Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

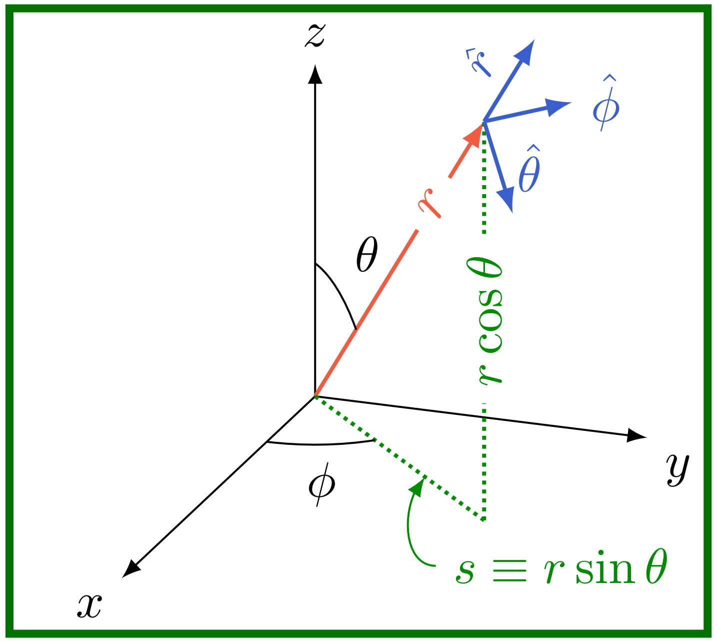
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

Aula de 5 de maio

Eletrostática

Coordenadas esféricas



$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

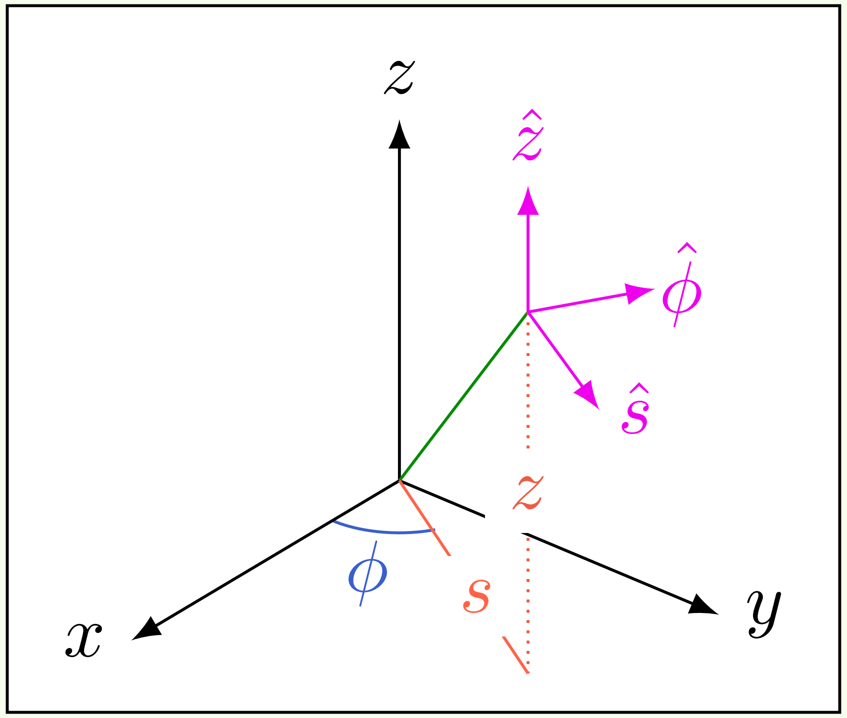
$$\vec{\nabla}\times v = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$

$$\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Coordenadas cilíndricas

$$d\vec{\ell} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$$



$$\vec{\nabla}T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{s}\frac{\partial(s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\nabla}\times\vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial(s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

