

PROBLEM SET #1— ANSWER KEY

Econ 490
International Economics
UIUC, Fall 2019
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QUESTION 1

a) Production Possibilities Frontiers

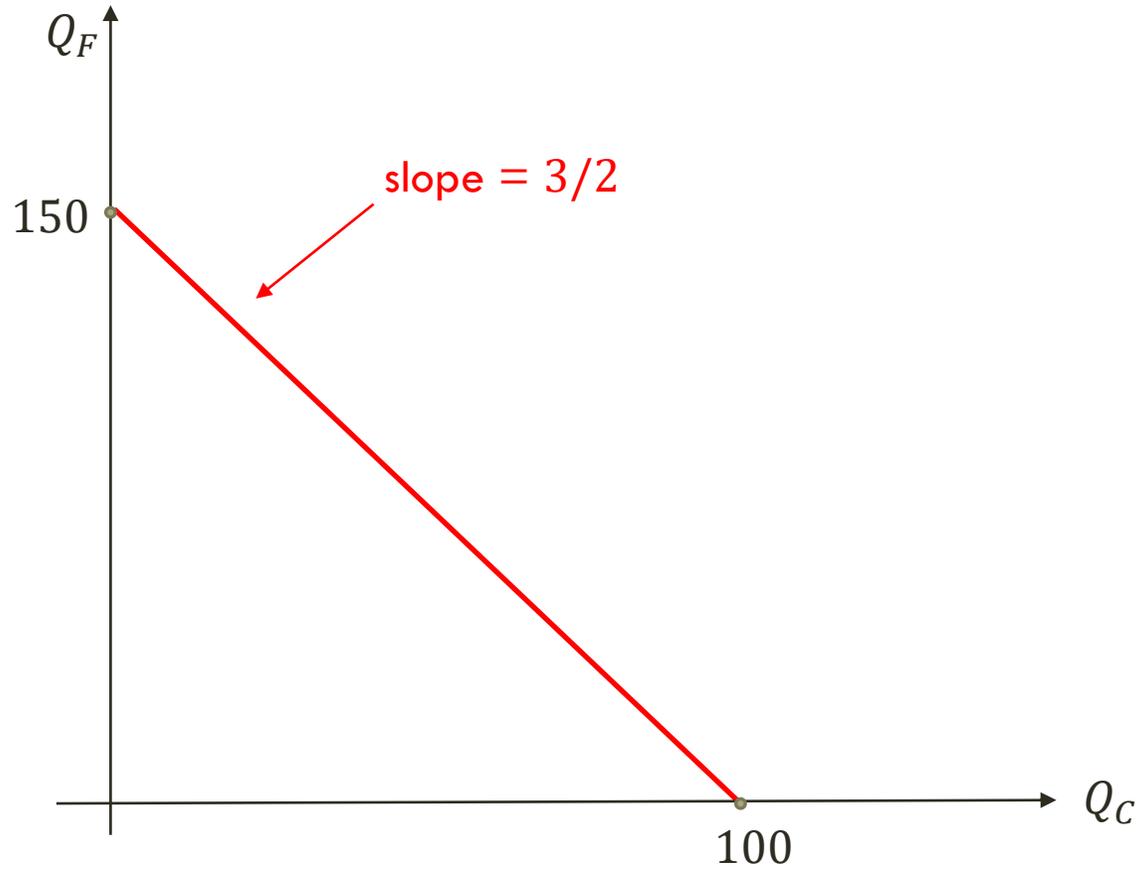
$$a_{LT} = 3; a_{LA} = 2$$

$$a_{LT}^* = 4; a_{LA}^* = 1$$

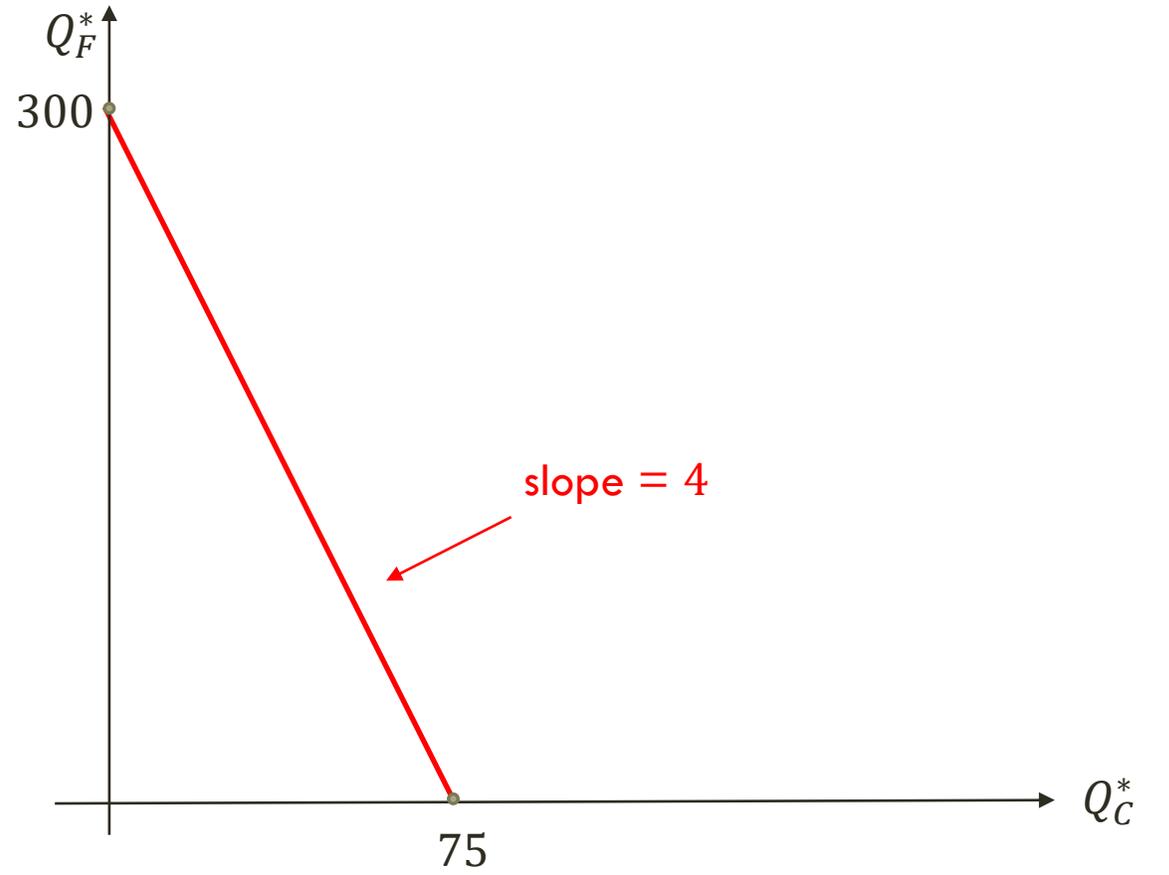
$$\text{BR: } 3Q_C + 2Q_F = 300$$

$$\text{ROW: } 4Q_C^* + Q_F^* = 300$$

BR



ROW



QUESTION 1

b) Autarchy prices

$$\frac{P_C}{P_F} = \frac{3}{2}$$

$$\frac{P_C^*}{P_F^*} = 4$$

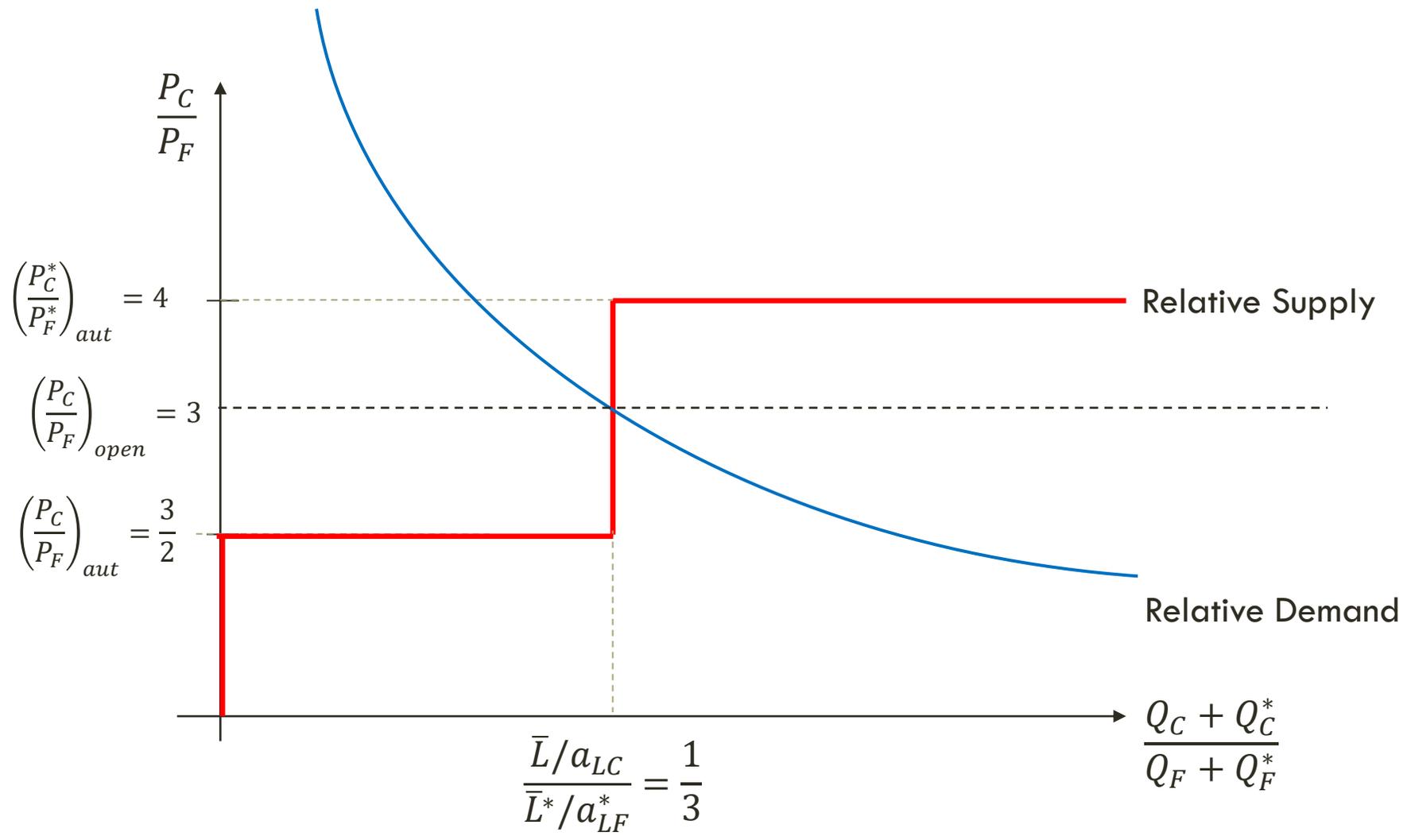
c) Yes. From the difference in relative prices, we can see that it is relatively cheaper to produce cloth in BR. Therefore, in an open economy, BR will export cloth and ROW will export food.

QUESTION 1

d) Relative supply curve (next slide)

There are 3 possibilities:

1. $\left(\frac{P_C}{P_F}\right)_{open} \in \left(\frac{3}{2}, 4\right)$: BR specializes in C, ROW specializes in F
2. $\left(\frac{P_C}{P_F}\right)_{open} = \frac{3}{2}$: BR produces both, ROW specializes in F
3. $\left(\frac{P_C}{P_F}\right)_{open} = 4$: BR specializes in C, ROW produces both



QUESTION 1

Let's guess that the RD curve crosses the RS curve in the vertical segment (case 1 above):

