat{n} \; \mathrm{d}A = \int_{\mathcal{V}} \vec{\nabla} \cdot \vec{E} \; \mathrm{d}\tau$$

$$\frac{q}{\epsilon_0} = \frac{1}{\epsilon_0} \int_{\mathcal{V}} \rho(\vec{r}) \; \mathrm{d}\tau$$

Eletrostática

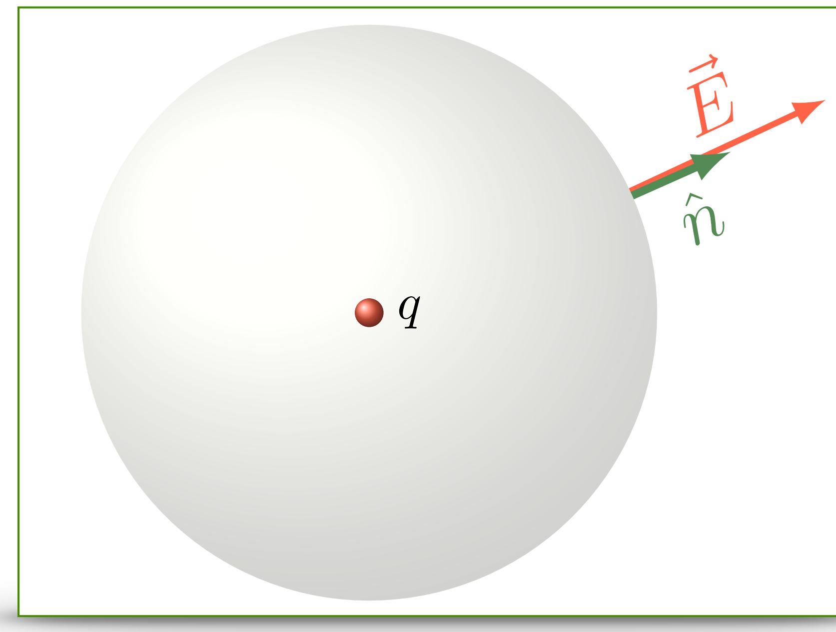
Lei de Gauss

$$\boxed{\int \vec{E}\cdot\hat{n}\ \mathrm{d}A = \frac{q}{\epsilon_0}}$$

$$\int_{\mathcal{S}} \vec{E}\cdot\hat{n}\ \mathrm{d}A = \int_{\mathcal{V}} \vec{\nabla}\cdot\vec{E}\ \mathrm{d}\tau$$

$$\frac{q}{\epsilon_0} = \frac{1}{\epsilon_0}\int_{\mathcal{V}} \rho(\vec{r})\ \mathrm{d}\tau$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}}$$

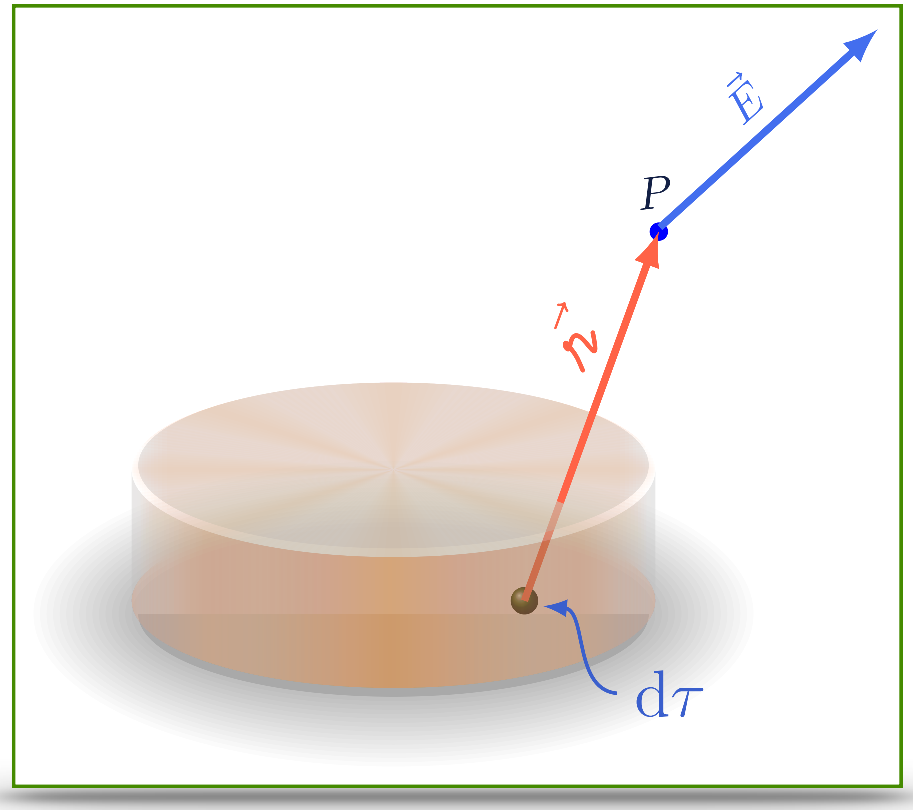


$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$$

Eletrostática

Outra visão

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \rho(\vec{r'}) \, \mathrm{d}\tau'$$



$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

Eletrostática

Outra visão

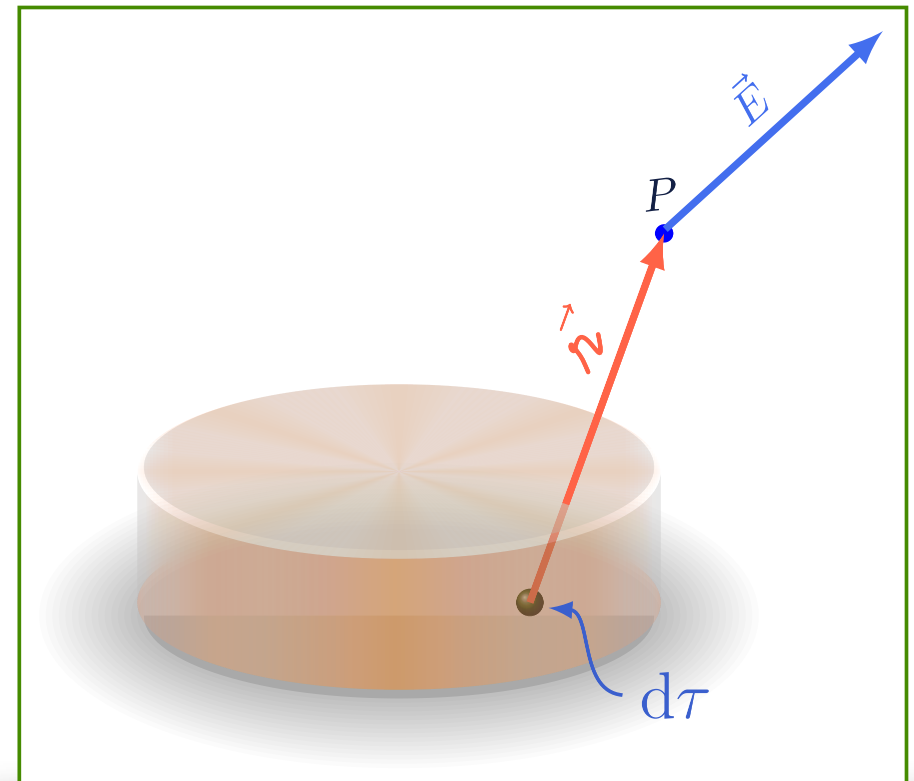
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\rho(\vec{r'})\,\mathrm{d}\tau'$$

$$\vec{\nabla}\cdot\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \vec{\nabla}\cdot\left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right)\rho(\vec{r'})\,\mathrm{d}\tau'$$

AGE SOBRE COORDENADA $\vec{r}$

$\vec{\mathscr{r}} = \vec{r} - \vec{r}'$

$\therefore \vec{\nabla}$ AGE SOBRE $\vec{\mathscr{r}}$, DENTRO DA INTEGRAL, MAS NÃO SOBRE $\vec{r}'$



Eletrostática

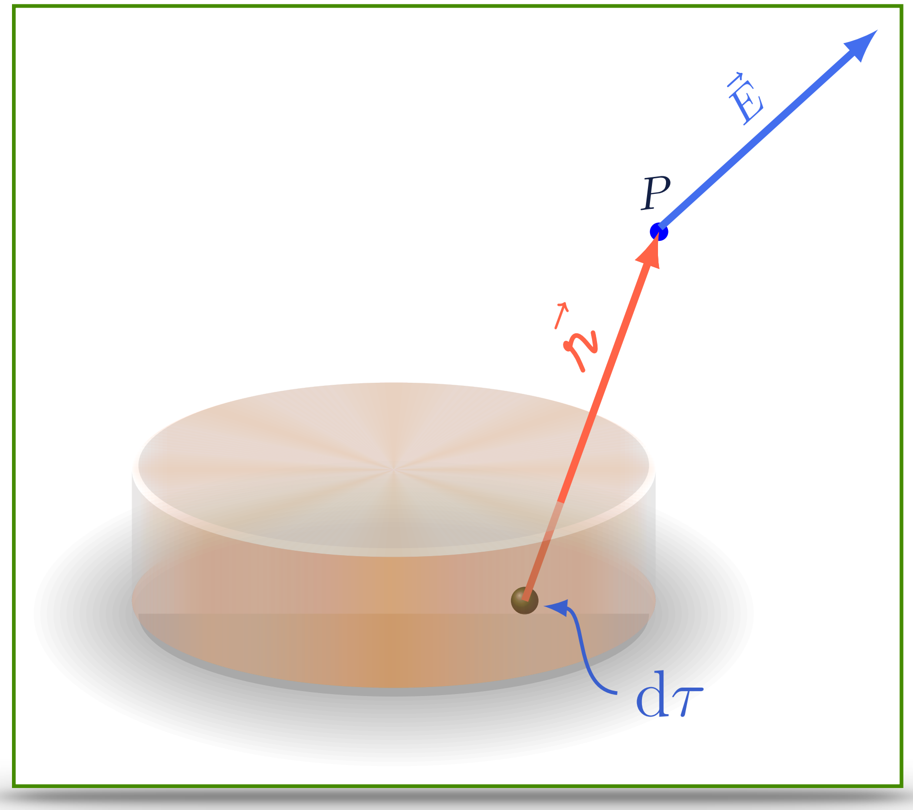
Outra visão

$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \rho(\vec{r'})\ \mathrm{d}\tau'$$

$$\vec{\nabla} \cdot \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \vec{\nabla} \cdot \left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) \rho(\vec{r'})\ \mathrm{d}\tau'$$

$$\vec{\nabla} \cdot \left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = 4\pi\delta^3(\mathscr{r}) \qquad (\vec{\mathscr{r}} = \vec{r} - \vec{r'})$$



