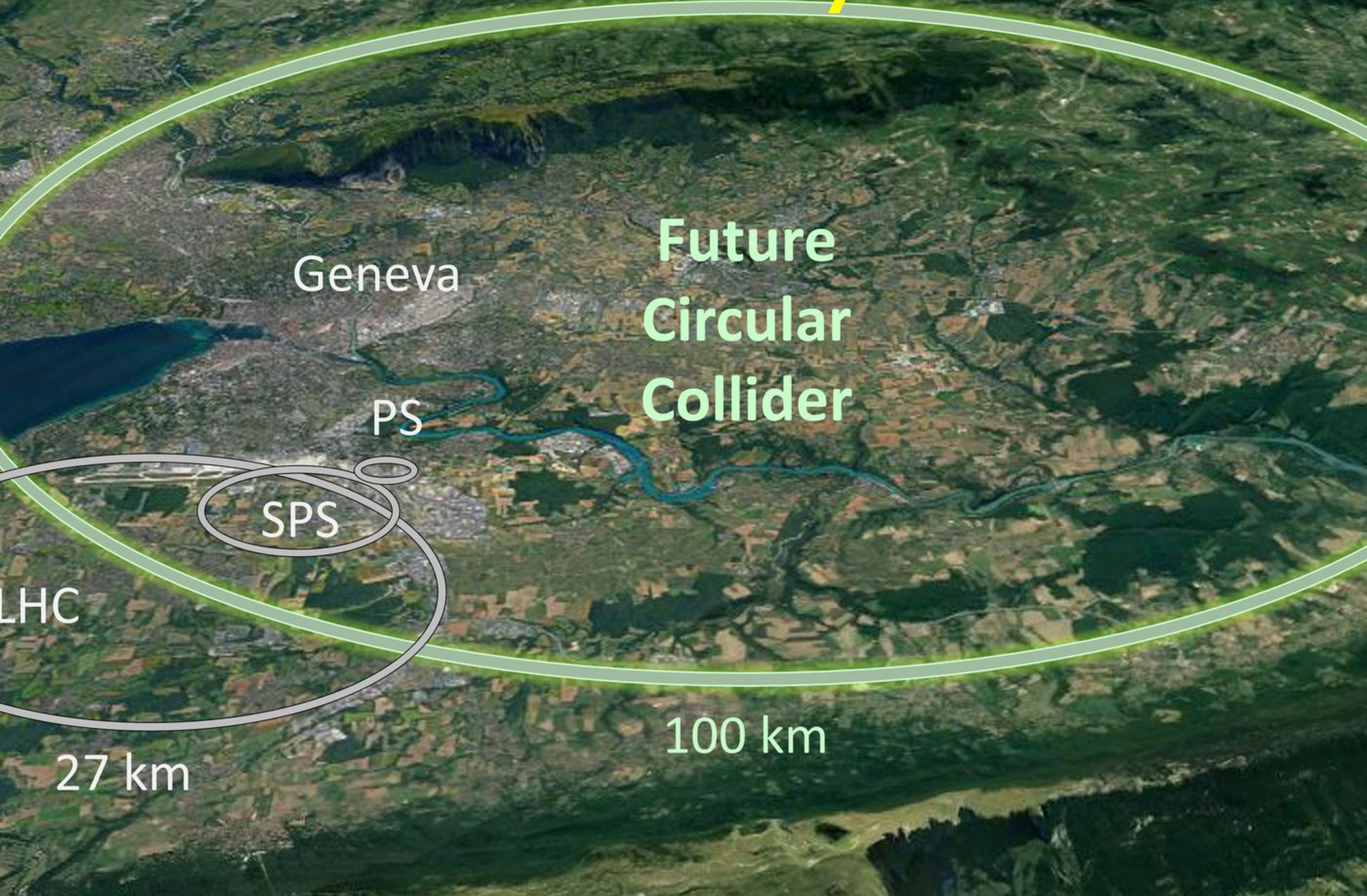
# Collider Physics



# Lightning review of QFT

- Our goal: practical review of perturbation theory in QFT

  - Wave equations
  - Propagators
  - Interactions/Vertices
  - Feynman diagrams

# I. Wave equations

- Given a Lagrangian density the EOM is

$$\partial_\mu \left( \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi_j} \right) = \frac{\partial \mathcal{L}}{\partial \varphi_j}$$

for instance

$$\mathcal{L} = i\psi^\star \partial_t \psi + \frac{1}{2m} \psi^\star \nabla^2 \psi - V(\vec{x}) \psi^\star \psi \implies i\partial_t \psi = -\frac{1}{2m} \nabla^2 \psi + V(\vec{x}) \psi$$

- The Lagrangian must reflect the symmetries of the problem.

