08/03

Pasta 16

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1ª lista MAE 5725 Galarito

1) $\det(B) = \det(B_{22}) \det(B_{11} - B_{12} \cdot B_{22}^{-1} B_{21})$.

$= \det(dI) \det\left(aI - bI(dI)^{-1} cI\right) =$

$= d^m \det\left(\left[a - \frac{bc}{d}\right] I\right) = d^m \left[\frac{ad-bc}{d}\right]^m$

$= (ad-bc)^m$ inversa na pag 1

S/F

2) B inv gen A $\Rightarrow$ BAB inv. gen. A

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$\boxed{ABA = A} \Rightarrow$

$A\,\underbrace{BAB}\,A = ABA = A \Rightarrow$ BAB é inv gen de A

3) $\frac{1}{m} A \frac{1}{m} A \frac{1}{m} A = \frac{1}{m^3} A^2 A = \frac{1}{m^3} m A A =$

$= \frac{1}{m^2} m A = \frac{1}{m} A$ $\therefore$ $\frac{1}{m} A$ é inv gen de

$\frac{1}{m} A$.

1'

$$B^{-1} = \begin{bmatrix} \left[aI - bI\frac{1}{d}I^{-1}cI\right]^{-1} & -\frac{1}{a}I^{-1}bI\left[dI - cI\frac{1}{a}I^{-1}bI\right]^{-1} \\ -\frac{1}{d}I^{-1}cI\left[aI - bI\frac{1}{d}I^{-1}cI\right]^{-1} & \left[dI - cI\frac{1}{a}I^{-1}bI\right]^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} \left[aI - \frac{b}{d}cI\right]^{-1} & -\frac{b}{a}\left[dI - \frac{bc}{a}I\right]^{-1} \\ -\frac{c}{d}\left[aI - \frac{b}{d}cI\right]^{-1} & \left[dI - \frac{bc}{a}I\right]^{-1} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{d}{ad-bc}I & \frac{-b}{ad-bc}I \\ \frac{-c}{ad-bc}I & \frac{a}{ad-bc}I \end{bmatrix}$$

$$= \frac{1}{ad-bc}\begin{bmatrix} dI & -bI \\ -cI & aI \end{bmatrix}$$

4) $A \frac{1}{k} B A = \frac{kA}{k} = A$

5) Sejam $B_{11} = I_2$ $\quad B_{12} = \begin{bmatrix} 0 & 3 \\ 0 & 4 \end{bmatrix}$

$B_{21} = \begin{bmatrix} 0 & 0 \\ 3 & 4 \end{bmatrix}$ $\quad B_{22} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$

$B_{11}^{-1} = I_2$ $\quad B_{22}^{-1} = \frac{1}{5-4} \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$

$$\left[B_{11} - B_{12} B_{22}^{-1} B_{21}\right]^{-1} = \begin{bmatrix} -8 & -12 \\ -12 & -15 \end{bmatrix}^{-1} = \frac{-1}{24} \begin{bmatrix} -15 & 12 \\ 12 & -8 \end{bmatrix}$$

$$-B_{11} B_{12} \left[B_{22} - B_{21} B_{11}^{-1} B_{12}\right]^{-1} = \frac{1}{24} \begin{bmatrix} -6 & 3 \\ -8 & 4 \end{bmatrix}$$

$$-B_{22}^{-1} B_{21} \left[B_{11} - B_{12} B_{22}^{-1} B_{21}\right]^{-1} = \frac{1}{24} \begin{bmatrix} -6 & -8 \\ 3 & 4 \end{bmatrix}$$

$$\left[B_{22} - B_{21} B_{11}^{-1} B_{12}\right]^{-1} = -\frac{1}{24} \begin{bmatrix} -20 & -2 \\ -2 & 1 \end{bmatrix}$$

↳ feito anteriormente

$$\therefore B^{-1} = \frac{1}{24}\begin{bmatrix} 15 & -12 & -6 & 3 \\ -12 & 8 & -8 & 4 \\ -6 & -8 & 20 & 2 \\ 3 & 4 & 2 & -1 \end{bmatrix}$$

6) a) $AXA = AA^{-}A + AUA - \underbrace{AA^{-}A}_{A}U\underbrace{AA^{-}A}_{A}$

$= A + AUA - AUA = A$

$\therefore$ X é in. gen. de A.

b) $AXA = AA^{-}A + AV(I - AA^{-})A + A(I - A^{-}A)WA$

$= A + AVA - AVAA^{-}A + AWA - AA^{-}AWA$

$= A + AVA - AVA + AWA - AWA = A$

$\therefore$ X é in. gen. de A.