Eletrostática

Outra visão

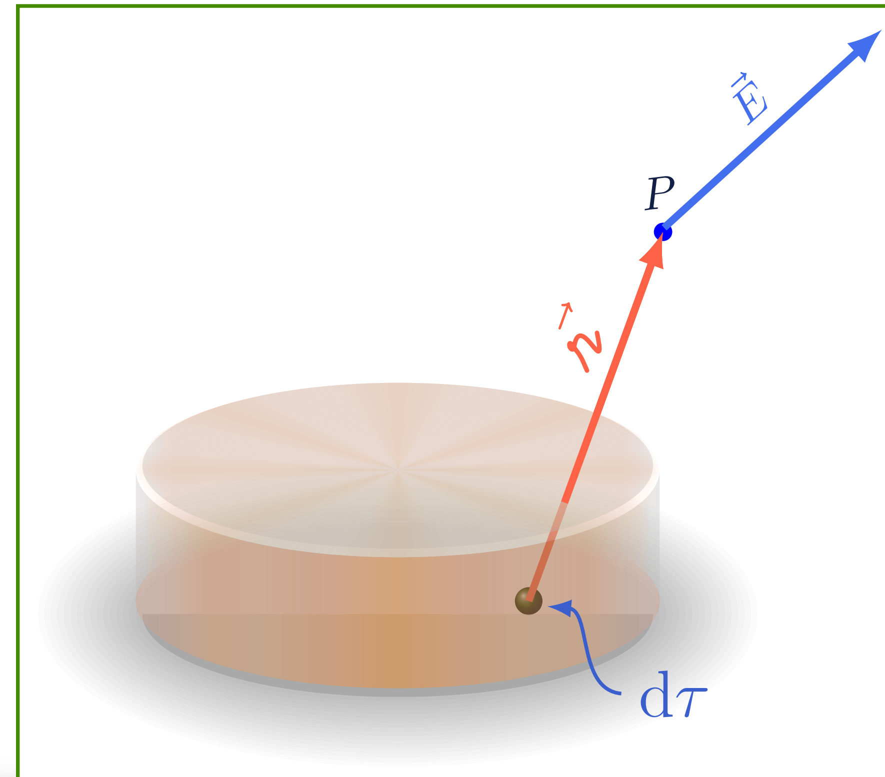
$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \rho(\vec{r}') \, \mathrm{d}\tau'$$

$$\vec{\nabla} \cdot \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \vec{\nabla} \cdot \left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) \rho(\vec{r}') \, \mathrm{d}\tau'$$

$$\vec{\nabla} \cdot \left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = 4\pi\delta^3(\mathscr{r}) \qquad (\vec{\mathscr{r}} = \vec{r} - \vec{r}')$$

$$\vec{\nabla} \cdot \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \cancel{4\pi}\delta^3(\vec{r} - \vec{r}')\rho(\vec{r}') \, \mathrm{d}\tau'$$



$$\int \delta^3(\vec{r} - \vec{r}') \rho(\vec{r}') d\tau'$$

$$= \int \delta^3(\vec{r}' - \vec{r}) \rho(\vec{r}') d\tau'$$

$$= \rho(\vec{r})$$

δ é função par

Eletrostática

Outra visão

$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$$

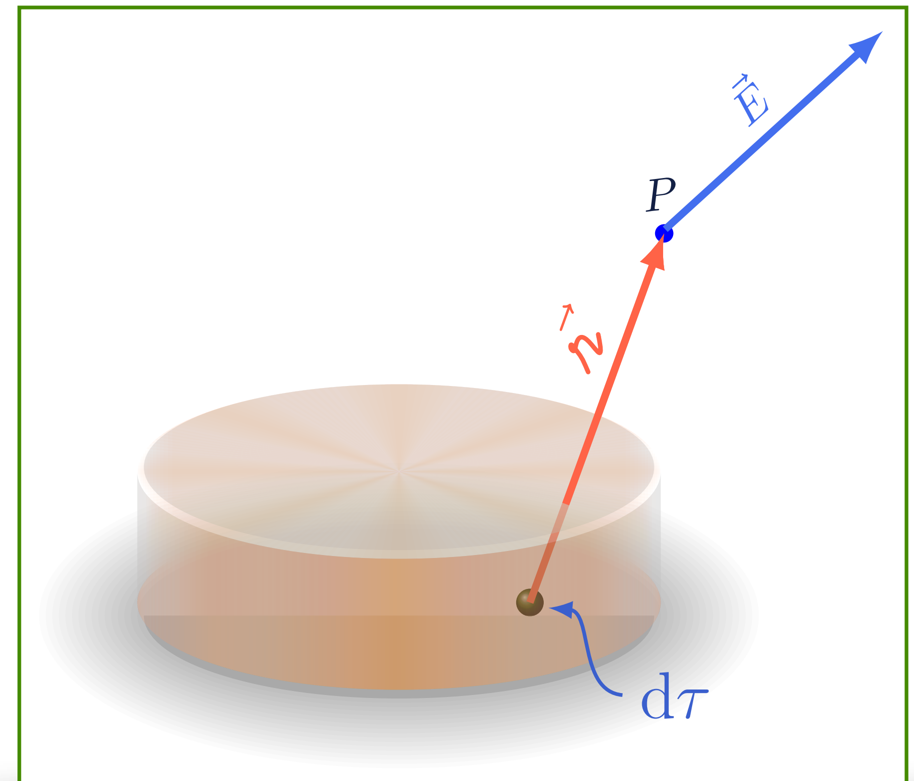
$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \rho(\vec{r'})\ \mathrm{d}\tau'$$

$$\vec{\nabla} \cdot \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \vec{\nabla} \cdot \left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) \rho(\vec{r'})\ \mathrm{d}\tau'$$

$$\vec{\nabla} \cdot \left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) = 4\pi\delta^3(\mathscr{r}) \qquad (\vec{\mathscr{r}} = \vec{r} - \vec{r'})$$

$$\vec{\nabla} \cdot \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int 4\pi\delta^3(\vec{r} - \vec{r'}) \rho(\vec{r'})\ \mathrm{d}\tau'$$

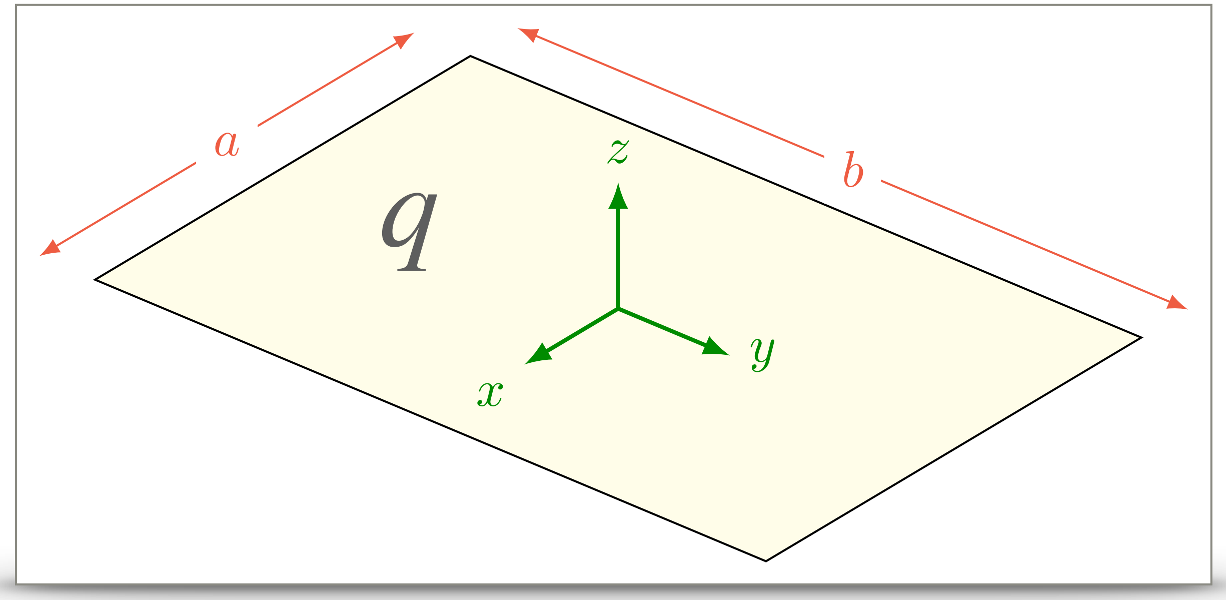
$$\vec{\nabla} \cdot \vec{E} = \frac{\rho(\vec{r})}{\epsilon_0}$$