- A symmetry is a transformation that leaves the action invariant

$$
\begin{aligned}
&x \to x' \\
&\varphi_j(x) \to \varphi_j(x') \\
&\mathcal{L} \to \mathcal{L} + \partial_\mu \Lambda^\mu
\end{aligned}
$$

for instance $$\mathcal{L} = i\psi^\star \partial_t \psi + \frac{1}{2m}\psi^\star \nabla^2 \psi - V(\vec{x})\psi^\star \psi$$

is invariant under

constant

$$x \to x' = x \quad \text{and} \quad \psi \to \psi'(x') = e^{i\alpha}\psi(x)$$

- Noether's theorem: for continuous symmetry there is a conserved current

$$x \to x'$$

$$\varphi_j(x) \to \varphi_j(x')$$

$$\mathcal{L} \to \mathcal{L} + \partial_\mu \Lambda^\mu$$

$$\delta\varphi_j(x) = \varphi'_j(x) - \varphi_j(x)$$

$$J_\mu = \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi_j}\,\delta\varphi_j - \Lambda_\mu$$

$$\partial_\mu J^\mu = 0$$

for the Schrödinger field $\quad J^\mu = \left(\psi^\star\psi, \frac{i}{2m}(\psi^\star\nabla\psi - \psi\nabla\psi^\star)\right)$

# Real scalar field

- Under Lorentz transformation its transformation is

$$x \to x' \quad \Longrightarrow \quad \phi'(x') = \phi(x)$$

- The relativistic free Lagrangian density is

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi - \frac{m^2}{2}\phi^2$$

the EOM is $$(\partial_\mu \partial^\mu + m^2)\phi = 0$$

with solutions $$e^{\pm ipx} \quad \text{with} \quad p_\mu p^\mu = m^2$$

# Complex scalar field

- Under Lorentz transformation its transformation is

$$x \to x' \quad \Longrightarrow \quad \phi'(x') = \phi(x)$$

- The relativistic free Lagrangian density is $\mathcal{L} = \partial_\mu \phi^\star \; \partial^\mu \phi - m^2 \phi^\star \phi$

the EOM is $$(\partial_\mu \partial^\mu + m^2)\phi = 0$$

with solutions $$e^{\pm ipx} \;\; \text{with} \;\; p_\mu p^\mu = m^2$$

constant

conserved current $$\phi(x) \to \phi'(x) = e^{i\alpha}\phi(x)$$

symmetry $$J_\mu = \phi \partial_\mu \phi^\star - \phi^\star \partial_\mu \phi$$

# Dirac field $\psi_j(x) \;,\; j = 1 - 4$

- Under Lorentz transformation its transformation is

$$x \to x' \implies \psi'(x') = S(\Lambda)\phi(x) \quad \text{with} \quad S = e^{-\frac{i}{4}\omega_{\mu\nu}\Sigma^{\mu\nu}}$$

- EOM $$(i\gamma^\mu\partial_\mu - m)\psi = 0$$

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad \text{one representation} \quad \gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^j = \begin{pmatrix} 0 & \sigma^j \\ -\sigma^j & 0 \end{pmatrix}$$

- Lagrangian density $$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi \qquad \bar{\psi} = \psi^\dagger\gamma^0$$

constant

- Invariance under $$\psi(x) \to \psi'(x) = e^{i\alpha}\psi(x) \implies J^\mu = \bar{\psi}\gamma^\mu\psi$$

- Free particle solutions $\psi(x) = u(p)\ e^{ipx}$ or $\psi(x) = v(p)\ e^{-ipx}$

$$p_\mu p^\mu = m^2 \quad \text{and} \quad p^0 = \sqrt{\vec{p}^2 + m^2}$$

where

$$u_1(p) = \sqrt{E+m}\begin{pmatrix} 1 \\ 0 \\ p_3/(E+m) \\ (p_1 - ip_2)/(E+m) \end{pmatrix} \quad u_2(p) = \sqrt{E+m}\begin{pmatrix} 0 \\ 1 \\ (p_1 - ip_2)/(E+m) \\ -p_3/(E+m) \end{pmatrix}$$

$$v_1(p) = \sqrt{E+m}\begin{pmatrix} p_3/(E+m) \\ (p_1 - ip_2)/(E+m) \\ 1 \\ 0 \end{pmatrix} \quad v_2(p) = \sqrt{E+m}\begin{pmatrix} (p_1 - ip_2)/(E+m) \\ -p_3/(E+m) \\ 0 \\ 1 \end{pmatrix}$$

