# Eletromagnetismo

$$\vec{\nabla} \cdot \vec{D} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

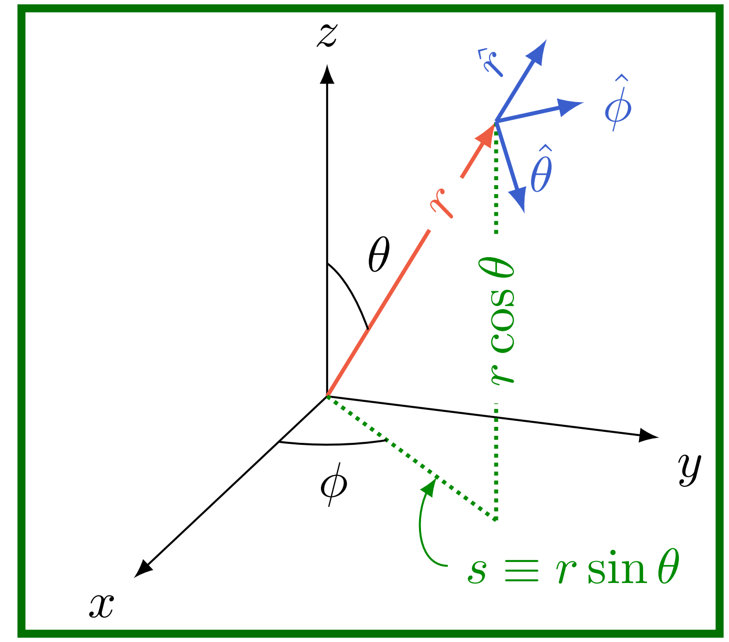
$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} = \vec{j} + \epsilon_0 \frac{\partial \vec{D}}{\partial t}$$

Aula de 28 de abril

Análise vetorial

# Coordenadas esféricas




$$d\vec{\ell} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

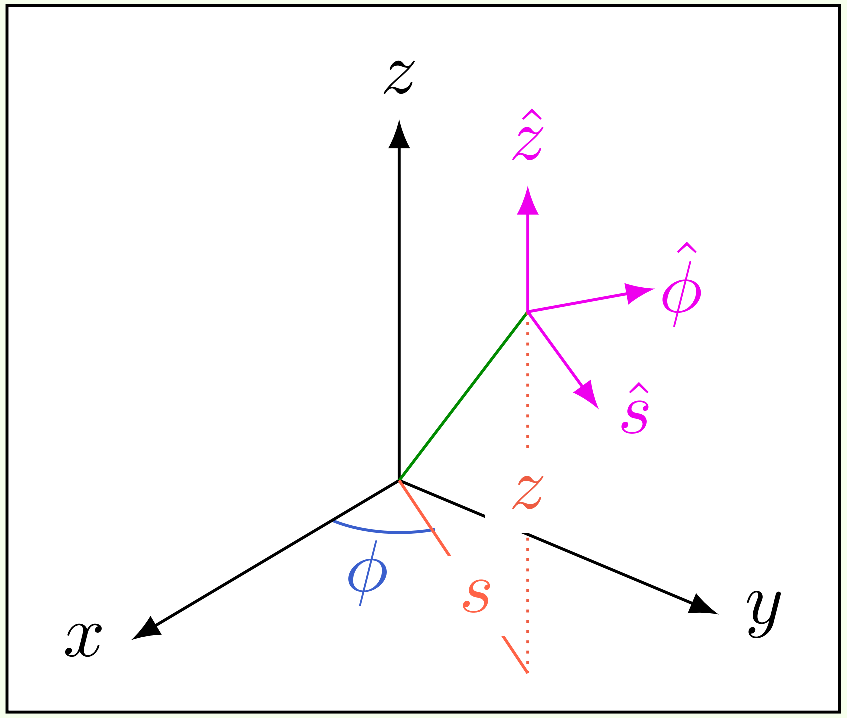
$$\vec{\nabla}\times v = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r}$$

$$\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial t}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial\phi^2}$$

# Coordenadas cilíndricas

$$\mathrm{d}\vec{\ell} = \mathrm{d}s\,\hat{s} + s\,\mathrm{d}\phi\,\hat{\phi} + \mathrm{d}z\,\hat{z}$$




$$\vec{\boldsymbol{\nabla}} T = \frac{\partial T}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial T}{\partial \phi}\hat{\phi} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\boldsymbol{\nabla}} \cdot \vec{v} = \frac{1}{s}\frac{\partial (s v_s)}{\partial s} + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}$$

$$\vec{\boldsymbol{\nabla}} \times \vec{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s v_\phi)}{\partial s} - \frac{\partial v_s}{\partial \phi}\right)\hat{z}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}$$

# Pratique o que aprendeu

$$\vec{\nabla} \cdot \left( \frac{1}{r^2} \hat{r} \right) = ?$$

MAIS FÁCIL EM COORDENADAS ESFÉRICAS

# Pratique o que aprendeu

$\boxed{\vec{v} = \frac{1}{r^2}\hat{r}}$

$v_r$

$$\vec{\nabla} \cdot \left(\frac{1}{r^2}\hat{r}\right) = ?$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

# Pratique o que aprendeu

$$\vec{\nabla} \cdot \left( \frac{1}{r^2} \hat{r} \right) = ?$$

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$$\vec{\nabla} \cdot \left( \frac{1}{r^2} \hat{r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{1}{r^2} \right) = \frac{1}{r^2} \frac{d}{dr} (1) = 0$$

# Pratique o que aprendeu

$$\vec{\nabla} \cdot \left( \frac{1}{r^2} \hat{r} \right) = ?$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2} \frac{\partial (r^2 v_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial (\sin\theta v_\theta)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial v_\phi}{\partial \phi}$$

$$\vec{\nabla} \cdot \left( \frac{1}{r^2} \hat{r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{1}{r^2} \right)$$

