MONITORIA MAT4 29/04

$f : \mathbb{R}^2 \to \mathbb{R}$

$$f(x,y) = \begin{cases} \dfrac{x^3 y}{x^4 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

$$\lim_{t \to 0} \frac{t^3 t}{t^4 + t^2}$$

$$M_{n \times n}(\mathbb{R}) = \left\{ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \right\}$$

$$M_{n \times n}(\mathbb{R}) \cong \mathbb{R}^{n^2}$$

Prop.: Se $\|\cdot\|_1$ e $\|\cdot\|_2$ são normas em $\mathbb{R}^p$ então $\exists\, c_1, c_2 > 0$ tais que

$$c_1 \|x\|_1 \leq \|x\|_2 \leq c_2 \|x\|_1$$

$\forall x \in \mathbb{R}^p$

$$f_1(x) = \sum_{i=1}^{n} x_i \qquad f_2(x) = \max_{i \in \{1,\dots,n\}} \{x_i\}$$

$$\|x\|_1 = \sum_{i=1}^{n} |x_i| \qquad \|x\|_\infty = \max_{j \in \{1,\dots,n\}} \{|x_j|\}$$

-/-

$$\|A\|_{op} = \sup\{\|Ax\| ; \|x\| = 1\}$$

$$T_A : \mathbb{R}^n \to \mathbb{R}^n \qquad g(x) = \|Ax\|$$
$$x \mapsto Ax$$

Ex. da lista: Verifique que

(i) $\|\cdot\|_{op}$ é norma;

(ii) $\|Ax\| \leq \|A\|_{op} \|x\|$;

(iii) $\|BA\|_{op} \leq \|B\|_{op} \|A\|_{op}$

$GL_n(\mathbb{R}) = \{A \in M_{n\times n}(\mathbb{R});$
A é invertível$\}$

Prop.: $GL_n(\mathbb{R})$ é aberto

Dem.: A f. $\det : M_{n\times n}(\mathbb{R}) \to \mathbb{R}$ é contínua (exercício) e $GL_n(\mathbb{R}) = \det^{-1}(\mathbb{R} - \{0\})$ $\square$

$$Inv : GL_n(\mathbb{R}) \to M_{n\times n}(\mathbb{R})$$
$$X \mapsto X^{-1}$$

$$f : \underset{ab.}{\Omega \subset} \mathbb{R}^p \to \mathbb{R}^q$$

$\mathrm{Inv}_{ij}(X) : Gl_n(\mathbb{R}) \to \mathbb{R}$

$X \mapsto X^{-1}_{ij}$

Prop.: Inv é contínua

Dem.: $A^{-1} = \frac{1}{\det A} \operatorname{cof} A^T$

$\operatorname{cof} A_{ij} = (-1)^{i+j} \det(A$ del. lin. i e col. $j)$

$A \mapsto \operatorname{cof} A$ é cont.

$X \mapsto X^T$ é cont.

$\square$

(i) $\|Ax\| \leq \|A\|_{op} \|x\|$

(ii) $\|BA\|_{op} \leq \|B\|_{op} \|A\|_{op}$

(iii) $Inv : GL_n(\mathbb{R}) \to M_{n \times n}(\mathbb{R})$ é cont.

—/—

Prop.: Inv é diferenciável em $X \in GL_n(\mathbb{R})$

Dem.: O que queremos: $T_X : M_{n\times n}(\mathbb{R}) \to M_{n\times n}(\mathbb{R})$ linear t.q.