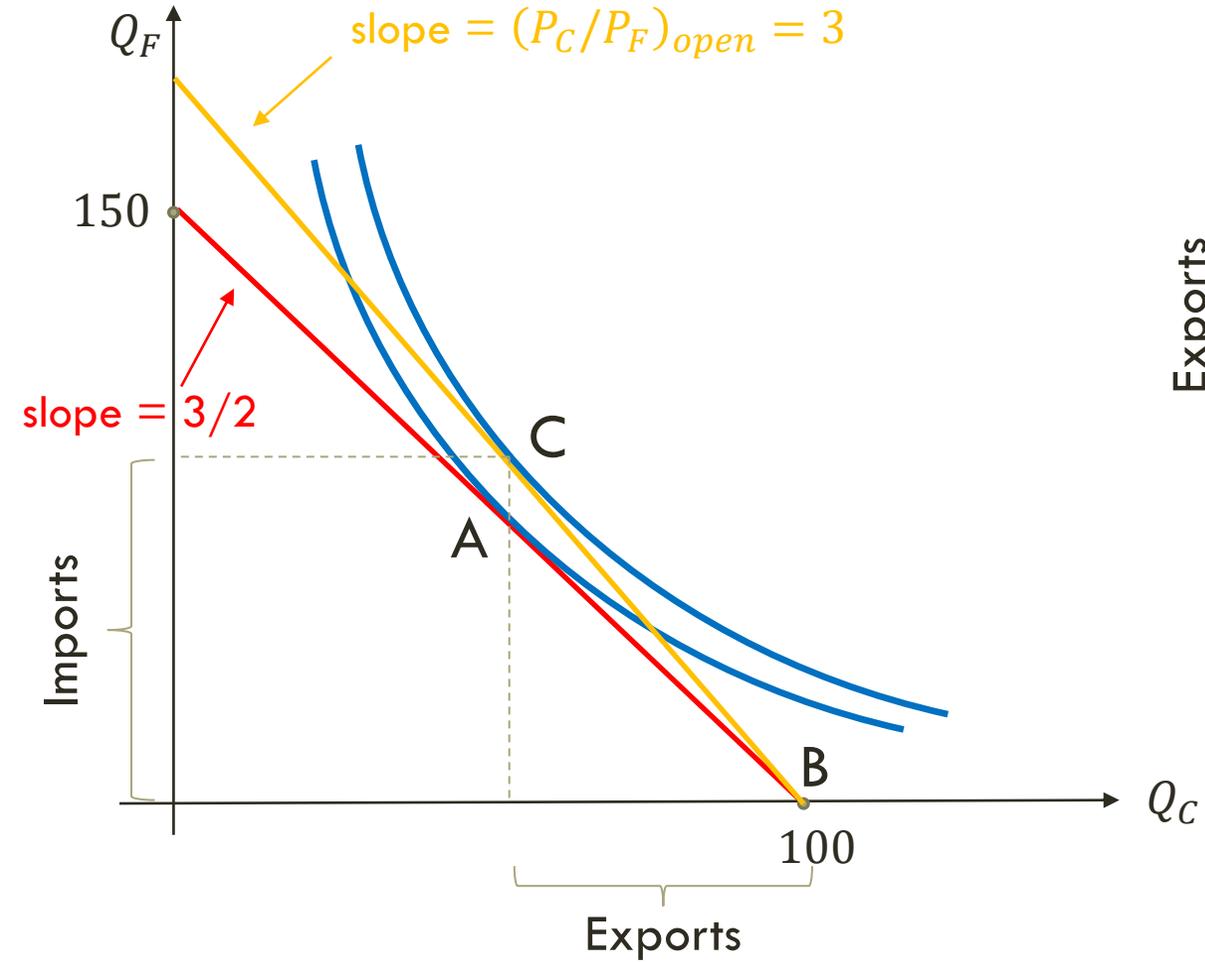
$$\frac{P_C}{P_F} = \frac{Q_F + Q_F^*}{Q_C + Q_C^*} = 3 \in \left(\frac{3}{2}, 4\right)$$

Our guess is therefore verified.

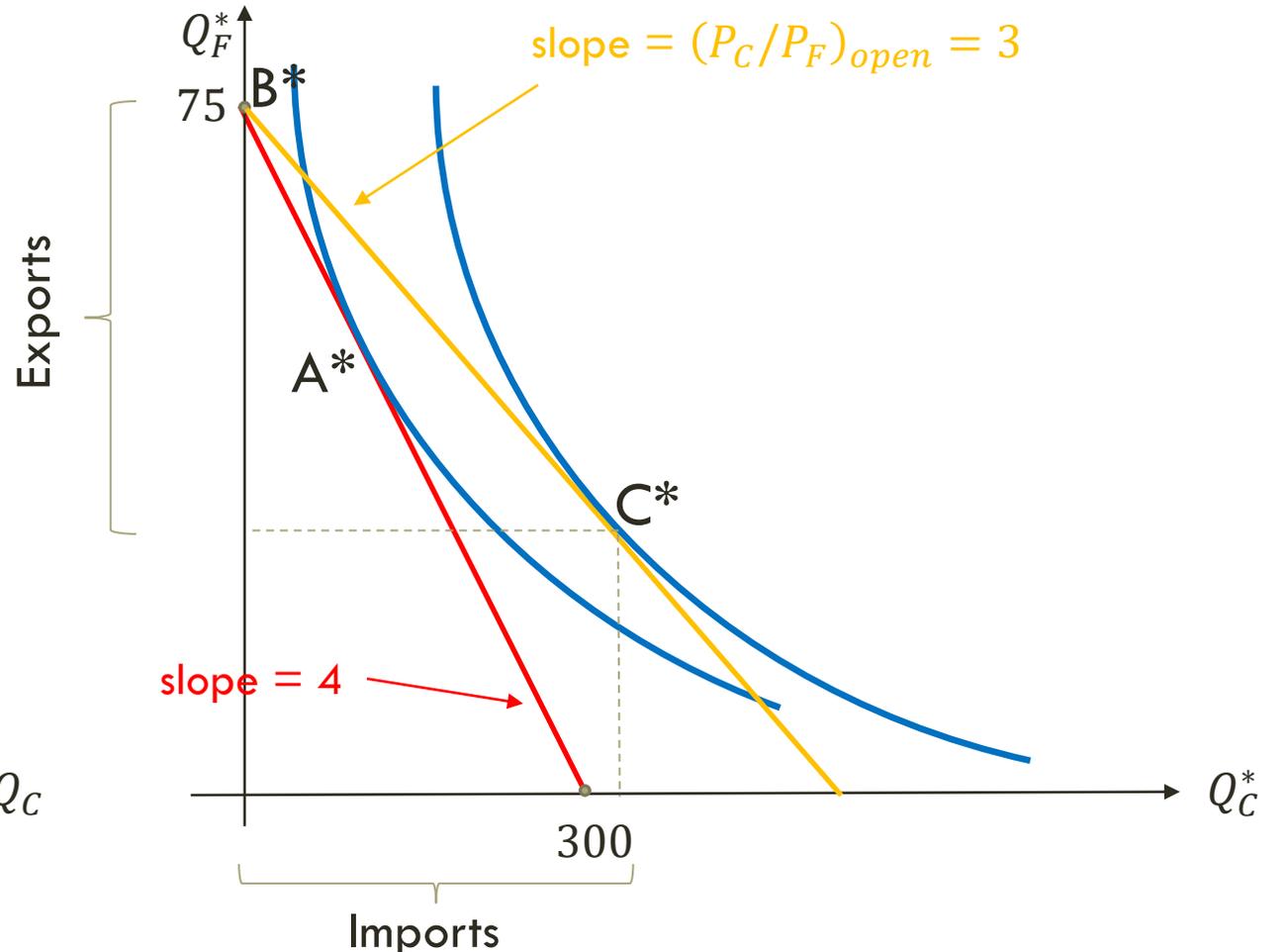
Open economy relative price:

$$\left(\frac{P_C}{P_F}\right)_{open} = 3$$

BR



ROW



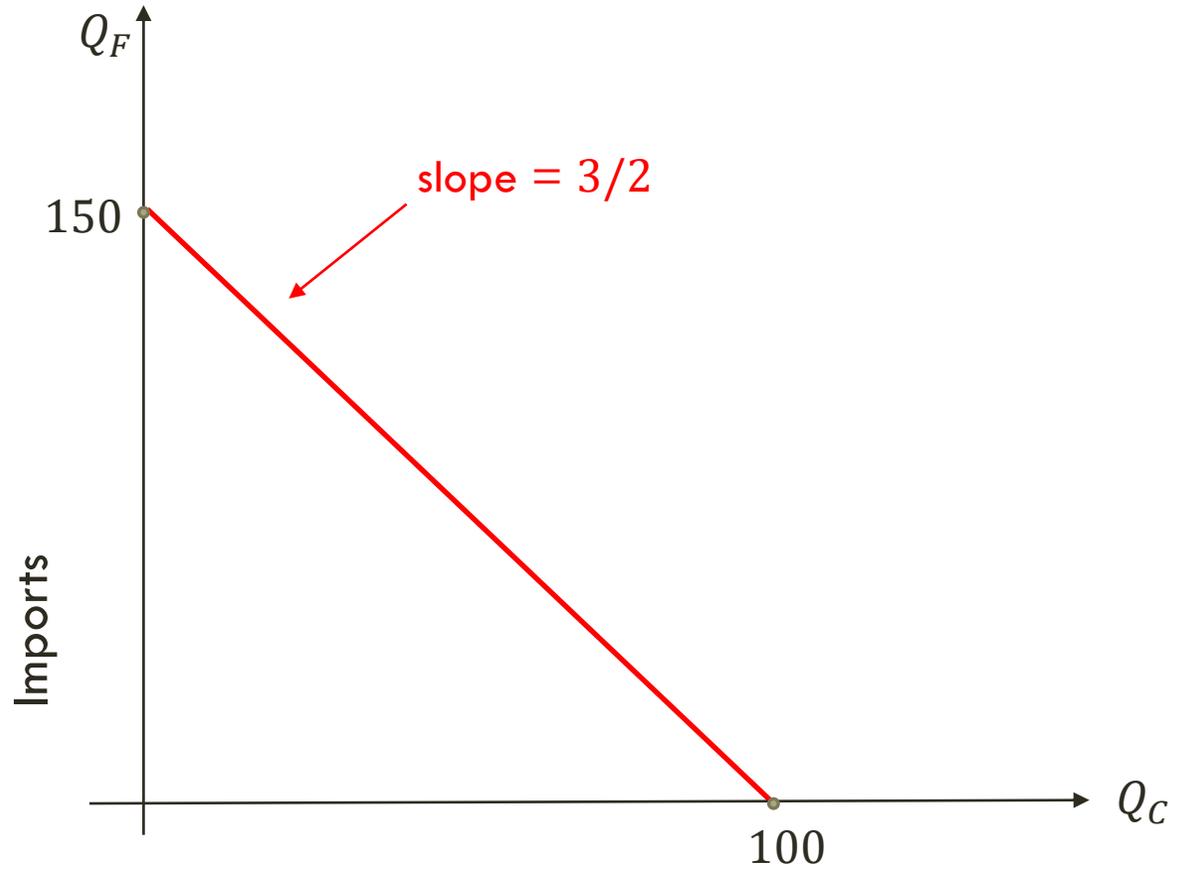
QUESTION 1

e) Production Possibilities Frontiers

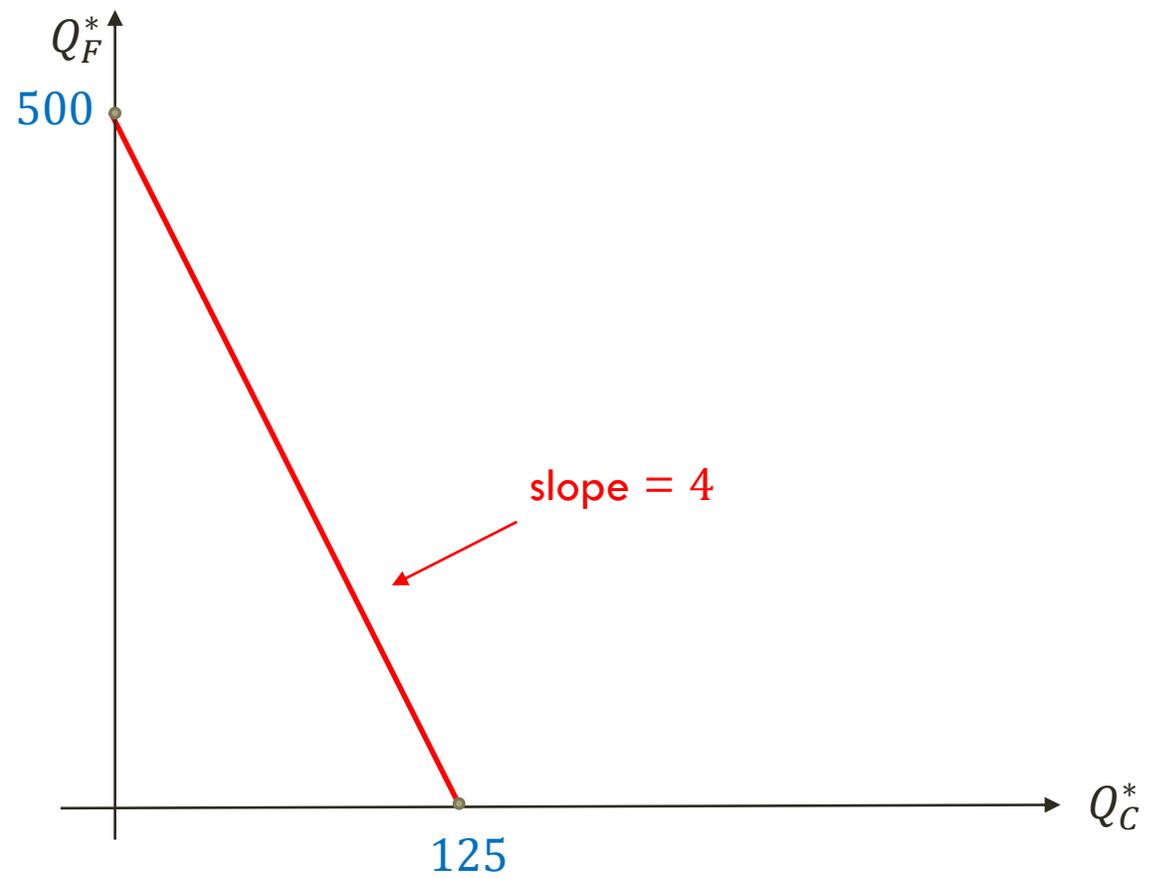
$$\text{BR: } 3Q_C + 2Q_F = 300$$