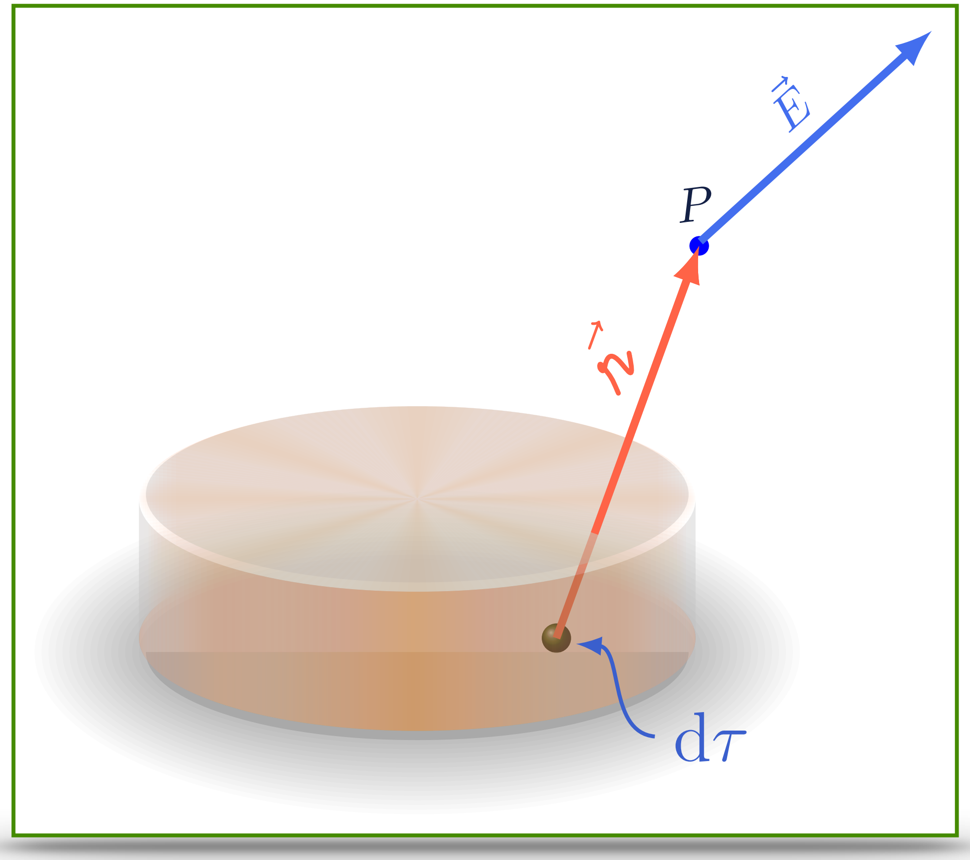
$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Eletrostática

Campo de distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{1}{\mathscr{r}^2} \hat{\mathscr{r}} \, dq$$

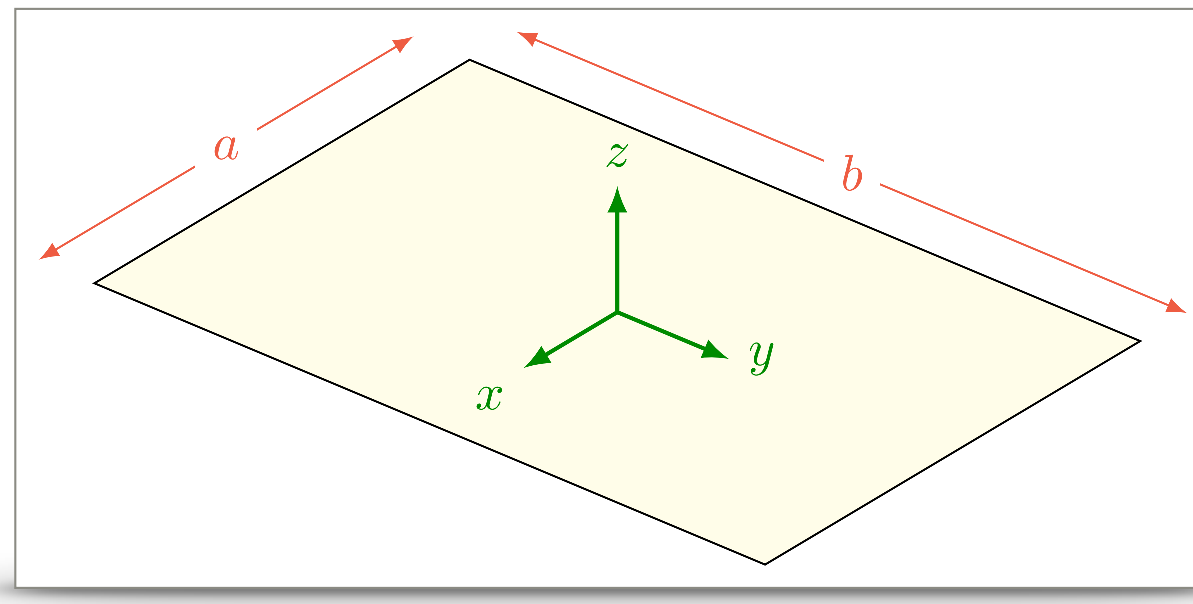
$$dq = \begin{cases} \lambda \, d\ell & \text{(linear)} \\ \sigma \, dA & \text{(superficial)} \\ \rho \, d\tau & \text{(volumétrica)} \end{cases}$$



