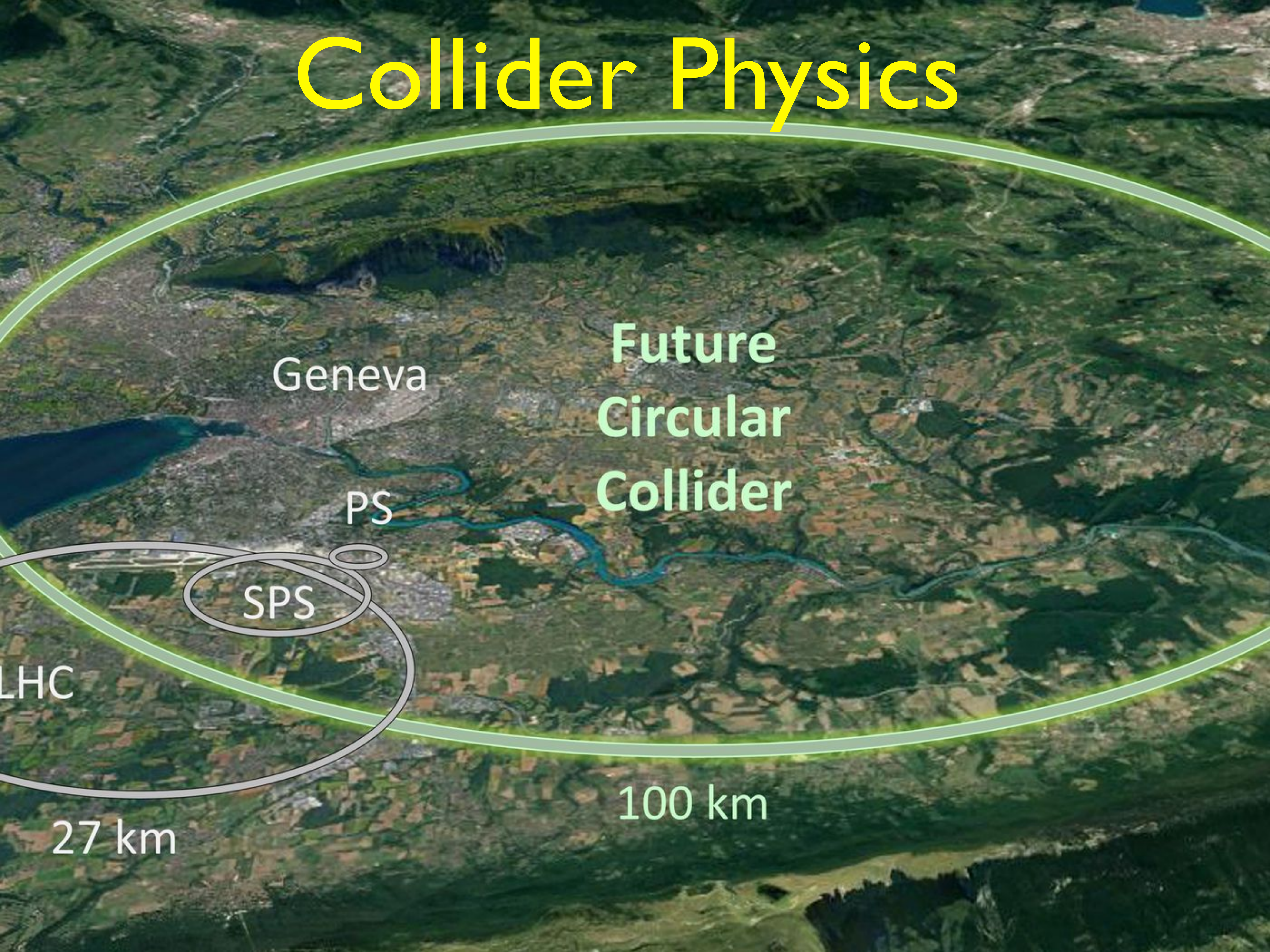
Collider Physics



II.C Deep Inelastic Scattering

- Elastic ep scattering
- DIS kinematics
- Inelastic ep scattering
- Bjorken scaling
- Parton model
- Extracting PDFs

Basic idea

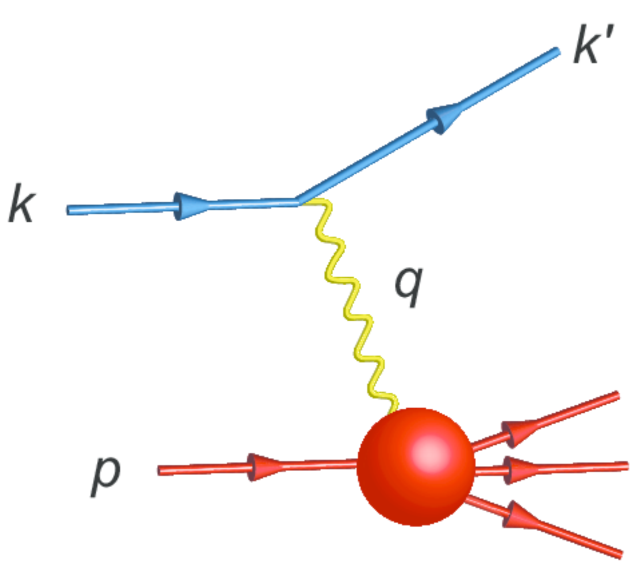
- Study the scattering of an elementary particle of a proton
- Extract the information about the composite object (p)
- From QM

form factor

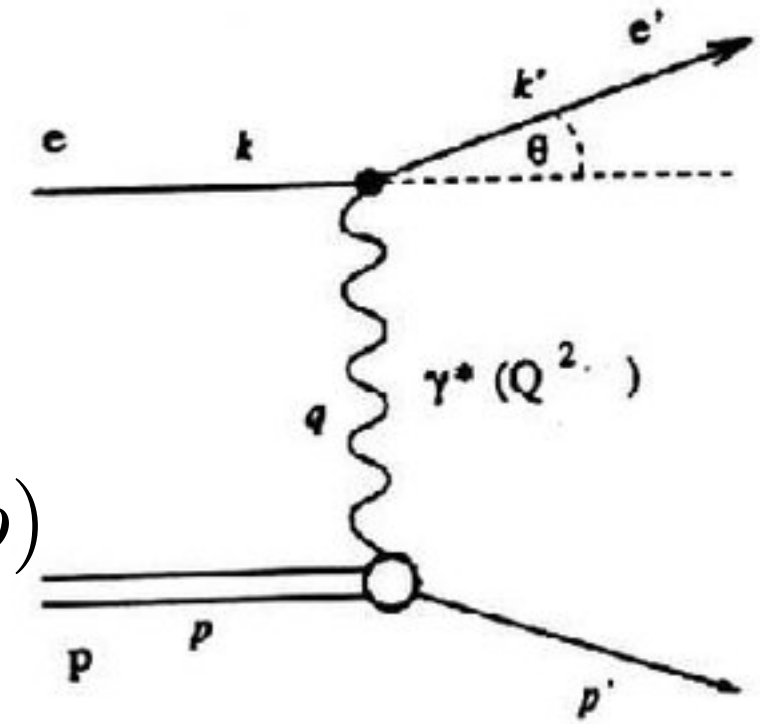
$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{point}} |F(q)|^2$$

- For continuous matter/charge distributions $F(q) \to 0 \ \text{ for } \ q \to \infty$
- For exponential charge distribution

$$F(q) \propto \left(1 - \frac{q^2}{m^2}\right)^{-2}$$



Elastic ep scattering



- We can parametrize the proton current as

$$J^{\mu} = e\bar{u}(p')\left[F_1(q^2)\gamma^{\mu} + \frac{\kappa}{2M}F_2(q^2)i\sigma^{\mu\nu}q_{\nu}\right]u(p)$$