- Useful relations:

$$\not{p} \equiv p_\mu \gamma^\mu$$

$$(\not{p} - m)u_j(p) = 0$$

$$(\not{p} + m)v_j(p) = 0$$

$$\sum_{j=1}^{2} u_j(\vec{p})\bar{u}_j(\vec{p}) = \not{p} + m$$

$$\sum_{j=1}^{2} v_j(\vec{p})\bar{v}_j(\vec{p}) = \not{p} - m$$

# Vector field $A_\mu(x)$

- Under Lorentz transformation its transformation is

$$x \to x'^{\,\mu} = \Lambda^\mu{}_\nu x^\nu \quad \Longrightarrow \quad A'^{\,\mu}(x') = \Lambda^\mu{}_\nu A^\nu(x)$$

- EOM $\quad \partial_\mu F^{\mu\nu} = 0 \quad \text{with} \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

- Lagrangian density $\quad \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

- Invariance under $\quad A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu \Lambda$

- Solutions

$$A_\mu(x) = \epsilon_\mu^{(j)} e^{ipx} \quad \text{with} \quad p^2 = 0 \quad , \quad p\epsilon^{(j)} = 0 \quad , \quad j = 1, 2$$

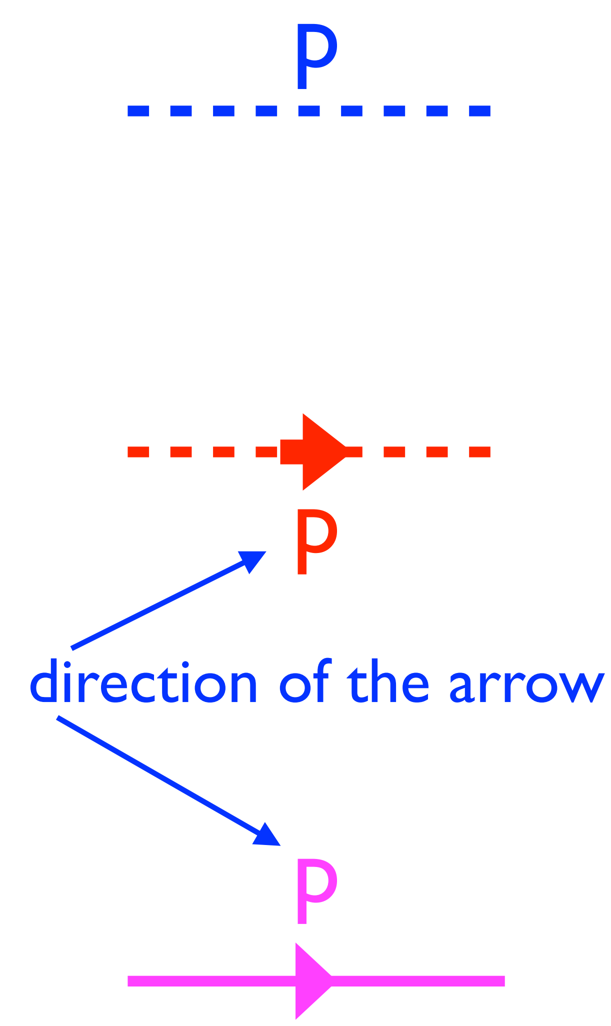
# II. Propagators

- The quadratic parts of the Lagrangian define the propagators
- Propagators are the inverse of the operators in the quadratic parts

$$\mathcal{L} = \frac{1}{2}\phi(-\partial_\mu \partial^\mu - m^2)\phi \quad \Longrightarrow \quad \frac{i}{p^2 - m^2 + i\epsilon}$$

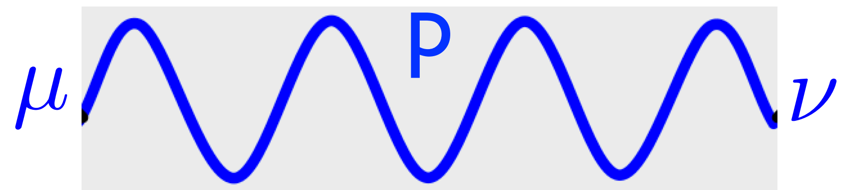
$$\mathcal{L} = \phi^\star(-\partial_\mu \partial^\mu - m^2)\phi \quad \Longrightarrow \quad \frac{i}{p^2 - m^2 + i\epsilon}$$