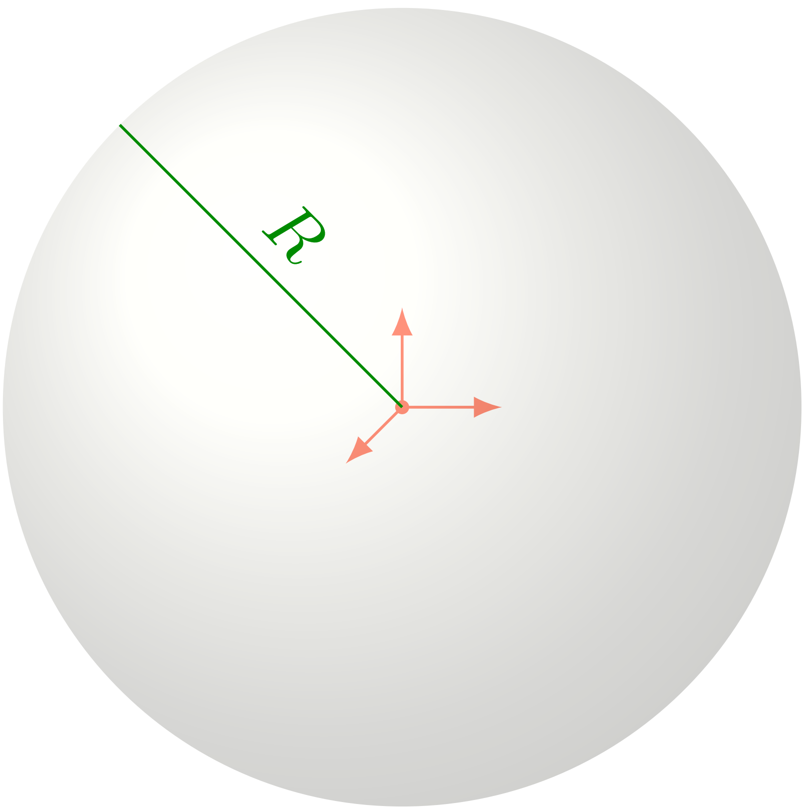
$$\boxed{\vec{\nabla} \cdot \left( \frac{1}{r^2} \hat{r} \right) = 0}$$

$$\vec{\nabla} \cdot \left(\frac{1}{r^2}\hat{r}\right) = 0$$

Pratique o que aprendeu

POR OUTRO LADO ...

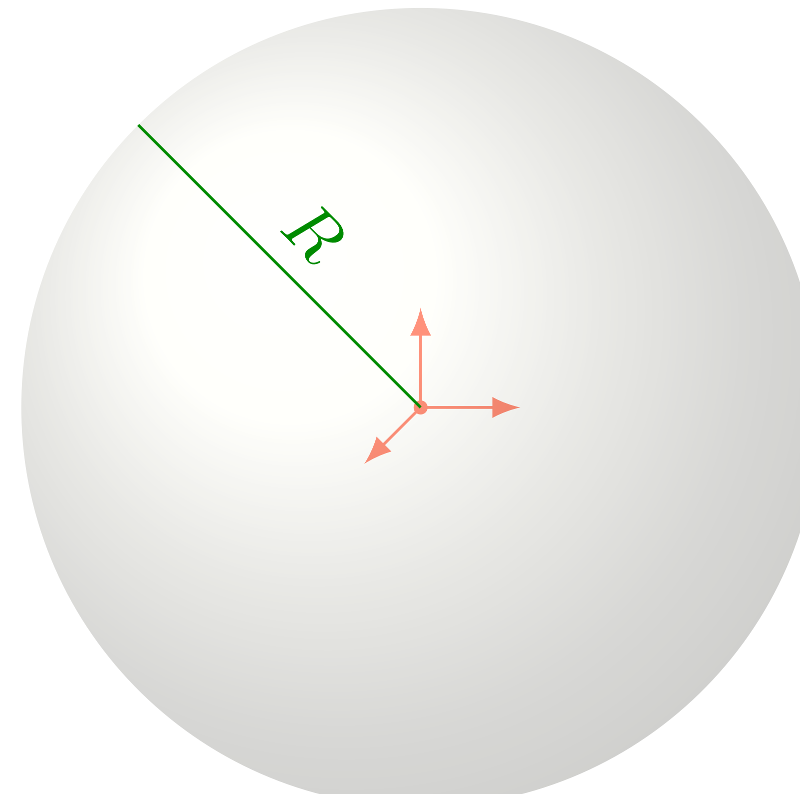
$$\int_{\mathcal{V}} (\vec{\nabla} \cdot \vec{v})\, \mathrm{d}\tau = \int_{\mathcal{S}} \vec{v} \cdot \hat{n}\, \mathrm{d}A$$



$$\vec{\nabla}\cdot\left(\frac{1}{r^2}\hat{r}\right) = 0$$

Pratique o que aprendeu

$$\int_{\mathcal{V}} (\vec{\nabla}\cdot\vec{v})\,\mathrm{d}\tau = \int_{\mathcal{S}} \vec{v}\cdot\hat{n}\,\mathrm{d}A$$




$$\int \vec{\nabla}\cdot\left(\frac{1}{r^2}\hat{r}\right)\,\mathrm{d}\tau = \int_0^{2\pi}\int_{-1}^{1} \frac{1}{R^2} R^2\,\mathrm{d}u\,\mathrm{d}\phi$$

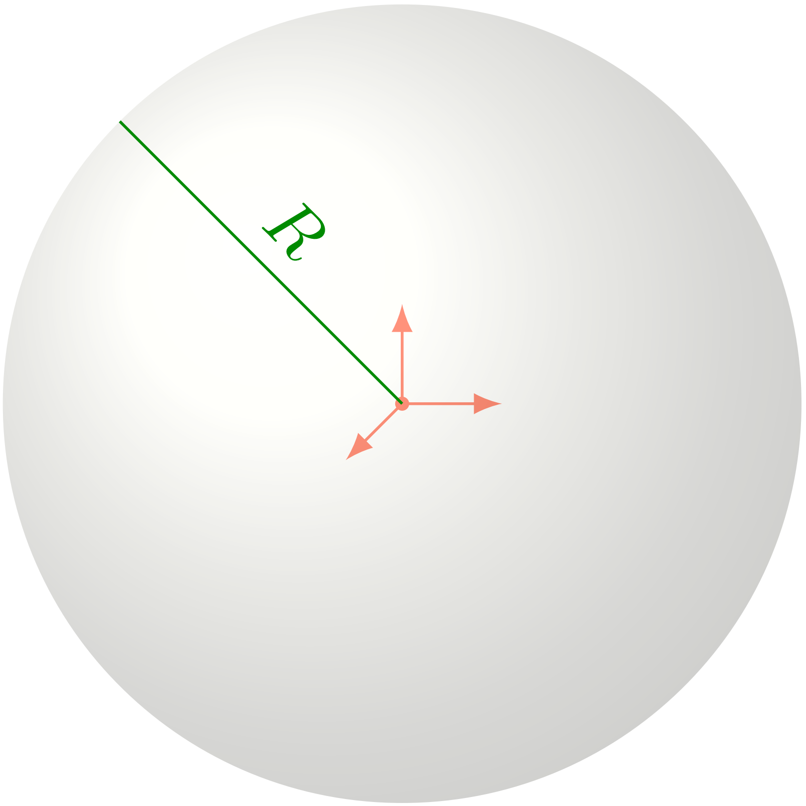
NA SUPERFÍCIE

$$\int_0^{\pi} \text{sen}\,\theta\, d\theta = \int_{-1}^{1} du$$

$$\vec{\boldsymbol{\nabla}} \cdot \left(\frac{1}{r^2}\hat{r}\right) = 0$$

Pratique o que aprendeu

$$\int_{\mathcal{V}} (\vec{\nabla} \cdot \vec{v})\, \mathrm{d}\tau = \int_{\mathcal{S}} \vec{v} \cdot \hat{n}\, \mathrm{d}A$$