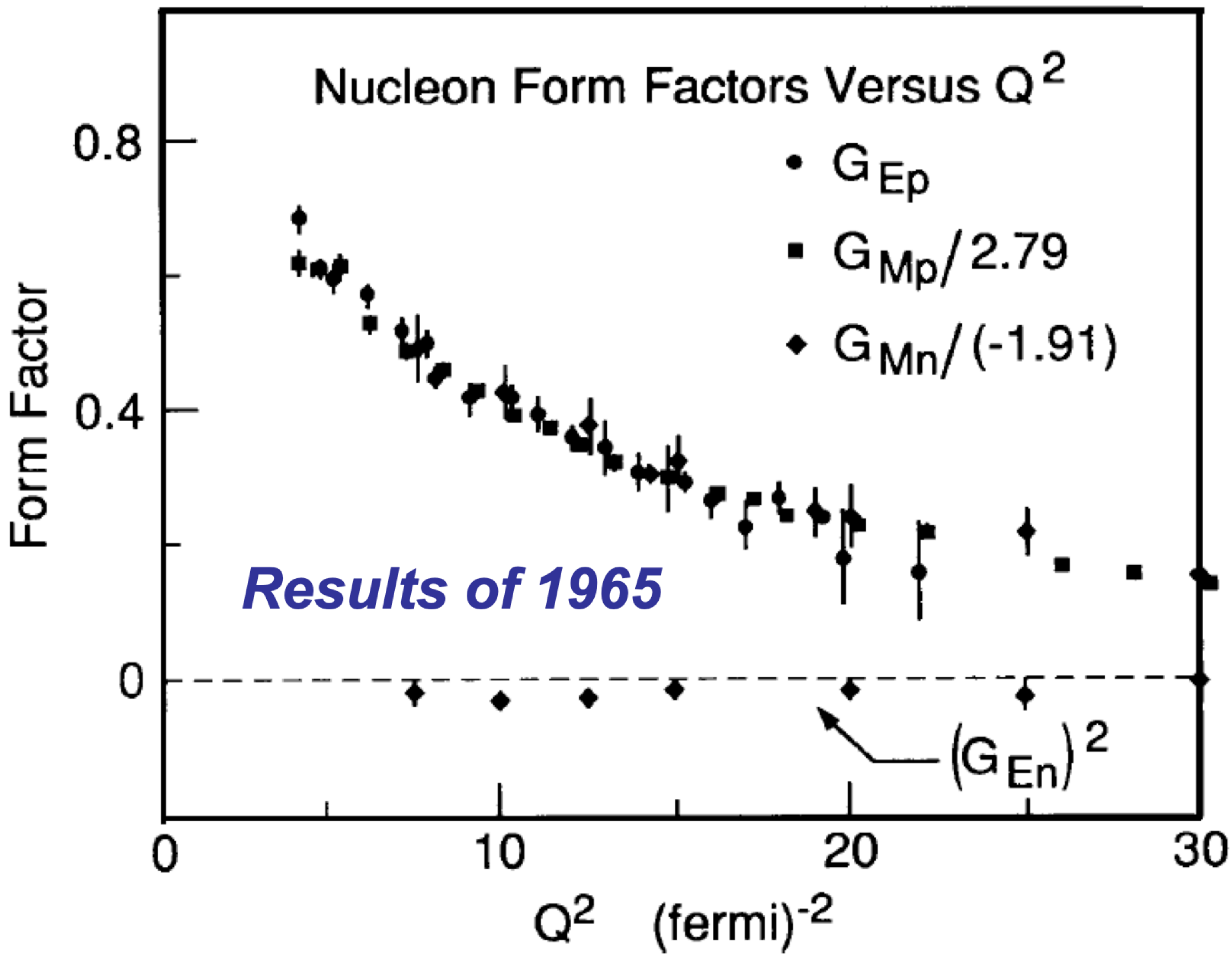
with $\quad F_1(0) = F_2(0) = 1$

$$\frac{E'}{E} = \frac{1}{1 + \frac{2E}{M}\sin^2\frac{\theta}{2}}$$

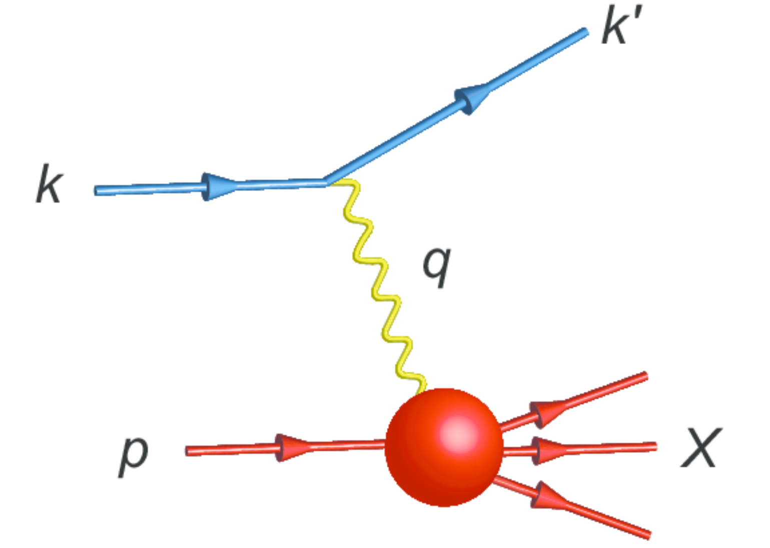
- The cross section is then

$$\left.\frac{d\sigma}{d\Omega}\right|_{lab} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\,\frac{E'}{E}\left(\frac{G_E^2 + \tau G_M^2}{1+\tau}\,\cos^2\frac{\theta}{2} + 2\tau G_M^2\sin^2\frac{\theta}{2}\right)$$

$$G_E = F_1 + \frac{\kappa q^2}{4M^2}F_2 \quad ; \quad G_M = F_1 + \kappa F_2 \quad ; \quad \tau = \frac{-q^2}{4M^2}$$


Nucleon Form Factors Versus Q 2
G Ep
G Mp/ 2.79
G Mn/ (-1.91)
0.8
0.4
0
Form Factor
Results of 1965
(G En) 2
0
10
20
30
Q2 (fermi)-2

DIS kinematical variables



$$M_X \equiv W$$

- Independent Lorentz invariant variables

$$q^2 \quad \text{and} \quad \nu = \frac{p \cdot q}{M}$$

in the proton resting (lab) frame $\nu = E - E'$

- We know that $W^2 = (p+q)^2 = M^2 + 2M\nu + q^2$

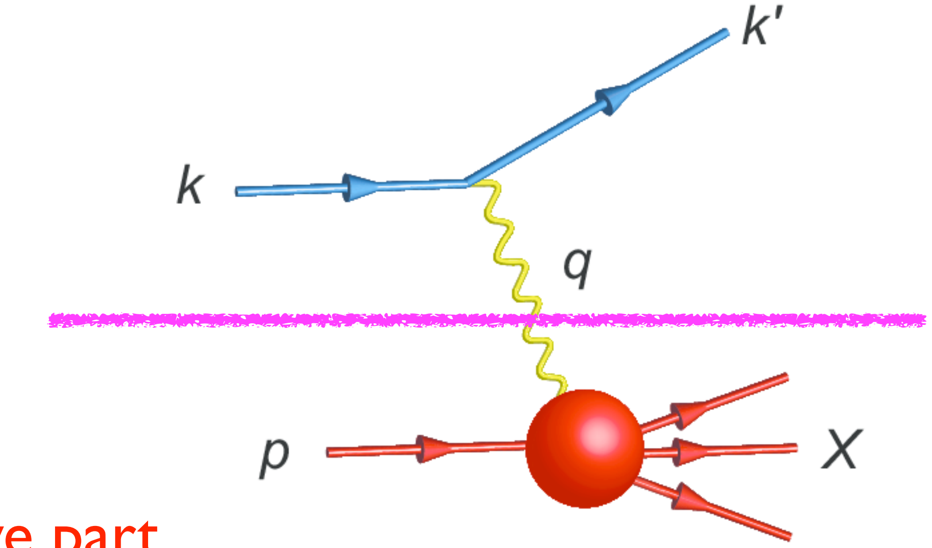
- Useful dimensionless variables

$$x = \frac{-q^2}{2p \cdot q} = \frac{-q^2}{2M\nu} \quad \text{and} \quad y = \frac{p \cdot q}{p \cdot k}$$