$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

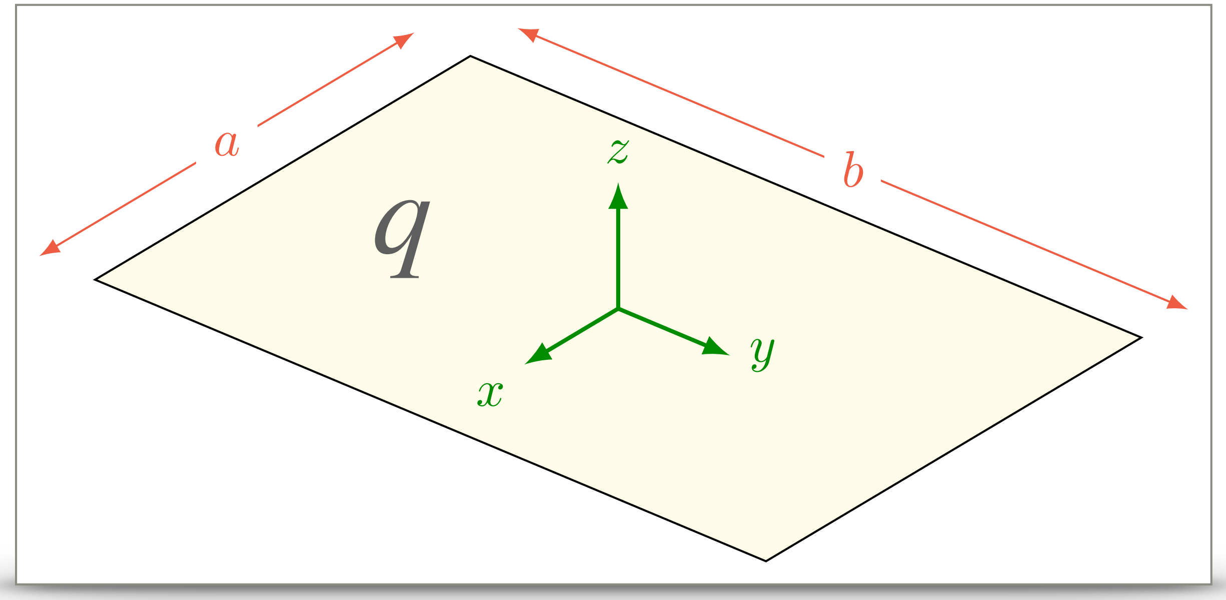
$$\overrightarrow{E} = ?$$



$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

Escolher superfície simétrica

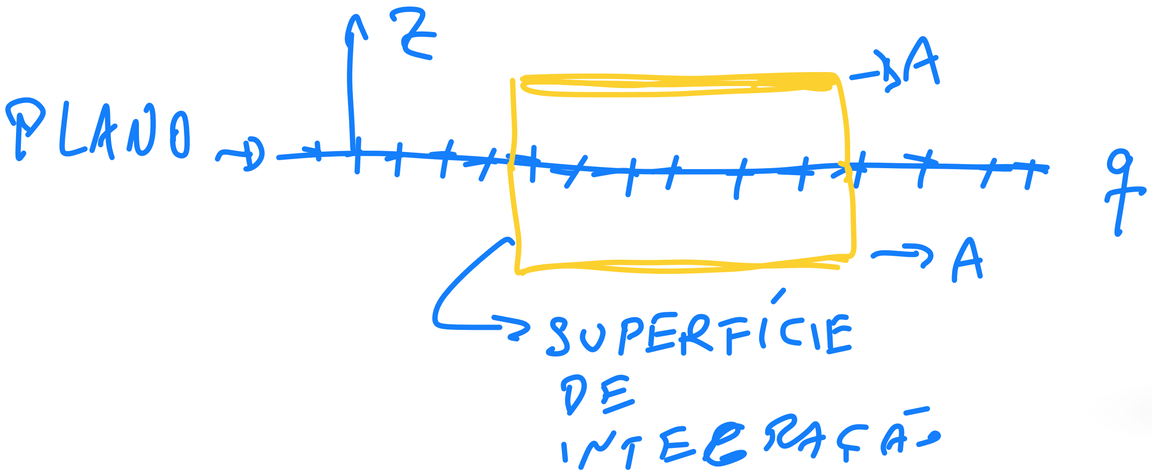


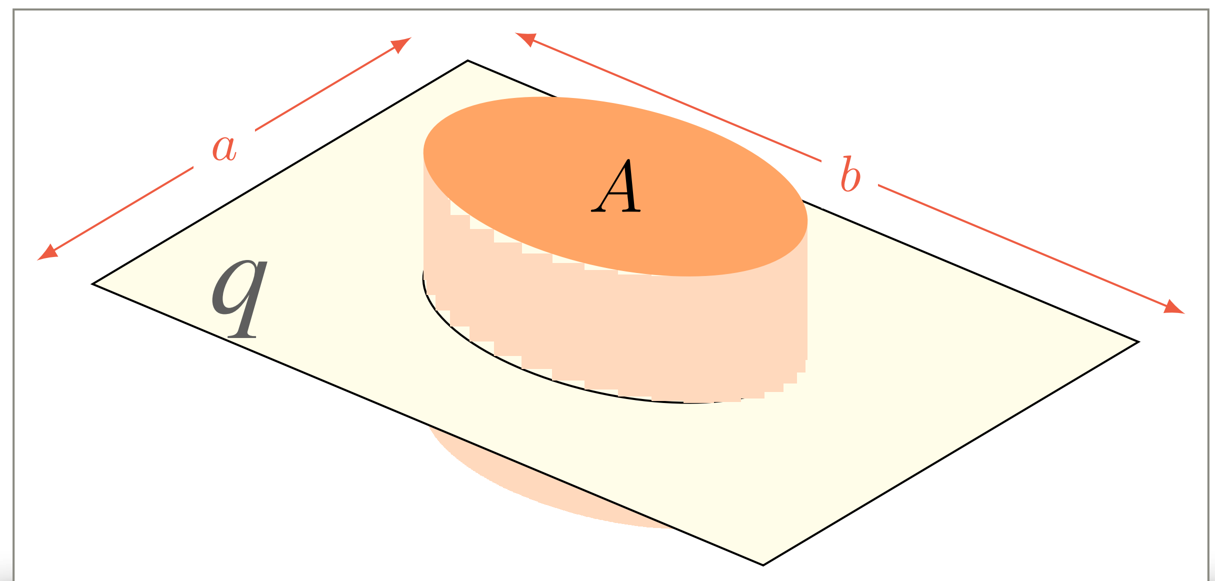
$$\int \vec{E} \cdot \hat{n}\, \mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

Escolher superfície simétrica

IDÊNTICA ACIMA E EMBAIXO DO PLANO





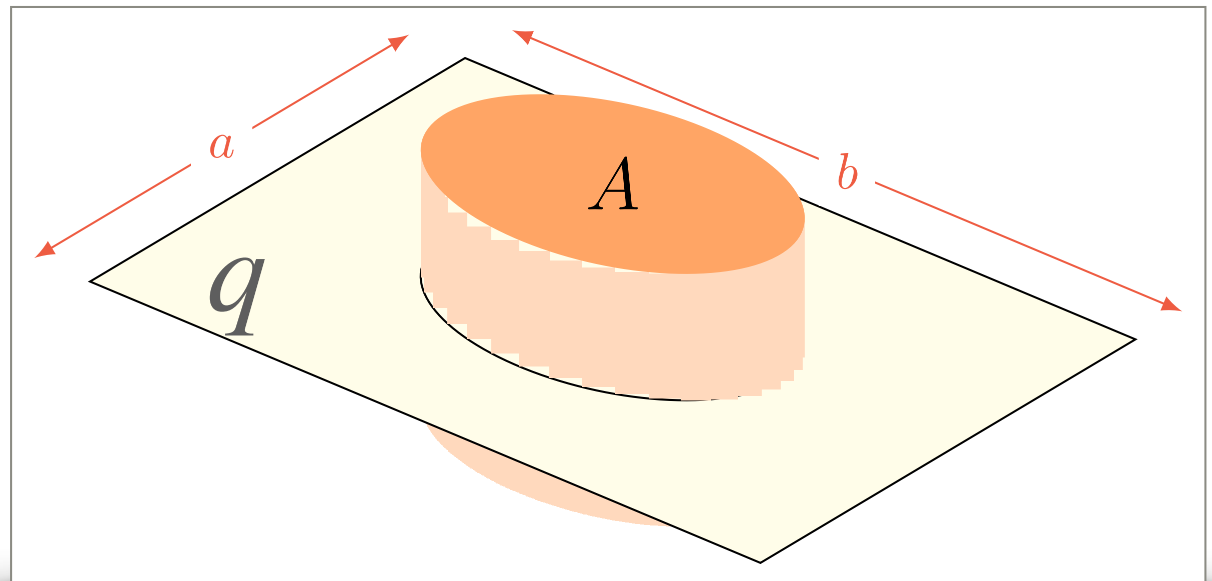
$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

Escolher superfície simétrica

$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = EA + EA$$

$\hat{n}$ TEM DIREÇÃO DE $\vec{E}$,
NA TAMPA DE CIMA
E NA DE BAIXO.



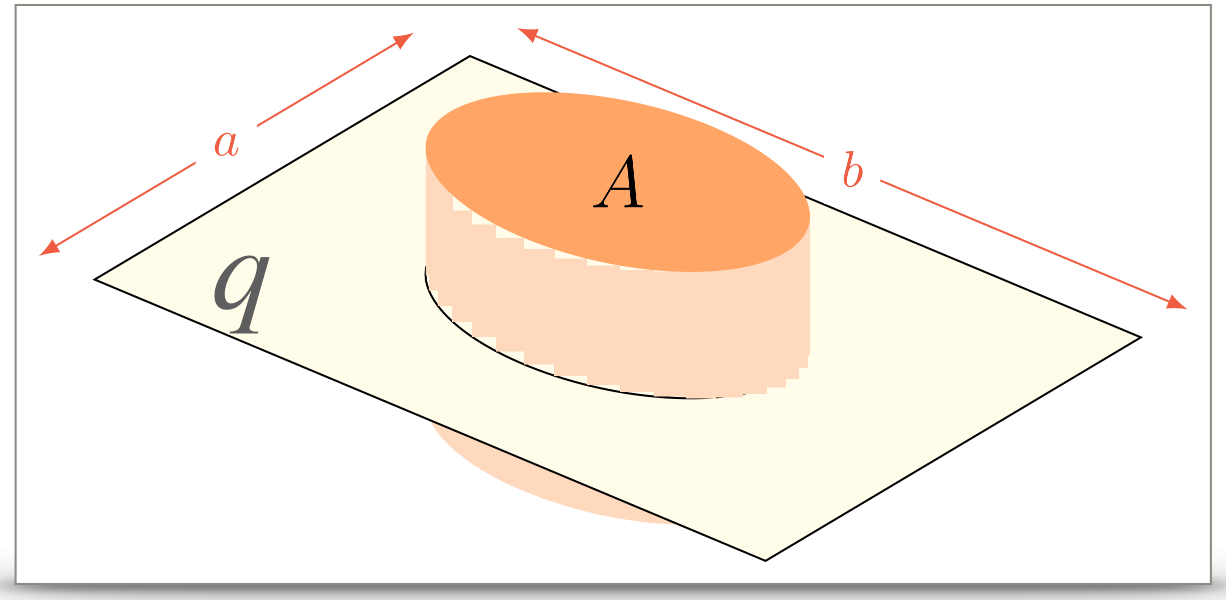
$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

Escolher superfície simétrica

$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = EA + EA$$

$$2EA = \Delta Q$$



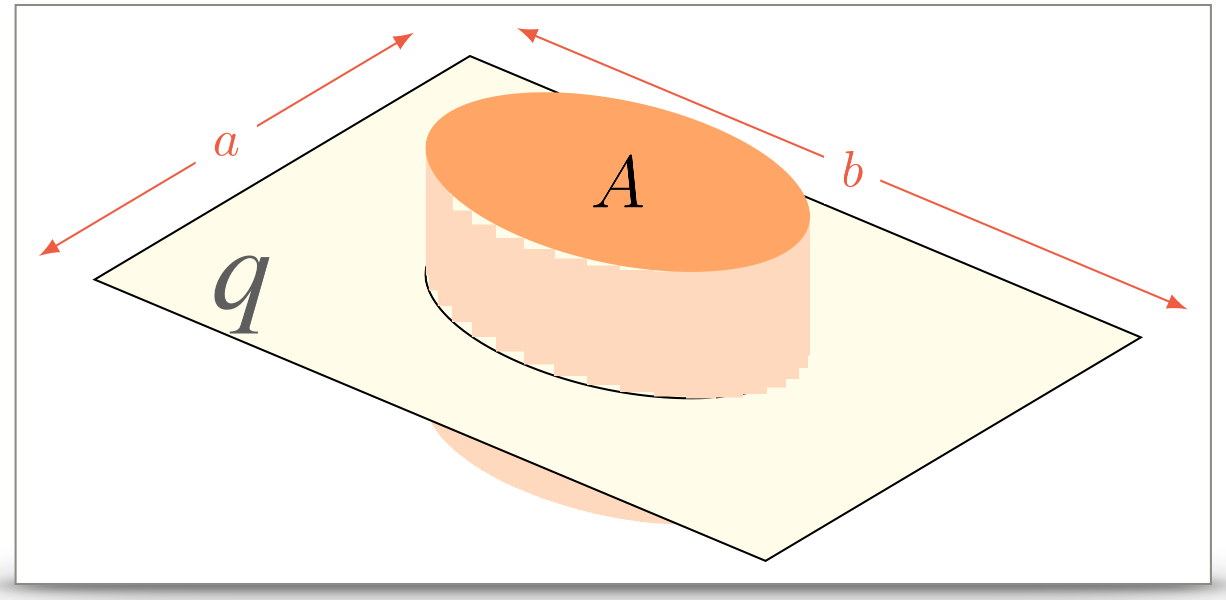
$$\int \vec{E}\cdot\hat{n}\;\mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