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi \quad \Longrightarrow \quad \frac{i}{p\!\!\!/ - m + i\epsilon}$$



- For massless vector fields we need to add a gauge fixing term

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\lambda}\partial_\mu A^\mu \quad \Longrightarrow \quad -i\left[\frac{g_{\mu\nu}}{p^2+i\epsilon} - \frac{(1-\lambda)p_\mu p_\nu}{(p^2+i\epsilon)^2}\right]$$



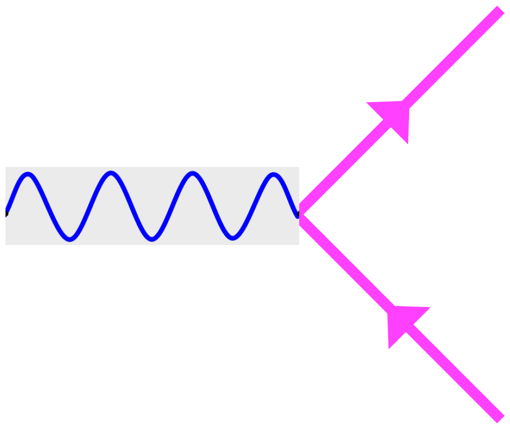

- For a massive vector field

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu \quad \Longrightarrow \quad -i\frac{g_{\mu\nu} - p_\mu p_\nu/m^2}{p^2 - m^2 + i\epsilon}$$

# III. Vertices

- The non quadratic part of the Lagrangian defines the vertices
- Each field gives rise to one line of the vertex

Example $\mathcal{L} = \bar{\psi}\gamma^{\mu} A_{\mu} \psi$



- Rules to determine the weight of a vertex

1. Start with a factor of i
2. To derivatives associate an incoming momentum $\partial_{\mu} \leftrightarrow -ip_{\mu}$
3. Remove fields and the remaining is a contribution to the weight
4. “daggers" lead to outgoing "arrows"

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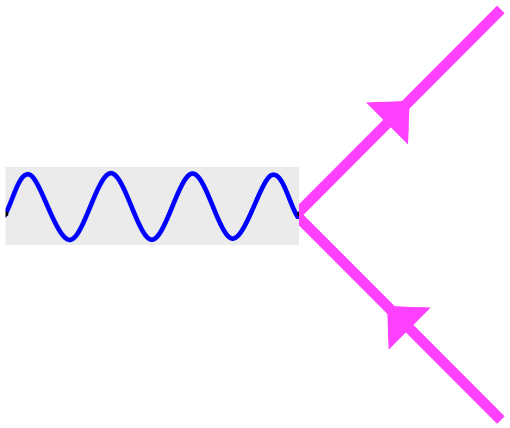

$i\gamma^\mu$

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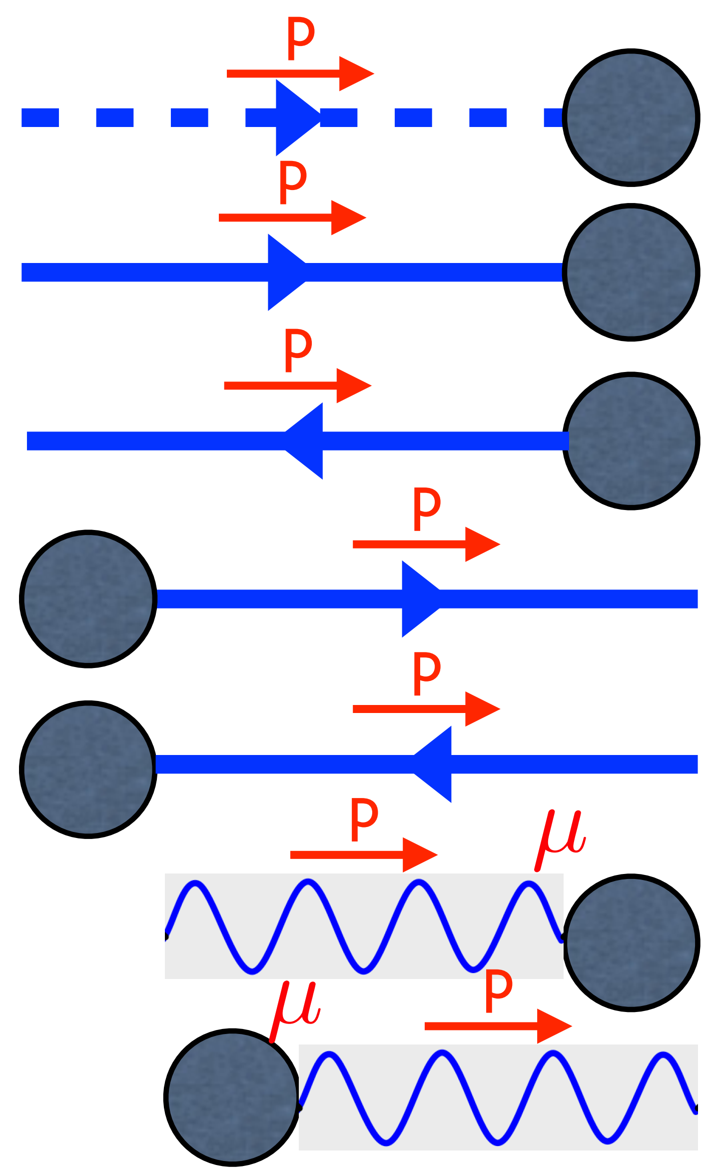
# IV. Feynman diagrams

