Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

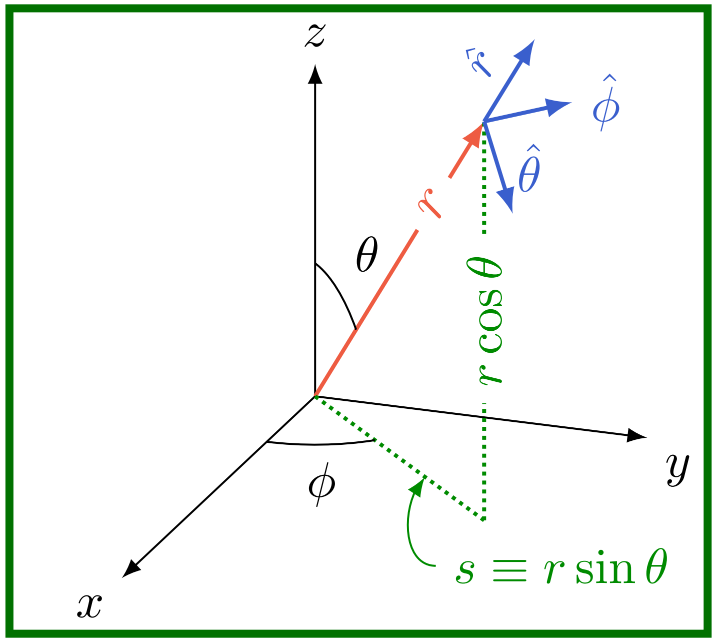
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

Aula de 3 de maio
Eletrostática

Coordenadas esféricas



$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

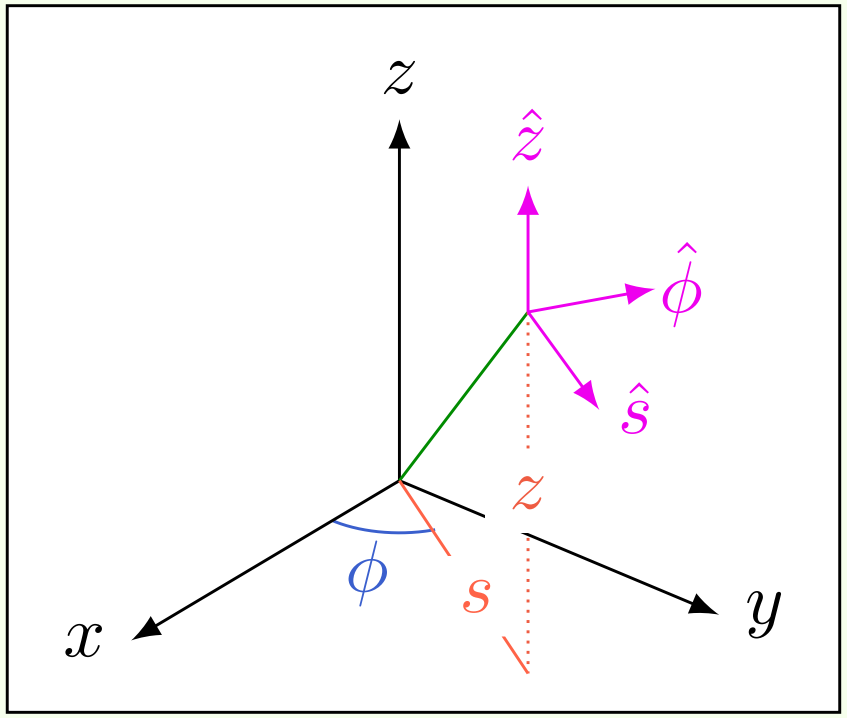
$$\vec{\nabla}\times v = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$

$$\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

Coordenadas cilíndricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}s\,\hat{s} + s\,\mathrm{d}\phi\,\hat{\phi} + \mathrm{d}z\,\hat{z}$$



$$\vec{\boldsymbol{\nabla}} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{v} = \frac{1}{s}\frac{\partial (s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

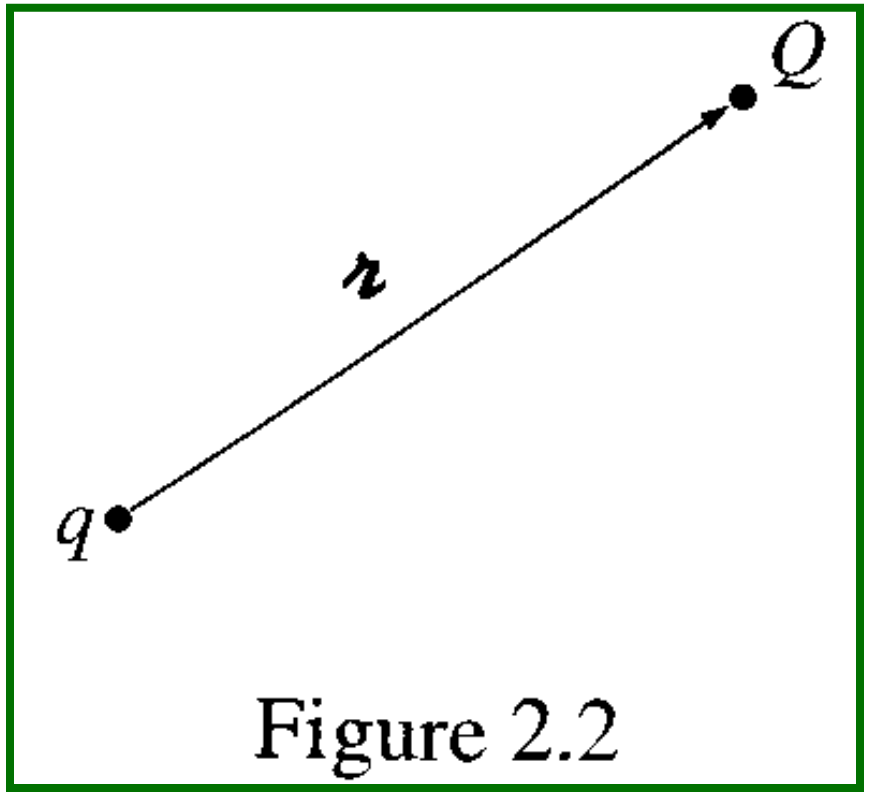
$$\vec{\boldsymbol{\nabla}} \times \vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Eletrostática

Lei de Coulomb

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{\mathscr{r}^2}\hat{\mathscr{r}}$$



$$\vec{\mathscr{r}} = \vec{R} - \vec{r}$$

$$\epsilon_0 = 8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{Nm}^2}$$

Eletrostática

Lei de Coulomb

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{qQ}{\mathscr{r}^2}\hat{\mathscr{r}}$$

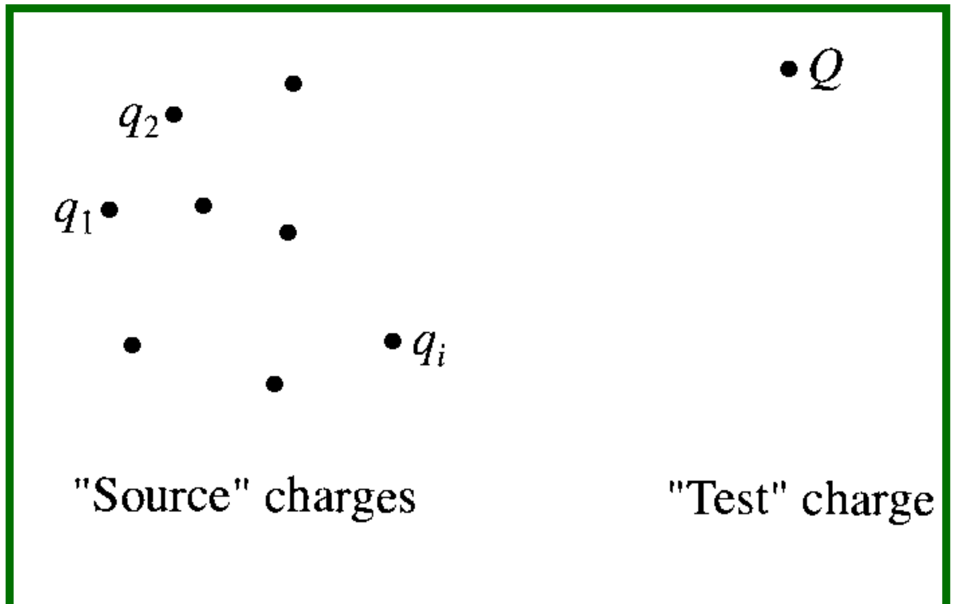


Figure 2.1

$$\epsilon_0 = 8.85 \times 10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{Nm}^2}$$

$$\vec{\mathscr{r}} = \vec{R} - \vec{r}$$

$$\vec{F} = \frac{1}{4\pi\epsilon_0} Q \sum_i \frac{q_i \vec{\mathscr{r}}_i}{\mathscr{r}_i^3}$$

Eletrostática

Campo elétrico

$$\vec{F} = \frac{1}{4\pi\epsilon_0} Q \sum_i \frac{q_i \vec{\mathscr{r}}_i}{\mathscr{r}_i^3}$$

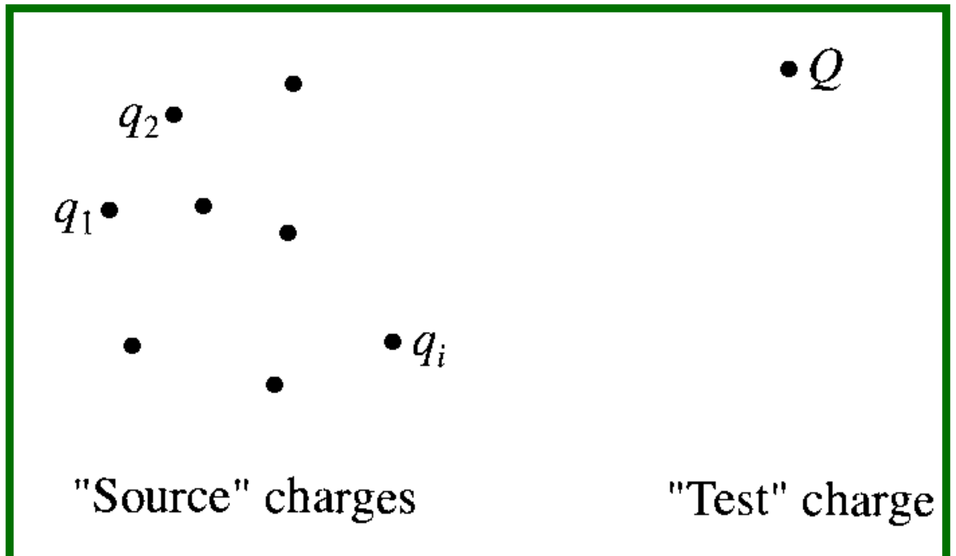


Figure 2.1

Eletrostática

Campo elétrico

$$\vec{F} = \frac{1}{4\pi\epsilon_0} Q \sum_i \frac{q_i \vec{\mathscr{r}}_i}{\mathscr{r}_i^3}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i \vec{\mathscr{r}}_i}{\mathscr{r}_i^3}$$