Escolher superfície simétrica

$$\int \vec{E}\cdot\hat{n}\,\mathrm{d}A = EA + EA$$

$$2EA = \frac{\Delta Q}{\epsilon_0}$$



$$\int \vec{E}\cdot\hat{n}\,\mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

Escolher superfície simétrica

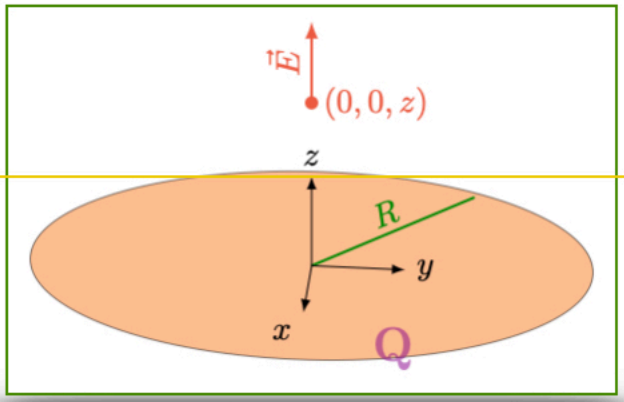
$$\int \vec{E}\cdot\hat{n}\,\mathrm{d}A = EA + E$$

$$2EA = \frac{\Delta Q}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

AULA DE 3/5

Pratique o que aprendeu



$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\vec{r}}{r^3}\,\mathrm{d}q$$

$$E_z = \frac{\sigma}{4\pi\epsilon_0}\int_0^R\int_0^{2\pi}\frac{z}{(z^2+s^2)^{3/2}}\,s\,\mathrm{d}\phi\,\mathrm{d}s$$

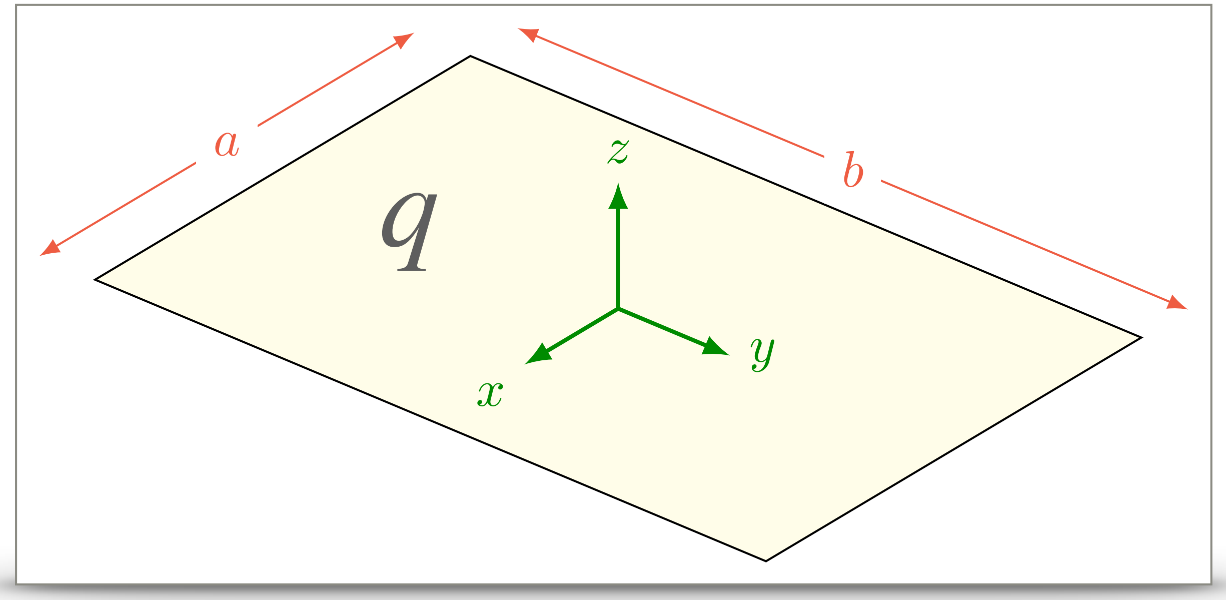
$$E_z = \frac{\sigma}{2\epsilon_0}\int_0^R\frac{z}{(z^2+s^2)^{3/2}}\,s\,\mathrm{d}s = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{\overset{\rightarrow 0}{z}}{\sqrt{z^2+R^2}}\right)$$

MUITO PERTO DO PLANO

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Pratique o que aprendeu

$$\overrightarrow{E} = ?$$

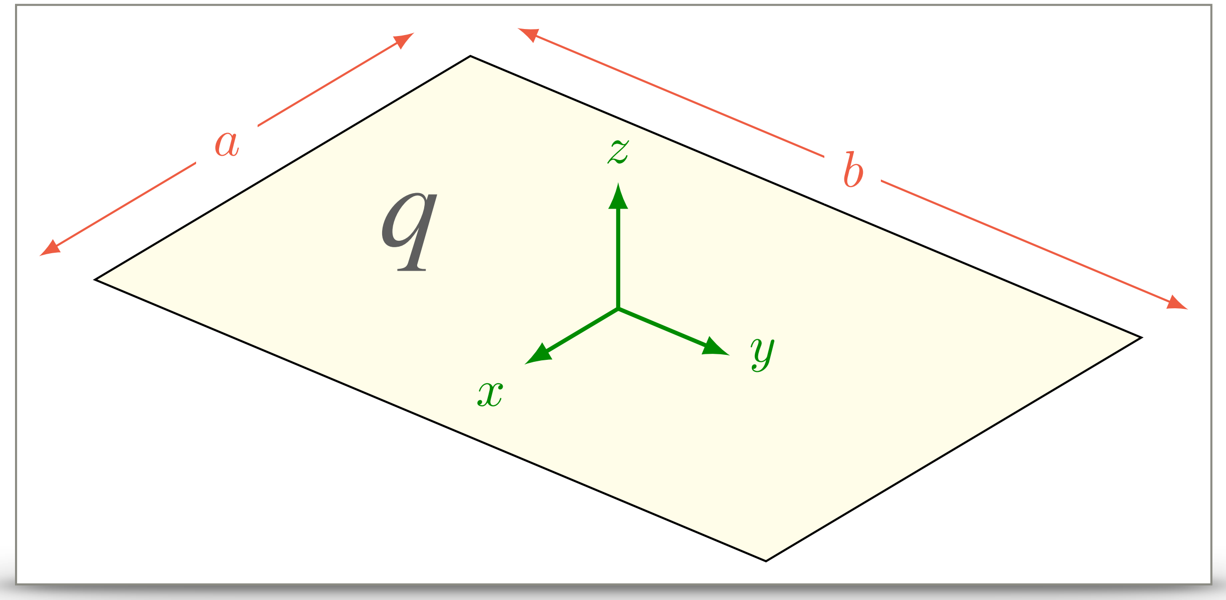


$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

Pratique o que aprendeu

$$\vec{E} = \begin{cases} E(z)\hat{z} & (z>0) \\ -E(z)\hat{z} & (z<0) \end{cases}$$

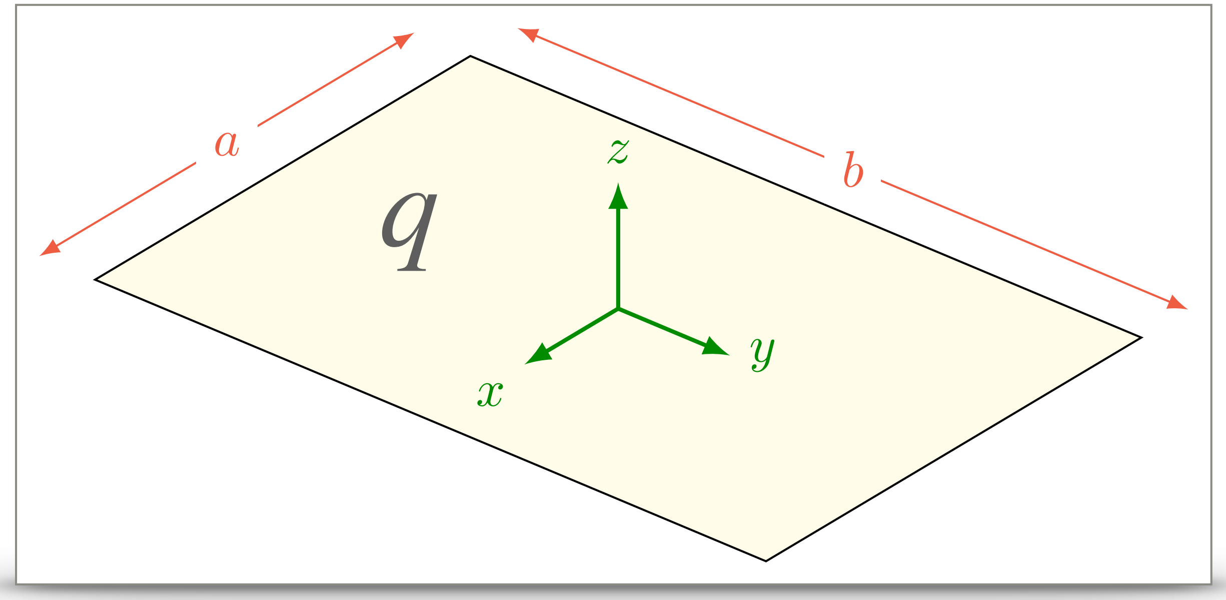
POR SIMETRIA



$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$$

Pratique o que aprendeu

$$\vec{E} = \begin{cases} E(z)\hat{z} & (z > 0) \\ -E(z)\hat{z} & (z < 0) \end{cases}$$