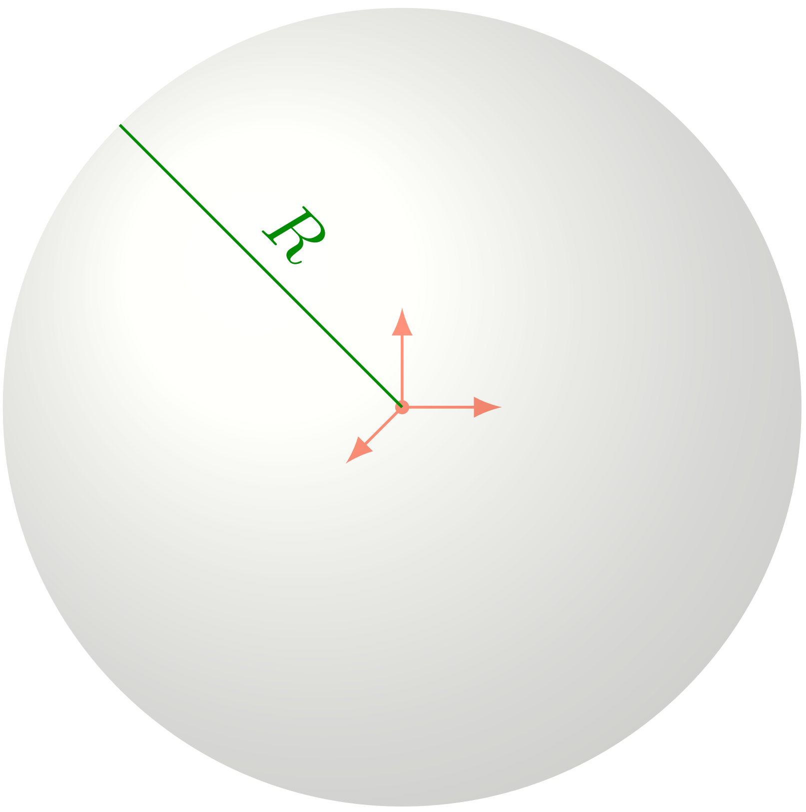

$$\int \vec{\boldsymbol{\nabla}} \cdot \left(\frac{1}{r^2}\hat{r}\right) \mathrm{d}\tau = \int_0^{2\pi} \int_{-1}^{1} \frac{1}{R^2} R^2 \mathrm{d}u\, \mathrm{d}\phi$$

$$\int \vec{\boldsymbol{\nabla}} \cdot \left(\frac{1}{r^2}\hat{r}\right) \mathrm{d}\tau = 4\pi$$

$$\boxed{\vec{\nabla}\cdot\left(\frac{1}{r^2}\hat{r}\right) = 0}$$

# Pratique o que aprendeu

$$\boxed{\int_{\mathcal{V}} (\vec{\nabla}\cdot\vec{v})\,\mathrm{d}\tau = \int_{\mathcal{S}} \vec{v}\cdot\hat{n}\,\mathrm{d}A}$$




$$\int \vec{\nabla}\cdot\left(\frac{1}{r^2}\hat{r}\right)\,\mathrm{d}\tau = \int_0^{2\pi}\int_{-1}^{1}\frac{1}{R^2}R^2\,\mathrm{d}u\,\mathrm{d}\phi$$

$$\int \vec{\nabla}\cdot\left(\frac{1}{r^2}\hat{r}\right)\,\mathrm{d}\tau = 4\pi$$

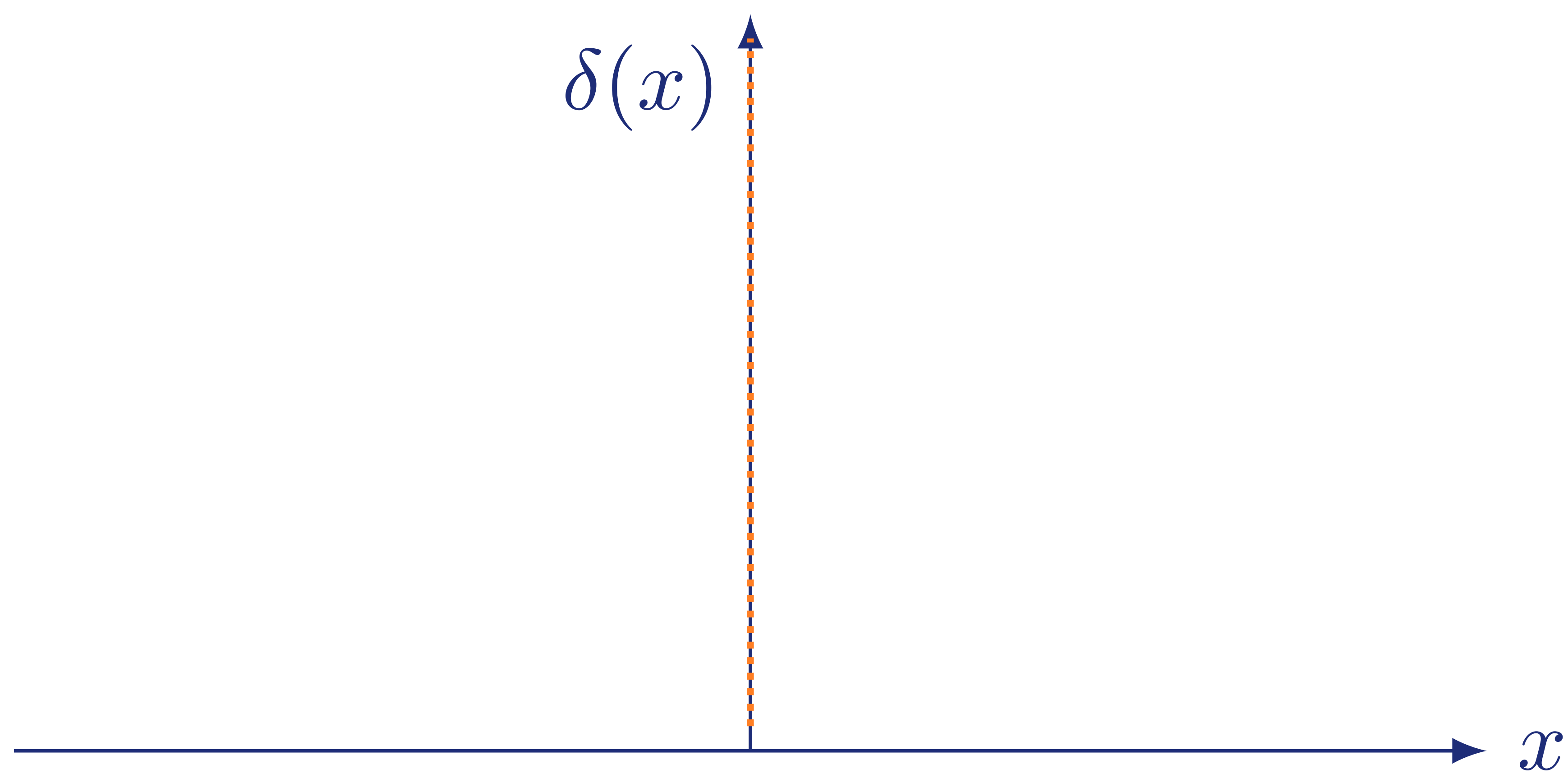
?