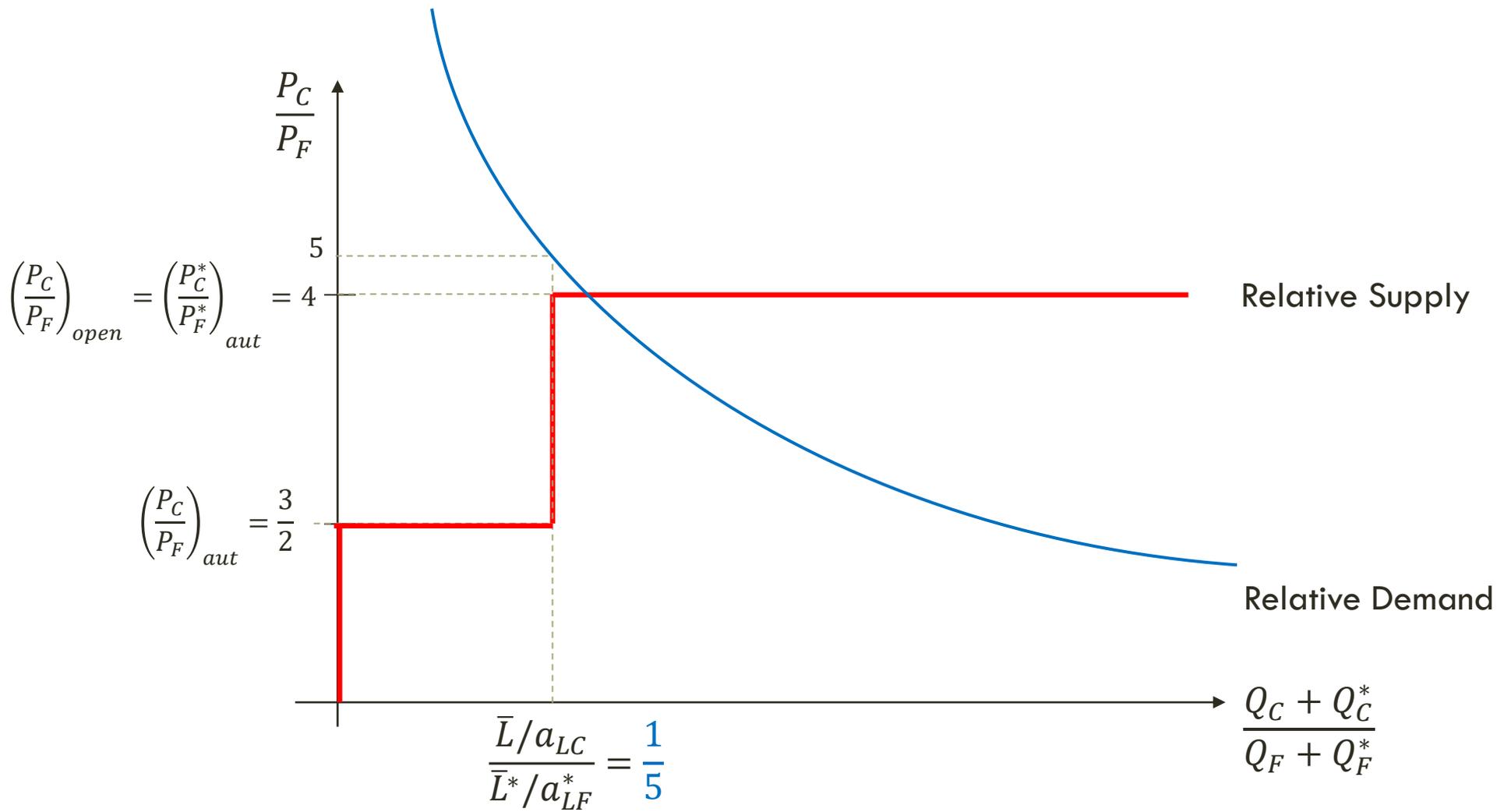
$$\text{ROW: } 4Q_C^* + Q_F^* = 500$$

BR



ROW





QUESTION 1

e) Again guess that both countries specialize:

$$\frac{P_C}{P_F} = \frac{Q_F + Q_F^*}{Q_C + Q_C^*} = 5 \notin \left(\frac{3}{2}, 4\right)$$