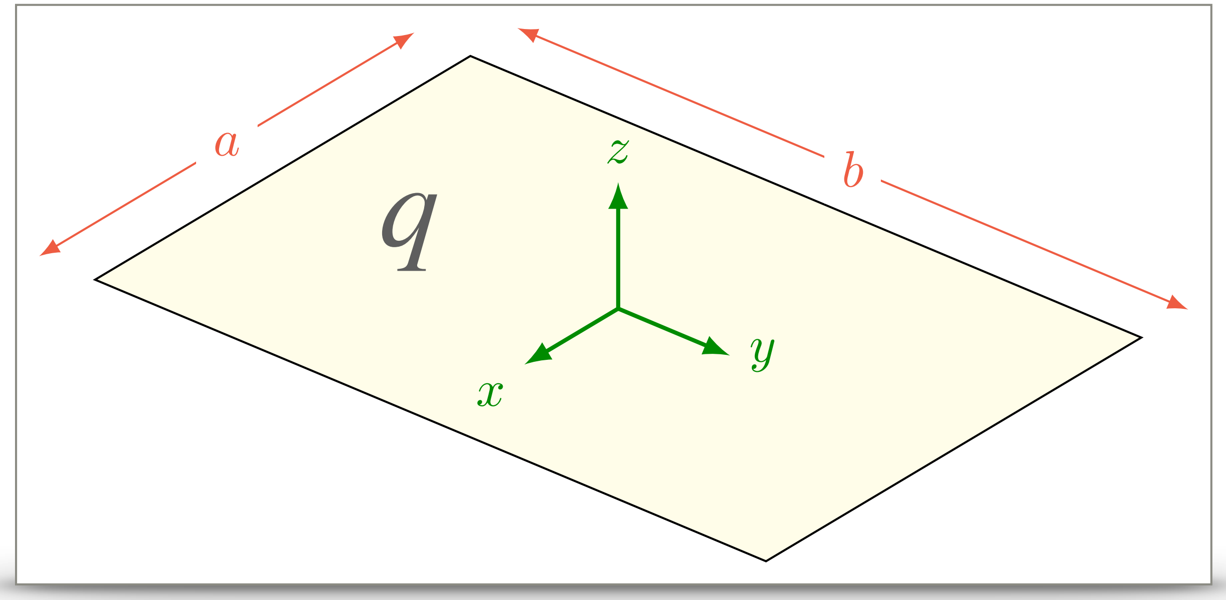
$$\frac{\mathrm{d}E}{\mathrm{d}z} = \frac{\rho}{\epsilon_0} = 0 \qquad (z \neq 0)$$

VEMOS QUE E Ñ
DEPENDE DE Z, PARA Z > 0 OU PARA Z < 0

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Pratique o que aprendeu

$$\vec{E} = \begin{cases} E(z)\hat{z} & (z > 0) \\ -E(z)\hat{z} & (z < 0) \end{cases}$$



$$\frac{\mathrm{d}E}{\mathrm{d}z} = \frac{\rho}{\epsilon_0} = 0 \qquad (z \neq 0)$$

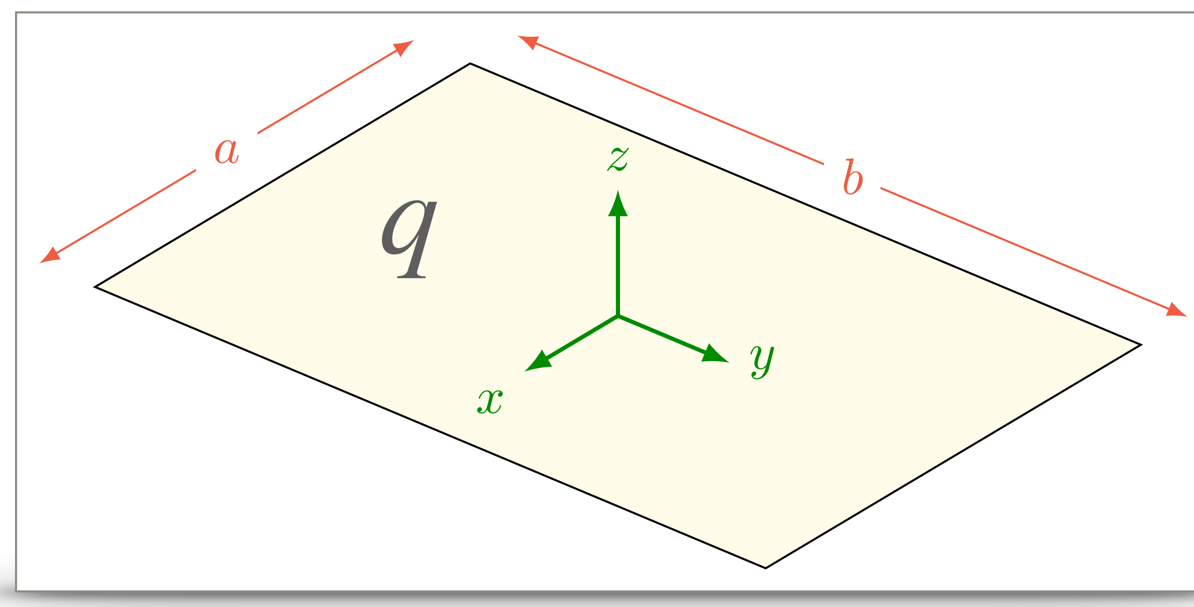
$$\vec{E} = \begin{cases} \varepsilon\hat{z} & (z > 0) \\ -\varepsilon\hat{z} & (z < 0) \end{cases}$$

→ CONSTANTE A DETERMINAR

$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$$

Pratique o que aprendeu

$$\vec{E} = \begin{cases} E(z)\hat{z} & (z > 0) \\ -E(z)\hat{z} & (z < 0) \end{cases}$$



$$\frac{\mathrm{d}E}{\mathrm{d}z} = \frac{\rho}{\epsilon_0} = 0 \qquad (z \neq 0)$$

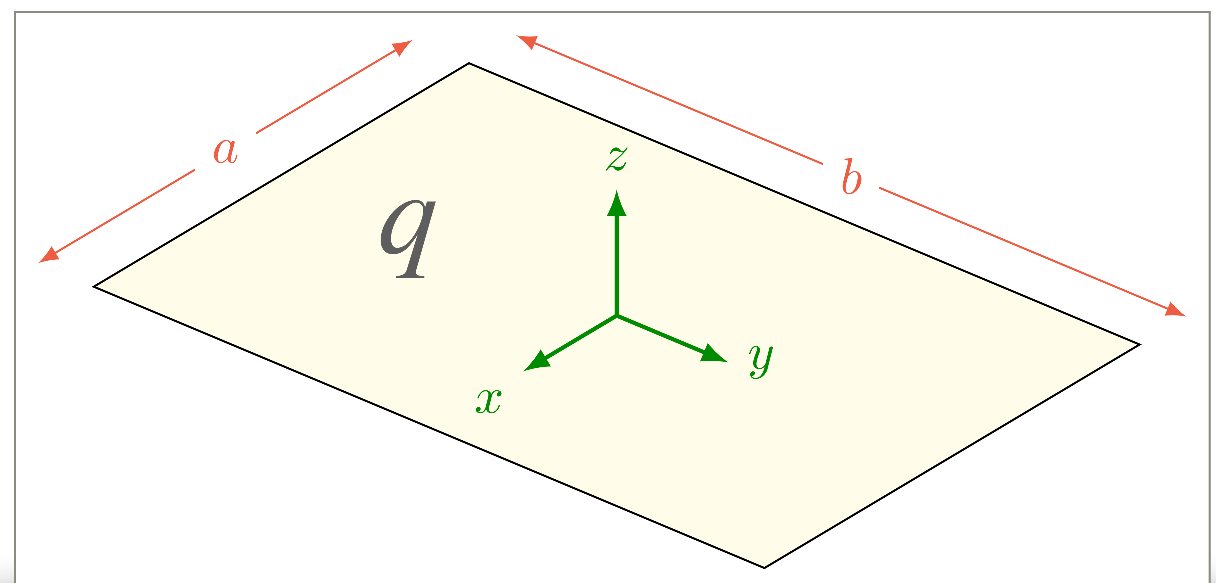
$$\vec{E} = \begin{cases} \varepsilon\hat{z} & (z > 0) \\ -\varepsilon\hat{z} & (z < 0) \end{cases}$$

$$\rho = \frac{q}{ab}\delta(z)$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

Pratique o que aprendeu

$$\frac{\mathrm{d}E}{\mathrm{d}z} = \frac{\rho}{\epsilon_0} = 0 \qquad (z \neq 0)$$



$$\vec{E} = \begin{cases} \varepsilon\hat{z} & (z > 0) \\ -\varepsilon\hat{z} & (z < 0) \end{cases}$$

$$\rho = \frac{q}{ab}\delta(z)$$

$E(z)$ $\qquad$ $E(-z)$

$$\int_{-z}^{z} \frac{\mathrm{d}E}{\mathrm{d}z}\,\mathrm{d}z = \frac{1}{\epsilon_0}\int_{-z}^{z}\rho(z)\,\mathrm{d}z \qquad \Rightarrow \varepsilon - (-\varepsilon) = \frac{1}{\epsilon_0}\frac{q}{ab}$$

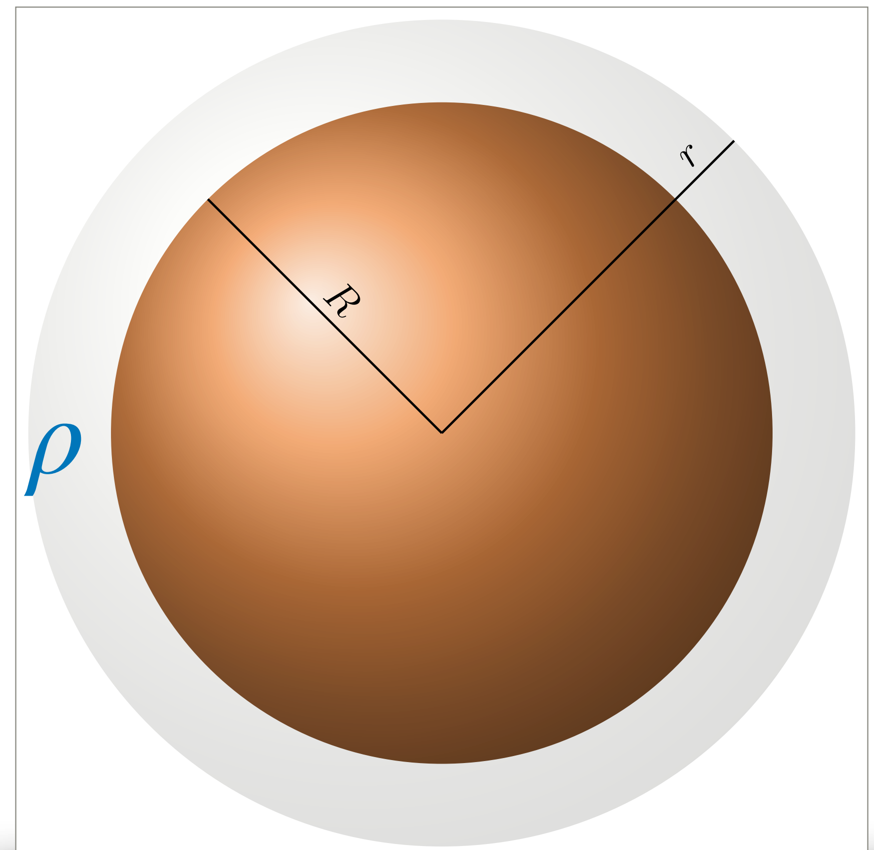
$$\Rightarrow \varepsilon = \frac{q}{ab}\,\frac{1}{2\epsilon_0} = \frac{\sigma}{2\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Pratique o que aprendeu

