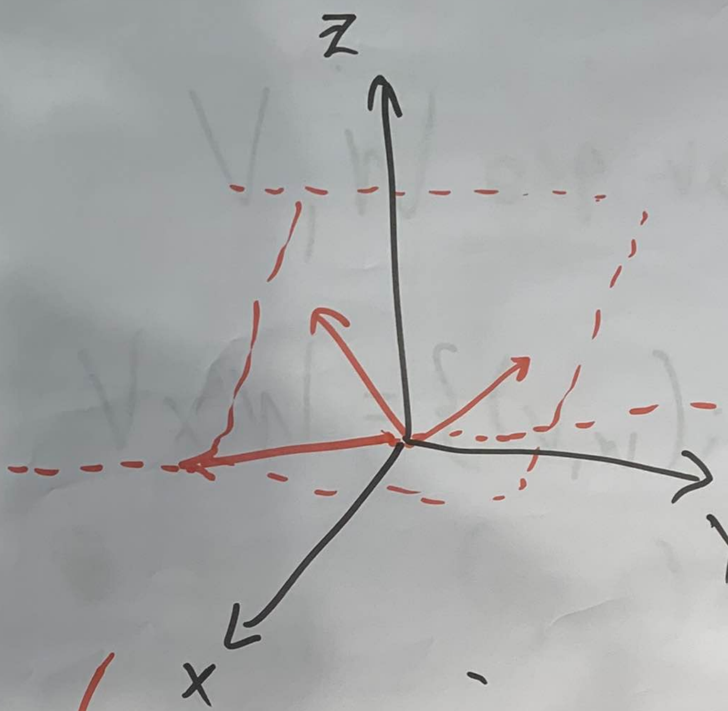
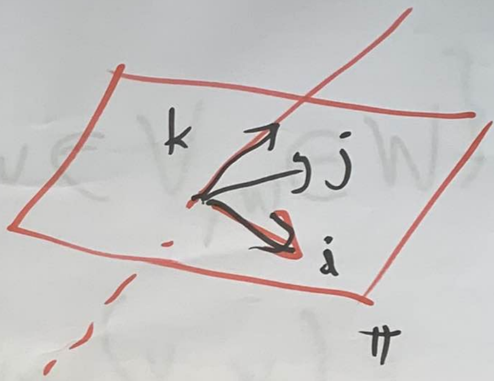
MONITORIA MAT 4 22/04



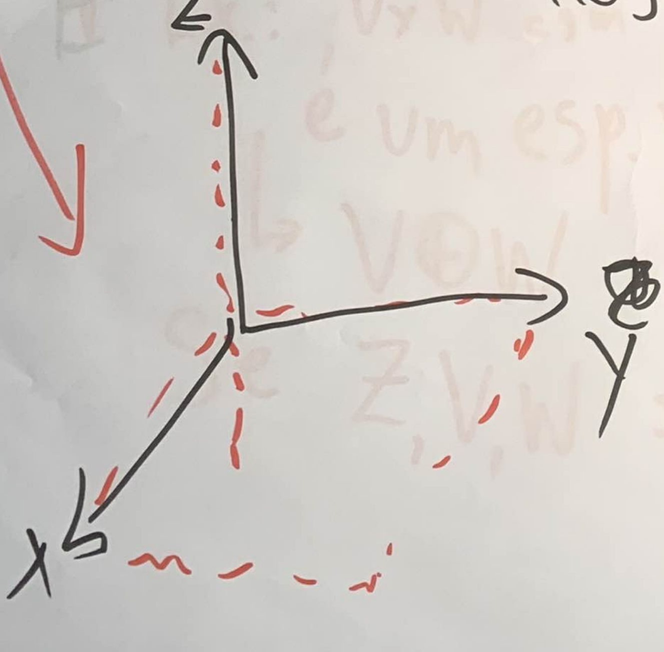


$$u = \underbrace{\alpha i + \beta j}_{v \in \pi} + \underbrace{\gamma k}_{w \in r}$$