This implies that the RD curve crosses the RS curve in the horizontal segment where the relative price is equal to 4 (see previous slide). Then:

$$\left(\frac{P_C}{P_F}\right)_{open} = 4$$

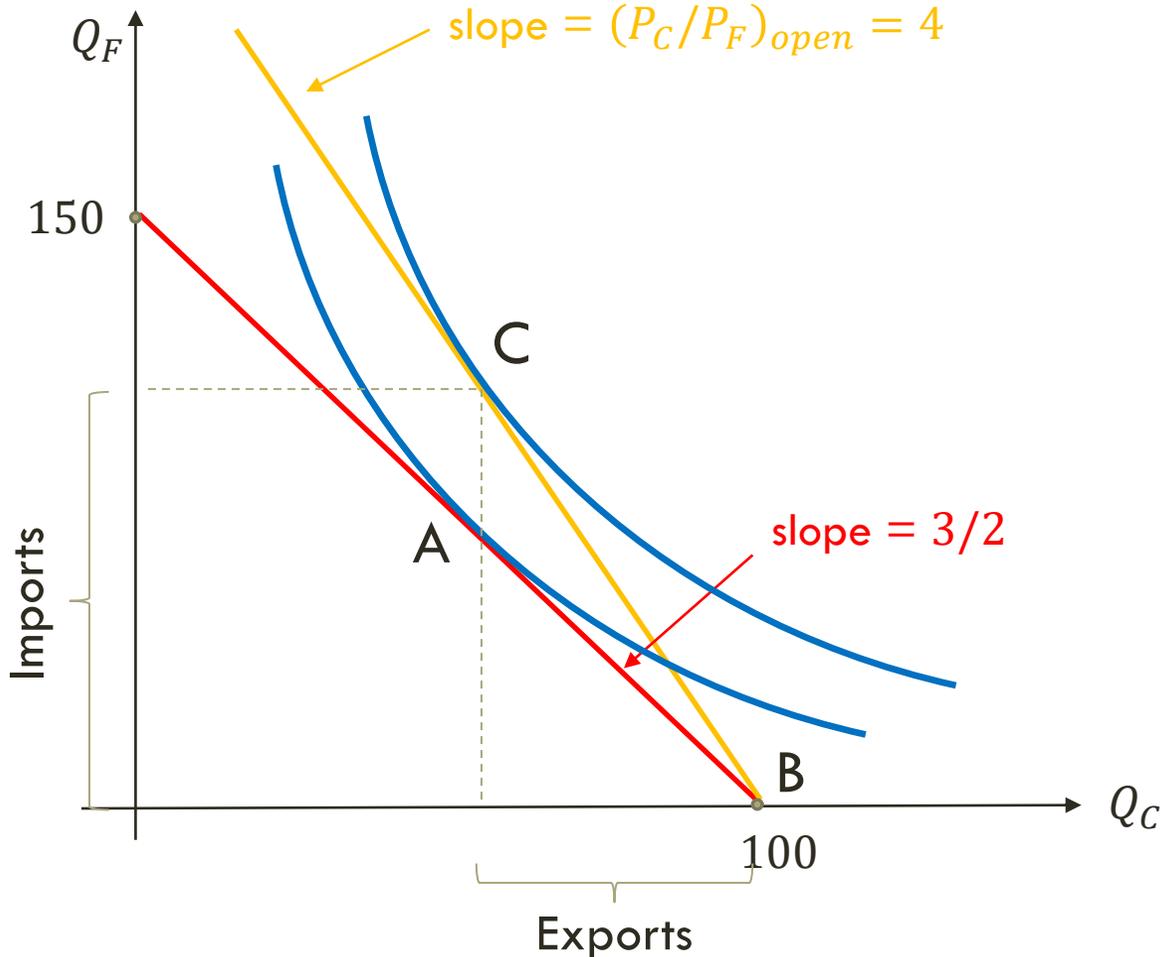
QUESTION 1

In this case, ROW is too large; BR's production is not enough to fulfill its demand for cloth.

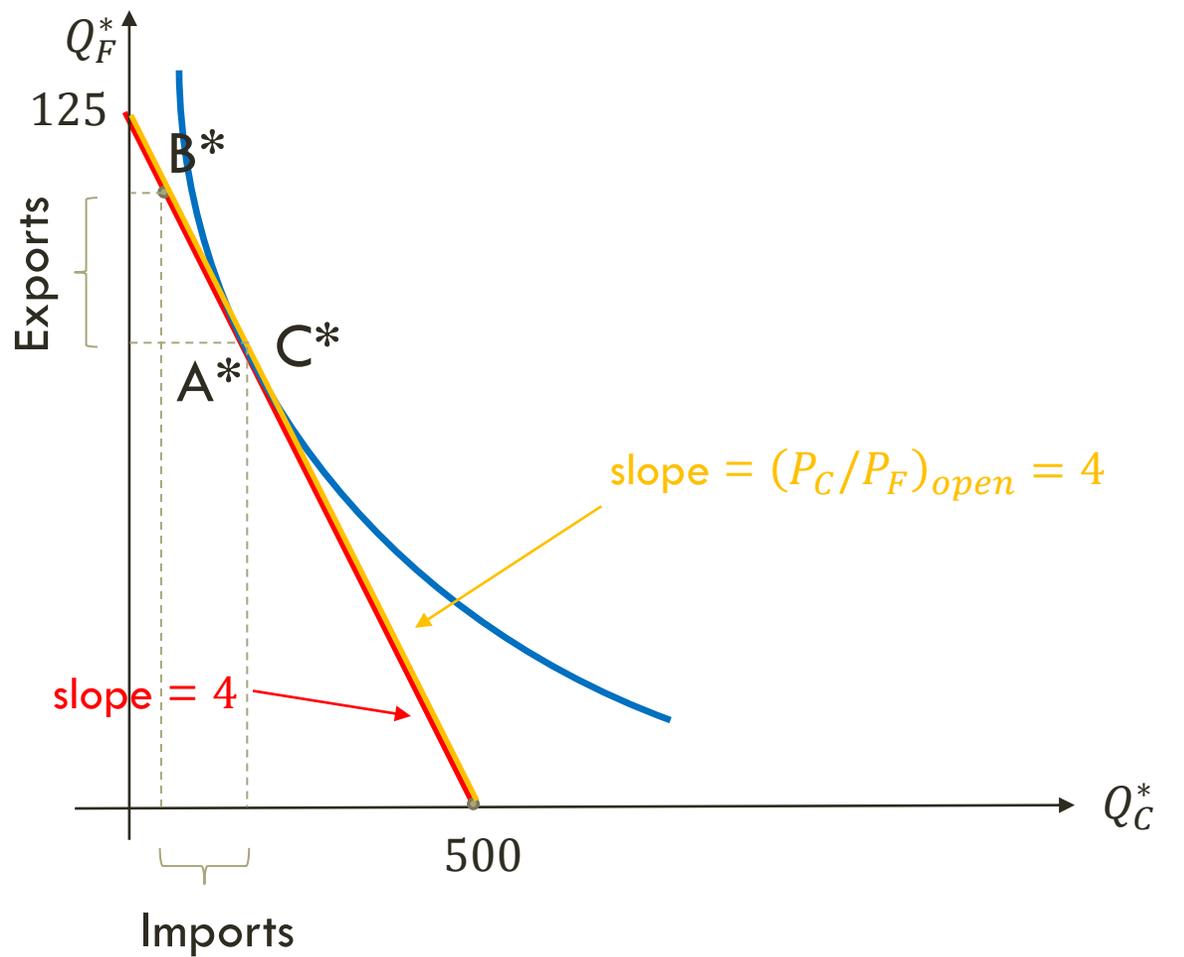
Therefore, ROW has to produce both goods; its relative price will not change relative to autarchy.