$$\lim_{H \to 0} \frac{\|Inv(X+H) - Inv(X) - T_X(H)\|}{\|H\|} = 0$$

$$[(X+H)^{-1} - X^{-1}](X+H) =$$

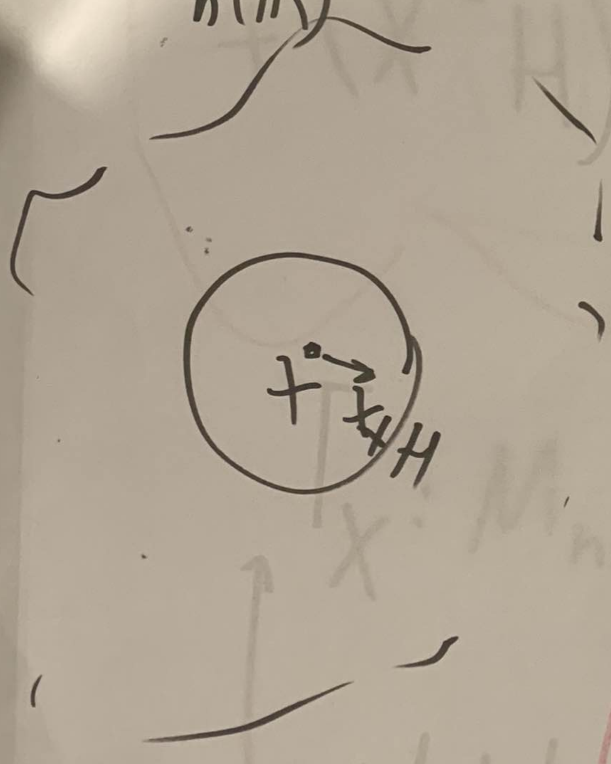
$$= (X+H)^{-1}(X+H) - X^{-1}(X+H) =$$

$$= \cancel{I_d - X^{-1}X} - X^{-1}H =$$

$$= -X^{-1}H = \cancel{-X^{-1}H(X+H)^{-1}}$$

$$T_X f(H) = -X^{-1}H$$

$GL_n(\mathbb{R})$



$$= -X^{-1}HX^{-1}X$$

$$- X^{-1}HX^{-1}H$$

$$+ X^{-1}HX^{-1}H$$

$$=$$

$$\left[(X+H)^{-1} - X^{-1}\right](X+H) =$$

$$= -X^{-1}HX^{-1}(X+H) +$$

$$+(X^{-1}H)(X^{-1}H)$$

$$(X+H)^{-1} - X^{-1} = -X^{-1}HX^{-1}$$

$$+(X^{-1}H)^{2}(X+H)^{-1}$$

$$T_X : M_{n\times n}(\mathbb{R}) \to M_{n\times n}(\mathbb{R})$$

$$H \longmapsto -X^{-1}HX^{-1}$$

candidata a $D\,\mathrm{inv}(X)(H)$

$$\frac{\|r(H)\|_{op}}{\|H\|_{op}} = \frac{\|(X^{-1}H)^2(X+H)^{-1}\|}{\|H\|}$$

$$\leq \frac{\|(X^{-1}H)^2\| \, \|(X+H)^{-1}\|}{\|H\|}$$

$$\leq \frac{\|X^{-1}H\| \, \|X^{-1}H\| \, \|(X+H)^{-1}\|}{\|H\|}$$

$$\leq \frac{(\|X^{-1}\| \, \|H\|)^2 \, \|(X+H)^{-1}\|}{\|H\|}$$

$$\leq \frac{\|X^{-1}\|^2 \, \|(X+H)^{-1}\| \, \|H\|^2}{\|H\|}$$

$n \geq 1 \Rightarrow \|X^n\| \leq \|X\|^n$

$n < 0$: falso!

$$\frac{\|r(H)\|}{\|H\|} \leq \|X^{-1}\|^2 \, \|(X+H)^{-1}\| \, \|H\|$$

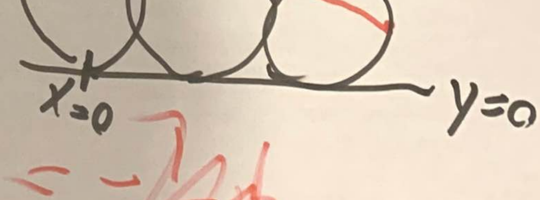
quando $H \to 0$ $\quad$ limitado $\quad \xrightarrow{\text{cont.}} \|X^{-1}\| \quad \to 0$

ou seja
$$\lim_{H \to 0} \frac{\|r(H)\|}{\|H\|} = 0$$

$$\boxed{\text{Conclusão: Inv é diferenciável em } X \text{ e } DInv(X)(H) = -X^{-1} H X^{-1}}$$

$f: \mathbb{R} - \{0\} \to \mathbb{R}$

$f(x) = \frac{1}{x}$



$Df(x)(h) = f'(x)h = \cancel{-\frac{1}{x^2}h}$

$= -\frac{1}{x^2} h$