- Draw all possible topologically distinct diagrams with the number of external lines given by the number of incoming and outgoing particles
- For each external line write

1. Real and complex scalars: $1$
2. Incoming fermion line: $u(p)$
3. Incoming anti-fermion line: $\bar{v}(p)$
4. Outgoing fermion line: $\bar{u}(p)$
5. Outgoing anti-fermion line: $v(p)$
6. Incoming neutral vector $\epsilon^{\mu}_{\lambda}(p)$
7. Outgoing vector particle $\epsilon^{\mu\,\star}_{\lambda}(p)$



- Write the contribution of a fermion line adding the elements going in the opposite direction of the arrow
- For each fermion loop take the trace and multiply by -1
- Impose energy-momentum conservation in each vertex
- For each momentum p not fixed add $\frac{d^4p}{(2\pi)^4}$
- Multiply the contribution of each diagram by:

1. A global minus sign for the external fermion lines if they are exchanged with respect to the first diagram
2. The symmetry factor 1/S where S is the number of permutations of the internal lines and vertices leaving the diagram unchanged with the external legs fixed.

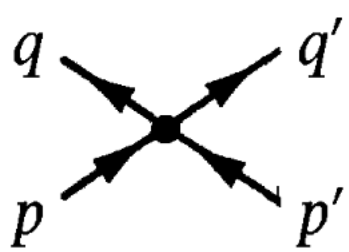
# V. Examples

## ✷ Scalar electrodynamics

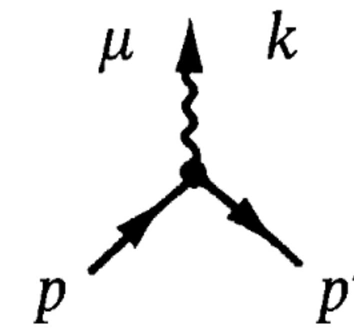
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2$$
$$+\left[(\partial_\mu + ieA_\mu)\varphi\right]^\dagger\left[(\partial^\mu + ieA^\mu)\varphi\right] - m^2\varphi^\dagger\varphi - \frac{\lambda}{4}\left(\varphi^\dagger\varphi\right)^2$$