$$\overrightarrow{F} = Q\overrightarrow{E}$$

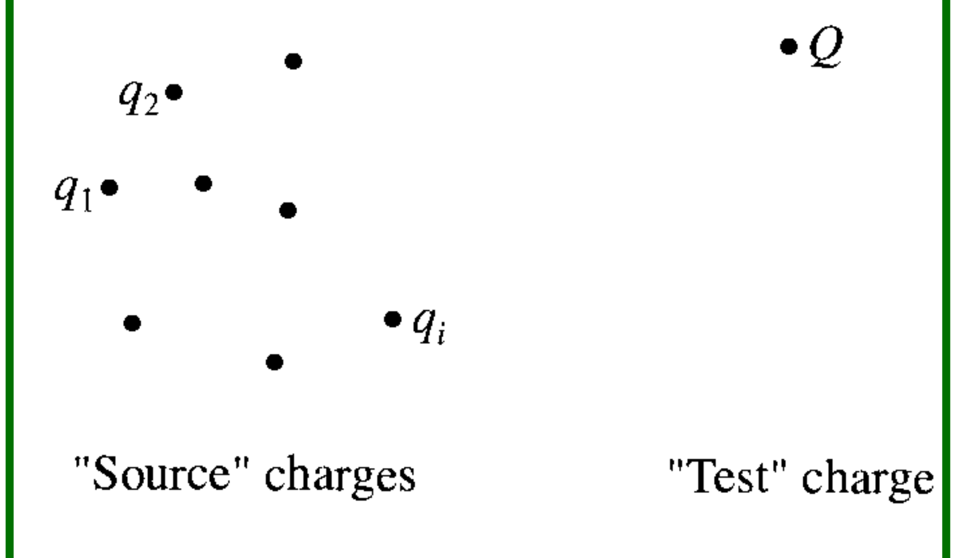
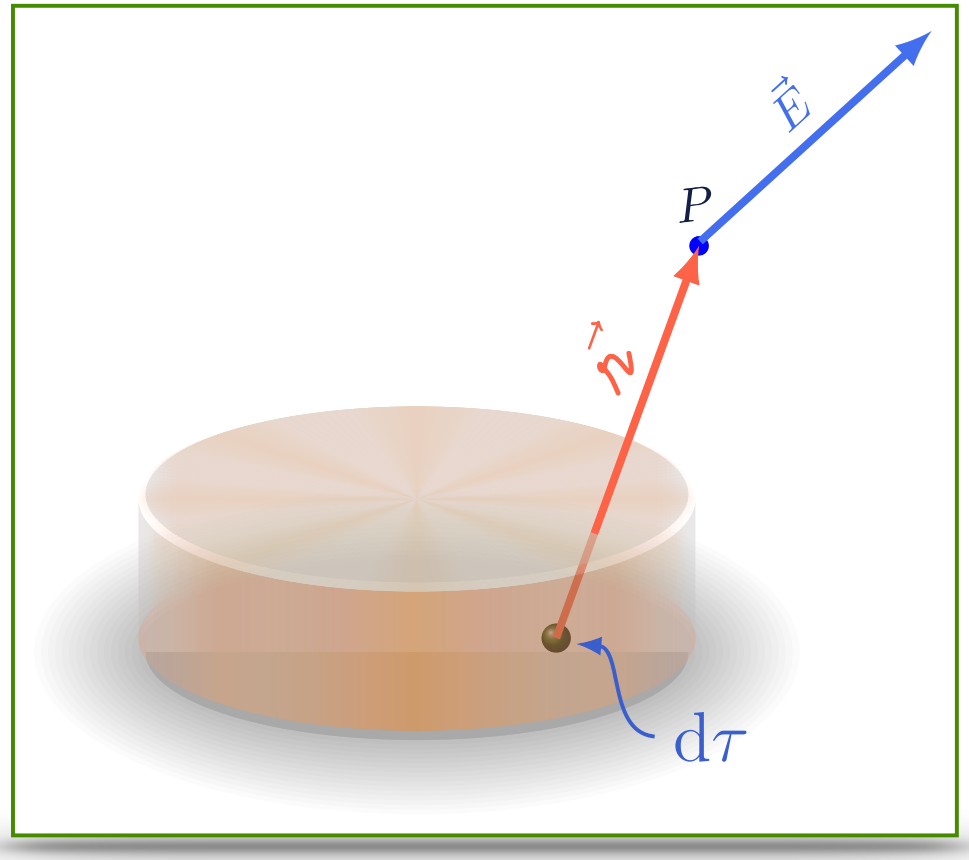


Figure 2.1

Eletrostática

Distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i \vec{\mathscr{r}}_i}{\mathscr{r}_i^3}$$

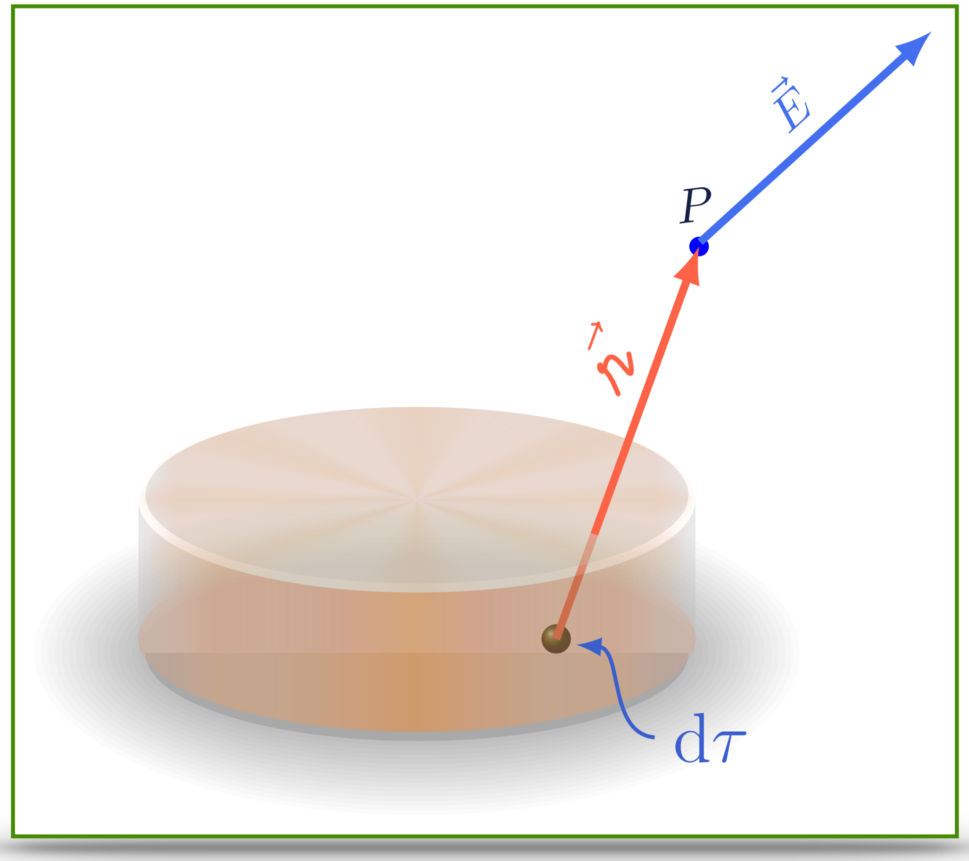


Eletrostática

Distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i \vec{\mathscr{r}}_i}{\mathscr{r}_i^3}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}\,\mathrm{d}q$$



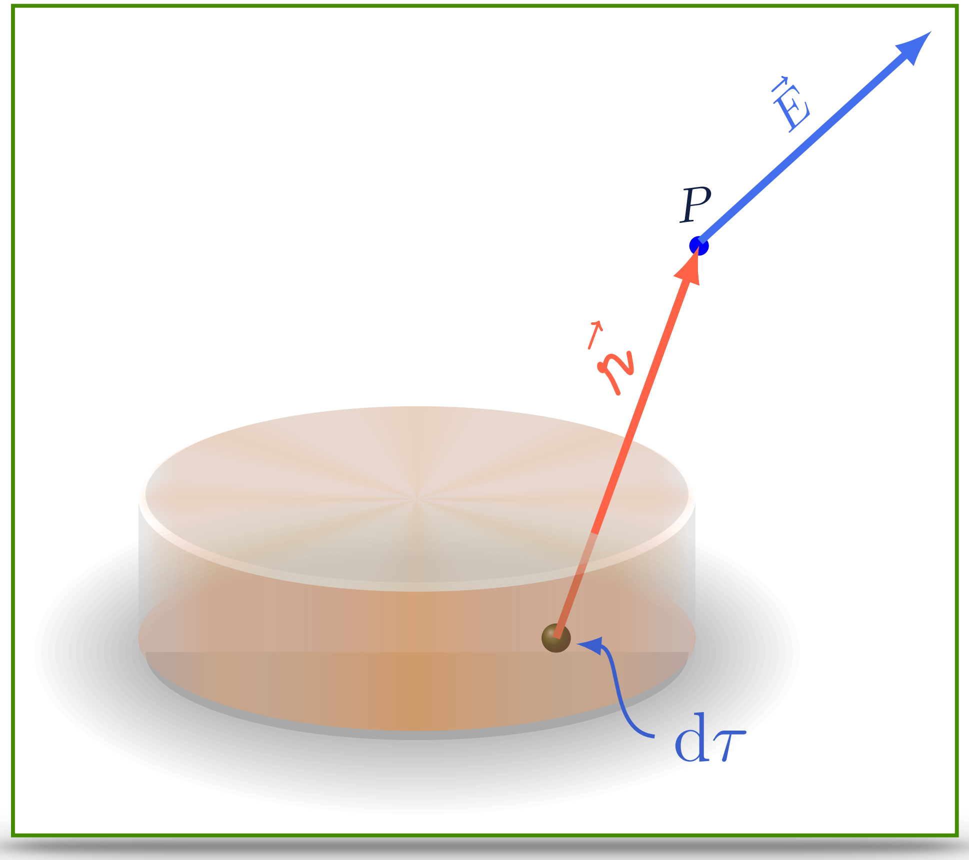
Eletrostática

Distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i \vec{\mathscr{r}}_i}{\mathscr{r}_i^3}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{1}{\mathscr{r}^2} \hat{\mathscr{r}} \, \mathrm{d}q$$

$$\mathrm{d}q = \begin{cases} \lambda \, \mathrm{d}\ell & \text{(linear)} \\ \sigma \, \mathrm{d}A & \text{(superficial)} \\ \rho \, \mathrm{d}\tau & \text{(volumétrica)} \end{cases}$$

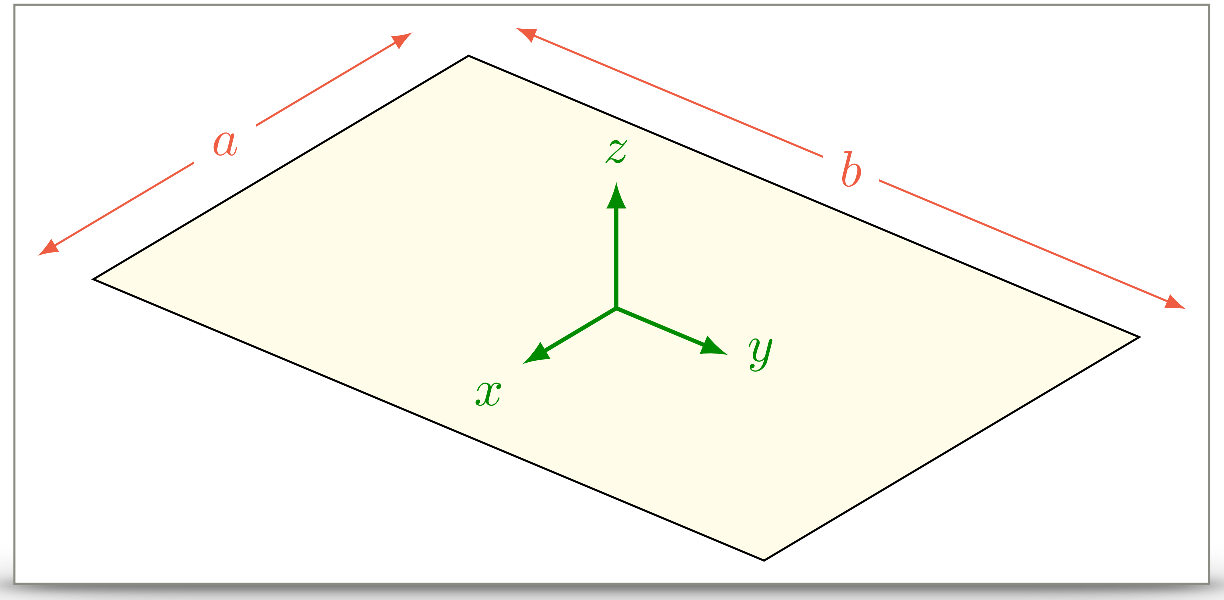


Eletrostática

Distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{1}{\mathscr{r}^2} \hat{\mathscr{r}} \, \mathrm{d}q$$

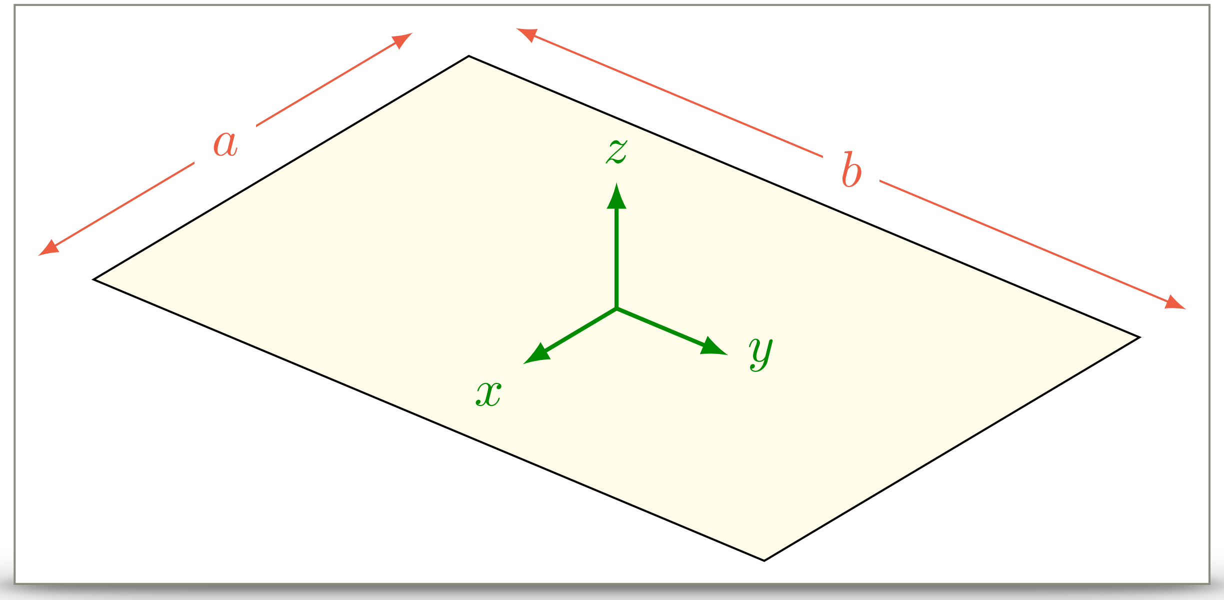
$$\mathrm{d}q = \rho(\vec{r}) \, \mathrm{d}\tau$$