Eletrostática

Distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{1}{\mathscr{r}^2} \hat{\mathscr{r}} \, \mathrm{d}q$$



$$\mathrm{d}q = \rho(\vec{r}) \, \mathrm{d}\tau$$

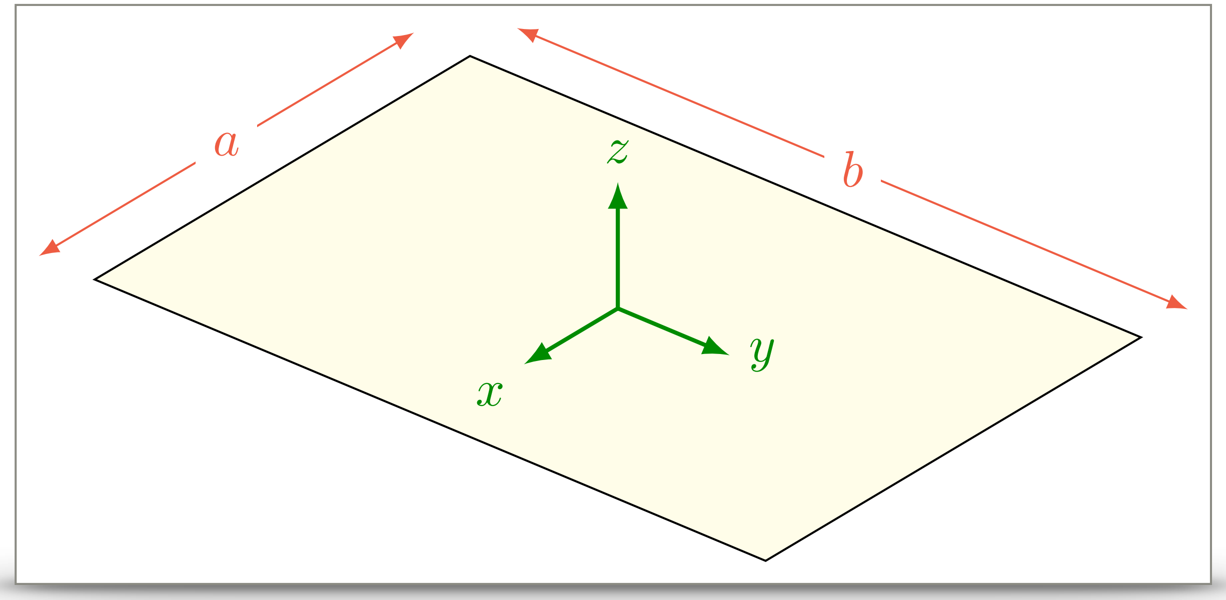
↳ EMBORA CARGA SEJA SUPERFICIAL

DEVE SATISFAZER $\int \mathrm{d}q = q$

Eletrostática

Distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{1}{\mathscr{r}^2} \hat{\mathscr{r}} \, \mathrm{d}q$$