- Physical range

$$0 \leq x \leq 1 \quad ; \quad 0 \leq y \leq 1 \quad ; \quad 0 \leq Q^2 \leq 2M\nu \quad ; \quad 0 \leq \nu \leq \frac{s - M^2}{2M}$$

Inelastic ep scattering $e + p \to e + X$



$$\mathcal{M} = [\bar{u}(k')(-ie\gamma^\mu)u(k)] \; \frac{-ie}{q^2} \; \langle X|J_\mu|P\rangle$$

non perturbative part

- Leading to the cross section

proton contribution

$$\frac{d\sigma}{dE'd\Omega} = \frac{4\alpha^2 M^2}{q^4} \; (L^e)^{\mu\nu} W_{\mu\nu}$$

electron contribution

$$W_{\mu\nu} = \frac{1}{4\pi M} \sum_N \frac{1}{2} \sum_s \int \prod_{n=1}^{N} \left(\frac{d^3 p'_n}{2E'_n (2\pi)^3} \right)$$

$$\sum_{s_n} \langle p, s|J^\dagger_\mu|X\rangle\langle X|J_\nu|p, s\rangle \delta^4(p + q - \sum_n P'_n)$$

UFA!

- Certainly we don't know how to calculate $W_{\mu\nu}$
- So we introduce form factors (why this form?)

$$W^{\mu\beta} = W_1(q^2,\nu)\left(-g^{\mu\beta} + \frac{q^\mu q^\beta}{q^2}\right) + W_2(q^2,\nu)\frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}q^\mu\right)\left(p^\beta - \frac{p\cdot q}{q^2}q^\beta\right)$$

we don't know

- Leading to

$$\left.\frac{d\sigma}{dE'd\Omega}\right|_{lab} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\left[W_2(q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(q^2,\nu)\sin^2\frac{\theta}{2}\right]$$

- For a point particle (Q=+1)

$$W_1^{point} = \frac{Q^2}{2m^2}\delta\left(\nu - \frac{Q^2}{2m}\right) \quad ; \quad W_2^{point} = \delta\left(\nu - \frac{Q^2}{2m}\right) \quad ; \quad Q^2 = -q^2$$

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- So we introduce form factors (why this form?)

$$W^{\mu\beta} = W_1(q^2,\nu)\left(-g^{\mu\beta}+\frac{q^\mu q^\beta}{q^2}\right)+W_2(q^2,\nu)\frac{1}{M^2}\left(p^\mu-\frac{p\cdot q}{q^2}q^\mu\right)\left(p^\beta-\frac{p\cdot q}{q^2}q^\beta\right)$$

we don't know

$$q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$$

- Leading to

$$\left.\frac{d\sigma}{dE' d\Omega}\right|_{lab} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\left[W_2(q^2,\nu)\cos^2\frac{\theta}{2}+2W_1(q^2,\nu)\sin^2\frac{\theta}{2}\right]$$

- For a point particle (Q=+1)

$$W_1^{point} = \frac{Q^2}{2m^2}\delta\left(\nu-\frac{Q^2}{2m}\right) \quad ; \quad W_2^{point} = \delta\left(\nu-\frac{Q^2}{2m}\right) \quad ; \quad Q^2 = -q^2$$

Bjorken scaling (DIS)

$$2mW_1^{point} = \frac{Q^2}{2m\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right) \quad ; \quad \nu W_2^{point} = \delta\left(1 - \frac{Q^2}{2m\nu}\right)$$

point form factors depend on a single dimensionless variable!