COMO PODE DAR 4π, SE O INTEGRANDO É ZERO?

RESPOSTA:
O INTEGRANDO NÃO É ZERO EM r=0!

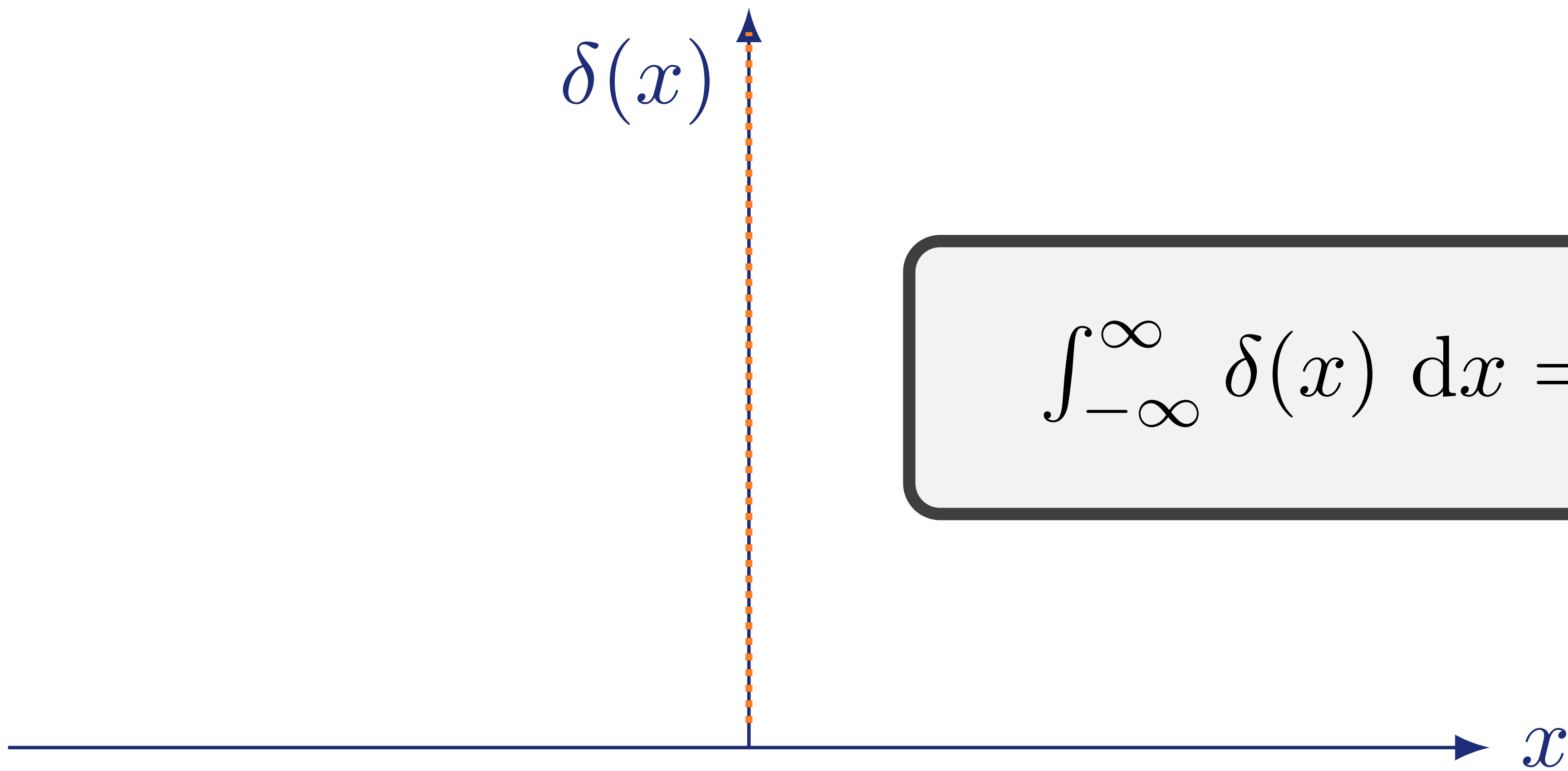
UM PONTO SÓ É SUFICIENTE PARA A INTEGRAL DAR 4π

# *Função* delta de Dirac




ZERO EM TODO O EIXO, EXCETO EM $x = 0$

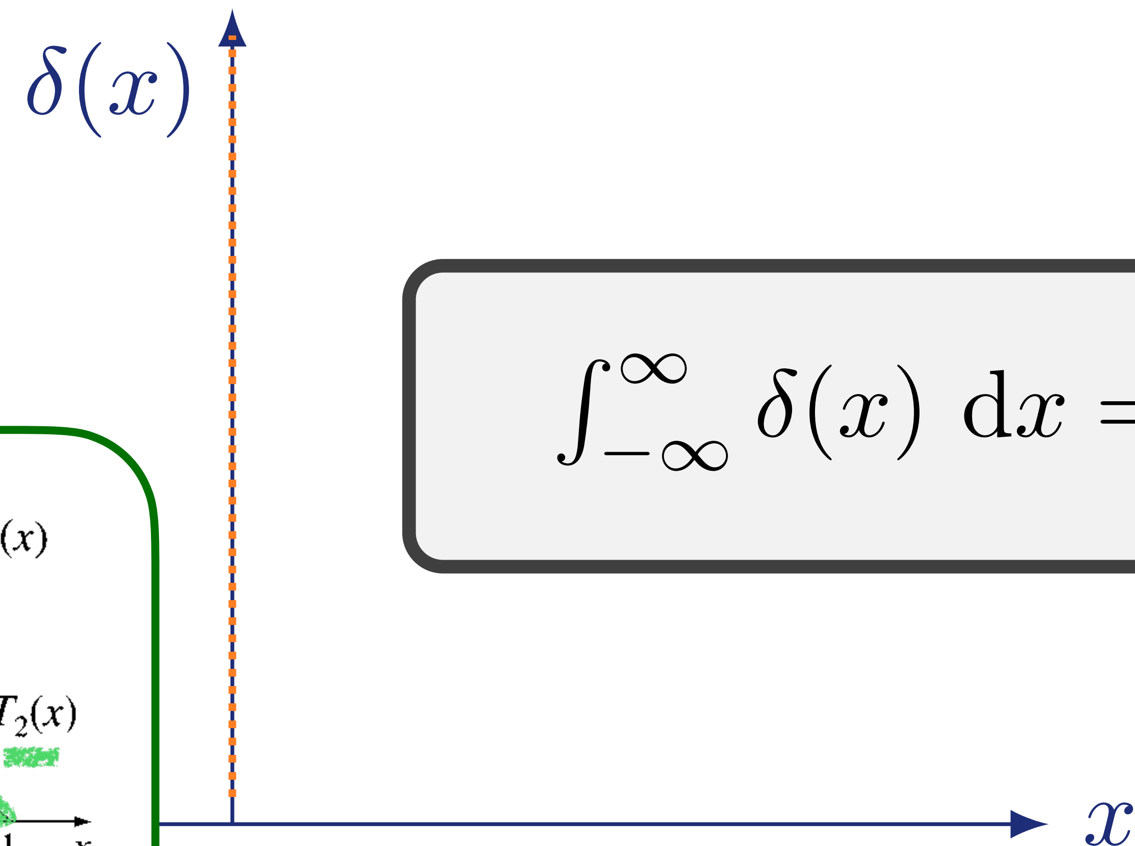