7) $ABA = A \Rightarrow (ABA)' = A' \Rightarrow$

$(BA)' A' = A' \Rightarrow A' B' A' = A' \Rightarrow$

B' é in. gen. de A'

8) Devemos mostrar que

$$\underbrace{\begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix}\begin{bmatrix} B^- & 0 \\ 0 & C^- \end{bmatrix}}\begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix} = \begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix}$$

$$\begin{bmatrix} BB^- & 0 \\ 0 & CC^- \end{bmatrix}\begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix} = \begin{bmatrix} BB^-B & 0 \\ 0 & CC^-C \end{bmatrix} = *$$

B^- e C^- in. gen. de B e C $\Rightarrow$

$$* = \begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix}$$ c.q.d.

9) $A = \begin{bmatrix} 3 & 0 \end{bmatrix}_{1\times 2}$ $\quad A^{-}_{(2\times 1)} = \begin{bmatrix} x \\ y \end{bmatrix}$

$$\begin{bmatrix} 3 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 \end{bmatrix}$$

$$3x \begin{bmatrix} 3 & 0 \end{bmatrix} = \begin{bmatrix} 9x & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 \end{bmatrix}$$

$\Rightarrow x = 3 \quad y \in \mathbb{R}$

$\therefore A^{-} = \begin{bmatrix} 3 & 7 \end{bmatrix}$ é uma in. gen. de A

$$A^{-} = \left\{ \begin{bmatrix} 3 & y \end{bmatrix}, \; y \in \mathbb{R} \right\}$$

10) $\begin{cases} 2x_1 + 3x_2 - x_3 = 1 \\ x_2 + x_3 = 2 \end{cases}$

a) $\begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -3/2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix} =$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

b) $A = \begin{array}{c} \begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix} \\ \begin{array}{ccc} \underset{\sim}{c}_1 & \underset{\sim}{c}_2 & \underset{\sim}{c}_3 \end{array} \end{array}$. O sistema é consistente pois

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = a_1 \begin{bmatrix} 2 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix} + a_3 \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ para, por ex.

$a_1 = -\frac{1}{2}$ $a_2 = 1$ $a_3 = 1$, que é uma possível solução.

a solução geral do sistema é

$$\underset{\sim}{x} = A^{-}\underset{\sim}{y} + (A^{-}A - I)\underset{\sim}{z}$$

$$\underset{\sim}{x} = \begin{bmatrix} 1/2 & -3/2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \left(\begin{bmatrix} 1/2 & -3/2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 0 & 1 & 1 \end{bmatrix} - I_3 \right) \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= \begin{bmatrix} -5/2 - 2z_3 \\ 2 + z_3 \\ -z_3 \end{bmatrix} \qquad z_3 \in \mathbb{R}$$

11) a) Posto 2 b) Posto 1

12) $$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$$

$$= (a_{11}x_1 + a_{12}x_2 + a_{13}x_3)x_1 + (a_{12}x_1 + a_{22}x_2 + a_{23}x_3)x_2$$

$$+ (a_{13}x_1 + a_{23}x_2 + a_{33}x_3)x_3 = 2x_1^2 - 2x_1x_2 + x_2^2 + 4x_1x_3 - 3x_3^2$$

$a_{11} = 2 \quad a_{22} = 1 \quad a_{33} = -3$

$a_{12} = -1 \quad a_{13} = 2 \quad a_{23} = 0$

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & 0 \\ 2 & 0 & -3 \end{bmatrix}$$