$\vec{E} = ?$ $\quad (r < R)$

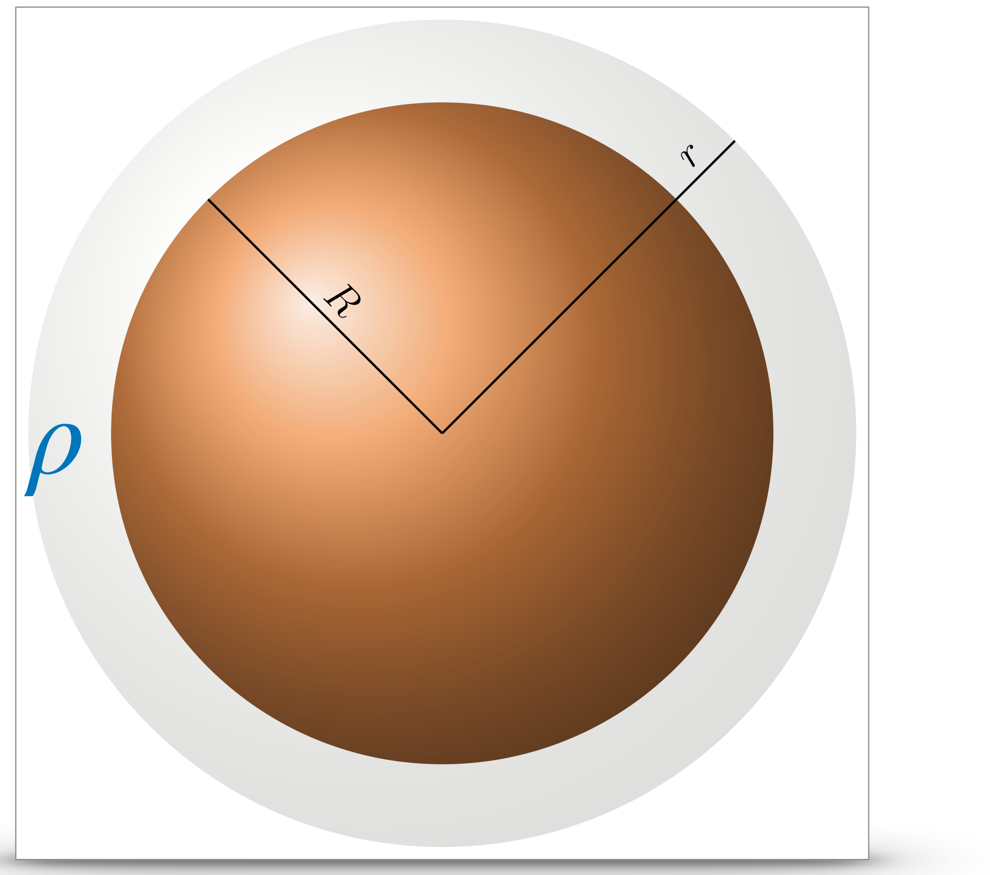
PODERÍAMOS CALCULAR FORA DA ESFERA $(r > R)$, MAS DENTRO É UM POUCO MAIS DIFÍCIL



$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

Pratique o que aprendeu

$\vec{E} = ?$ $(r < R)$



COORDENADAS ESFÉRICAS

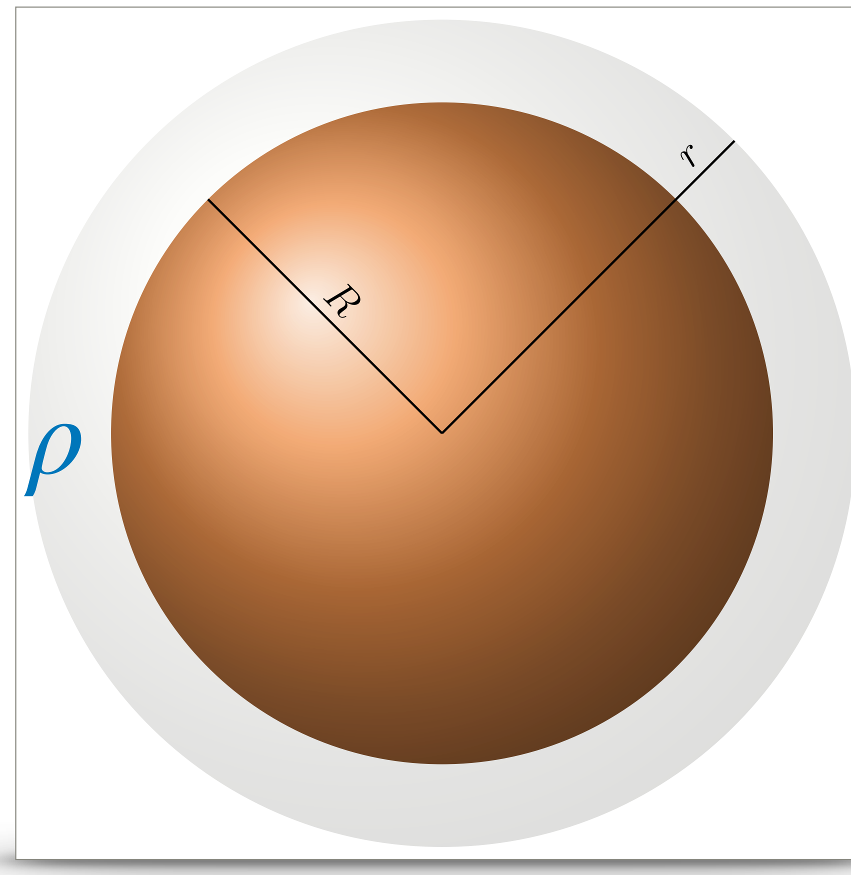
$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$$

Pratique o que aprendeu

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 E(r)\right) = \frac{\rho}{\epsilon_0}$$

↳ CAMPO SOMENTE DEPENDE DE r



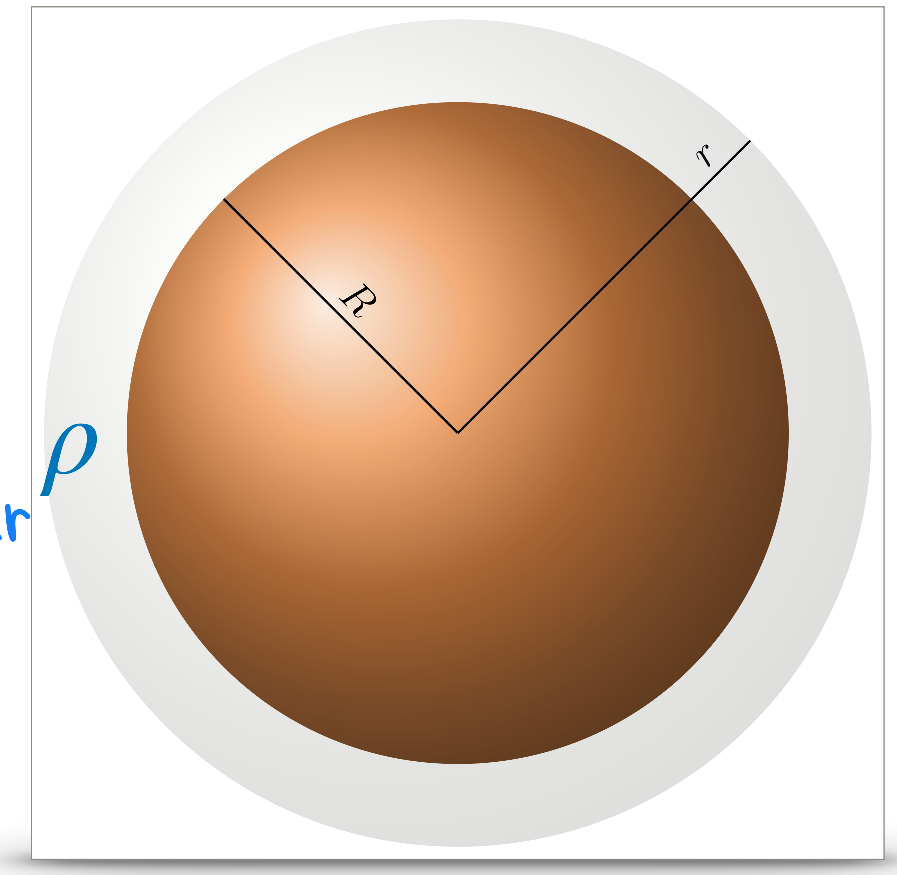
$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

Pratique o que aprendeu

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\Big(r^2E(r)\Big) = \frac{\rho}{\epsilon_0}$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\Big(r^2E(r)\Big) = \frac{\rho}{\epsilon_0}r^2 \implies \int \frac{d}{dr}\left[r^2 E\right] dr = \frac{\rho}{\varepsilon_0}\int r^2 dr$$



$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

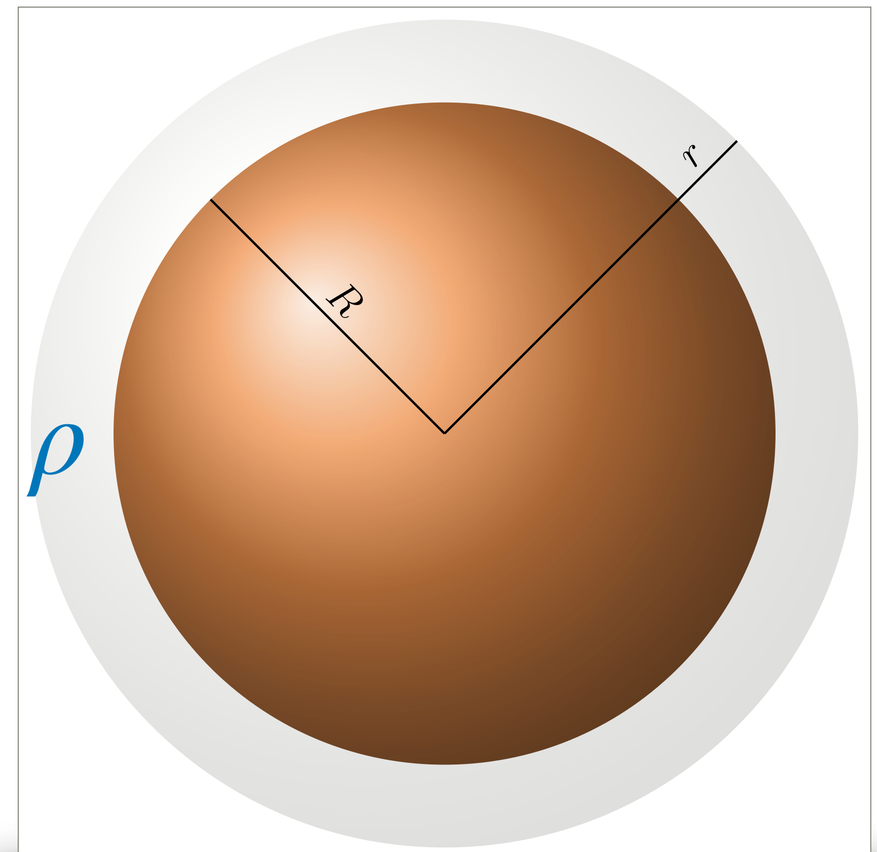
Pratique o que aprendeu

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 E(r)\right) = \frac{\rho}{\epsilon_0}$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 E(r)\right) = \frac{\rho}{\epsilon_0} r^2$$

$$r^2 E(r) - 0\,E(0) = \frac{\rho}{\varepsilon_0}\int_0^r r'^2\,dr'$$

$$E(r) - \underbrace{\cancel{E(0)}}_{0} = \frac{\rho}{\epsilon_0 r^2}\int_0^r r'^2\,\mathrm{d}r'$$