Since the relative price does not change, trade does not bring any welfare gain for ROW. Only BR benefits from trade (see next slide).

BR



ROW



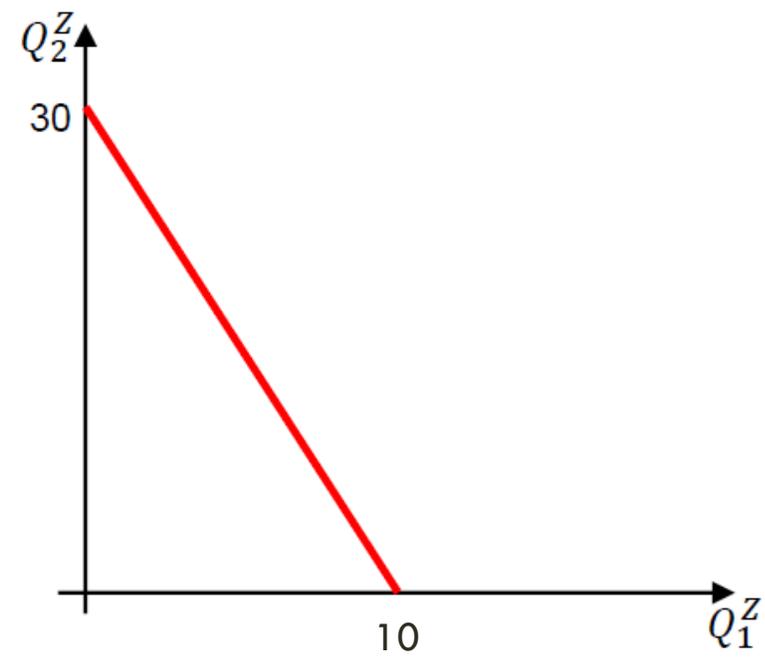
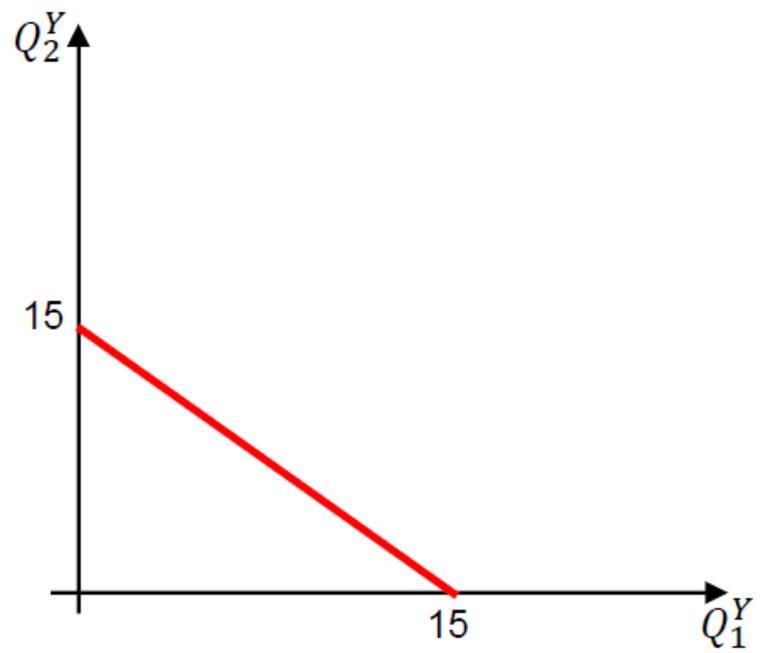
QUESTION 2

a) PPF for country $j \in \{X, Y, Z\}$

$$a_{L1}^j Q_1^j + a_{L2}^j Q_2^j = \bar{L}_j$$

Then:

$$\begin{aligned} Q_1^X + 3Q_2^X &= 30 \\ 2Q_1^Y + 2Q_2^Y &= 30 \\ 3Q_1^Z + Q_2^Z &= 30 \end{aligned}$$



QUESTION 2

b) Autarchy prices:

$$\left(\frac{P_1}{P_2}\right)_{aut}^X = \frac{a_{L1}^X}{a_{L2}^X} = \frac{1}{3}$$

$$\left(\frac{P_1}{P_2}\right)_{aut}^Y = \frac{a_{L1}^Y}{a_{L2}^Y} = 1$$

$$\left(\frac{P_1}{P_2}\right)_{aut}^Z = \frac{a_{L1}^Z}{a_{L2}^Z} = 3$$

QUESTION 2

c) Using the rule:

$$P_1/P_2 > a_{L1}/a_{L2} \implies Q_1 = \bar{L}/a_{L1}, \quad Q_2 = 0$$

$$P_1/P_2 < a_{L1}/a_{L2} \implies Q_1 = 0, \quad Q_2 = \bar{L}/a_{L2}$$

$$P_1/P_2 = a_{L1}/a_{L2} \implies Q_1 \geq 0, \quad Q_2 \geq 0$$

We then have the following cases:

$$\bullet \quad P_1/P_2 < 1/3 : \quad \begin{cases} Q_1^X = 0, Q_2^X = 15 \\ Q_1^Y = 0, Q_2^Y = 15 \\ Q_1^Z = 0, Q_2^Z = 30 \end{cases} \implies \frac{Q_1^X + Q_1^Y + Q_1^Z}{Q_2^X + Q_2^Y + Q_2^Z} = 0$$