Eletrostática

Distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}\,\mathrm{d}q$$



$$\mathrm{d}q = \rho(\vec{r})\ \mathrm{d}\tau$$

$$\int \rho(\vec{r})\ \mathrm{d}\tau = q$$

Eletrostática

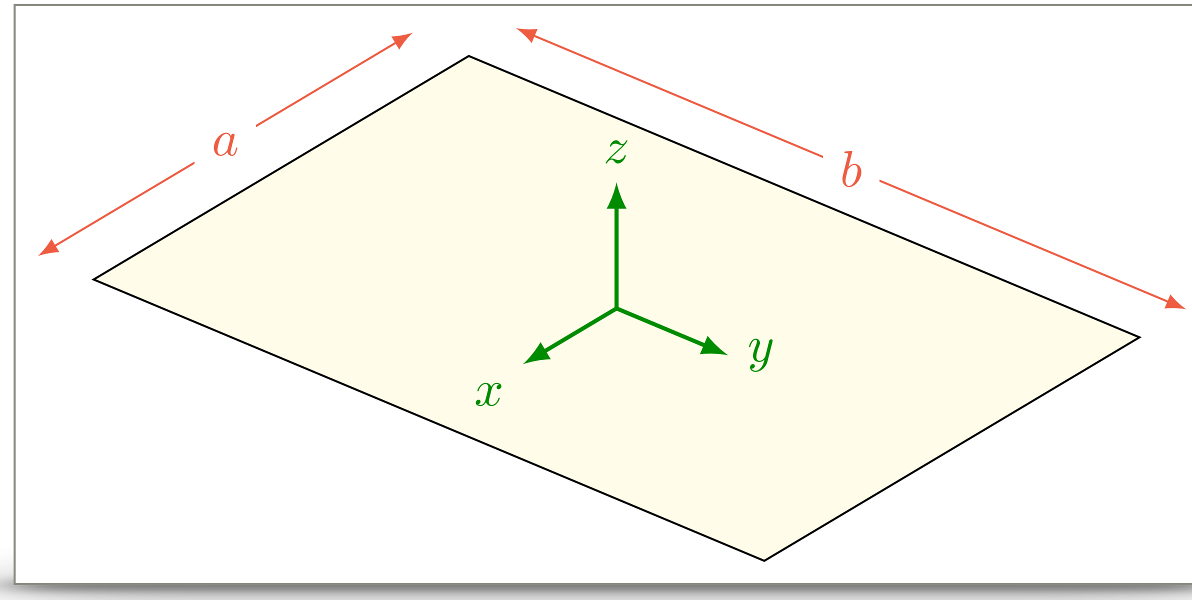
Distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}\,\mathrm{d}q$$

$$\mathrm{d}q = \rho(\vec{r})\ \mathrm{d}\tau$$

$$\int \rho(\vec{r})\ \mathrm{d}\tau = q$$

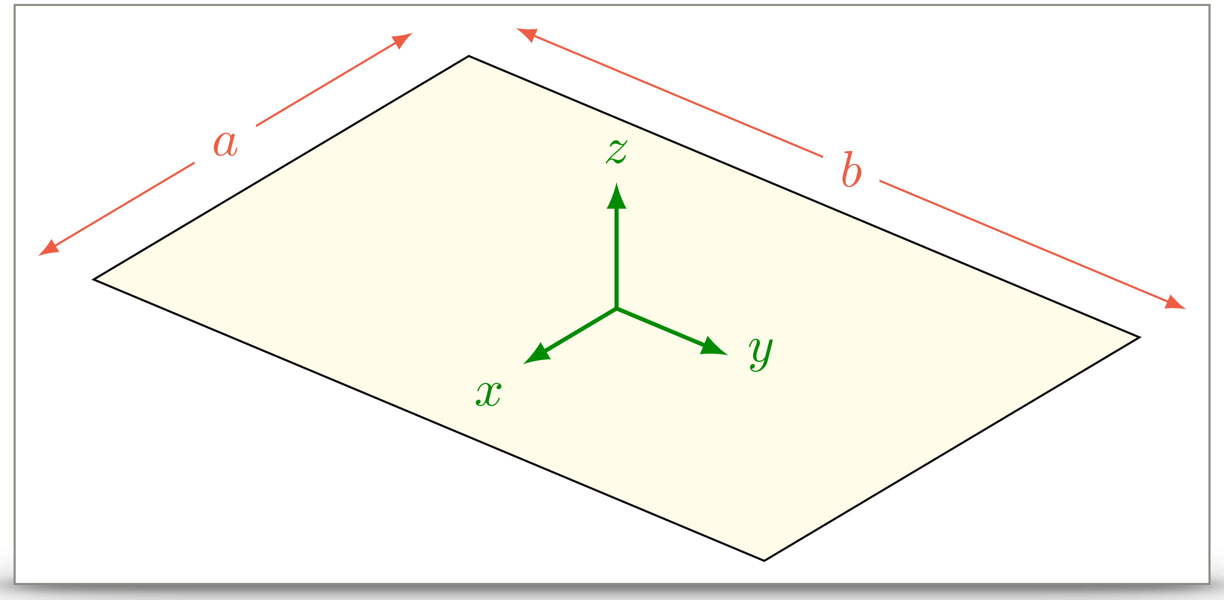
$$\rho(\vec{r}) = \alpha\delta(z)$$



Eletrostática

Distribuição de cargas

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}\,\mathrm{d}q$$



$$\mathrm{d}q = \rho(\vec{r})\ \mathrm{d}\tau$$

$$\int \rho(\vec{r})\ \mathrm{d}\tau = q$$

$$\rho(\vec{r}) = \alpha\delta(z)$$

$$\alpha ab = q$$

Eletrostática

Distribuição de cargas

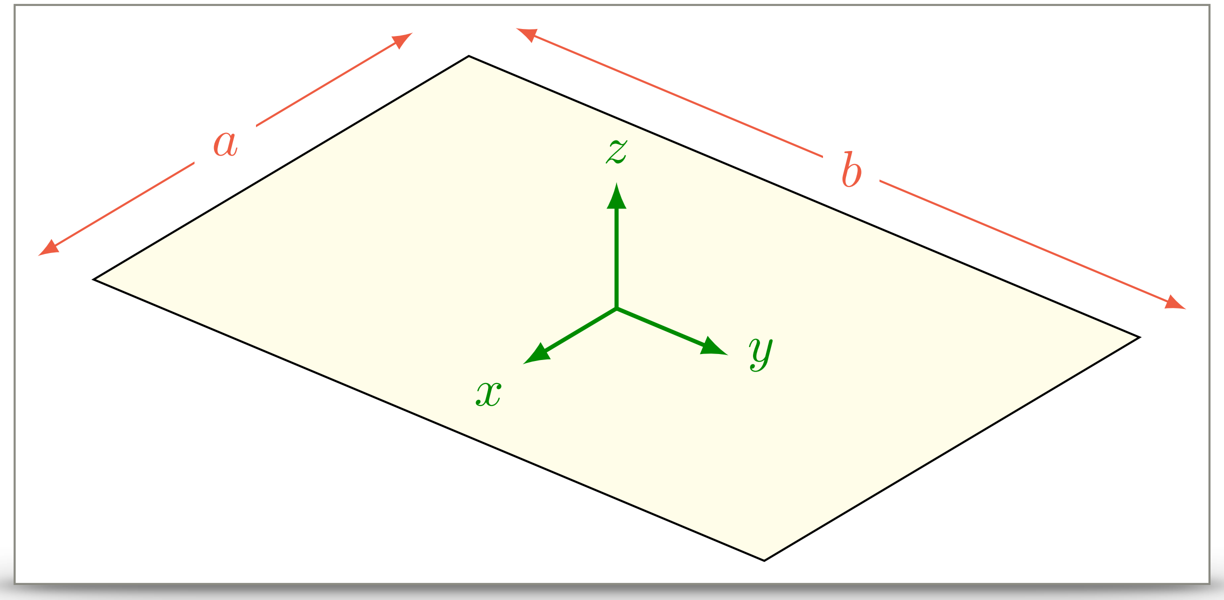
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}\,\mathrm{d}q$$

$$\mathrm{d}q = \rho(\vec{r})\ \mathrm{d}\tau$$

$$\int \rho(\vec{r})\ \mathrm{d}\tau = q$$

$$\rho(\vec{r}) = \alpha\delta(z)$$

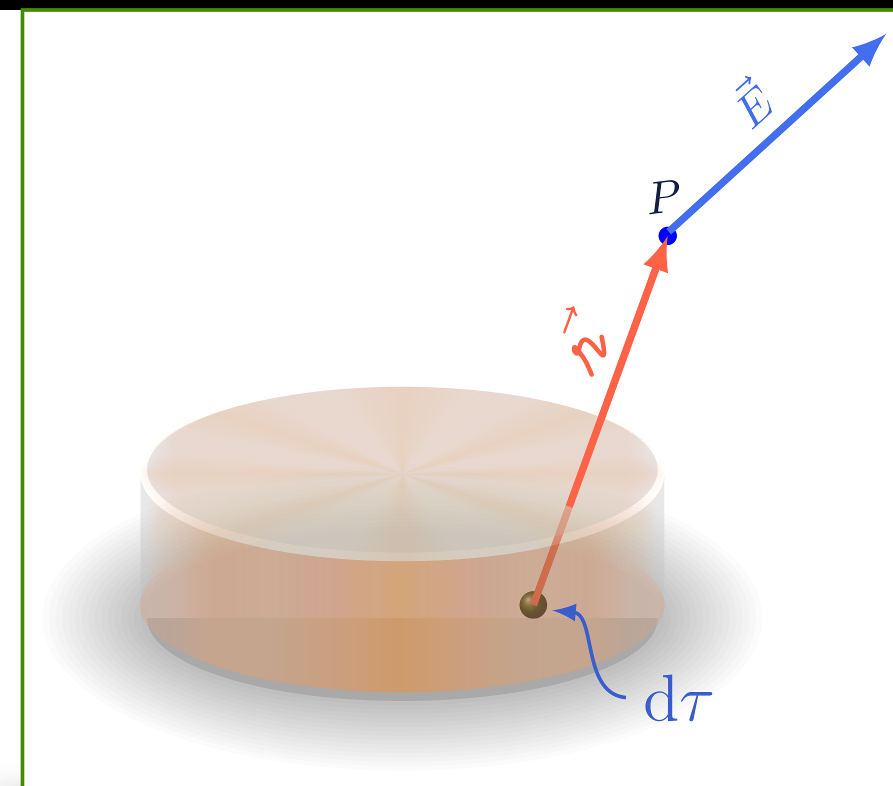
$$\alpha ab = q \quad \Rightarrow \quad \rho(\vec{r}) = \frac{q}{ab}\delta(z)$$



Pratique o que aprendeu

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{1}{\mathscr{r}^2}\hat{\mathscr{r}}\,\mathrm{d}q$$

$$\mathrm{d}q = \begin{cases} \lambda\,\mathrm{d}\ell & \text{(linear)} \\ \sigma\,\mathrm{d}A & \text{(superficial)} \\ \rho\,\mathrm{d}\tau & \text{(volumétrica)} \end{cases}$$