$$\mathrm{d}q = \rho(\vec{r}) \, \mathrm{d}\tau$$

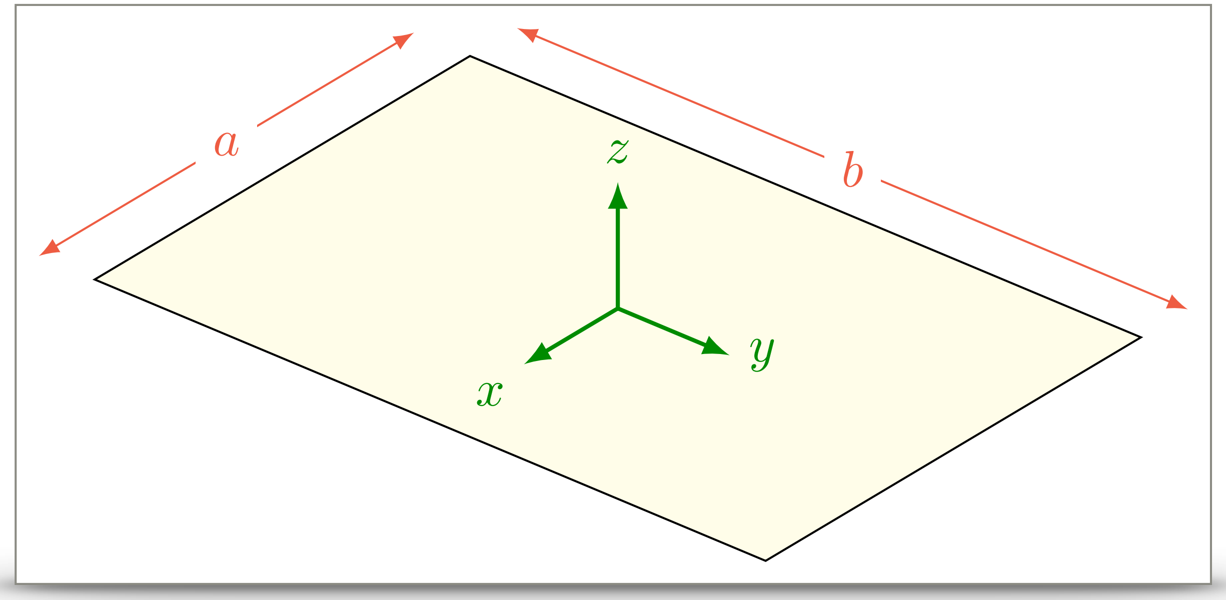
$$dq(\vec{r}) = \frac{q}{ab} \delta(z) \, \mathrm{d}\tau \quad \Rightarrow \quad \int dq = \frac{q}{ab} \iiint \delta(z) \, dz \, dx \, dy$$

$$= \frac{q}{ab} \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} dx \, dy = \frac{q}{ab} \cdot a \cdot b = q$$

Eletrostática

Distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}\,\mathrm{d}q$$



$$\mathrm{d}q = \rho(\vec{r})\ \mathrm{d}\tau$$

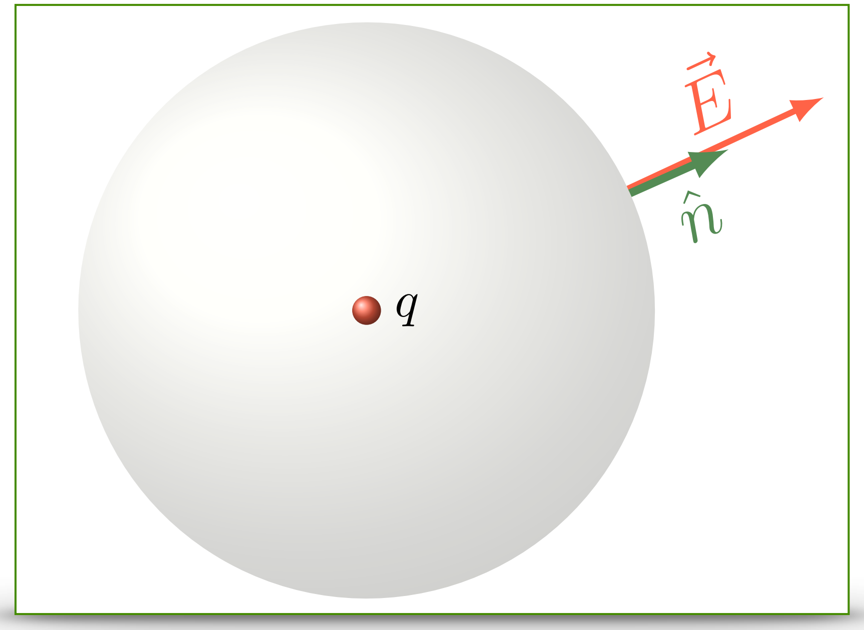
$$dq(\vec{r}) = \frac{q}{ab}\delta(z)\ \mathrm{d}\tau \iff dq(\vec{r}) = \frac{q}{ab}\ \mathrm{d}A$$