whose vertices are

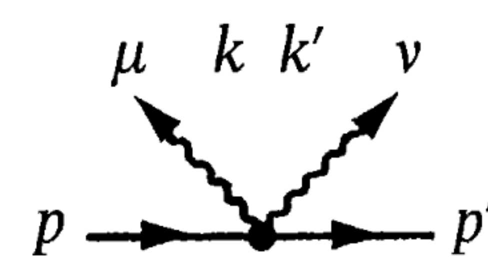
$-i\lambda$




$-ie(p+p')_\mu$




$2ie^2 g_{\mu\nu}$

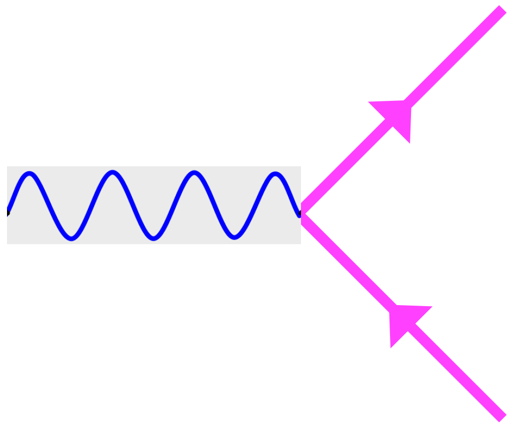


## Electrodynamics

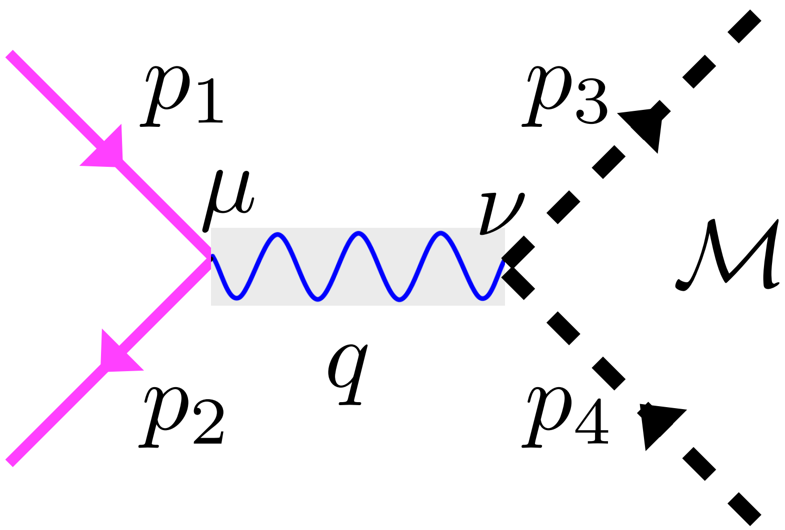
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}\left(\partial_\mu A^\mu\right)^2 + \bar{\psi}\left(i\partial\!\!\!/ - eA\!\!\!/ - m\right)\psi$$

whose vertex is

$$-ie\gamma^\mu$$



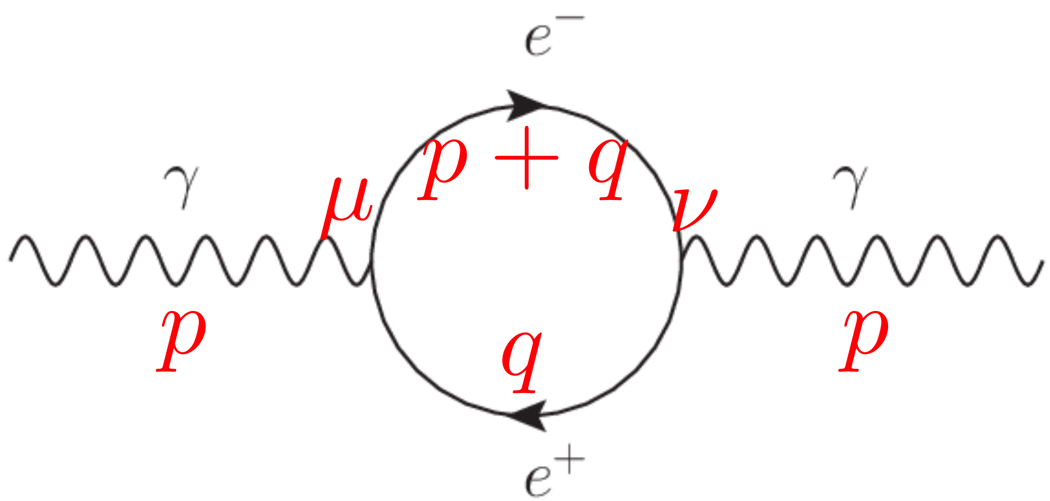
## Mixing both models $e^+e^- \to s^+s^-$




$$\mathcal{M} = \bar{v}_j(p_2)(-ie\gamma^\mu)u_k(p_1)\ \frac{-ig_{\mu\nu}}{q^2}\ \left[-ie(p_3-p_4)^\nu\right]$$

spin

# ✷ Vacuum polarization QED




$$-\int \frac{d^4q}{(2\pi)^4}\ \mathrm{Tr}\left[(-ie\gamma^\mu)\frac{i}{\not{q}-m}(-ie\gamma^\nu)\frac{i}{\not{q}+\not{p}-m}\right]$$

Denner, Hahn, Küblbeck, PLB 291 (1992) 278

# VI. Majorana fermions

- Let's go out of the box: Majorana fermions $\psi^c \equiv C\bar{\psi}^T = \psi$
- Consider the interaction $\mathcal{L}_I = \bar{\chi}\Gamma\chi = h^i_{abc}\bar{\chi}_a\Gamma_i\chi_b\Phi_c$
- Then write the interaction in terms of

$$\tilde{\chi} = C\bar{\chi}^T \quad , \quad \bar{\tilde{\chi}} = -\chi^T C^\dagger \quad , \quad \Gamma' = C\Gamma^T C^\dagger$$