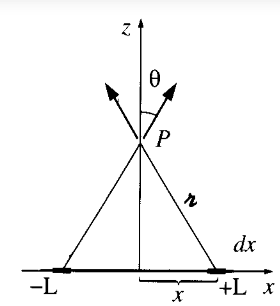
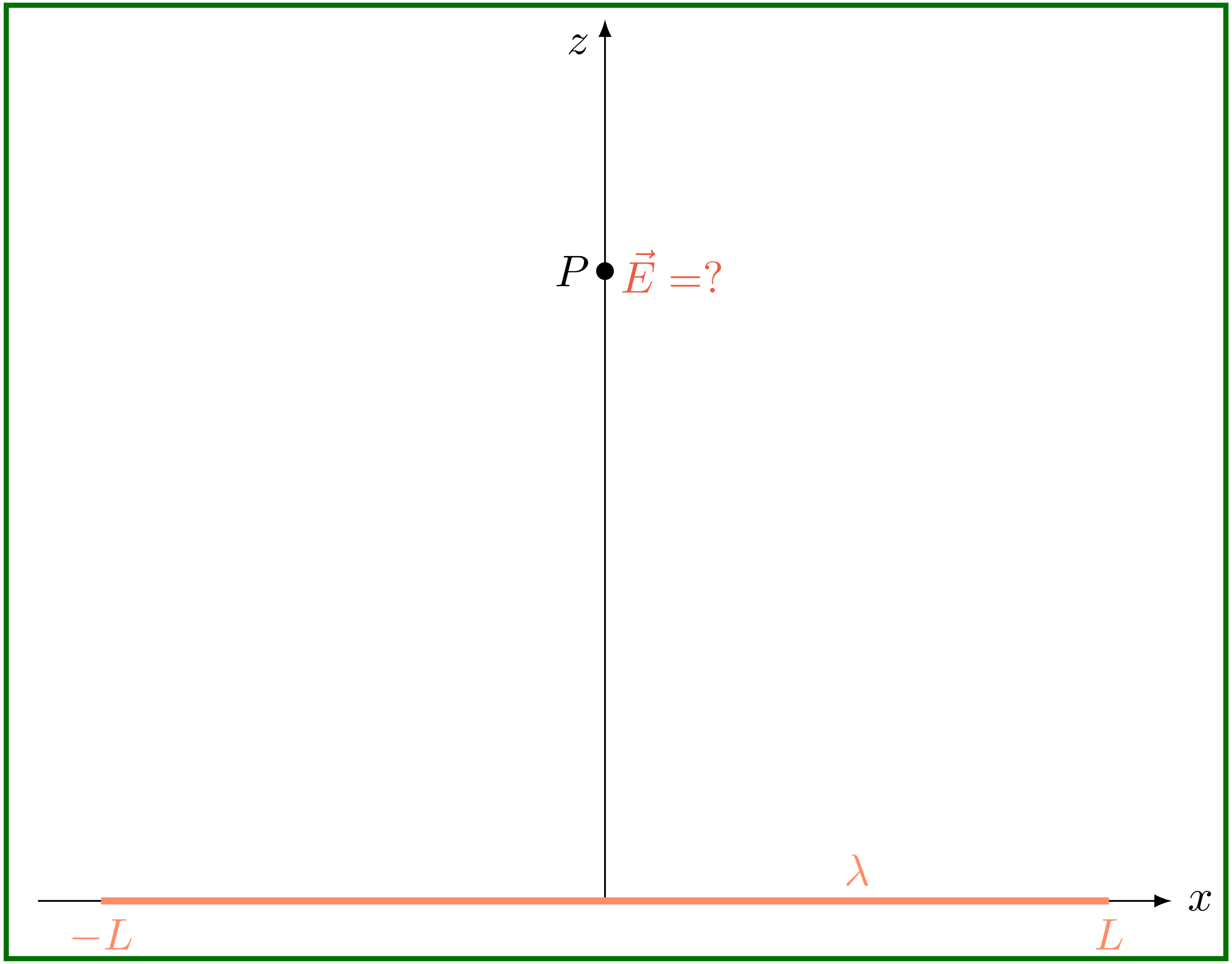


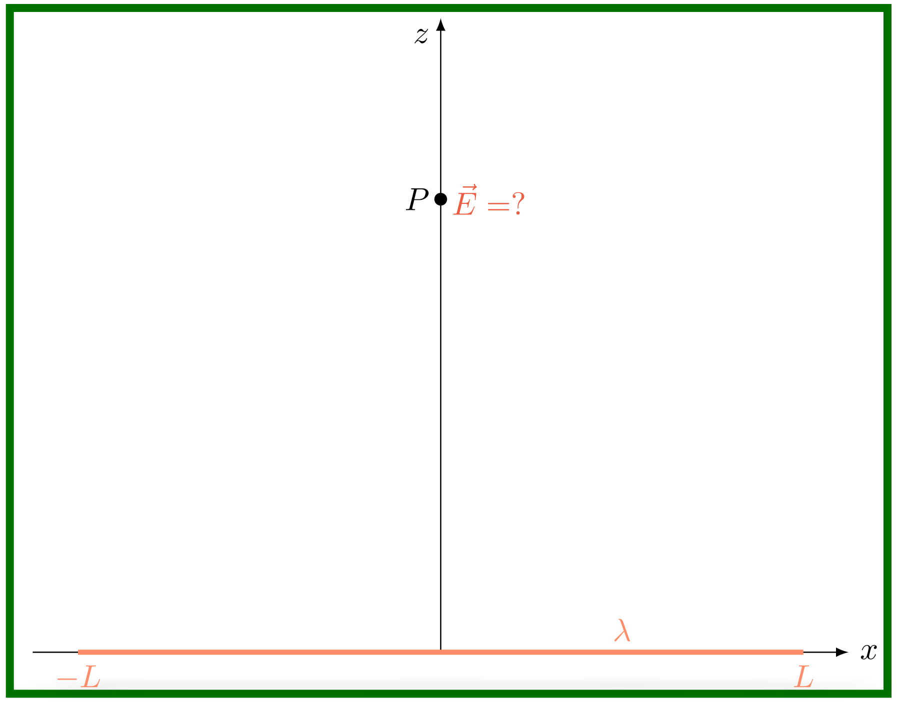
Figure 2.6

Pratique o que aprendeu



Pratique o que aprendeu

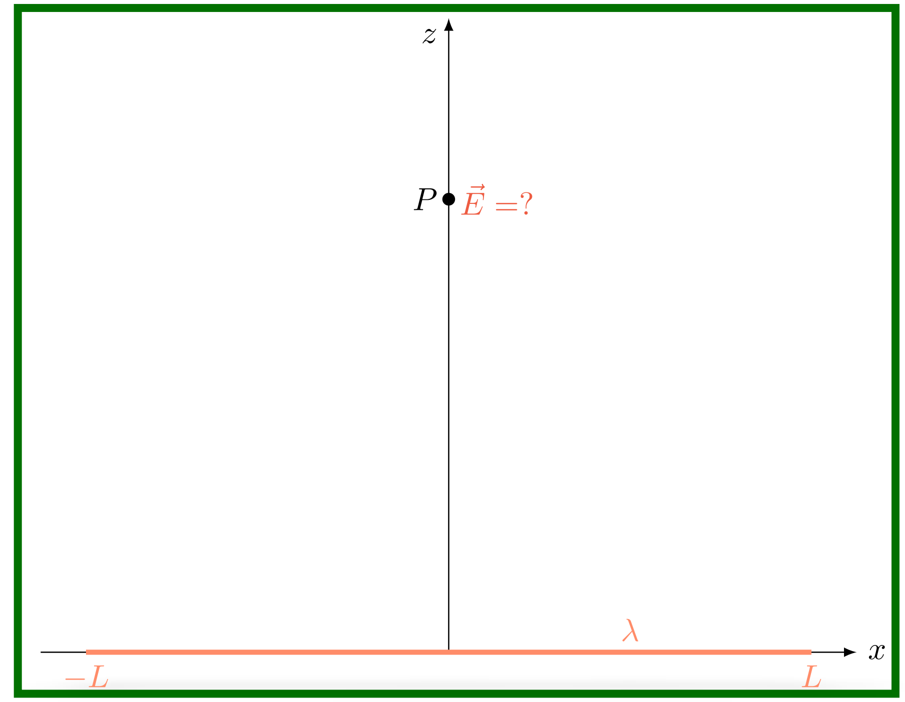
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_{-L}^{L}\frac{\vec{\mathscr{r}}}{\mathscr{r}^3}\lambda\,\mathrm{d}x$$



Pratique o que aprendeu

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_{-L}^{L} \frac{\vec{r}}{r^3}\lambda\,\mathrm{d}x$$

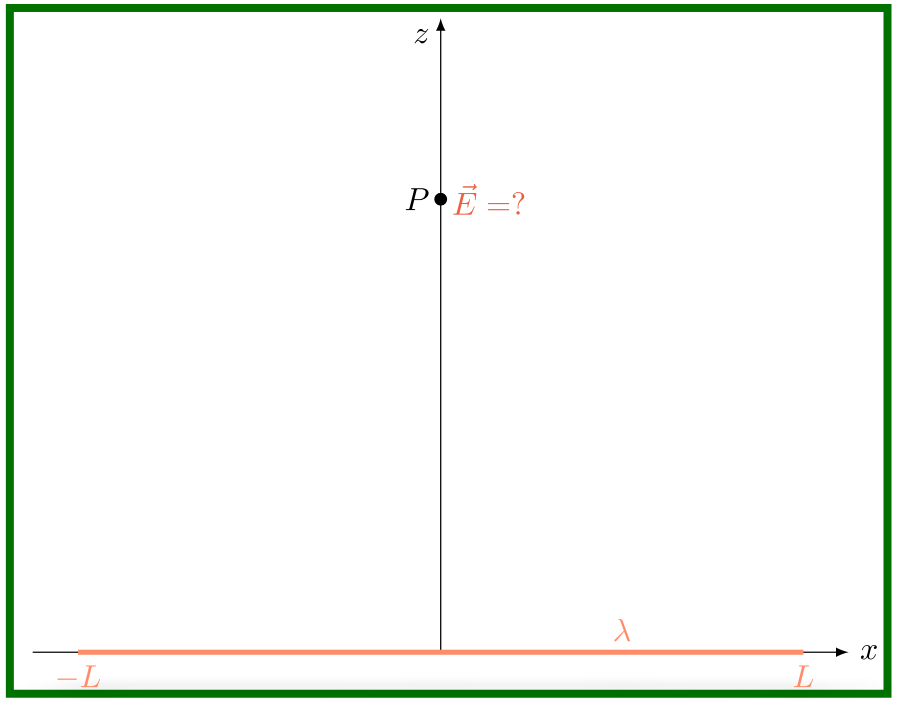
$$E_z = \frac{1}{4\pi\epsilon_0}\lambda\int_{-L}^{L} \frac{z\hat{z}}{(x^2+z^2)^{3/2}}\,\mathrm{d}x$$



Pratique o que aprendeu

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_{-L}^{L}\frac{\vec{\mathscr{r}}}{\mathscr{r}^3}\lambda\,\mathrm{d}x$$

$$E_z = \frac{1}{4\pi\epsilon_0}\lambda\int_{-L}^{L}\frac{z\hat{z}}{(x^2+z^2)^{3/2}}\,\mathrm{d}x$$

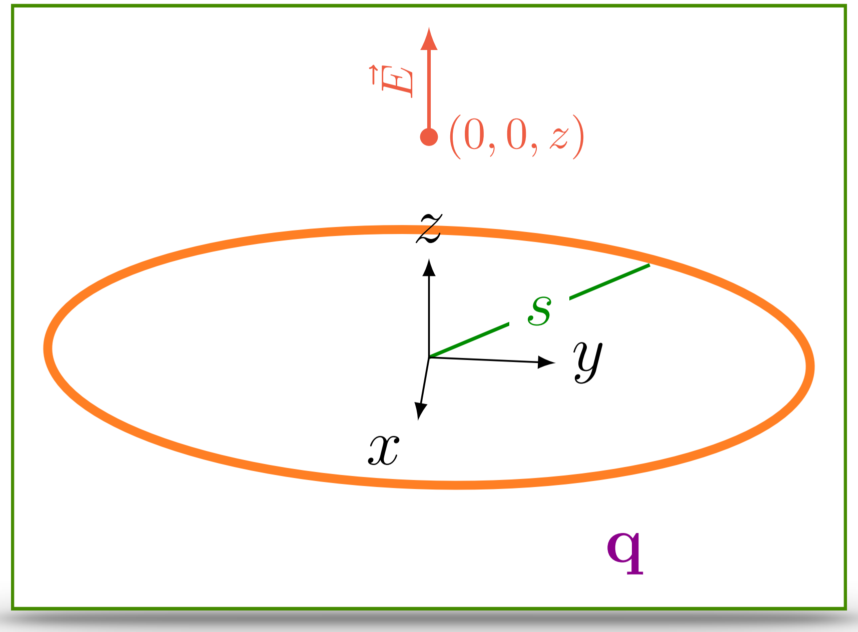


$$E_z = \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z\sqrt{z^2+L^2}}$$

Pratique o que aprendeu

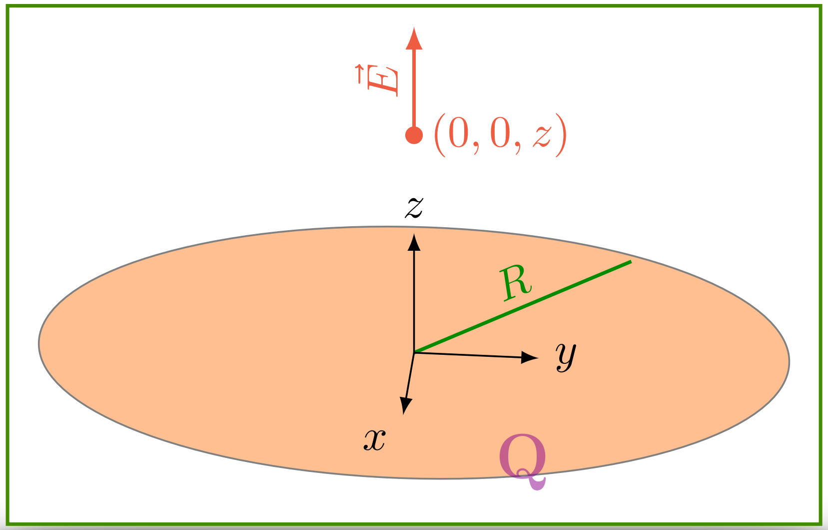
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\vec{r}}{r^3}\, \mathrm{d}q$$