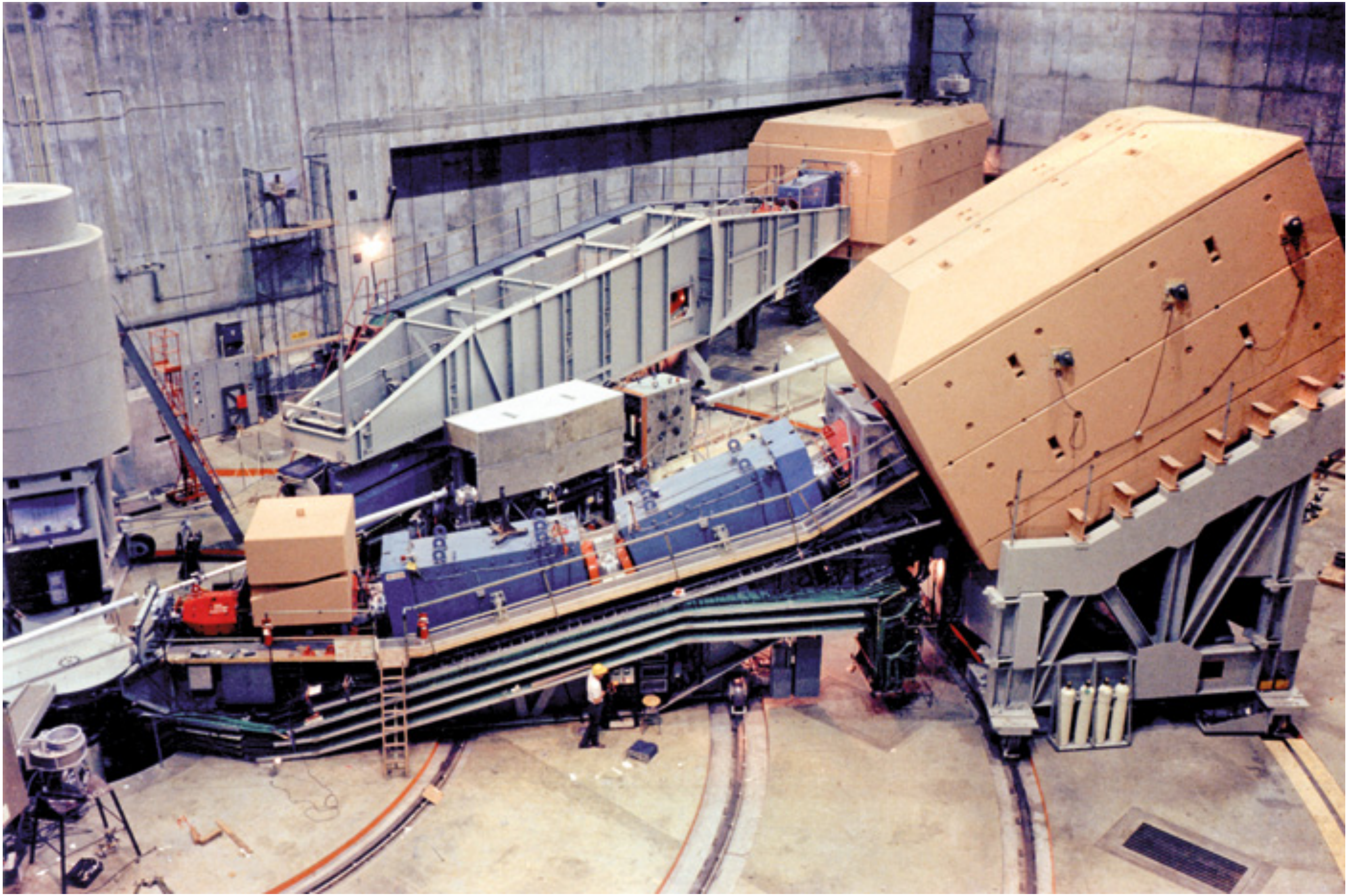
- For elastic scattering $(\kappa = 0)$

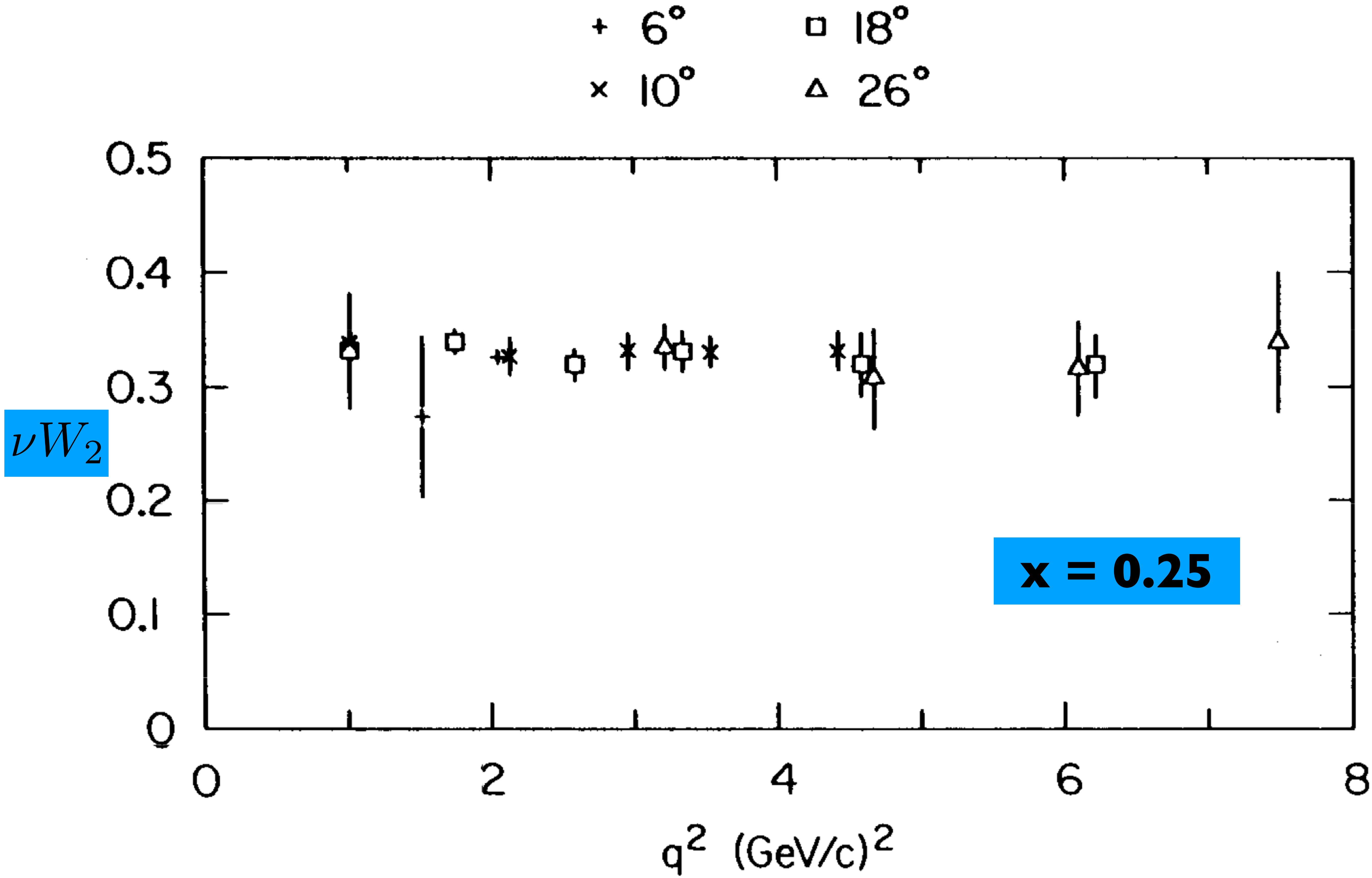
$$W_1^{elastic} = \frac{Q^2}{4M^2} G^2(Q^2)\ \delta\left(\nu - \frac{Q^2}{2M}\right)$$

- If there are elementary constituents at large Q^2 we should have

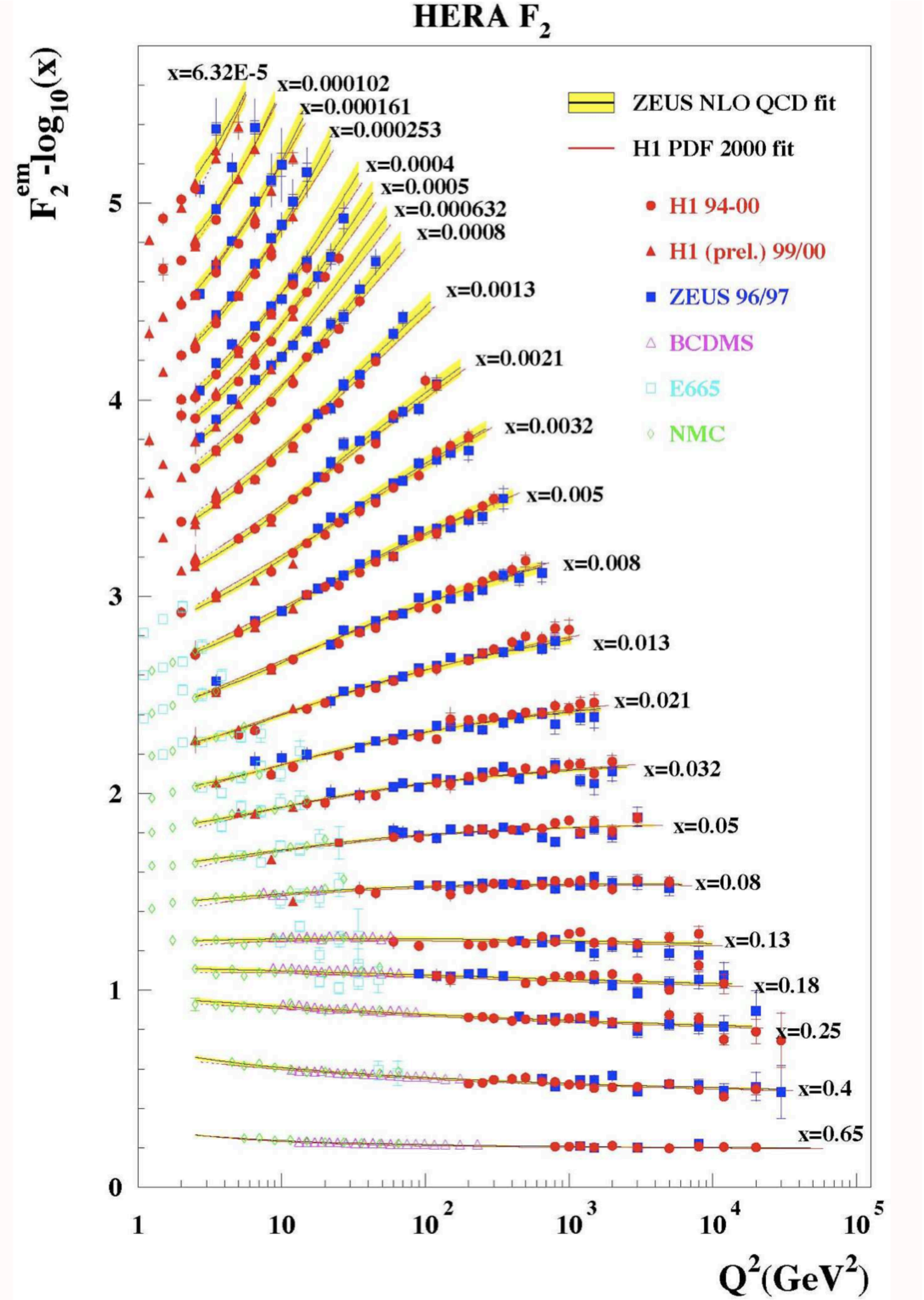
$$MW_1(Q^2,\nu) \to F_1(x)$$
$$\nu W_2(Q^2,\nu) \to F_2(x)$$