integrar sobre z

Eletrostática

Lei de Gauss

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$$



Eletrostática

Lei de Gauss

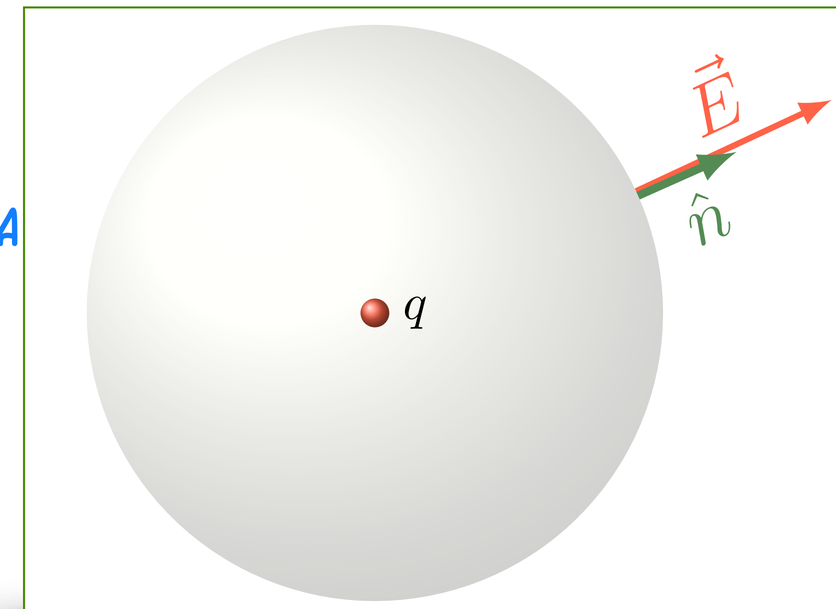
$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2} \Rightarrow \vec{E}\cdot\hat{n} = \frac{q}{4\pi\epsilon_0}\frac{1}{R^2}$$

NA SUPERFÍCIE $\hat{n} = \hat{r}$

$$\int \vec{E}\cdot\hat{n}\ \mathrm{d}A = \frac{q}{\epsilon_0}$$

$$\Leftarrow \int \vec{E}\cdot\hat{n}\, dA = \frac{q}{4\pi\epsilon_0 R^2}\int dA$$

$$\int dA = 4\pi R^2$$



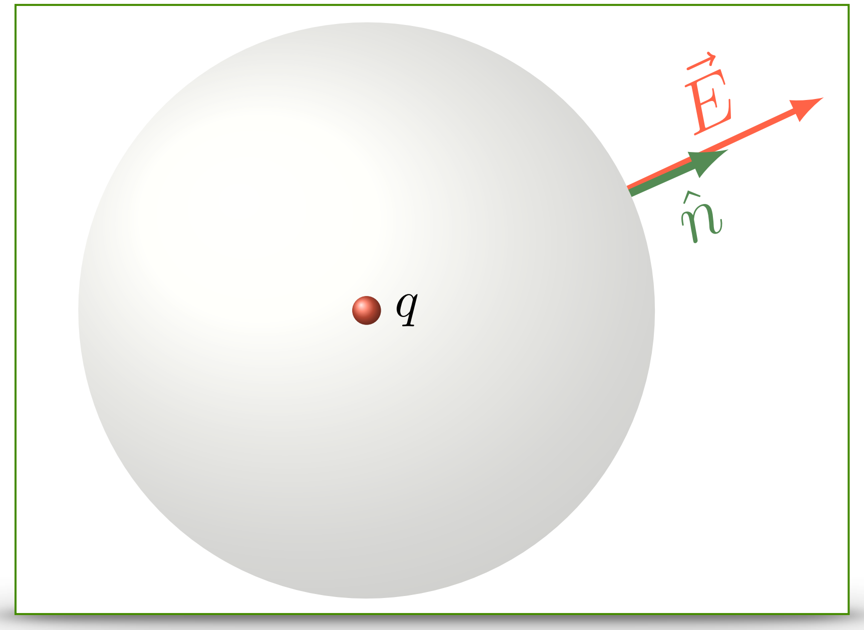
- Superfície não precisa ser esférica.
- Carga pode estar distribuída dentro da superfície.