$$E_z = \frac{1}{4\pi\epsilon_0} \frac{qz}{(z^2 + s^2)^{3/2}}$$



Pratique o que aprendeu

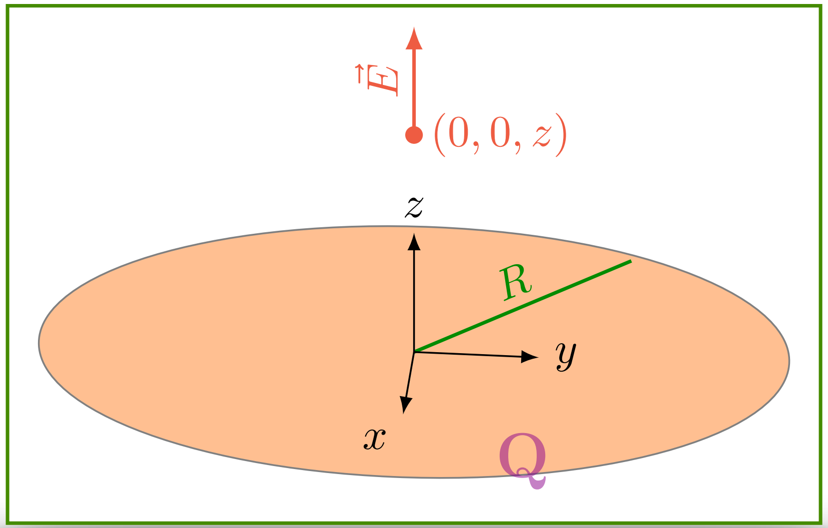
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\vec{r}}{r^3}\, \mathrm{d}q$$



$$E_z = \frac{\sigma}{4\pi\epsilon_0} \int_0^R \int_0^{2\pi} \frac{z}{(z^2 + s^2)^{3/2}} s\, \mathrm{d}\phi\, \mathrm{d}s$$

Pratique o que aprendeu

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\vec{r}}{r^3}\, \mathrm{d}q$$

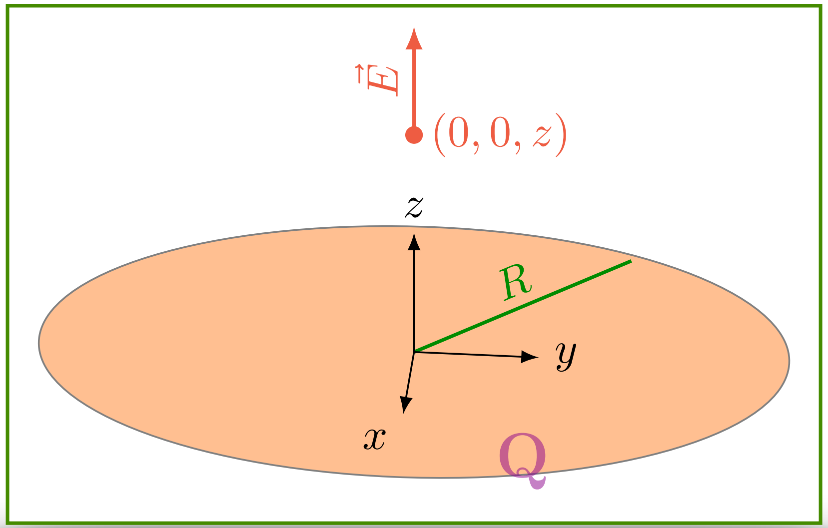


$$E_z = \frac{\sigma}{4\pi\epsilon_0} \int_0^R \int_0^{2\pi} \frac{z}{(z^2+s^2)^{3/2}}\, s\,\mathrm{d}\phi\,\mathrm{d}s$$

$$E_z = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{z}{(z^2+s^2)^{3/2}}\, s\,\mathrm{d}s$$

Pratique o que aprendeu

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\vec{r}}{r^3}\,\mathrm{d}q$$



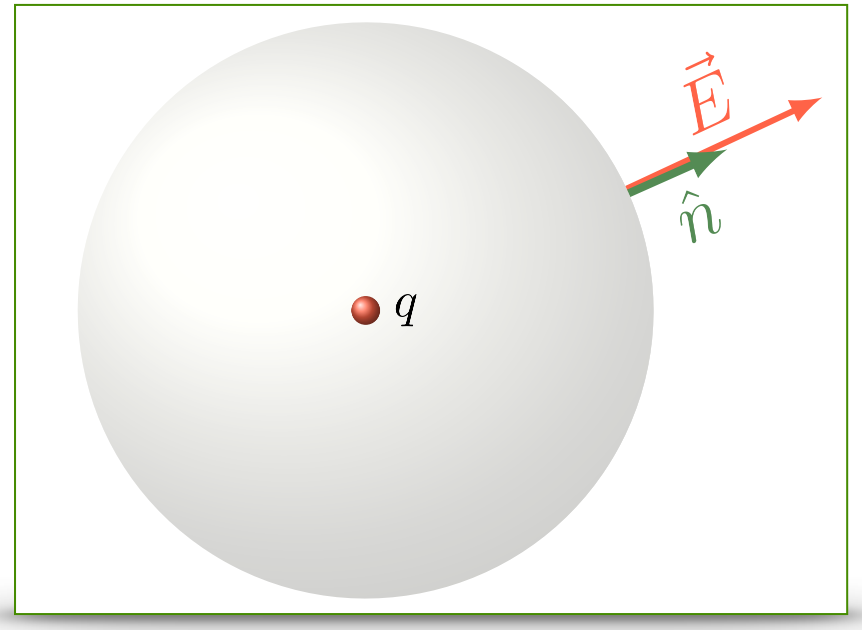
$$E_z = \frac{\sigma}{4\pi\epsilon_0}\int_0^R\int_0^{2\pi}\frac{z}{(z^2+s^2)^{3/2}}\,s\,\mathrm{d}\phi\,\mathrm{d}s$$

$$E_z = \frac{\sigma}{2\epsilon_0}\int_0^R\frac{z}{(z^2+s^2)^{3/2}}\,s\,\mathrm{d}s = \frac{\sigma}{2\epsilon_0}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)$$

Eletrostática

Lei de Gauss

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$$

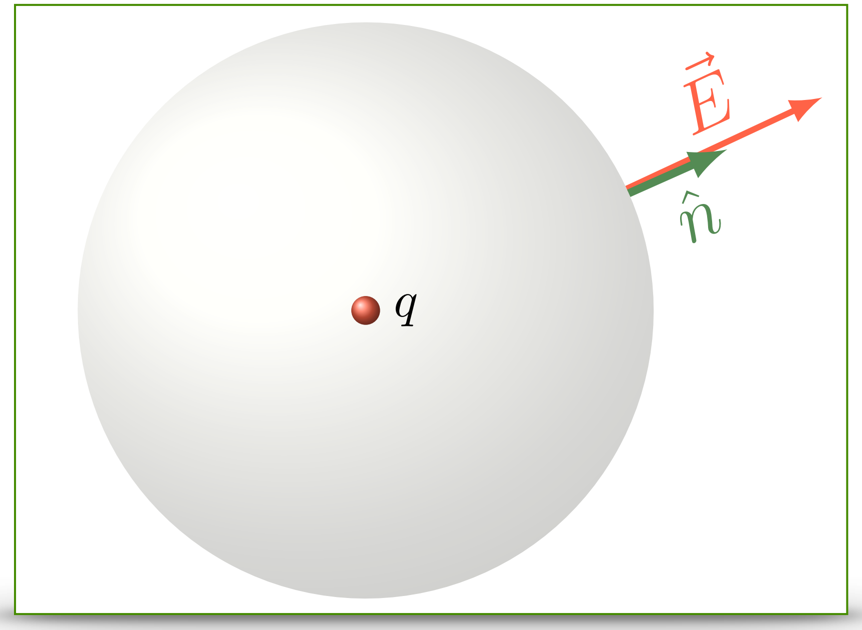


Eletrostática

Lei de Gauss

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$$

$$\int \vec{E}\cdot\hat{n}\;\mathrm{d}A = \frac{q}{4\pi\epsilon_0}\int_{-1}^{1}\int_{0}^{2\pi}\frac{1}{R^2}R^2\,\mathrm{d}\phi\,\mathrm{d}u$$

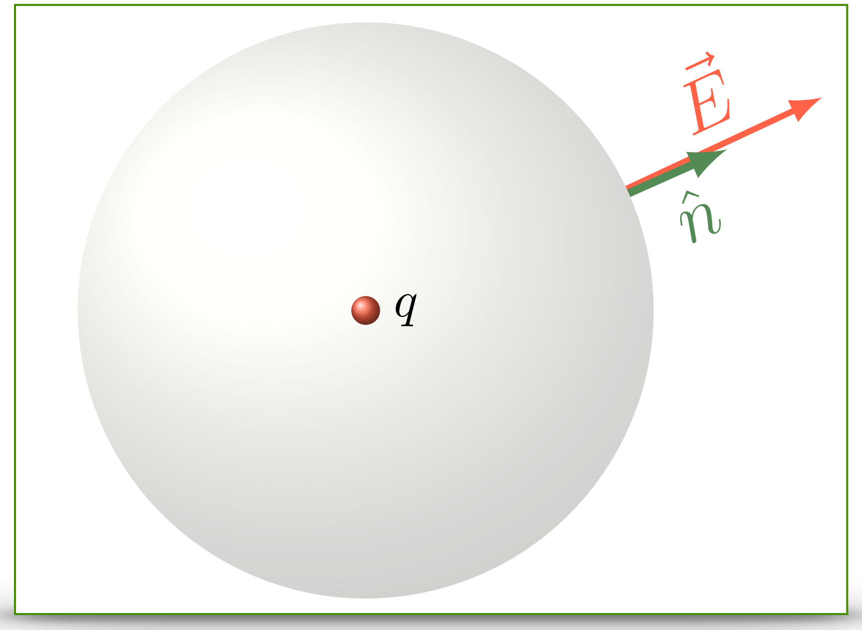


Eletrostática

Lei de Gauss

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$$

$$\int \vec{E}\cdot\hat{n}\ \mathrm{d}A = \frac{q}{4\pi\epsilon_0}\int_{-1}^{1}\int_{0}^{2\pi}\frac{1}{R^2}R^2\,\mathrm{d}\phi\,\mathrm{d}u$$



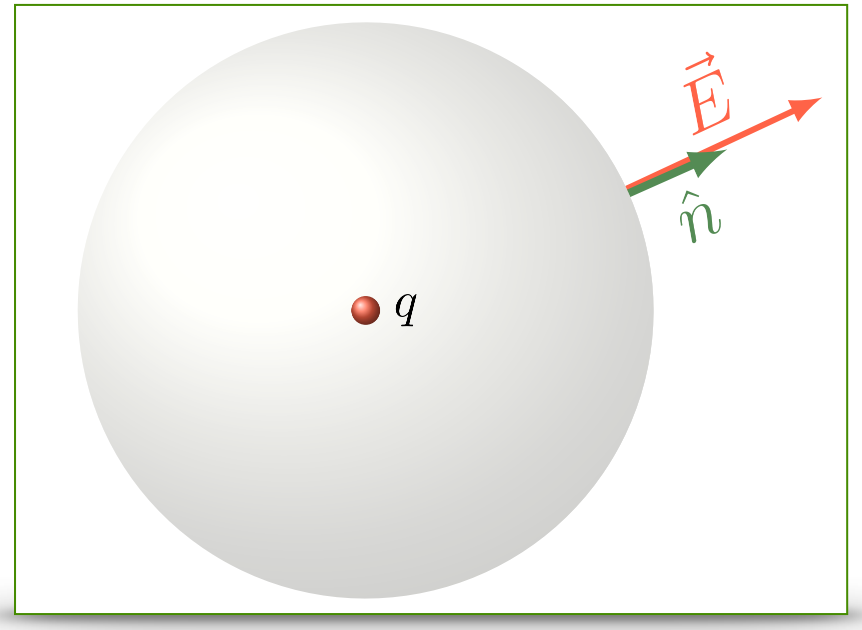
$$\int \vec{E}\cdot\hat{n}\ \mathrm{d}A = \frac{q}{\epsilon_0}$$

Eletrostática

Lei de Gauss

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$$

$$\int \vec{E}\cdot\hat{n}\;\mathrm{d}A = \frac{q}{4\pi\epsilon_0}\int_{-1}^{1}\int_{0}^{2\pi}\frac{1}{R^2}R^2\,\mathrm{d}\phi\,\mathrm{d}u$$