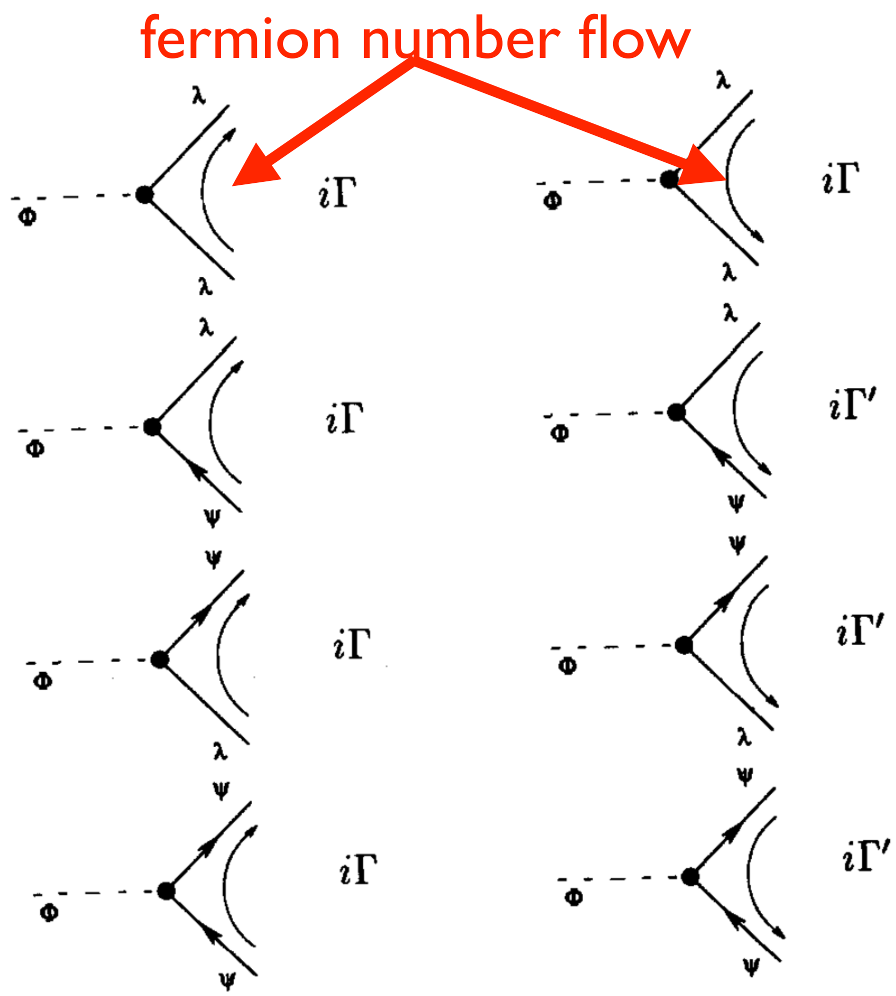
$$C\Gamma_i^{\mathrm{T}}C^{-1} = \eta_i\Gamma_i \qquad \begin{aligned} \eta_i &= \phantom{-}1 \quad \text{for } \Gamma_i = 1,\ \mathrm{i}\gamma_5,\ \gamma_\mu\gamma_5\,, \\ &= -1 \quad \text{for } \Gamma_i = \gamma_\mu,\ \sigma_{\mu\nu}\,, \end{aligned}$$

$$\mathcal{L}_I = \bar{\tilde{\chi}}\Gamma'\tilde{\chi} = h^i_{abc}\eta_i\bar{\tilde{\chi}}_b\Gamma_i\tilde{\chi}_a\Phi_c$$

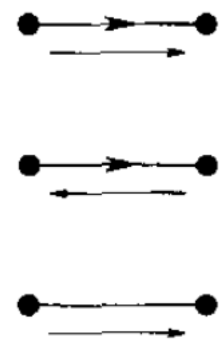
- Obtain the Feynman rules for the two forms of the interaction

- Feynman rules

1. Fermions are represented by solid lines. Dirac fermions carry an arrow while Majorana ones don't.
2. Vertices are read of the two forms of the interaction $\Gamma$ and $\Gamma'$