13) a) $\det = 36$

Posto 3 , não pode ser positiva semi definida

$\therefore$ Positiva definida

b) det = 0

posto < 3, não pode ser positiva definida

∴ Positiva semi definida

c) det = 0 ∴ Positiva semi definida

14) Supondo A triangular superior

$$\left| \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ & a_{22} & & a_{2n} \\ & & & \\ & & & a_{nn} \end{bmatrix} - \lambda \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix} \right| = 0$$

$$\Rightarrow \begin{vmatrix} a_{11}-\lambda & a_{12} & \cdots & a_{1n} \\ & a_{22}-\lambda & & a_{2n} \\ & & & \\ & & & a_{nn}-\lambda \end{vmatrix} = 0$$

$\prod_{i=1}^{n} (a_{ii} - \lambda) = 0$ p.q. det. de matriz triangular é o produto dos elem. da diagonal principal

$$\prod_{i=1}^{n} (a_{ii} - \lambda) = 0 \implies \lambda_1 = a_{11} \quad \lambda_2 = a_{22} \cdots$$

$$\lambda_n = a_{nn}$$

15) $A = \begin{bmatrix} 5 & 2 \\ 2 & 4 \end{bmatrix}$ é a matriz da forma quadrática

$5x_1^2 + 4x_1x_2 + 4x_2^2$.

A é positiva definida pois $a_{11} > 0$ e $\det\left(\begin{bmatrix} 5 & 2 \\ 2 & 4 \end{bmatrix}\right) > 0$

$\therefore \; 5x_1^2 + 4x_1x_2 + 4x_2^2 > 0 \quad \forall \; (x_1, x_2) \in \mathbb{R}^2$.

16) $B = I + \underset{\sim}{b}\,\underset{\sim}{b}' \qquad \underset{\sim}{b}' = [1 \; 2 \; 1 \; 3] \qquad \underset{\sim}{b} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}$

$$B^{-1} = I^{-1} - \frac{(I^{-1}\underset{\sim}{b})(\underset{\sim}{b}'I^{-1})}{1 + \underset{\sim}{b}'I^{-1}\underset{\sim}{b}} =$$

$$= I_4 - \frac{1}{16} \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 4 & 2 & 6 \\ 1 & 2 & 1 & 3 \\ 3 & 6 & 3 & 9 \end{bmatrix} = \begin{bmatrix} 15/16 & 2/16 & 1/16 & 3/16 \\ 2/16 & 12/16 & 2/16 & 6/16 \\ 1/16 & 2/16 & 15/16 & 3/16 \\ 3/16 & 6/16 & 3/16 & 7/16 \end{bmatrix}$$

17) $$\mathrm{Var}\begin{bmatrix} \underset{\sim}{\epsilon}_1 \\ \underset{\sim}{\epsilon}_2 \\ \vdots \\ \underset{\sim}{\epsilon}_n \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} & \underset{2\times 2}{0} & \cdots & \underset{2\times 2}{0} \\ 0 & \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} & & \\ & & \ddots & \\ 0 & & & \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \end{bmatrix}_{2n\times 2n}$$

$$= I_n \otimes \Sigma \qquad \Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$$

18) $$\mathrm{Var}\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{bmatrix} \mathrm{Var}(Y_1) & \mathrm{Cov}(Y_1, Y_2) \\ \mathrm{Cov}(Y_1, Y_2) & \mathrm{Var}(Y_2) \end{bmatrix} = \begin{bmatrix} \sigma^2 & -\sigma^2 \\ -\sigma^2 & \sigma^2 \end{bmatrix}$$

$$\mathrm{Cov}(Y_1, Y_2) = \mathrm{Cov}(X, 1-X) = E(X(1-X)) - E(X)\,E(1-X)$$
$$= \mu - [\sigma^2 + \mu^2] - \mu(1-\mu) = -\sigma^2$$
$$\mathrm{Var}(1-X) = \mathrm{Var}(X) = \sigma^2$$

$$(x, y)\begin{bmatrix}\sigma^2 & -\sigma^2\\ -\sigma^2 & \sigma^2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = (x^2 - 2xy + y^2)\sigma^2$$

$= (x-y)^2\sigma^2 \geq 0$ | pode ser nulo para $\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}1\\ +1\end{bmatrix} \neq \underset{\sim}{0}$

$\therefore$ Var $\begin{pmatrix}Y_1\\ Y_2\end{pmatrix}$ é positiva semi-definida.