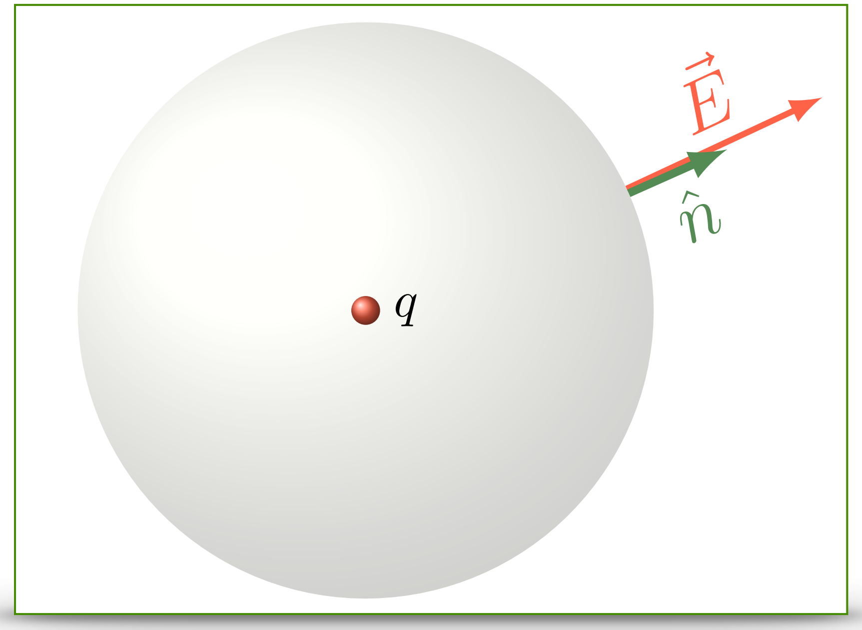
$$\int \vec{E}\cdot\hat{n}\;\mathrm{d}A = \frac{q}{\epsilon_0}$$

- Superfície não precisa ser esférica.
- Carga pode estar distribuída dentro da superfície.

Eletrostática

Lei de Gauss

$$\int \vec{E}\cdot\hat{n}\;\mathrm{d}A = \frac{q}{\epsilon_0}$$



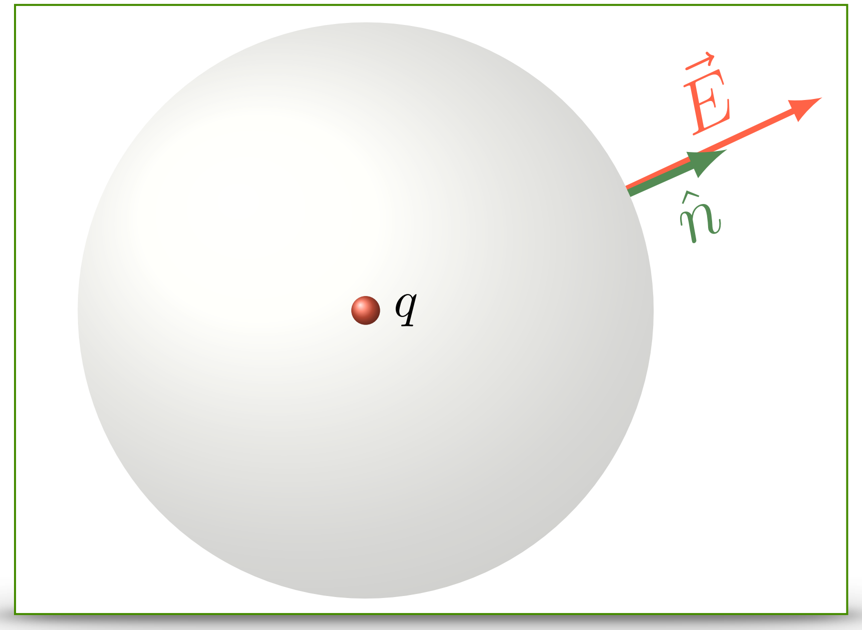
$$\int_{\mathcal{S}} \vec{E}\cdot\hat{n}\;\mathrm{d}A = \int_{\mathcal{V}} \vec{\boldsymbol{\nabla}}\cdot\vec{E}\;\mathrm{d}\tau$$

Eletrostática

Lei de Gauss

$$\int \vec{E} \cdot \hat{n} \; \mathrm{d}A = \frac{q}{\epsilon_0}$$

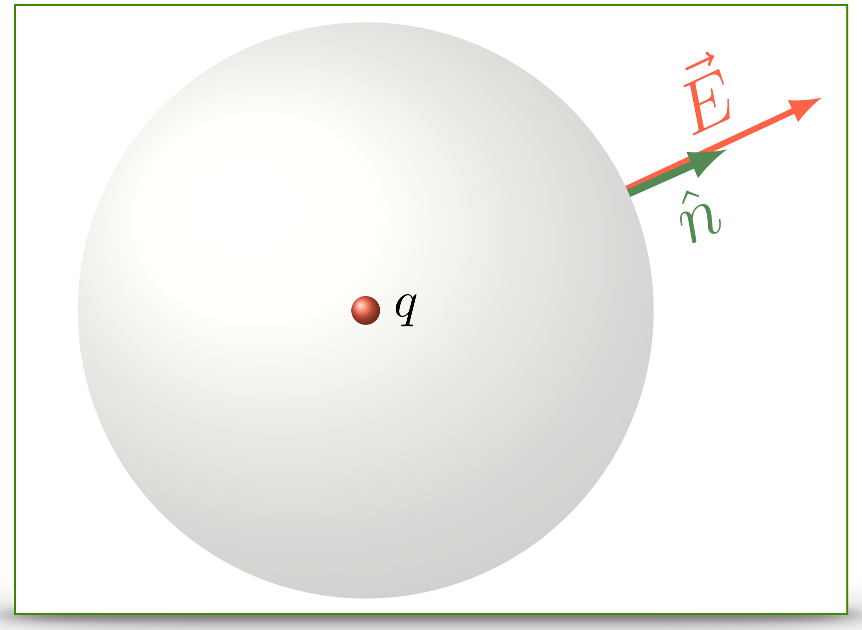
$$\int_{\mathcal{S}} \vec{E} \cdot \hat{n} \; \mathrm{d}A = \int_{\mathcal{V}} \vec{\nabla} \cdot \vec{E} \; \mathrm{d}\tau$$



Eletrostática

Lei de Gauss

$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = \frac{q}{\epsilon_0}$$



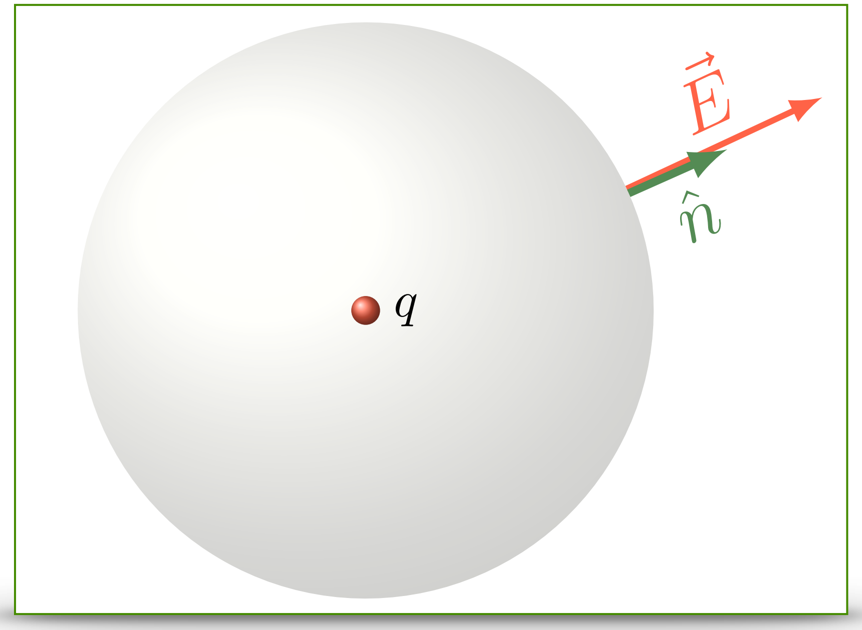
$$\int_{\mathcal{S}} \vec{E} \cdot \hat{n} \, \mathrm{d}A = \int_{\mathcal{V}} \vec{\boldsymbol{\nabla}} \cdot \vec{E} \, \mathrm{d}\tau$$

$$\frac{q}{\epsilon_0} = \frac{1}{\epsilon_0} \int_{\mathcal{V}} \rho(\vec{r}) \, \mathrm{d}\tau$$

Eletrostática

Lei de Gauss

$$\int \vec{E}\cdot\hat{n}\ \mathrm{d}A = \frac{q}{\epsilon_0}$$



$$\int_{\mathcal{S}} \vec{E}\cdot\hat{n}\ \mathrm{d}A = \int_{\mathcal{V}} \vec{\boldsymbol{\nabla}}\cdot\vec{E}\ \mathrm{d}\tau$$

$$\frac{q}{\epsilon_0} = \frac{1}{\epsilon_0}\int_{\mathcal{V}} \rho(\vec{r})\ \mathrm{d}\tau$$

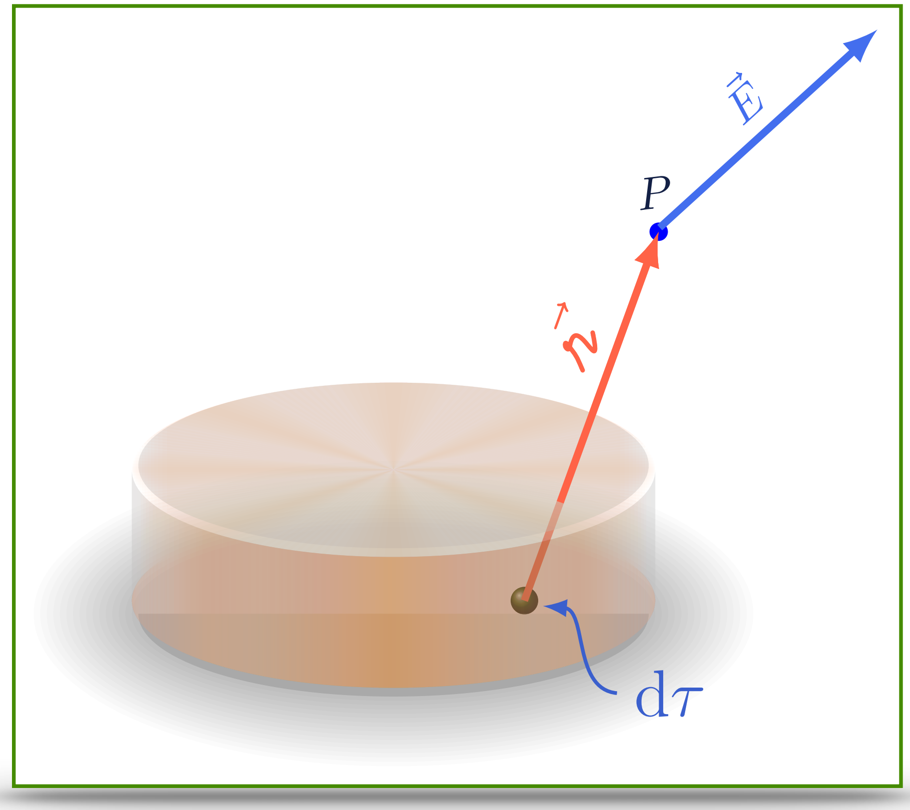
$$\vec{\boldsymbol{\nabla}}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

Eletrostática

Outra visão

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \rho(\vec{r}\,)\, \mathrm{d}\tau$$