# *Função* delta de Dirac




$$\boxed{\int_{-\infty}^{\infty} \delta(x)\ \mathrm{d}x = 1}$$

APESAR DE SER ZERO EM QUASE TODO O EIXO

# Função delta de Dirac




$$\int_{-\infty}^{\infty} \delta(x)\ \mathrm{d}x = 1$$

$$\lim_{N\to\infty} R_N(x) = \delta(x)$$

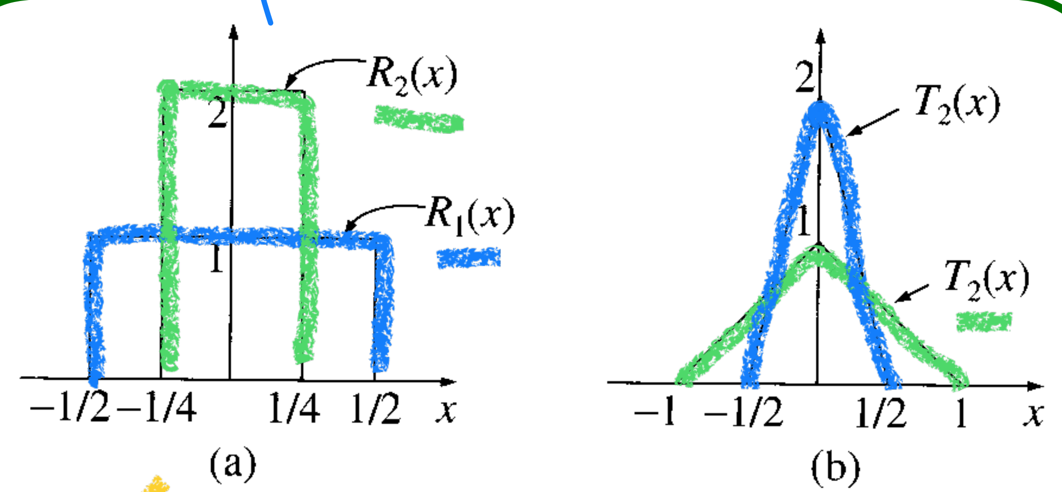
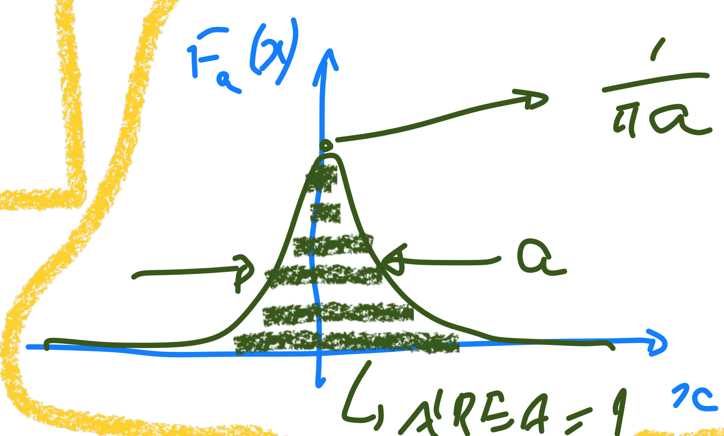