19) $[x_1 \quad x_2]\begin{bmatrix}a_{11} & a_{12}\\ a_{12} & a_{22}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} = a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2$

$A = \begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}$ $\quad a_{11} = 3 > 0$ $\quad \begin{vmatrix}3 & 1\\ 1 & 3\end{vmatrix} = 8 > 0$ $\quad$ A é positiva definida

20) a) $A'A$ é simétrica pois $(A'A)' = A'(A')' = A'A$

b) $\underset{\sim}{x}'A'A\underset{\sim}{x} = \underset{\sim}{y}'\underset{\sim}{y} \geq 0 \qquad \underset{\sim}{y} = A\underset{\sim}{x}$

Pode ser nula sem que $\underset{\sim}{x} = \underset{\sim}{0}$

Ex $\quad A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \qquad A\underset{\sim}{x} = \underset{\sim}{0}$

$\Longleftrightarrow \begin{cases} x_1 + 2x_2 = 0 \\ 2x_1 + 4x_2 = 0 \end{cases}$ infinitas sol.

ou

se A não é de posto completo, A'A não é de posto completo $\Rightarrow$ A'A é positiva semi-definida

21 a) $A = E\,D\,E'$

$E = \begin{bmatrix} \underset{\sim}{x}_1 & \underset{\sim}{x}_2 \cdots & \underset{\sim}{x}_p \end{bmatrix}$ $\quad \underset{\sim}{x}_1, \underset{\sim}{x}_2 \cdots \underset{\sim}{x}_p$ vetores

característicos de A padronizados $[\ \underset{\sim}{x}_1' \underset{\sim}{x}_1 = 1\ ]$

$$E'E = \begin{bmatrix} \underset{\sim}{x}_1' \\ \underset{\sim}{x}_2' \\ \vdots \\ \underset{\sim}{x}_p' \end{bmatrix} \begin{bmatrix} \underset{\sim}{x}_1 & \underset{\sim}{x}_2 \cdots & \underset{\sim}{x}_p \end{bmatrix} = \begin{bmatrix} \underset{\sim}{x}_1'\underset{\sim}{x}_1 & \underset{\sim}{x}_1'\underset{\sim}{x}_2 & \cdots & \underset{\sim}{x}_1'\underset{\sim}{x}_p \\ \underset{\sim}{x}_2'\underset{\sim}{x}_1 & \underset{\sim}{x}_2'\underset{\sim}{x}_2 & & \underset{\sim}{x}_2'\underset{\sim}{x}_p \\ & & & \\ \underset{\sim}{x}_p'\underset{\sim}{x}_1 & \underset{\sim}{x}_p'\underset{\sim}{x}_2 & \cdots & \underset{\sim}{x}_p'\underset{\sim}{x}_p \end{bmatrix}$$

$\underset{\sim}{x}_i' \underset{\sim}{x}_j = 0 \quad i \neq j$

$\Rightarrow E'E = I_p$

E é ortogonal (pois $\underset{\sim}{x}_i' \underset{\sim}{x}_i = 1$ e $\underset{\sim}{x}_i' \underset{\sim}{x}_j = 0$) $\Rightarrow$

$E^{-1} = E' \Rightarrow EE' = EE^{-1} = I_p$

b) Seja $P = ED^{1/2}E'$,

$$PP = ED^{1/2}\underbrace{E'E}_{I}D^{1/2}E' = E\underbrace{D^{1/2}D^{1/2}}E' = EDE'$$

$D = \text{diag}(\lambda_1, \lambda_2 \ldots \lambda_p)$

c) $A^{-1} = ED^{-1}E'$

$$A^{-1}A = ED^{-1}\underbrace{E'E}_{I}DE' = ED^{-1}DE' = EE' = I$$

Ex 22 - 1ª lista $A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$

a)

$\det(A - \lambda I) = 0$

$$\begin{vmatrix} 9-\lambda & -2 \\ -2 & 6-\lambda \end{vmatrix} = (9-\lambda)(6-\lambda) - 4 = 54 - 9\lambda - 6\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 15\lambda + 50 = 0$$