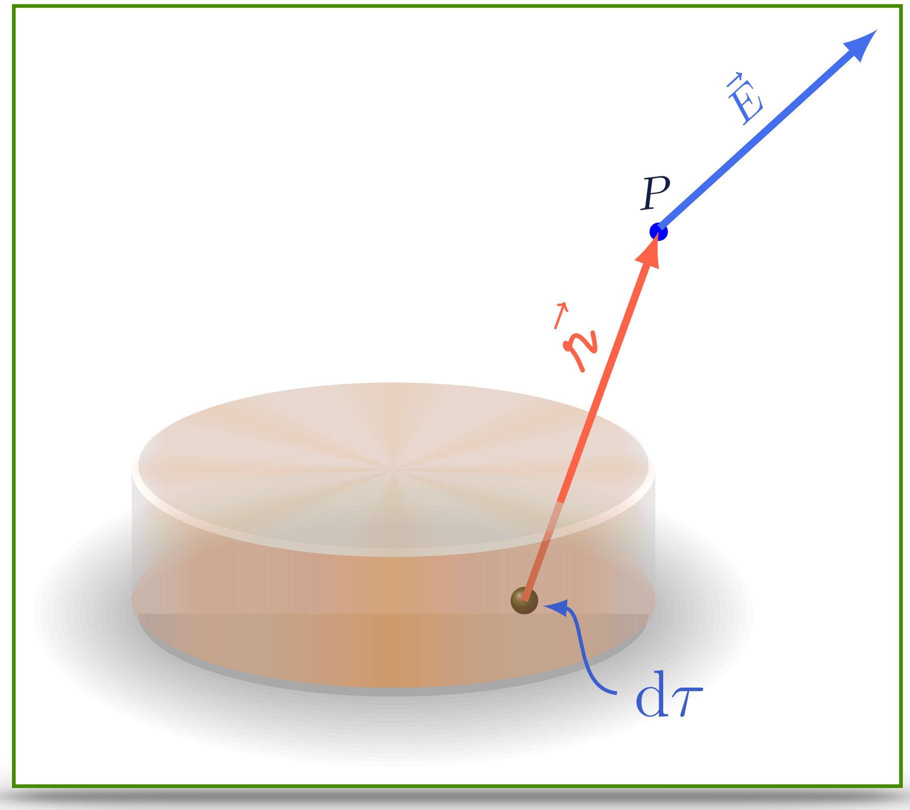
Eletrostática

Outra visão

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \rho(\vec{r}) \, \mathrm{d}\tau$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{4\pi\epsilon_0} \int \vec{\nabla} \cdot \left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) \rho(\vec{r}) \, \mathrm{d}\tau$$



Eletrostática

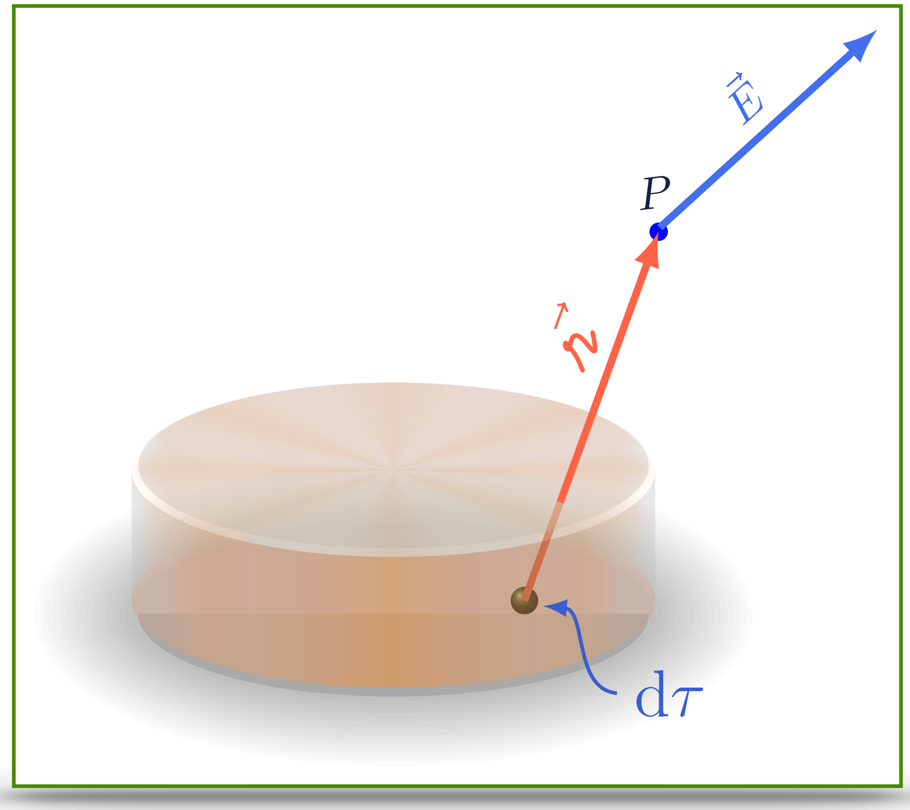
Outra visão

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \rho(\vec{r}) \, \mathrm{d}\tau$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{4\pi\epsilon_0} \int \vec{\nabla} \cdot \left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \right) \rho(\vec{r}) \, \mathrm{d}\tau$$

$$\vec{\nabla} \cdot \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} = 4\pi\delta(\vec{\mathscr{r}}) \qquad (\mathscr{r} = \vec{r} - \vec{r'})$$



Eletrostática

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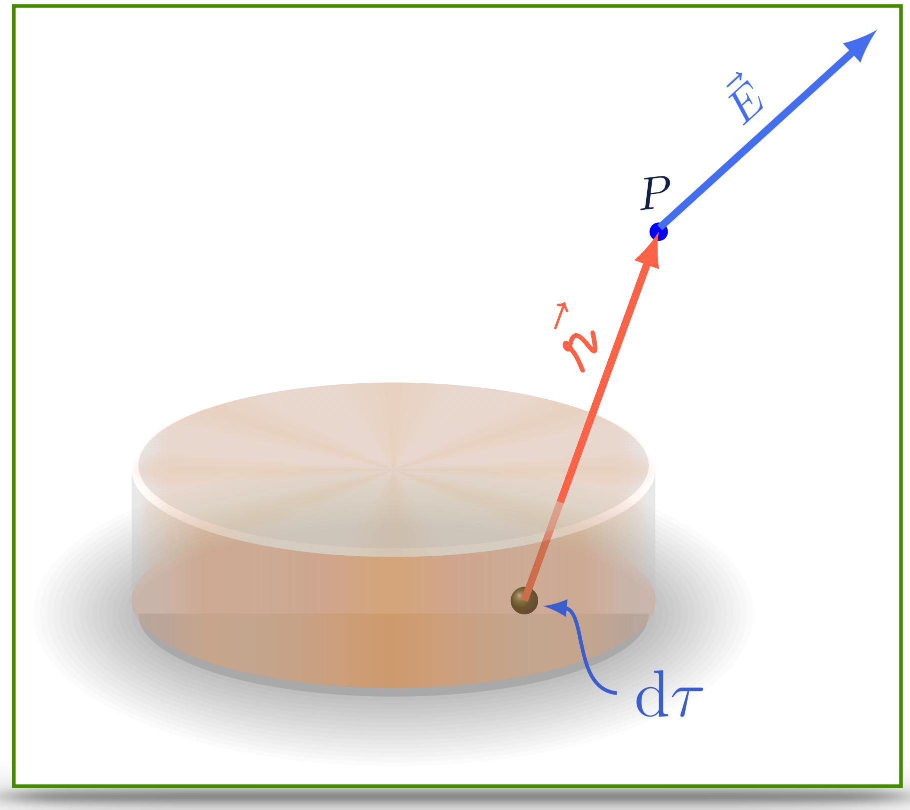

Eletrostática

Outra visão

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\mathscr{r}}}{\mathscr{r}^2} \rho(\vec{r}') \, \mathrm{d}\tau$$

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{4\pi\epsilon_0} \int \vec{\nabla} \cdot \left(\frac{\hat{\mathscr{r}}}{\mathscr{r}^2}\right) \rho(\vec{r}') \, \mathrm{d}\tau$$

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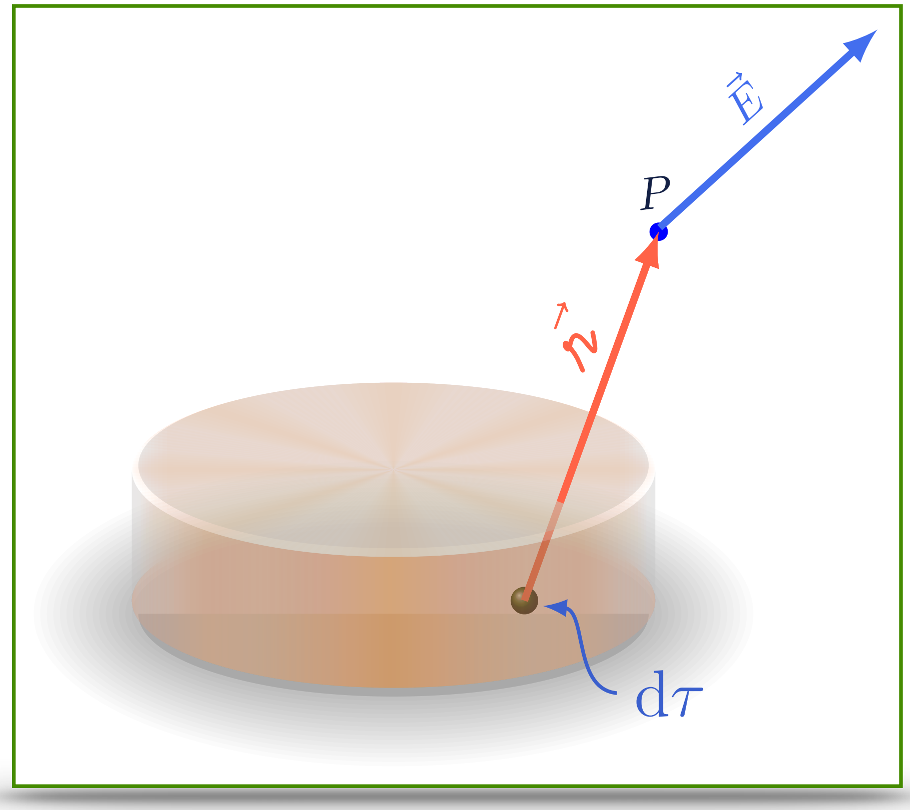
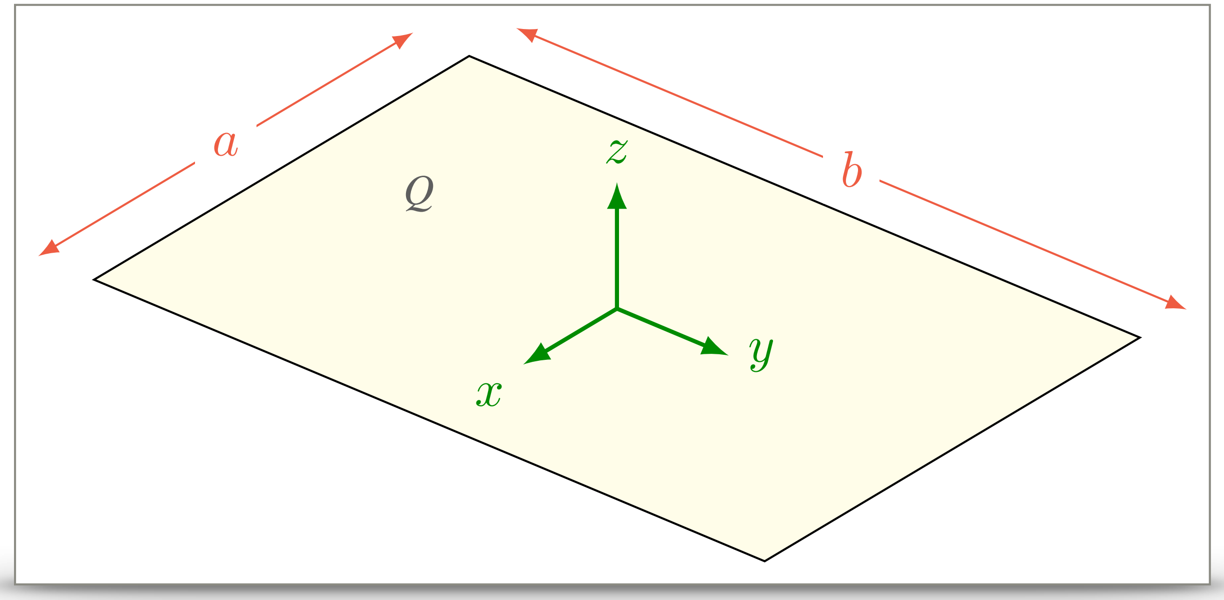
$$\vec{\nabla} \cdot \vec{E} = \frac{\rho(\vec{r})}{\epsilon_0}$$