$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

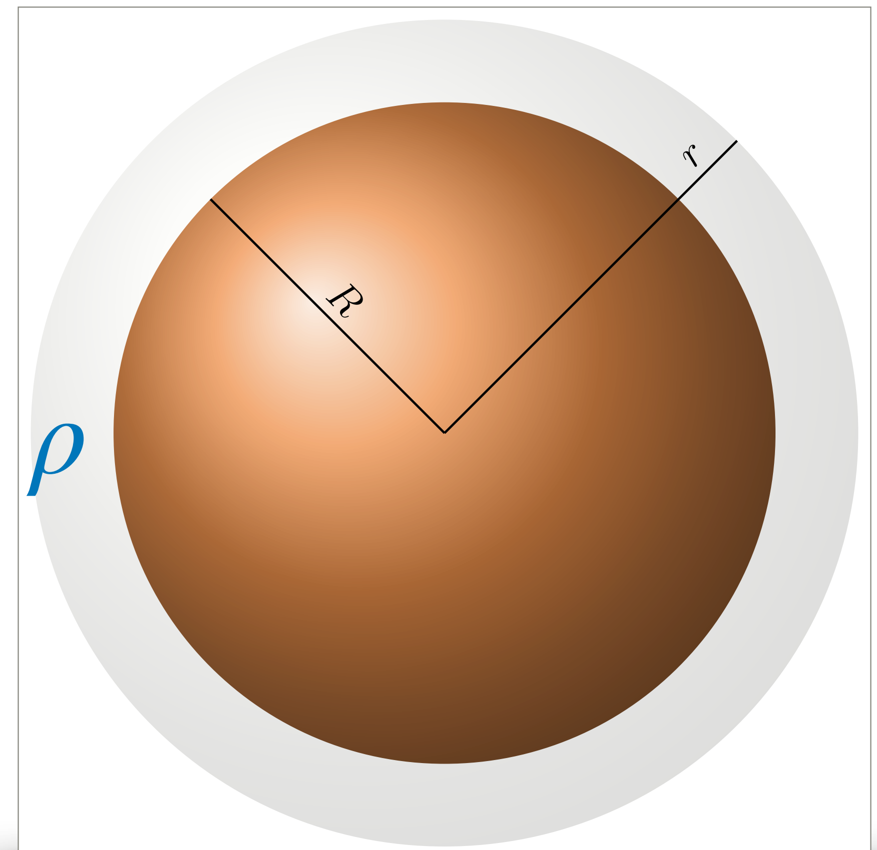
Pratique o que aprendeu

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\Big(r^2 E(r)\Big) = \frac{\rho}{\epsilon_0}$$

$$\frac{\mathrm{d}}{\mathrm{d}r}\Big(r^2 E(r)\Big) = \frac{\rho}{\epsilon_0} r^2$$

$$E(r) - E(0) = \frac{\rho}{\epsilon_0 r^2}\int_0^r r'^2 \, \mathrm{d}r'$$

$$\Rightarrow E(r) = \frac{\rho}{\epsilon_0}\frac{r}{3}$$



FORA DA ESFERA É, MAIS FÁCIL,
PQ $\rho = 0$
$\Rightarrow \frac{1}{r^2}\frac{d}{dr}(r^2 E) = 0 \Rightarrow r^2 E = \text{constant}$

Rotacional do campo

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \Rightarrow$$

COORDENADAS ESFÉRICAS $\Rightarrow$ FÁCIL VER QUE $\vec{\nabla}\times\vec{E} = 0$.

MAS VAMOS SEGUIR OUTRO RACIOCÍNIO

Rotacional do campo

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r})\cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{\vec{a}}^{\vec{b}} \frac{q}{r^2}\hat{r}\cdot \mathrm{d}\vec{\ell}$$

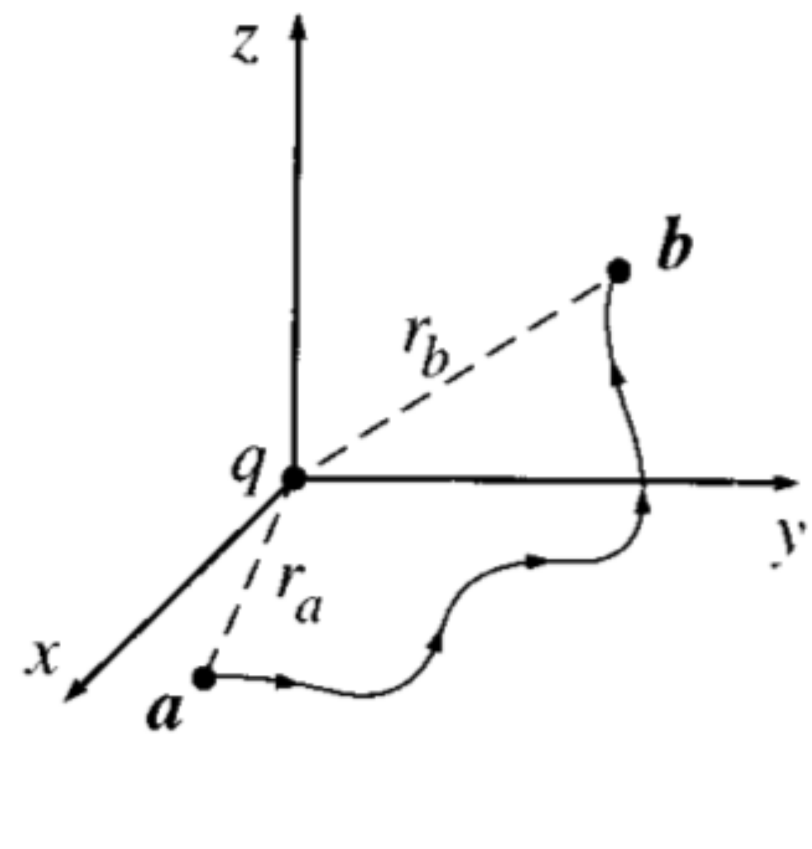


Figure 2.29

Rotacional do campo

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{\vec{a}}^{\vec{b}} \frac{q}{r^2}\hat{r} \cdot \mathrm{d}\vec{\ell},$$

MAS

$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\,\mathrm{sen}\,\theta\,d\phi\,\hat{\phi}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{q}{r^2}\,\mathrm{d}r$$

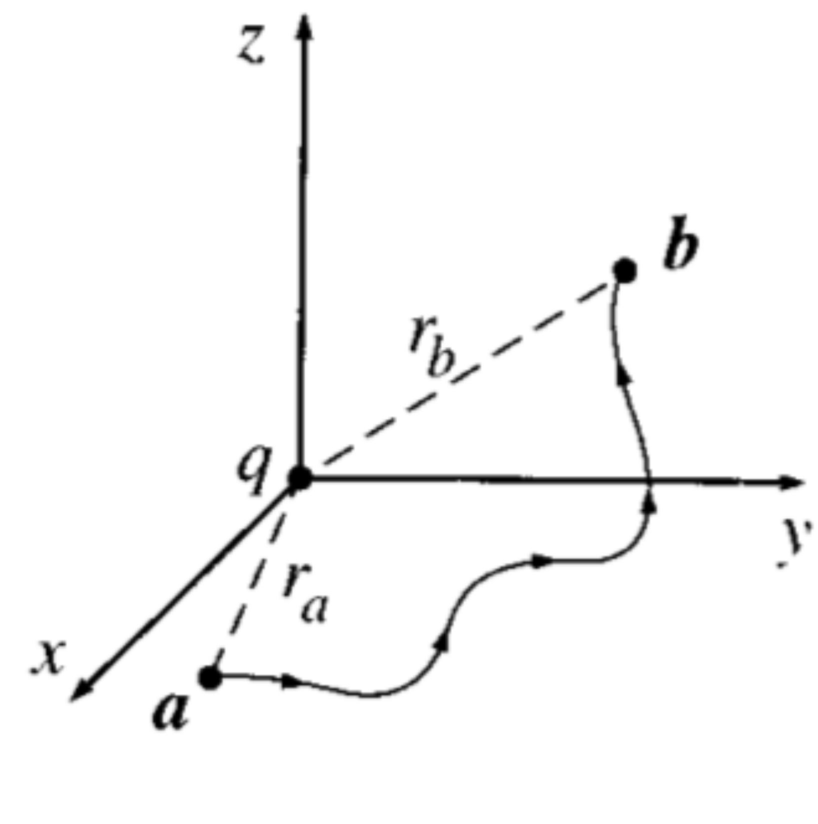


Figure 2.29

Rotacional do campo

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{\vec{a}}^{\vec{b}} \frac{q}{r^2}\hat{r} \cdot \mathrm{d}\vec{\ell}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{q}{r^2}\,\mathrm{d}r$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r}) \cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right)$$