$\lambda_1 = 10 \qquad \lambda_2 = 5$

$\lambda_1 = 10$

$$\begin{bmatrix} 9-10 & -2 \\ -2 & 6-10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 \\ -2 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{aligned} -x_1 - 2x_2 &= 0 \\ -2x_1 - 4x_2 &= 0 \end{aligned}$$

$x_1 = -2x_2 \qquad x_2 = 1 \qquad x_1 = -2$

$\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ Normalizando: $\begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$

$\lambda_2 = 5$

$$\begin{bmatrix} +4 & -2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_1 = 2 \quad x_2 = -4$

$$\begin{bmatrix} 2 \\ +4 \end{bmatrix}$$

normalizando : $\begin{bmatrix} 2/\sqrt{20} \\ +4/\sqrt{20} \end{bmatrix}$

b)

$$\begin{array}{ccc} E & D & E' \end{array}$$

$$\begin{bmatrix} -2/\sqrt{5} & 2/\sqrt{20} \\ 1/\sqrt{5} & +4/\sqrt{20} \end{bmatrix} \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \\ 2/\sqrt{20} & +4/\sqrt{20} \end{bmatrix} =$$

$$\begin{bmatrix} -20/\sqrt{5} & 10/\sqrt{20} \\ 10/\sqrt{5} & +20/\sqrt{20} \end{bmatrix} \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \\ 2/\sqrt{20} & +4/\sqrt{20} \end{bmatrix} =$$

$$\begin{bmatrix} +8+1 & -4+2 \\ -4+2 & 2+4 \end{bmatrix} = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$$

c) $A^{-1} = E D^{-1} E'$

$$A^{-1} = \begin{bmatrix} -2/\sqrt{5} & 2/\sqrt{20} \\ 1/\sqrt{5} & 4/\sqrt{20} \end{bmatrix} \begin{bmatrix} 1/10 & 0 \\ 0 & 1/5 \end{bmatrix} \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \\ 2/\sqrt{20} & 4/\sqrt{20} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{5\sqrt{5}} & \frac{2}{5\sqrt{20}} \\ \frac{1}{10\sqrt{5}} & \frac{4}{5\sqrt{20}} \end{bmatrix} \begin{bmatrix} \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{20}} & \frac{4}{\sqrt{20}} \end{bmatrix} \Rightarrow$$

$$= \begin{bmatrix} \frac{2}{25} + \frac{2}{50} & -\frac{2}{50} + \frac{8}{5.20} \\ -\frac{2}{50} + \frac{4}{50} & \frac{2}{100} + \frac{16}{100} \end{bmatrix} = \begin{bmatrix} \frac{6}{50} & \frac{4}{100} \\ \frac{2}{50} & \frac{18}{50} \end{bmatrix}$$

d) $P = E D^{1/2} E'$

$$P = \begin{bmatrix} -2/\sqrt{5} & 2/\sqrt{20} \\ 1/\sqrt{5} & 4/\sqrt{20} \end{bmatrix} \begin{bmatrix} \sqrt{10} & 0 \\ 0 & \sqrt{5} \end{bmatrix} \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \\ 2/\sqrt{20} & 4/\sqrt{20} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4\sqrt{2}+1}{\sqrt{5}} & \frac{-2\sqrt{2}+2}{\sqrt{5}} \\ \frac{-2\sqrt{2}+2}{\sqrt{5}} & \frac{\sqrt{2}+4}{\sqrt{5}} \end{bmatrix}$$

23) $\underset{\sim}{x}' A \underset{\sim}{x} = \underbrace{\underset{\sim}{x}' E}_{\underset{\sim}{y}'} D \underbrace{E' \underset{\sim}{x}}_{\underset{\sim}{y}}$ para $\underset{\sim}{y} = E' \underset{\sim}{x}$

24) $\underset{\sim}{x}' A \underset{\sim}{x} = \underbrace{\underset{\sim}{x}' E D^{1/2}}_{y'} \underbrace{D^{1/2} E' \underset{\sim}{x}}_{y} = \underset{\sim}{y}' \underset{\sim}{y}$

$\underset{\sim}{y} = B \underset{\sim}{x}$ para $B = D^{1/2} E'$

25) $\underbrace{A A^{-1}}_{I} A = A$; A^{-1} é inv. gen. de A

Suponhamos que exista outra in. gen. de A, A^{-}.
Então
$A A^{-} A = A$

A inversível $\Rightarrow A^{-1} A A^{-} A = A^{-1} A \Rightarrow A^{-} A = I$

$\Rightarrow A^{-} A A^{-1} = A^{-1} \Rightarrow A^{-} = A^{-1}$.