$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

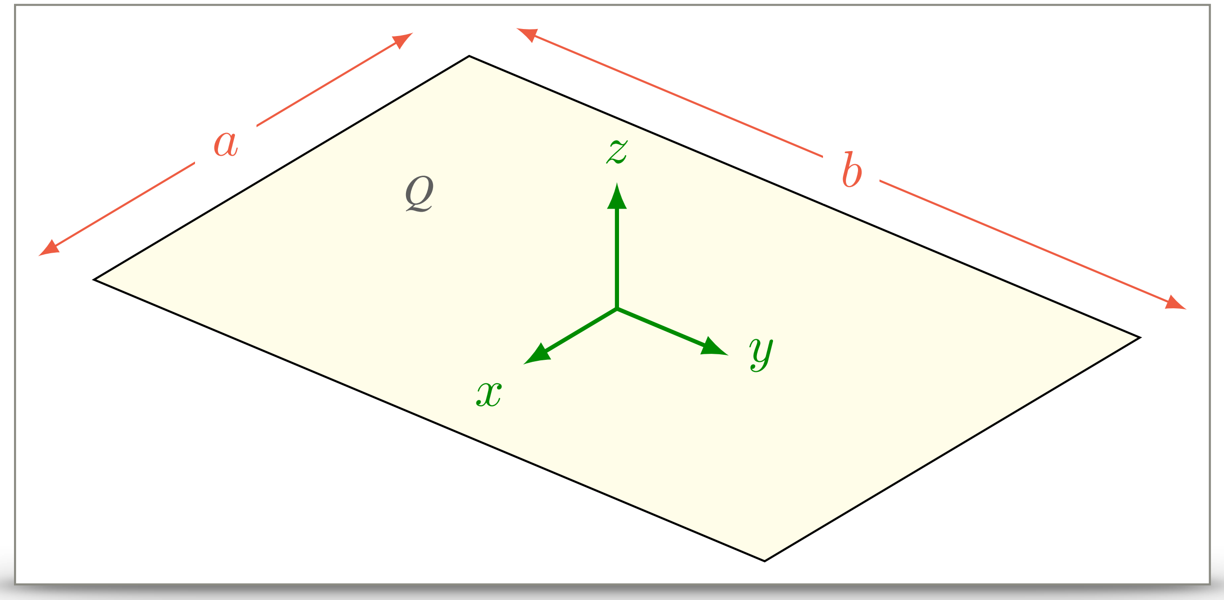
$$\overrightarrow{E} = ?$$



$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

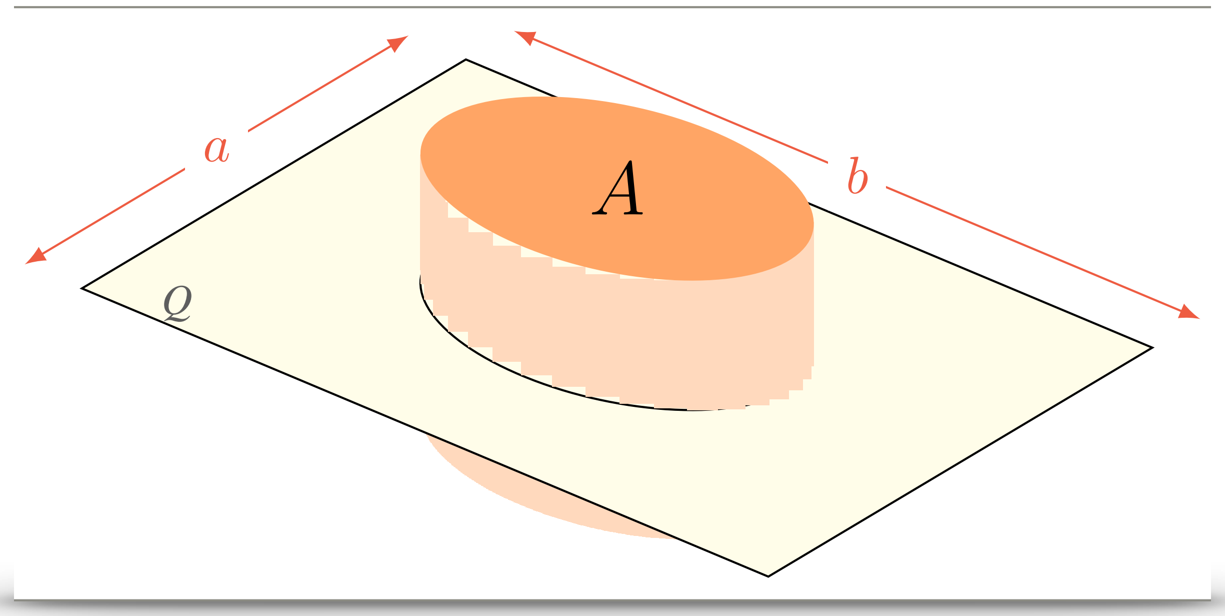
Escolher superfície simétrica



$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

Escolher superfície simétrica

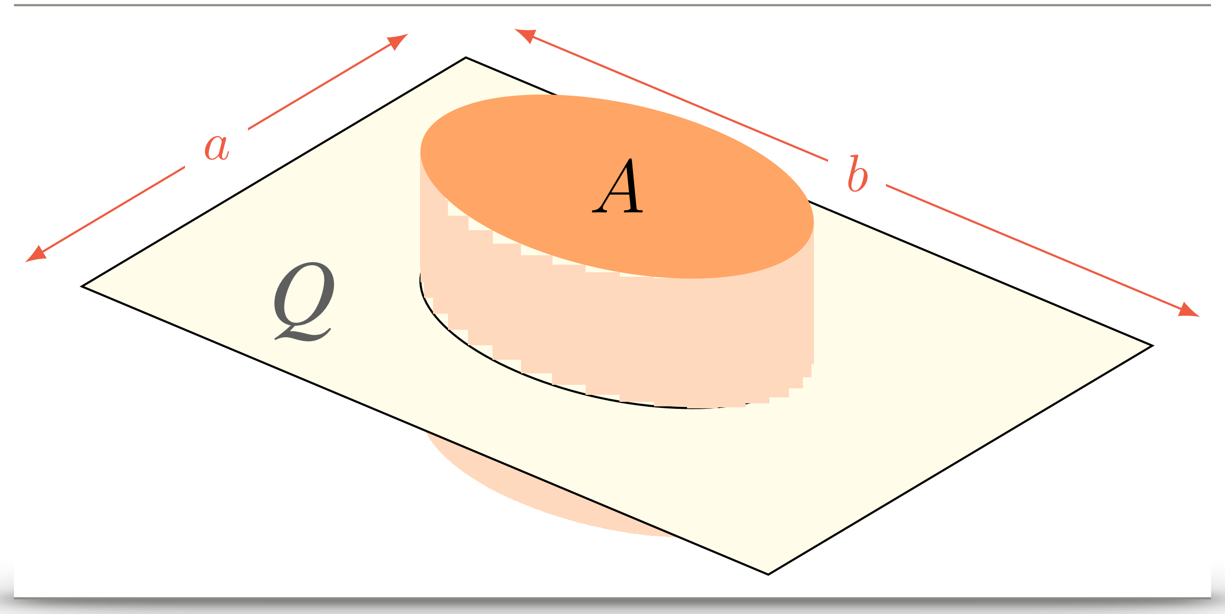


$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

Escolher superfície simétrica

$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = EA + EA$$



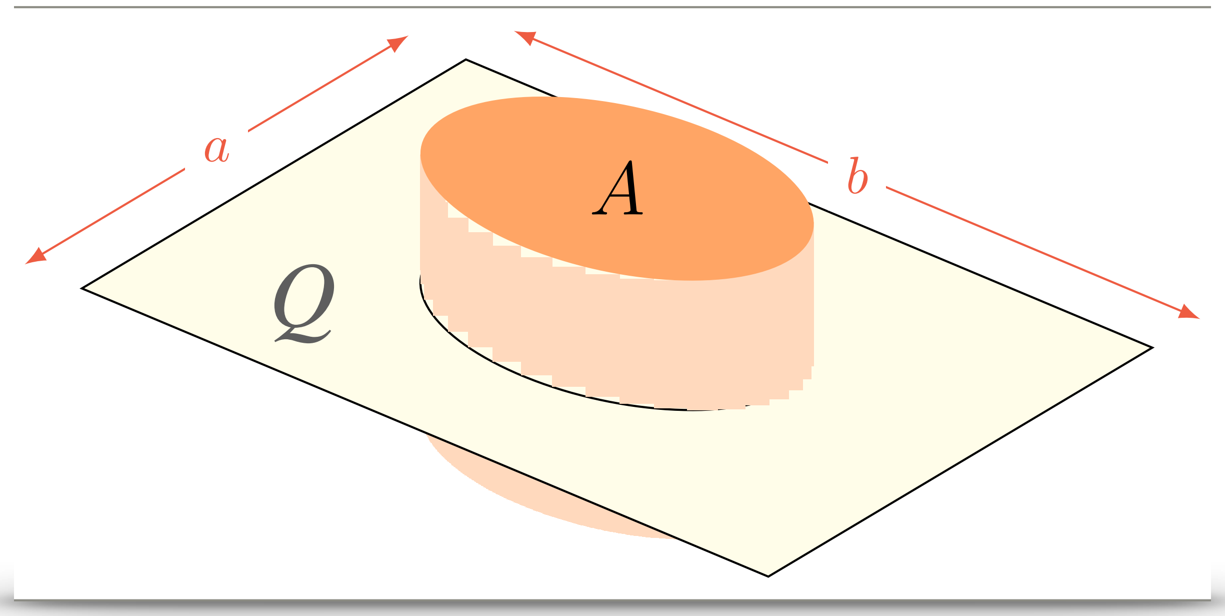
$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

Escolher superfície simétrica

$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = EA + EA$$

$$2EA = \Delta Q$$



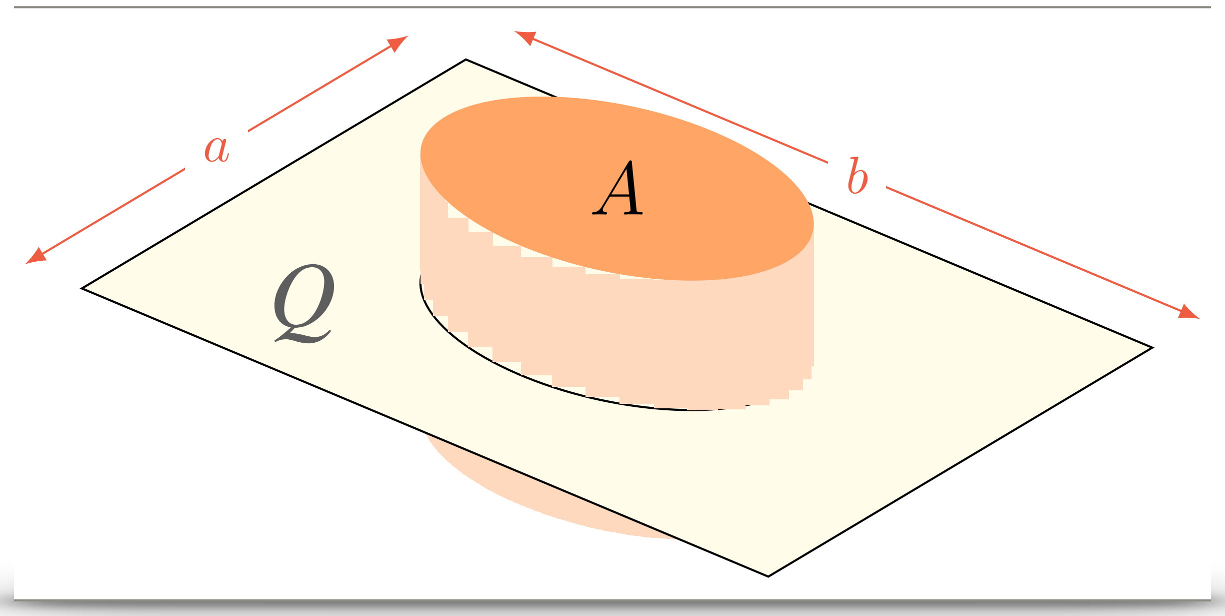
$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

Escolher superfície simétrica

$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = EA + EA$$

$$2EA = \frac{\Delta Q}{\epsilon_0}$$



$$\int \vec{E}\cdot\hat{n}\,\mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu

Escolher superfície simétrica

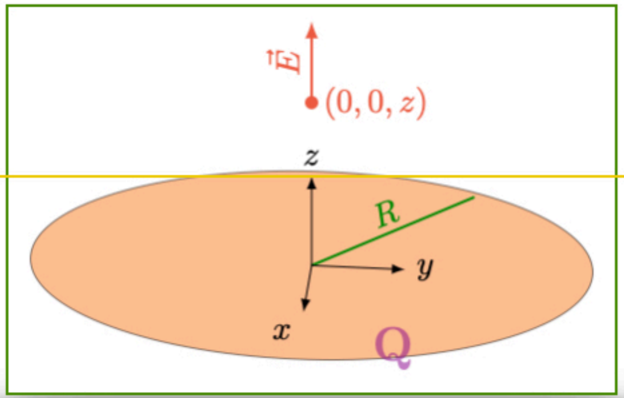
$$\int \vec{E}\cdot\hat{n}\,\mathrm{d}A = EA + E$$

$$2EA = \frac{\Delta Q}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

Pratique o que aprendeu

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\vec{r}}{r^3}\,\mathrm{d}q$$



$$E_z = \frac{\sigma}{4\pi\epsilon_0}\int_0^R\int_0^{2\pi}\frac{z}{(z^2+s^2)^{3/2}}\,s\,\mathrm{d}\phi\,\mathrm{d}s$$

$$E_z = \frac{\sigma}{2\epsilon_0}\int_0^R\frac{z}{(z^2+s^2)^{3/2}}\,s\,\mathrm{d}s = \frac{\sigma}{2\epsilon_0}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)$$

$$\int \vec{E} \cdot \hat{n} \, \mathrm{d}A = \frac{q}{\epsilon_0}$$

Pratique o que aprendeu