3. The propagators follow the following rule with respect to the fermion number flow

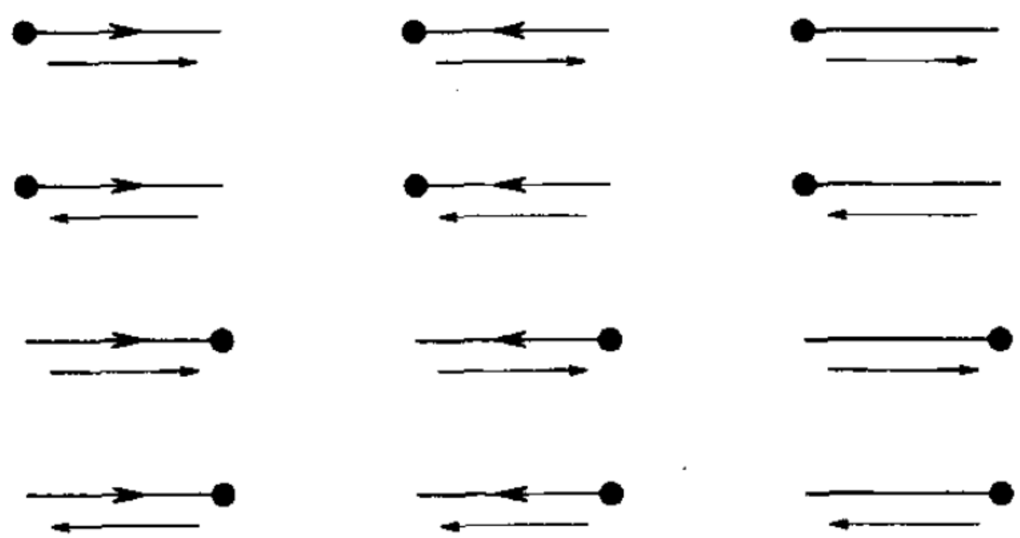


$iS(p)$

$iS(-p)$

$iS(p)$

4. The initial state obey the rule:



$\overline{u}(p,s)$

$v(p,s)$

$u(p,s)$

$\overline{v}(p,s)$

5. Draw all possible diagrams for a given process

6. Fix and arbitrary orientation (fermion flow) for each fermion chain

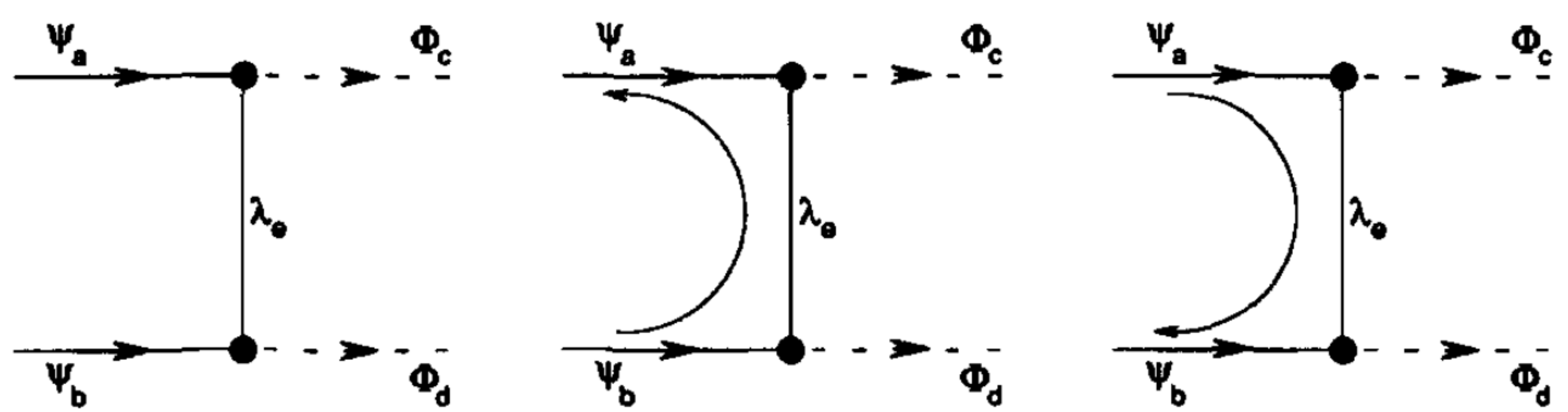
7. Follow the rules 2, 3 and 4 to write the fermion contributions

8. Multiply by (-1) each closed fermion loop

9. Consider the -1 factors associated to fermion permutations

10. Majorana fermions behave like scalars and vectors to obtain the combinatoric factors

Example




$$i\mathcal{M} = -i\overline{v}_a \Gamma_i' S(p_c - p_a)\Gamma_j u_b h^i_{eac} h^j_{ebd}$$