$$Q^2 \to \infty \text{ and } x = \frac{Q^2}{2m\nu} \text{ fixed}$$




+ 6°
□ 18°
× 10°
△ 26°
0.5
0.4
0.3
0.2
0.1
0
νW_2
x = 0.25
0
2
4
6
8
q² (GeV/c)²

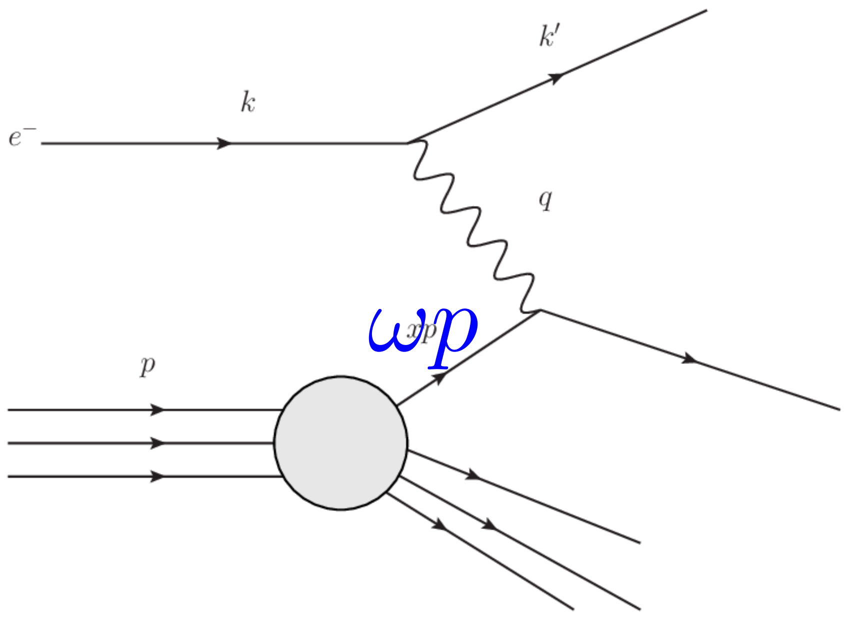
- QCD predicts scaling violation



Parton Model

$$\sigma(e(k)p(p) \to e(k')X) = \sum_j \int d\omega\ f_j(\omega)\ e_j^2\ \hat{\sigma}(e(k)q(\omega p) \to e(k')\dots)$$

there is the sum rule $\sum_j \int d\omega\ f_j(\omega) = 1$



- Let's apply the parton model to DIS

$$p_j = \omega p \to m = \omega M \quad \text{and} \quad \nu = \frac{p_j \cdot q}{m} = \frac{p \cdot q}{M}$$

$$\nu W_2^{point} = \delta\left(1 - \frac{Q^2}{2m\nu}\right) = \delta\left(1 - \frac{Q^2}{2M\nu\omega}\right) = \delta\left(1 - \frac{x}{\omega}\right)$$

$$\nu W_2^{point} = \delta\left(1 - \frac{Q^2}{2m\nu}\right) = \delta\left(1 - \frac{Q^2}{2M\nu\omega}\right) = \delta\left(1 - \frac{x}{\omega}\right)$$

$$F_2 = \nu W_2 = \sum_j \int d\omega\ f_j(\omega)\ e_j^2\ \nu W_2^{point} = \sum_j \int d\omega\ f_j(\omega)\ e_j^2\ \delta\left(1 - \frac{x}{\omega}\right)$$

$$= \sum_j e_j^2 x f_j(x)$$

- Analogously,

$$MW_1^{point} = \frac{Q^2 M}{4m^2\nu}\delta\ \left(1 - \frac{Q^2}{2m\nu}\right) = \frac{x}{2\omega^2}\delta\left(1 - \frac{x}{\omega}\right)$$

$$F_1 = MW_1 = \sum_j \int d\omega\ f_j(\omega)\ e_j^2\ MW_1^{point}$$

$$= \frac{1}{2}\sum_j e_j^2 f_j(x)$$

$$F_2(x) = 2xF_1(x)$$

Callan-Gross relation

Extracting PDFs

- First let's rewrite in terms of Lorentz invariant variables

$$y = \frac{q \cdot p}{k \cdot p} = \frac{E' - E}{E} \Longrightarrow E' = E(1-y)$$

$$Q^2 = -(k-k')^2 = 4EE' \sin^2 \frac{\theta}{2}$$

$$x = \frac{Q^2}{2p \cdot q} = \frac{E(1-y)}{My}\,(1-\cos\theta) \Longrightarrow \sin^2\frac{\theta}{2} = \frac{Mxy}{2E(1-y)}$$

- From this we have

$$\frac{d\sigma}{dxdy} = \frac{2\pi M y}{1-y} \left.\frac{d\sigma}{dE' d\Omega}\right|_{lab}$$

- Starting from

$$\left.\frac{d\sigma}{dE' d\Omega}\right|_{lab} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left[W_2(q^2, \nu) \cos^2 \frac{\theta}{2} + 2W_1(q^2, \nu) \sin^2 \frac{\theta}{2}\right]$$

we get

$$\frac{d\sigma}{dx dy} = \frac{4\pi\alpha^2 s}{Q^4} \left\{F_1 x y^2 + F_2(1-y) = \frac{xM}{2E} F_2\right\}$$

with

$$F_1 = MW_1 \quad \text{and} \quad F_2 = \nu W_2$$

$$F_1 = \frac{1}{2}\sum_j e_j^2 f_j(x) = \frac{1}{2}\left[\frac{4}{9}(u(x) + \bar{u}(x) + c(x) + \bar{c}(x)) + \frac{2}{9}(d(x) + \bar{d}(x) + s(x) + \bar{s}(x))\right]$$

$$F_2 = \sum_j e_j^2 x f_j(x) = x\left[\frac{4}{9}(u(x) + \bar{u}(x) + c(x) + \bar{c}(x)) + \frac{2}{9}(d(x) + \bar{d}(x) + s(x) + \bar{s}(x))\right]$$

- Starting from

$$\left.\frac{d\sigma}{dE' d\Omega}\right|_{lab} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left[W_2(q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(q^2,\nu)\sin^2\frac{\theta}{2}\right]$$

we get

$$\frac{d\sigma}{dx\,dy} = \frac{4\pi\alpha^2 s}{Q^4}\left\{F_1 x y^2 + F_2(1-y) = \frac{xM}{2E}F_2\right\}$$

with

$$F_1 = MW_1 \quad \text{and} \quad F_2 = \nu W_2$$

$$F_1 = \frac{1}{2}\sum_j e_j^2 f_j(x) = \frac{1}{2}\left[\frac{4}{9}(u(x)+\bar{u}(x)+c(x)+\bar{c}(x)) + \frac{2}{9}(d(x)+\bar{d}(x)+s(x)+\bar{s}(x))\right]$$

$$F_2 = \sum_j e_j^2 x f_j(x) = x\left[\frac{4}{9}(u(x)+\bar{u}(x)+c(x)+\bar{c}(x)) + \frac{2}{9}(d(x)+\bar{d}(x)+s(x)+\bar{s}(x))\right]$$

We need further information to extract PDFs

- Let's use neutrino scattering

$$\nu_\mu + \text{nucleon} \to \mu^- + \text{ hadrons}$$
$$\bar{\nu}_\mu + \text{nucleon} \to \mu^+ + \text{ hadrons}$$