Figure 1.46

FUNÇÕES QUE TÊM ÁREA = 1 E QUE SÃO = 0, EXCETO NUM PEQUENO INTERVALO

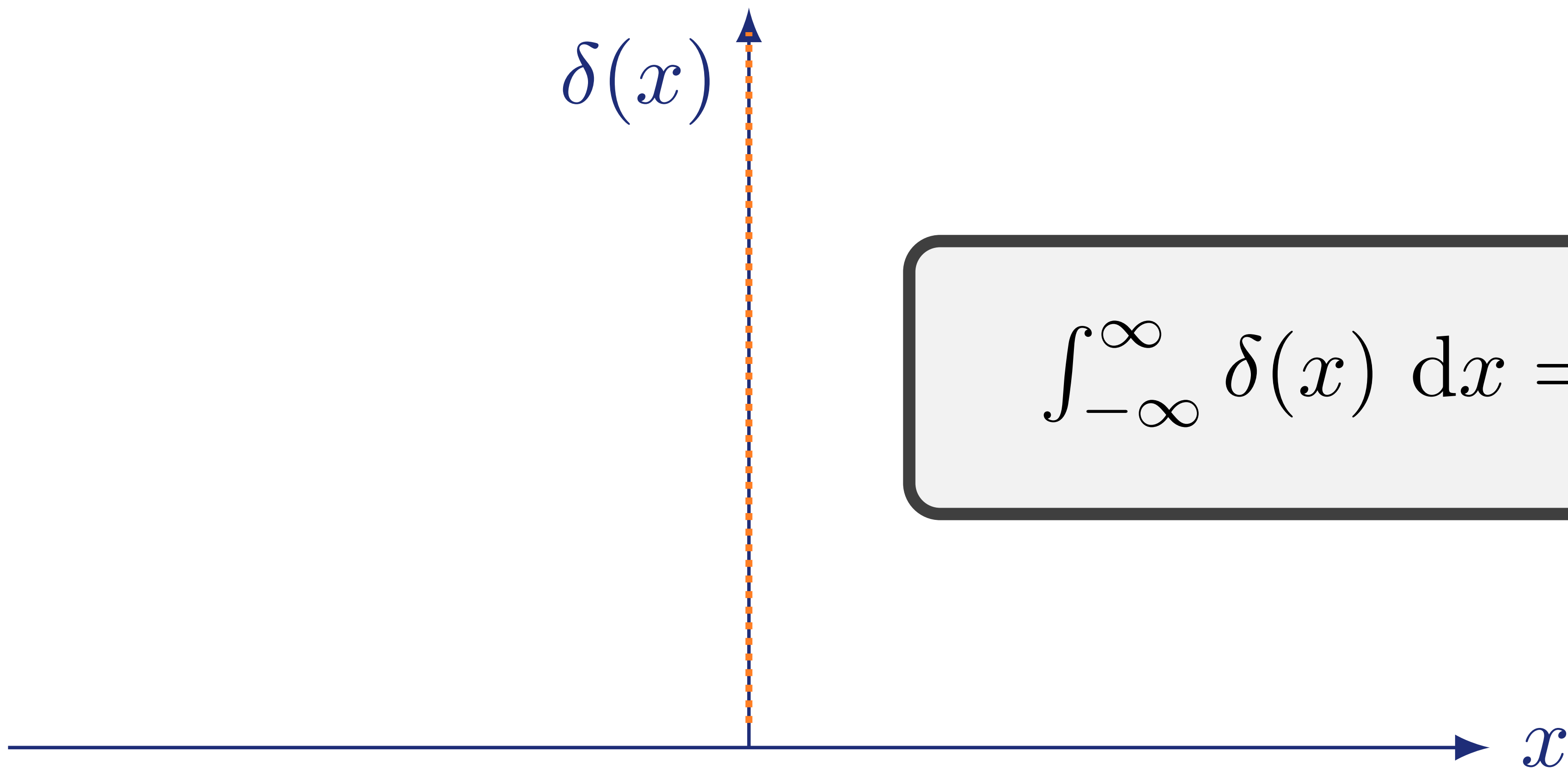
OUTRO EXEMPLO:

$$F_a(x) = \frac{1}{\pi}\frac{a}{a^2+x^2}$$




$$\lim_{a\to 0} F_a(x) = \delta(x)$$

# *Função* delta de Dirac



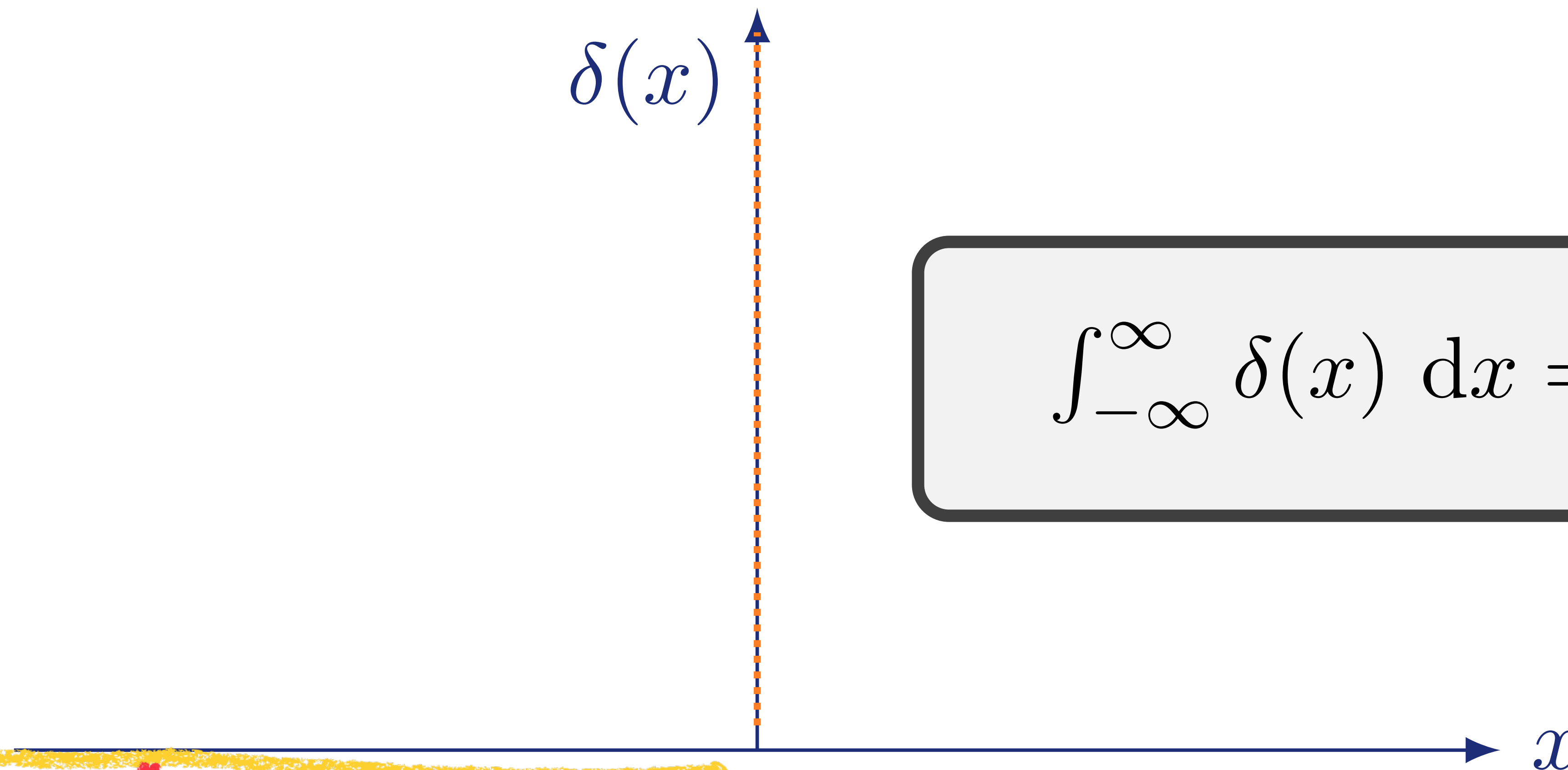

$$\int_{-\infty}^{\infty} \delta(x)\ \mathrm{d}x = 1$$

$$\int_{-\infty}^{\infty} f(x)\,\delta(x)\,dx = \int_{-\infty}^{\infty} f(0)\,\delta(x)\,dx$$

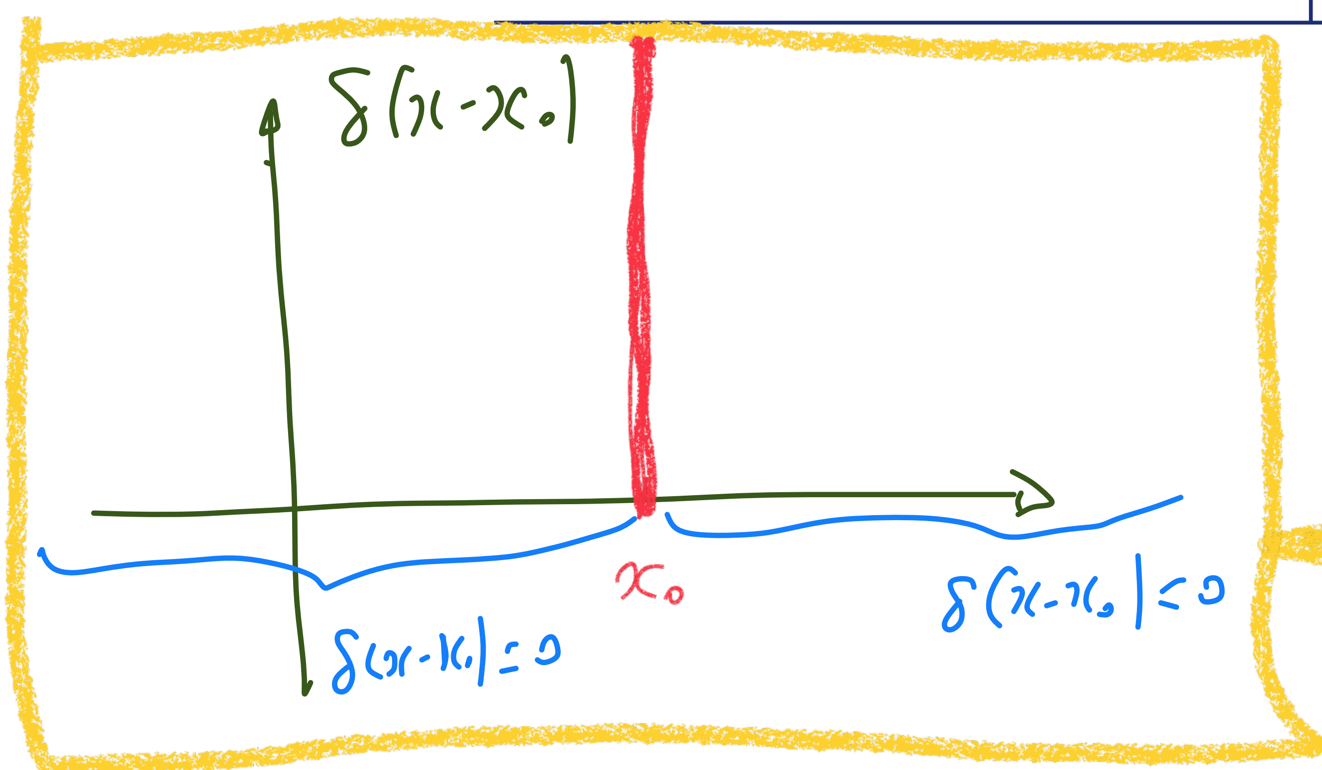
PODEMOS FAZER $f(x) \to f(0)$
PORQUE $\delta(x) = 0$, EXCETO $x = 0$

$$\int_{-\infty}^{\infty} f(x)\delta(x)\,\mathrm{d}x = f(0)\underbrace{\int_{-\infty}^{\infty} \delta(x)\,dx}_{1}$$

# Função delta de Dirac




$$\int_{-\infty}^{\infty} \delta(x)\ \mathrm{d}x = 1$$




$$\int_{-\infty}^{\infty} f(x)\delta(x)\,\mathrm{d}x = f(0)$$

$$\int_{-\infty}^{\infty} f(x)\delta(x-x_0)\,\mathrm{d}x = f(x_0)$$

$$\int_{-\infty}^{\infty} f(x_0)\,\delta(x-x_0)\,dx$$

$$\delta[f(x)] = ?$$

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$$\int_{-\infty}^{\infty} \delta\Big(f(x)\Big)\,\mathrm{d}x = \int_{f(-\infty)}^{f(\infty)} \delta(u)\Big|\frac{\mathrm{d}x}{\mathrm{d}f}\Big|\,\mathrm{d}u$$