LIMITES DE INTEGRAÇÃO INVERTIDOS, PARA ELIMINAR SINAL NEGATIVO

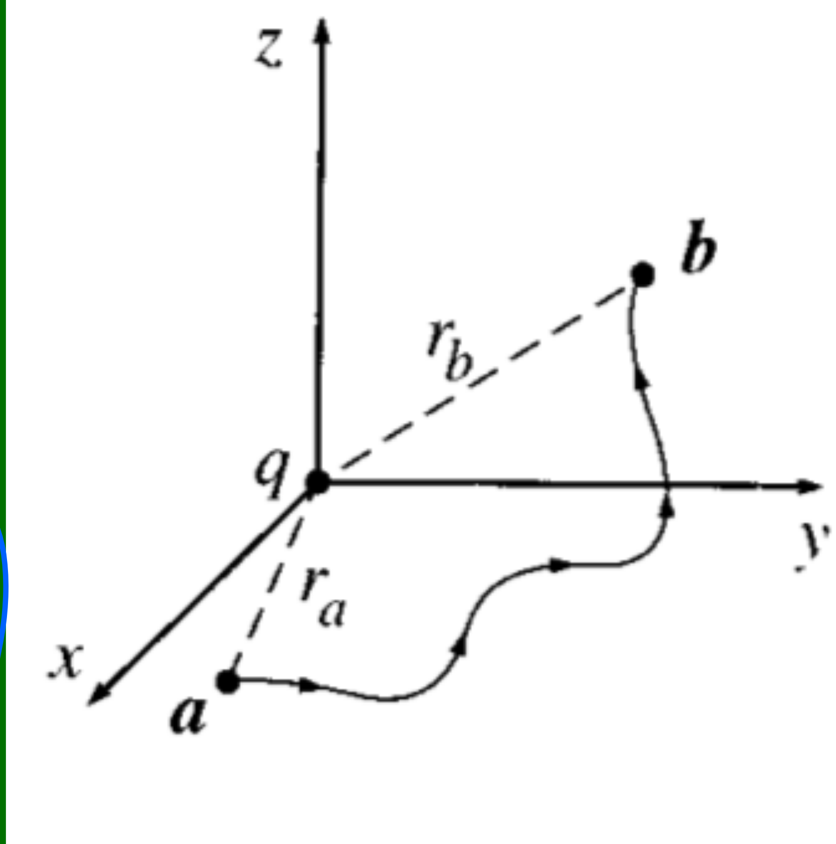


Figure 2.29

Rotacional do campo

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r})\cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{\vec{a}}^{\vec{b}} \frac{q}{r^2}\hat{r}\cdot \mathrm{d}\vec{\ell}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r})\cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{q}{r^2}\,\mathrm{d}r$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r})\cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_a} - \frac{q}{r_b}\right)$$

$$\oint \vec{E}(\vec{r})\cdot \mathrm{d}\vec{\ell} = 0$$

(BASTA FAZER $\vec{a} = \vec{b}$
$\Rightarrow r_a = r_b$)

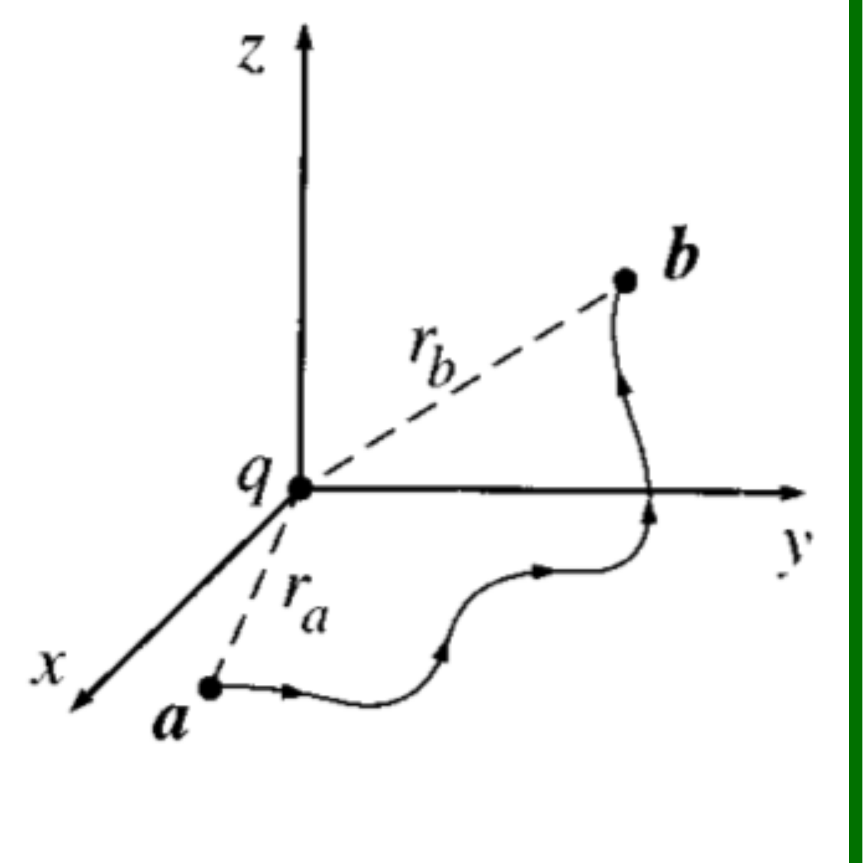


Figure 2.29

Rotacional do campo

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

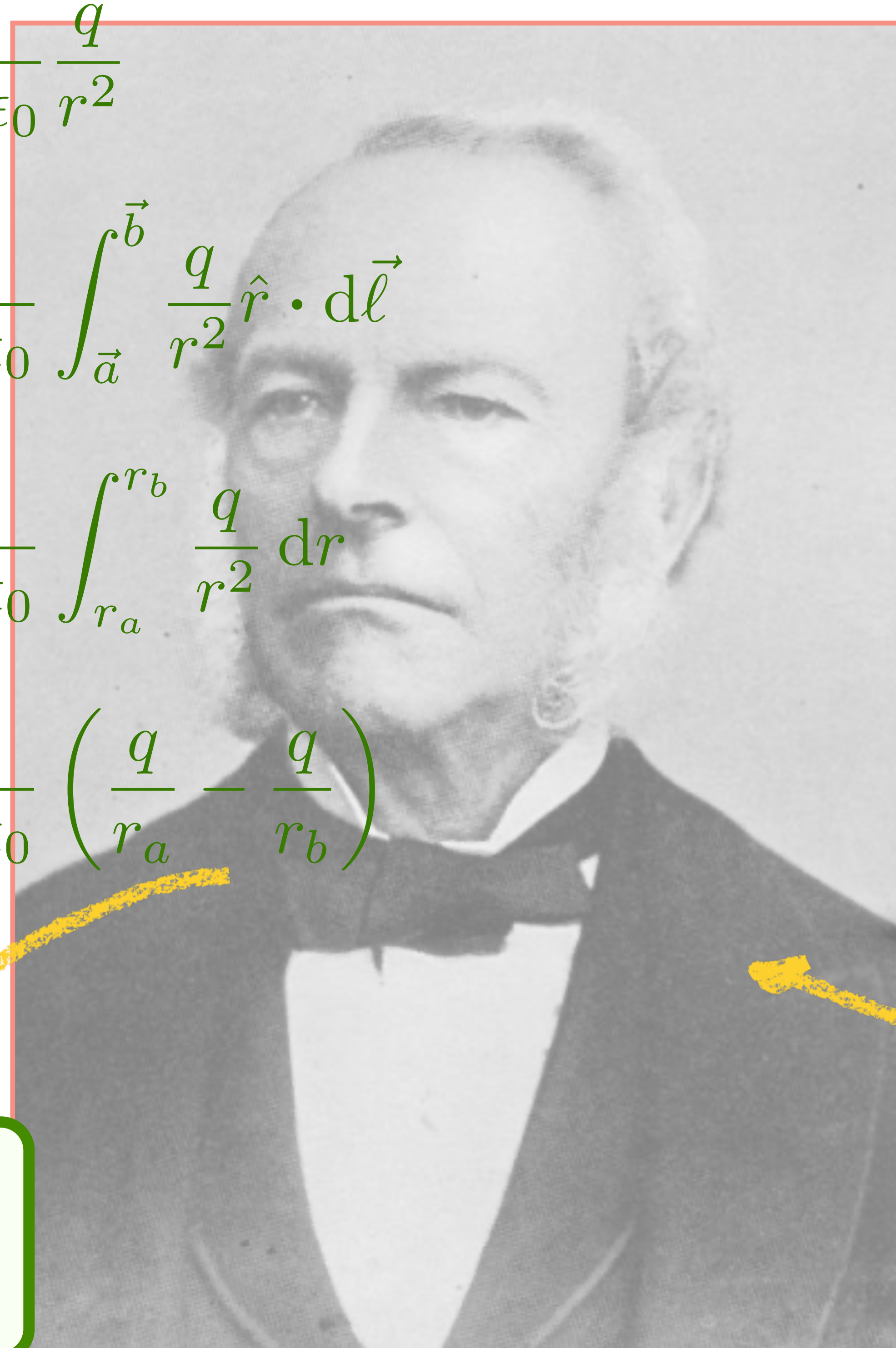
$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r})\cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{\vec{a}}^{\vec{b}} \frac{q}{r^2}\hat{r}\cdot \mathrm{d}\vec{\ell}$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r})\cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\int_{r_a}^{r_b} \frac{q}{r^2}\,\mathrm{d}r$$

$$\int_{\vec{a}}^{\vec{b}} \vec{E}(\vec{r})\cdot \mathrm{d}\vec{\ell} = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_a}-\frac{q}{r_b}\right)$$

$$\oint \vec{E}(\vec{r})\cdot \mathrm{d}\vec{\ell} = 0$$

$$\boxed{\vec{\nabla}\times\vec{E}=0}$$



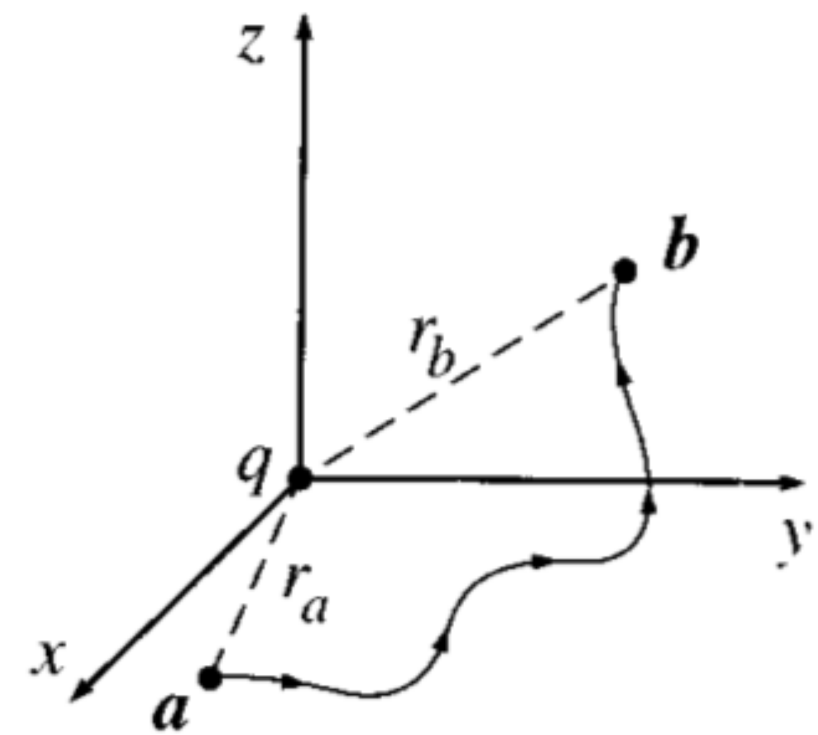


Figure 2.29

GEORGE G. STOKES