Eletrostática

Lei de Gauss

$$\int \vec{E} \cdot \hat{n}\ \mathrm{d}A = \frac{q}{\epsilon_0}$$

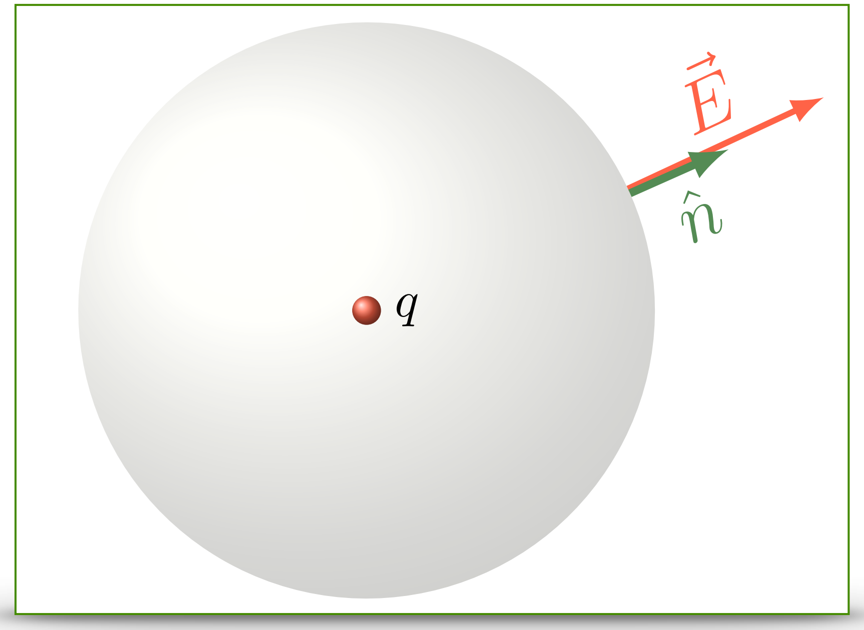
$$\int_{\mathcal{S}} \vec{E} \cdot \hat{n}\ \mathrm{d}A = \int_{\mathcal{V}} \vec{\boldsymbol{\nabla}} \cdot \vec{E}\ \mathrm{d}\tau$$



Eletrostática

Lei de Gauss

$$\int \vec{E} \cdot \hat{n} \; \mathrm{d}A = \frac{q}{\epsilon_0}$$



Gauss

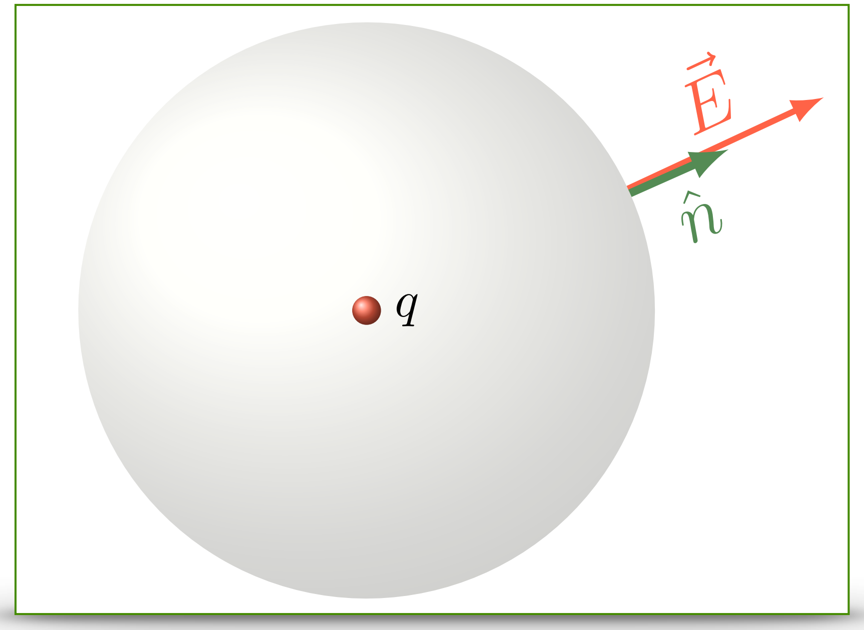
$$\int_{\mathcal{S}} \vec{E} \cdot \hat{n} \; \mathrm{d}A = \int_{\mathcal{V}} \vec{\boldsymbol{\nabla}} \cdot \vec{E} \; \mathrm{d}\tau$$

$$\frac{q}{\epsilon_0} = \frac{1}{\epsilon_0} \underbrace{\int_{\mathcal{V}} \rho(\vec{r}) \; \mathrm{d}\tau}_{q}$$

Eletrostática

Lei de Gauss

$$\int \vec{E} \cdot \hat{n} \; \mathrm{d}A = \frac{q}{\epsilon_0}$$



$$\int_{\mathcal{S}} \vec{E} \cdot \hat{n} \; \mathrm{d}A = \int_{\mathcal{V}} \vec{\nabla} \cdot \vec{E} \; \mathrm{d}\tau$$

$$\frac{q}{\epsilon_0} = \frac{1}{\epsilon_0} \int_{\mathcal{V}} \rho(\vec{r}) \; \mathrm{d}\tau$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$