26) - Modelo de Regressão Linear Múltipla $k_1 + k_2$ v. indep

$$\underset{\sim}{Y} = X \underset{\sim}{\beta} + \underset{\sim}{\epsilon}$$

$$X = \begin{bmatrix} \underset{n \times (k_1+1)}{X_1} & \underset{n \times k_2}{X_2} \end{bmatrix}$$

$\hat{\underset{\sim}{\beta}} = (X'X)^{-1} X' \underset{\sim}{Y}$ → estimador de mínimos quadrados usual

$$\hat{\underset{\sim}{\beta}} = \begin{bmatrix} \hat{\underset{\sim}{\beta}}_1 \\ \hat{\underset{\sim}{\beta}}_2 \end{bmatrix} \qquad \hat{\underset{\sim}{\beta}}_1 = \begin{bmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_{k_1} \end{bmatrix} \qquad \hat{\underset{\sim}{\beta}}_2 = \begin{bmatrix} \hat{\beta}_{k_1+1} \\ \vdots \\ \hat{\beta}_{k_1+k_2} \end{bmatrix}$$

Determinar $\hat{\underset{\sim}{\beta}}_2$ na forma matricial

$$X'X = \begin{bmatrix} X_1' \\ X_2' \end{bmatrix} \begin{bmatrix} X_1 & X_2 \end{bmatrix} = \begin{bmatrix} \overset{(k_1+1)\times(k_1+1)}{X_1' X_1} & \overset{(k_1+1)\times k_2}{X_1' X_2} \\ X_2' X_1 & X_2' X_2 \end{bmatrix} \qquad (X'X)^{-1} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\hat{\underset{\sim}{\beta}} = (X'X)^{-1} X' \underset{\sim}{Y} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X_1' \\ X_2' \end{bmatrix} \underset{\sim}{Y} =$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X_1' \underset{\sim}{Y} \\ X_2' \underset{\sim}{Y} \end{bmatrix} = \begin{bmatrix} \overset{(k_1+1)\times 1}{A X_1' \underset{\sim}{Y} + B X_2' \underset{\sim}{Y}} \\ \underset{k_2 \times 1}{C X_1' \underset{\sim}{Y} + D X_2' \underset{\sim}{Y}} \end{bmatrix} = \begin{bmatrix} \hat{\underset{\sim}{\beta}}_1 \\ \hat{\underset{\sim}{\beta}}_2 \end{bmatrix}$$

Obs:

Se $X_1' X_2 = 0$, $(X'X)^{-1} = \begin{bmatrix} (X_1' X_1)^{-1} & 0 \\ 0 & (X_2' X_2)^{-1} \end{bmatrix}$ e

$$\hat{\underset{\sim}{\beta}} = \begin{bmatrix} (X_1' X_1)^{-1} X_1' \underset{\sim}{Y} \\ (X_2' X_2)^{-1} X_2' \underset{\sim}{Y} \end{bmatrix}$$

27) O estimador de mínimos quadrados de $\underset{\sim}{\beta}$ minimiza

$$S = (\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}})'(\underset{\sim}{Y} - X\hat{\underset{\sim}{\beta}})$$

$$= \underset{\sim}{Y}'\underset{\sim}{Y} - 2\underset{\sim}{Y}'X\hat{\underset{\sim}{\beta}} + \hat{\underset{\sim}{\beta}}'X'X\hat{\underset{\sim}{\beta}}$$

$$\frac{\partial S}{\partial \hat{\underset{\sim}{\beta}}} = -2X'\underset{\sim}{Y} + 2X'X\hat{\underset{\sim}{\beta}} = \underset{\sim}{0}$$

$$X'X\hat{\underset{\sim}{\beta}} = X'\underset{\sim}{Y}$$

$$\hat{\underset{\sim}{\beta}} = (X'X)^{-1}X'\underset{\sim}{Y}$$