- We parametrize the cross section as

$$\frac{d\sigma^\nu}{dxdy} = \frac{G_F^2 M E}{\pi}\left[\left(1 - y - \frac{M}{2E}xy\right)\ F_2^\nu + xy^2\ F_1^\nu + \left(y + \frac{y^2}{2}\right)\ xF_3^\nu\right]$$

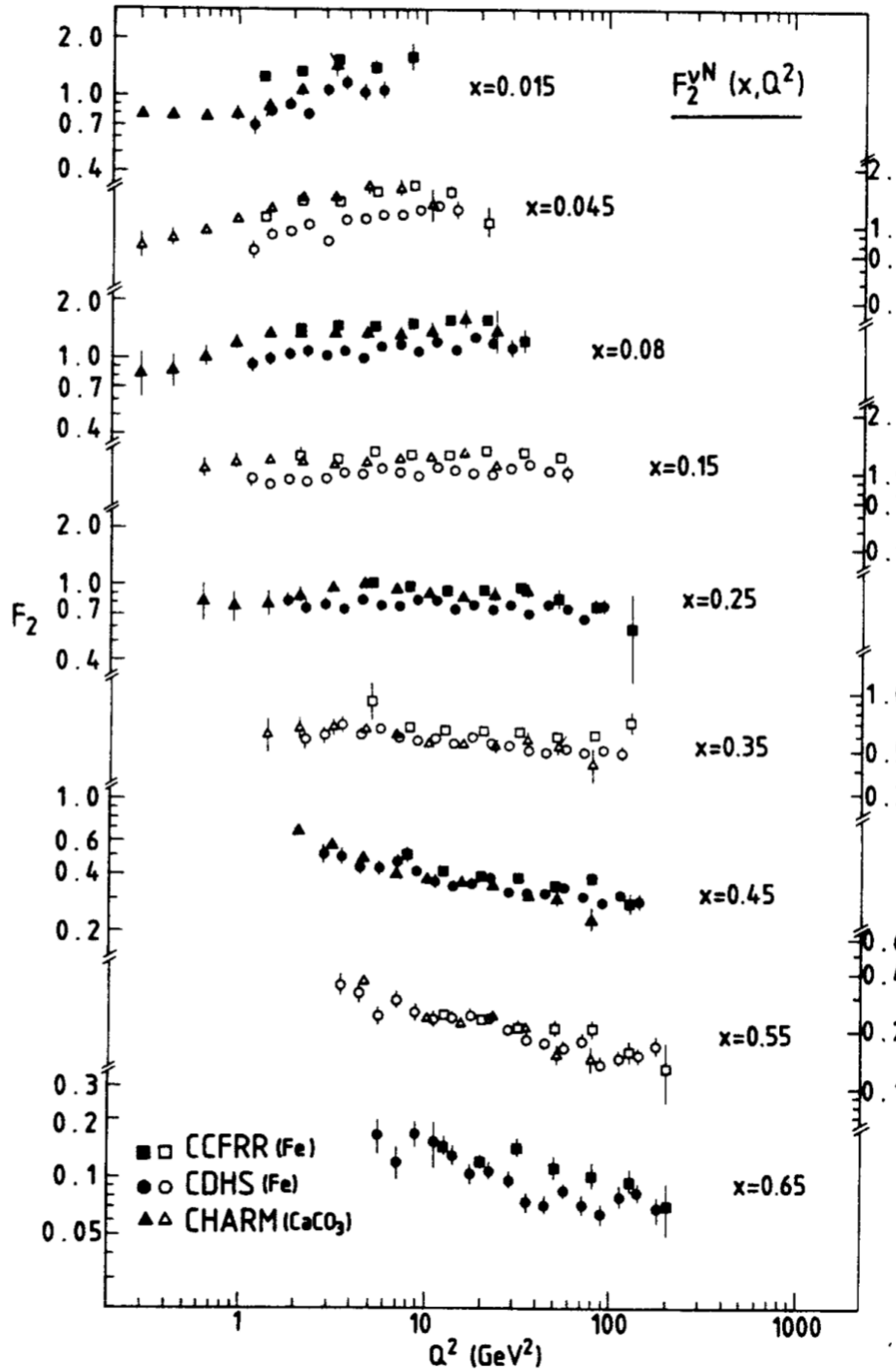
$$\frac{d\sigma^{\bar{\nu}}}{dxdy} = \frac{G_F^2 M E}{\pi}\left[\left(1 - y - \frac{M}{2E}xy\right)\ F_2^{\bar{\nu}} + xy^2\ F_1^\nu + \left(y - \frac{y^2}{2}\right)\ xF_3^{\bar{\nu}}\right]$$

with

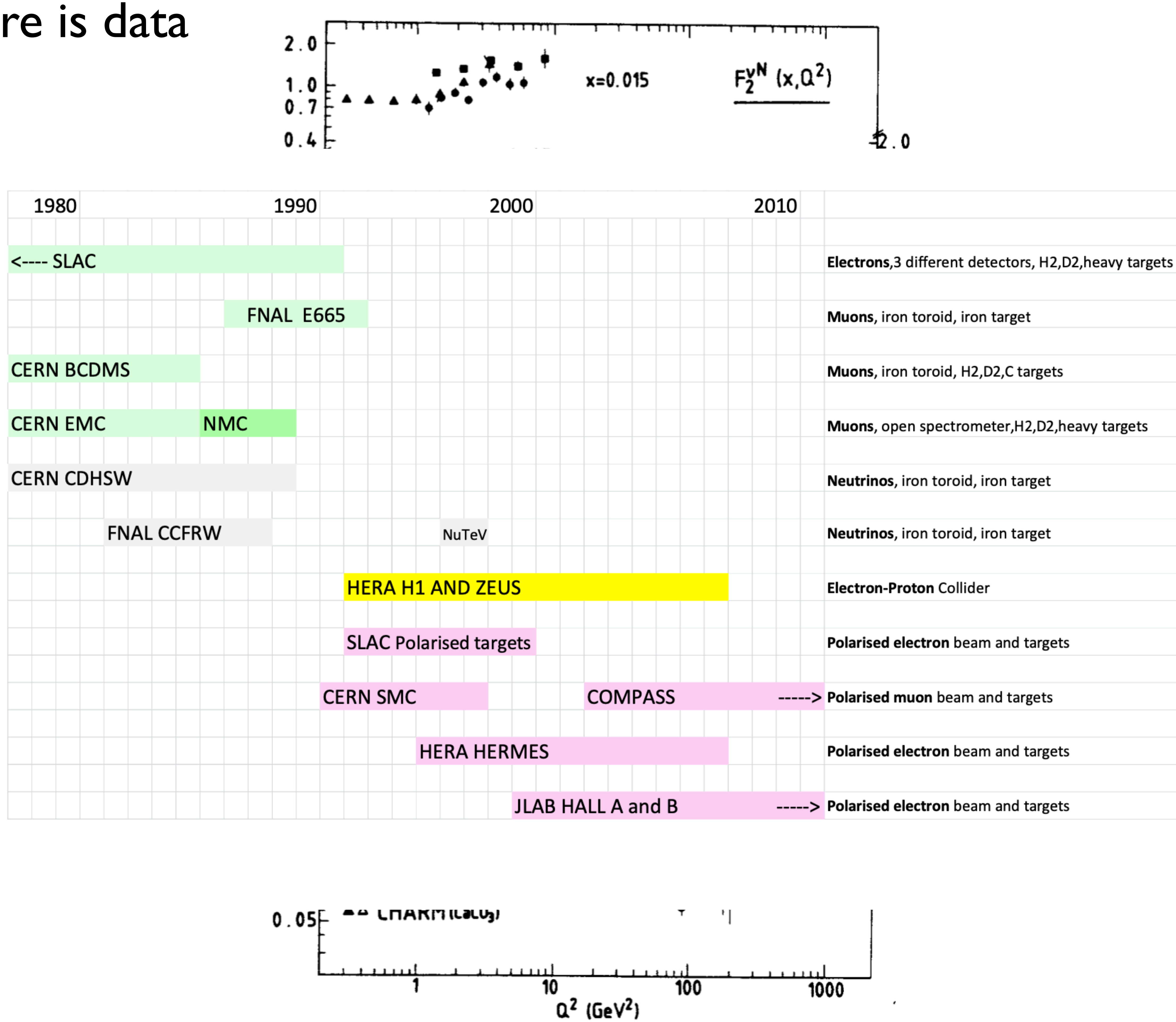
$$\begin{aligned} F_2^\nu &= 2x\left[d(x) + s(x) + \bar{u}(x) + \bar{c}(x)\right] \\ xF_3^\nu &= 2x\left[d(x) + s(x) - \bar{u}(x) - \bar{c}(x)\right] \\ F_2^{\bar{\nu}} &= 2x\left[u(x) + c(x) + \bar{d}(x) + \bar{s}(x)\right] \\ xF_3^{\bar{\nu}} &= 2x\left[u(x) + c(x) - \bar{d}(x) - \bar{s}(x)\right] \end{aligned}$$