$x : -\infty \to \infty$

$u : \infty \to -\infty$

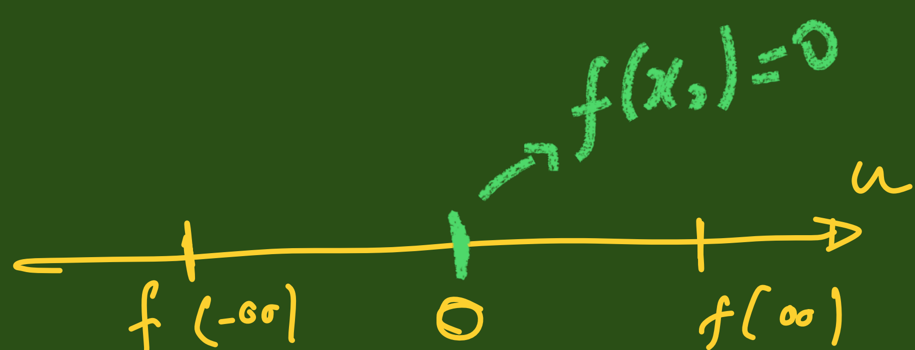
$\hookrightarrow -u : -\infty \to \infty$

MUDANÇA DE VARIÁVEL

$x \to f(x) = u$

$$\int_{f(-\infty)}^{f(\infty)} \delta(u)\,\frac{dx}{du}\,du = \int_{f(-\infty)}^{f(\infty)} \delta(u)\,\frac{dx}{du}\Big|_{x_0}\,du$$

PORQUE $\delta(u) = 0$, EXCETO EM $x = x_0 \Rightarrow u = 0$



$$\delta[f(x)] = ?$$

$$\int_{-\infty}^{\infty} \delta\Big(f(x)\Big)\,\mathrm{d}x = \int_{f(-\infty)}^{f(\infty)} \delta(u)\left|\frac{\mathrm{d}x}{\mathrm{d}f}\right|\mathrm{d}u$$

CASO f(x) NUNCA SE ANULE

$$\int_{-\infty}^{\infty} \delta\Big(f(x)\Big)\,\mathrm{d}x = \begin{cases} 0 & [f(x) \neq 0] \\ \dfrac{1}{\left|\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x_0}} & [f(x_0) = 0] \end{cases}$$

CASO f(x) SE ANULE EM $x = x_0$

# 3 dimensões

$$\delta^3(\vec{r}) = \delta(x)\delta(y)\delta(z)$$

$$\int_{-\infty}^{\infty} \delta\Big(f(x)\Big)\ \mathrm{d}x = \int_{f(-\infty)}^{f(\infty)} \delta(u) \left|\frac{\mathrm{d}x}{\mathrm{d}f}\right| \mathrm{d}u$$

$$\int_{-\infty}^{\infty} \delta\Big(f(x)\Big)\ \mathrm{d}x = \begin{cases} 0 & [f(x) \neq 0] \\ \dfrac{1}{\left|\frac{\mathrm{d}f}{\mathrm{d}x}\right|_{x_0}} & [f(x_0) = 0] \end{cases}$$

# 3 dimensões

$$\delta^3(\vec{r}) = \delta(x)\delta(y)\delta(z) \Rightarrow$$

$$\int \delta^3(\vec{r})\, d\tau = \int_{-\infty}^{\infty} \delta(x)\, dx \int_{-\infty}^{\infty} \delta(y)\, dy \int_{-\infty}^{\infty} \delta(z)\, dz$$

$$= 1 \times 1 \times 1$$

$$\int_{\text{espaço}} \delta^3(\vec{r})\, \mathrm{d}\tau = 1$$

# 3 dimensões

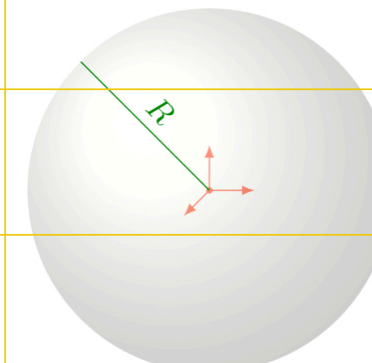
$$\delta^3(\vec{r}) = \delta(x)\delta(y)\delta(z)$$

$$\int_{\text{espaço}} \delta^3(\vec{r})\,\mathrm{d}\tau = 1$$

$$\vec{\nabla}\cdot\left(\frac{1}{r^2}\hat{r}\right) = 0$$

Pratique o que aprendeu

$$\int_{\mathcal{V}}(\vec{\nabla}\cdot\vec{v})\,\mathrm{d}\tau = \int_{\mathcal{S}}\vec{v}\cdot\hat{n}\,\mathrm{d}A$$




$$\int\vec{\nabla}\cdot\left(\frac{1}{r^2}\hat{r}\right)\,\mathrm{d}\tau = \int_0^{2\pi}\int_{-1}^{1}\frac{1}{R^2}R^2\,\mathrm{d}u\,\mathrm{d}\phi$$

$$\int\vec{\nabla}\cdot\left(\frac{1}{r^2}\hat{r}\right)\,\mathrm{d}\tau \overset{?}{=} 4\pi$$

PODEMOS AGORA
ESCREVER MELHOR

# 3 dimensões

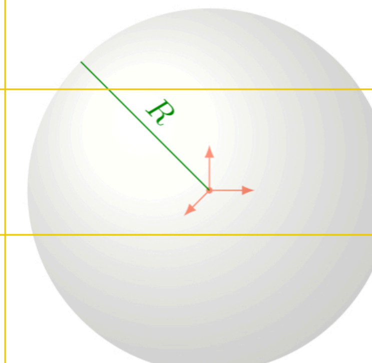
$$\delta^3(\vec{r}) = \delta(x)\delta(y)\delta(z)$$

$$\int_{\text{espaço}} \delta^3(\vec{r})\,\mathrm{d}\tau = 1$$

$$\vec{\nabla}\cdot\left(\frac{1}{r^2}\hat{r}\right) = 0$$

Pratique o que aprendeu

$$\int_{\mathcal{V}}(\vec{\nabla}\cdot\vec{v})\,\mathrm{d}\tau = \int_{\mathcal{S}}\vec{v}\cdot\hat{n}\,\mathrm{d}A$$




$$\int\vec{\nabla}\cdot\left(\frac{1}{r^2}\hat{r}\right)\,\mathrm{d}\tau = \int_0^{2\pi}\int_{-1}^{1}\frac{1}{R^2}R^2\,\mathrm{d}u\,\mathrm{d}\phi$$

$$\int\vec{\nabla}\cdot\left(\frac{1}{r^2}\hat{r}\right)\,\mathrm{d}\tau \overset{?}{=} 4\pi$$

$$\vec{\nabla}\cdot\left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$$

$$\int\vec{\nabla}\cdot\left(\frac{\hat{r}}{r^2}\right)d\tau = \int 4\pi\,\delta^3(\vec{r})\,d\tau$$

$$= 4\pi\int\delta^3(\vec{r})\,d\tau$$

$$= 4\pi$$