$\mathbb{R}^n, V, W$

(i) $\mathbb{R}^n = V + W$

(ii) $V \cap W = \{\vec{0}\}$



$\pi \to xy$

$r \to z$

(x, y, z)

$(x, y)\ z$

V, W esp. vetoriais

$$V \times W = \{(v, w);\ v \in V, w \in W\}$$

$+_{V\times W}: (v_1, w_1) +_{V\times W} (v_2, w_2) :=$

$$:= (v_1 +_V v_2,\ w_1 +_W w_2)$$

$\cdot_{V\times W}: \alpha \cdot_{V\times W} (v_1, w_1) = (\alpha \cdot_V v_1, \alpha \cdot_W w_1)$

Ex.: $V \times W$ com $+_{V\times W}$ e $\cdot_{V\times W}$ é um esp. vetorial

↳ $V \oplus W$

Se Z, V, W satisfazem (i), (ii)

~~então~~

Ex.: Se Z, V, W satisfazem:

(i) $Z = V + W$

(ii) $V \cap W = \{\vec{0}\}$

então $Z \simeq V \oplus W$, i.e.,

$\exists\, T : Z \to V \oplus W$ linear bijetora

$\{ f : \mathbb{R} \to \mathbb{R} \, ; \, f(0) = a \}$

$$\left\| T\left(\frac{x-\bar{x}}{\|x-\bar{x}\|}\right) - L\left(\frac{x-\bar{x}}{\|x-\bar{x}\|}\right)\right\| \xrightarrow[x \to \bar{x}]{} 0$$

$y \in S^{n-1} = S(0,1)$ qualquer

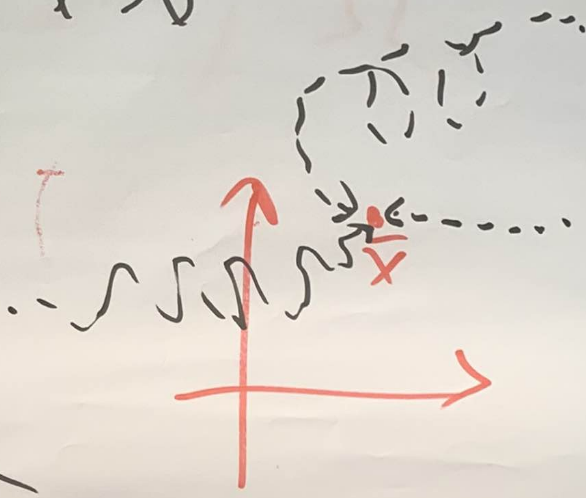
$$g : [0, \cancel{x}A) \to \mathbb{R}^n$$
$$t \longmapsto \bar{x} + ty$$

$$\frac{g(t)-\bar{x}}{\|g(t)-\bar{x}\|} = \frac{\bar{x}+ty-\bar{x}}{\|\bar{x}+ty-\bar{x}\|} = \frac{ty}{\|ty\|} =$$

$$= \frac{\cancel{t}y}{\cancel{t}\cdot 1} = y$$

$$\|T(y)-L(y)\| \underset{\substack{\forall t \\ \in (0,A)}}{=} \left\| T\left(\frac{g(t)-\bar{x}}{\|g(t)-\bar{x}\|}\right) - L\left(\frac{g(t)-\bar{x}}{\|g(t)-\bar{x}\|}\right) \right\| \xrightarrow[t\to 0]{} 0$$

$$g(t)=\bar{x}+ty$$



$$x\in\mathbb{R}^n \qquad x\to\bar{x}$$

$$\forall \varepsilon>0 \quad \exists t^* \quad \text{t.q.}$$

$$\|T(y)-L(y)\| = \underbrace{\left\| L\left(\frac{g(t^*)-\bar{x}}{\|g(t^*)-\bar{x}\|}\right) - T\left(\frac{g(t^*)-\bar{x}}{\|g(t^*)-\bar{x}\|}\right) \right\|}_{=\alpha>0} < \varepsilon$$

$$\varepsilon = \frac{\alpha}{2} \quad \Rightarrow\Leftarrow$$

Conclusão: $\| T(y) - L(y) \| = 0$