$$2xF_1 = F_2$$

• There is data



- There is data



Doing the job

- Variable count 6:

$$u(x)\ ,\ \bar{u}(x)\ ,\ d(x)\ ,\ \bar{d}(x)\ ,\ s(x)=\bar{s}(x)\ ,\ c(x)=\bar{c}(x)$$

- Some constraints:

$$\int_0^1 dx[u(x)-\bar{u}(x)]=2 \quad ; \quad \int_0^1 dx[d(x)-\bar{d}(x)]=1$$

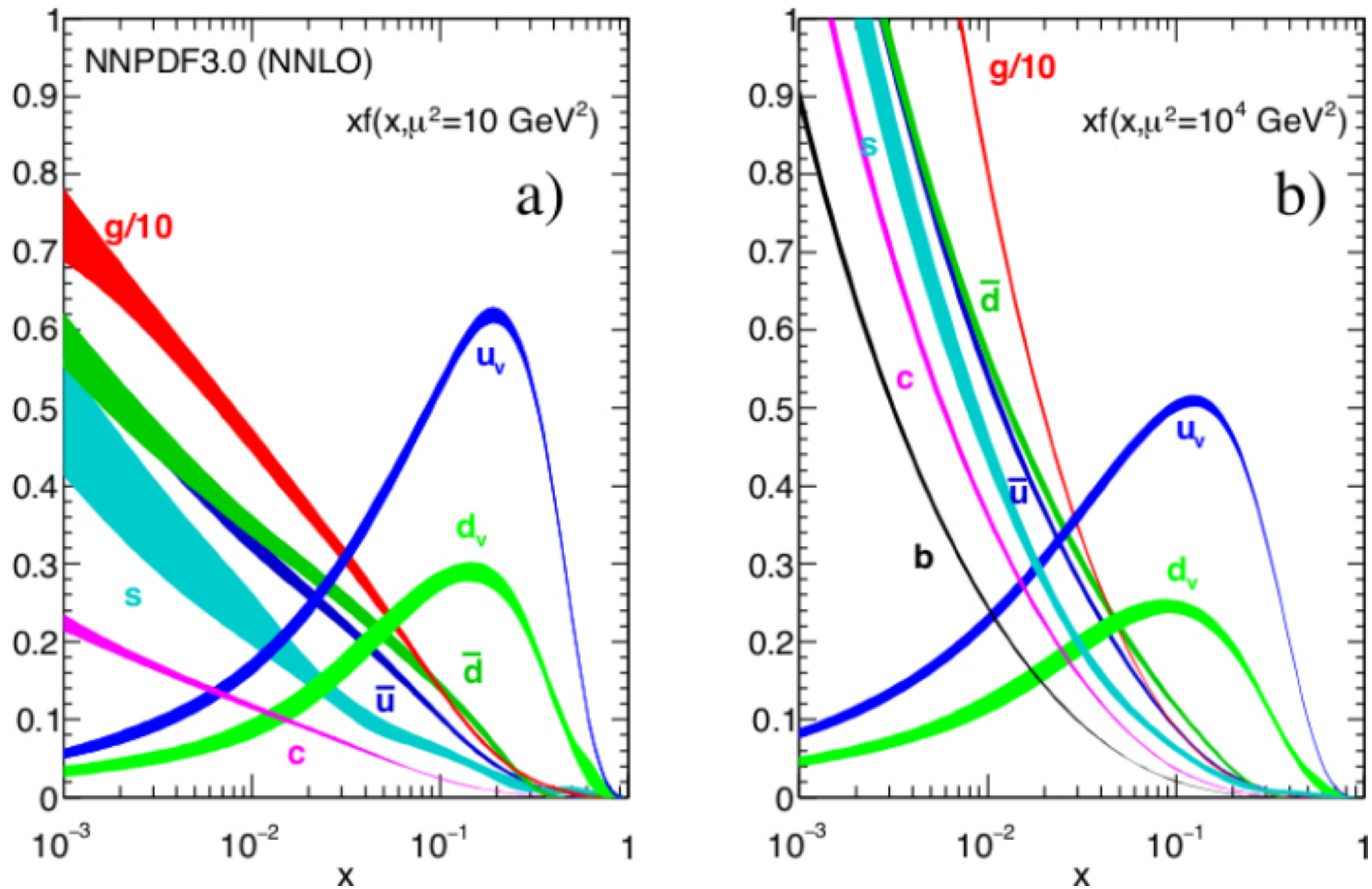
$$\int_0^1 dx[s(x)-\bar{s}(x)]=0 \quad ; \quad \int_0^1 dx[c(x)-\bar{c}(x)]=0$$

- Reasonable hypothesis

$$u(x)=u_v(x)+\underline{u_s(x)} \quad ; \quad d(x)=d_v(x)+\underline{d_s(x)} \quad ; \quad u_s(x)=d_s(x)$$

$$\bar{u}(x)=\underline{u_s(x)} \quad ; \quad \bar{d}(x)=\underline{d_s(x)}$$

Now we can fit!


NNPDF3.0 (NNLO)
xf(x,μ²=10 GeV²)
a)
g/10
s
c
ū
d̄
u_v
d_v
x
xf(x,μ²=10⁴ GeV²)
b)
g/10
s
c
d̄
ū
b
u_v
d_v
x

Further tests

- The neutron PDF is obtained from the p ones by $u(x) \Longleftrightarrow d(x)$

$$F_2^{\nu n} = 2x\left[u(x) + s(x) + \bar{d}(x) + \bar{c}(x)\right]$$

$$xF_3^{\nu n} = 2x\left[u(x) + s(x) - \bar{d}(x) - \bar{c}(x)\right]$$

$$F_2^{\bar{\nu} n} = 2x\left[d(x) + c(x) + \bar{u}(x) + \bar{s}(x)\right]$$

$$xF_3^{\bar{\nu} n} = 2x\left[d(x) + c(x) - \bar{u}(x) - \bar{s}(x)\right]$$

$$F_2^{emn} = x\left[\frac{4}{9}(d(x) + \bar{d}(x) + c(x) + \bar{c}(x)) + \frac{2}{9}(u(x) + \bar{u}(x) + s(x) + \bar{s}(x))\right]$$

- Consider

ignoring s, c..

$$\frac{\int dx\ [F_2^{\nu p}(x) + F_2^{\nu n}(x)]}{\int dx\ [F_2^{ep}(x) + F_2^{en}(x)]} = \frac{2}{Q_u^2 + Q_d^2} = \frac{18}{5}$$

- Gargamelle collaboration: 3.4 ± 0.7 evidence for fractional charges

- Gross Llewellyn Smith sum rule:

$$\int dx\ F_3^{\nu N}(x) = \int dx\ [u(x) + d(x) + s(x) - \bar{u}(x) - \bar{d}(x) - \bar{s}(x)]$$

- Gargamelle: 3.2 ± 0.6

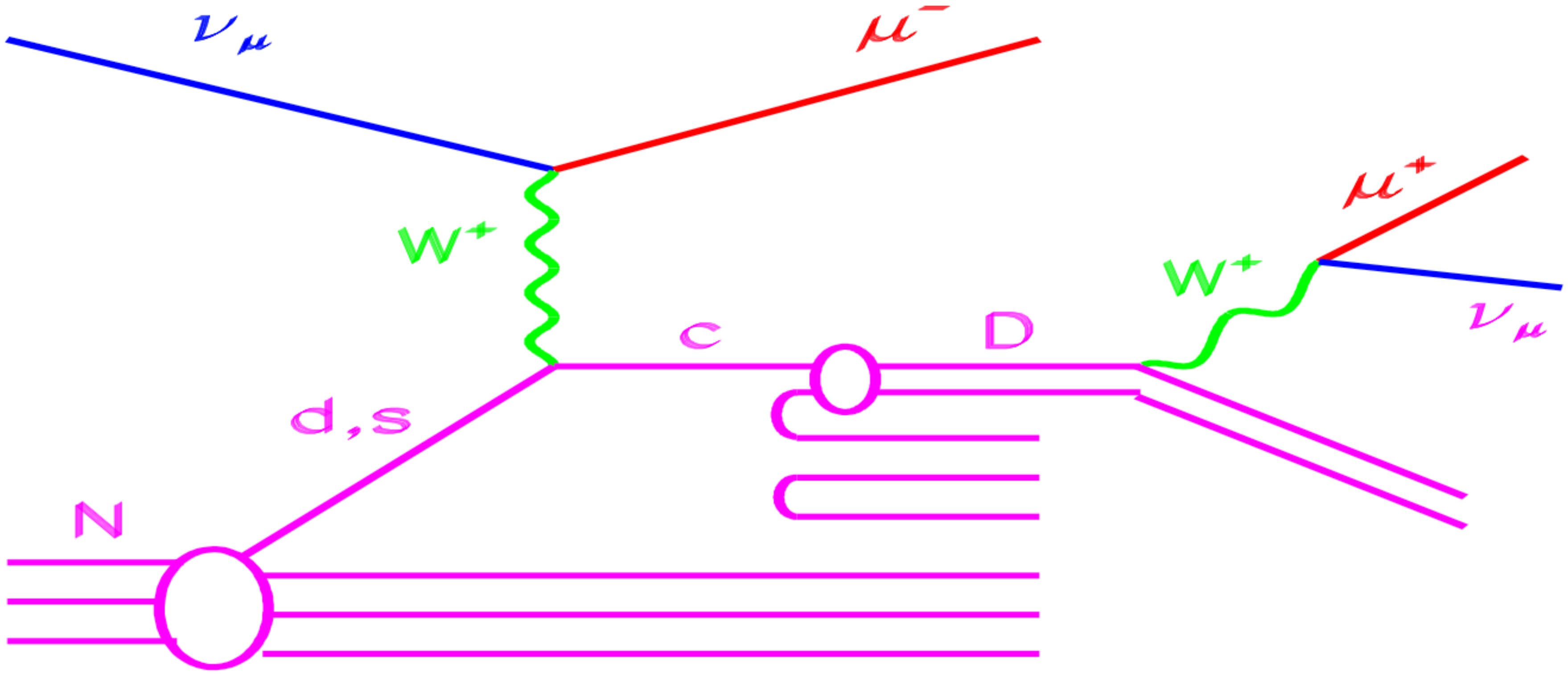
- Momentum sum rule:

$$\int dx\ [F_2^{\nu p}(x) + F_2^{\nu n}(x)] = \int dx\ x[u(x) + d(x) + s(x) + \bar{u}(x) + \bar{d}(x) + \bar{s}(x)]$$

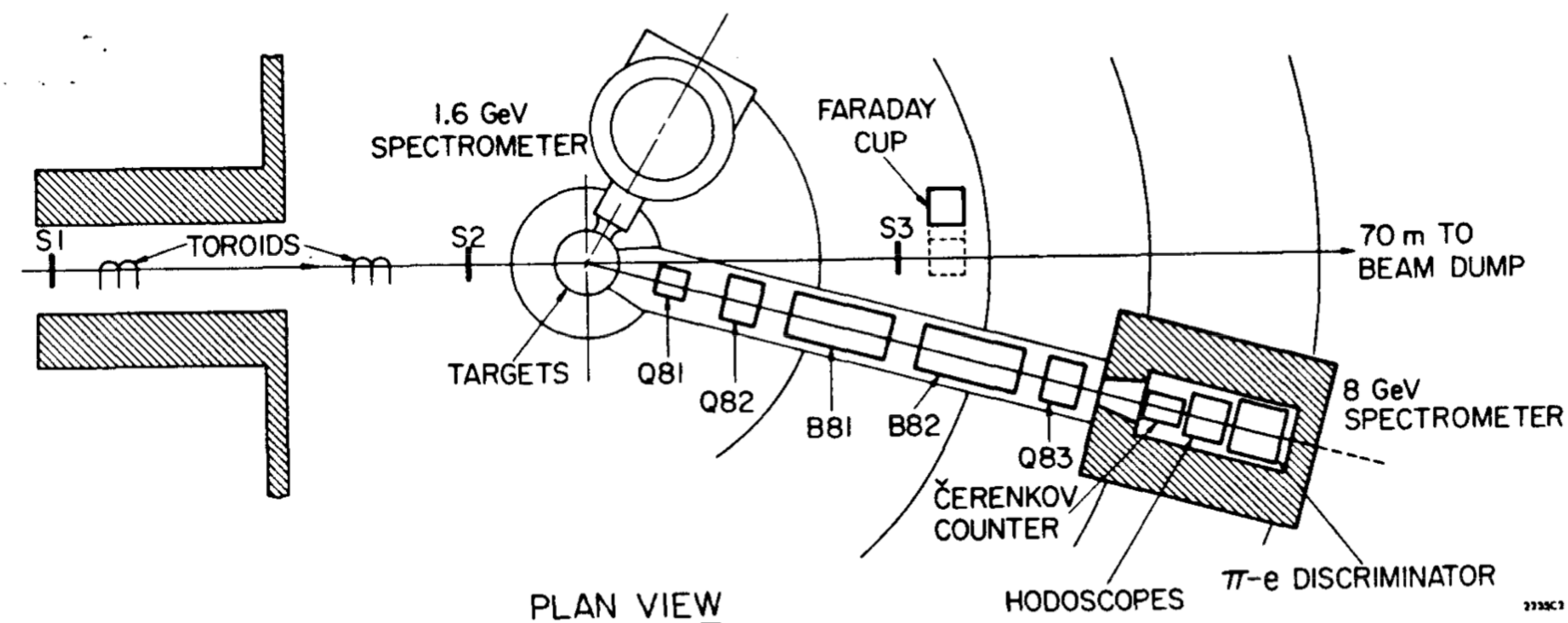
- Gargamelle: 0.49 ± 0.07

References:

1. Halzen & Martin, chapter 8 and 9
2. Barger & Phillips, chapter 5
3. Cahn & Goldhaber, chapter 8
4. Ellis & Stirling & Webber, chapter 4



ν_μ
μ^-
W^+
μ^+
W^+
ν_μ
c
D
d,s
N



1.6 GeV SPECTROMETER
FARADAY CUP
S1
TOROIDS
S2
S3
70 m TO BEAM DUMP
TARGETS
Q81
Q82
B81
B82
Q83
8 GeV SPECTROMETER
ČERENKOV COUNTER
HODOSCOPES
π-e DISCRIMINATOR
PLAN VIEW