X, Y and Z specialize in good 2

QUESTION 2

- $P_1/P_2 = 1/3$:
$$\begin{cases} Q_1^X \geq 0, Q_2^X \geq 0 \\ Q_1^Y = 0, Q_2^Y = 15 \\ Q_1^Z = 0, Q_2^Z = 30 \end{cases}$$

X produces both goods; Y and Z specialize in good 2

- $P_1/P_2 \in (1/3, 1)$:
$$\begin{cases} Q_1^X = 30, Q_2^X = 0 \\ Q_1^Y = 0, Q_2^Y = 15 \\ Q_1^Z = 0, Q_2^Z = 30 \end{cases} \implies \frac{Q_1^X + Q_1^Y + Q_1^Z}{Q_2^X + Q_2^Y + Q_2^Z} = \frac{30}{45} = \frac{2}{3}$$

X specializes in good 1; Y and Z specialize in good 2

QUESTION 2

- $P_1/P_2 = 1$:
$$\begin{cases} Q_1^X = 30, Q_2^X = 0 \\ Q_1^Y \geq 0, Q_2^Y \geq 0 \\ Q_1^Z = 0, Q_2^Z = 30 \end{cases}$$

X specializes in good 1; Y produces both; Z specializes in good 2

- $P_1/P_2 \in (1, 3)$:
$$\begin{cases} Q_1^X = 30, Q_2^X = 0 \\ Q_1^Y = 15, Q_2^Y = 0 \\ Q_1^Z = 0, Q_2^Z = 30 \end{cases} \implies \frac{Q_1^X + Q_1^Y + Q_1^Z}{Q_2^X + Q_2^Y + Q_2^Z} = \frac{45}{30} = \frac{3}{2}$$

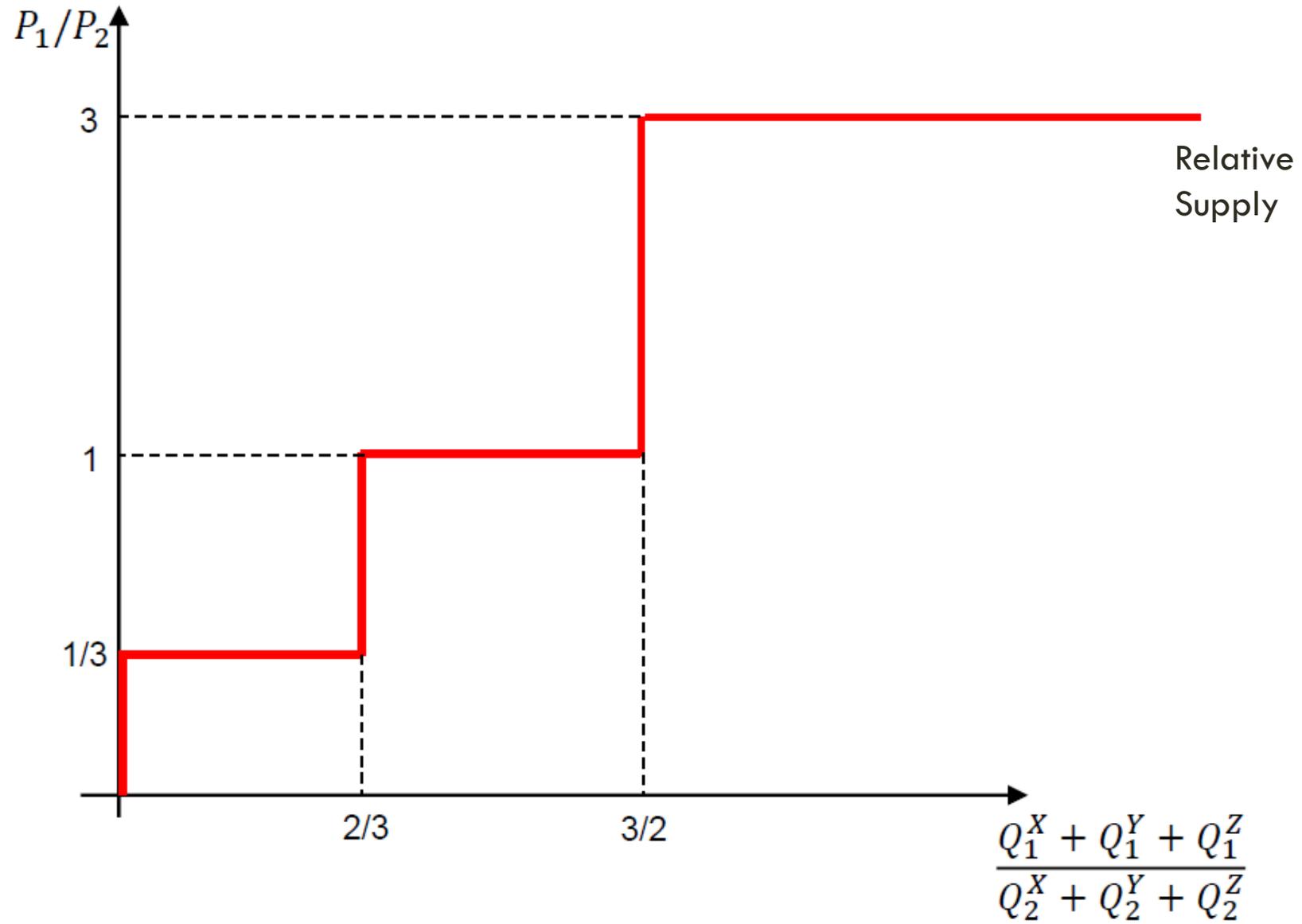
X and Y specialize in good 1; Z specializes in good 2

QUESTION 2

- $P_1/P_2 = 3$:
$$\begin{cases} Q_1^X = 30, Q_2^X = 0 \\ Q_1^Y = 15, Q_2^Y = 0 \\ Q_1^Z \geq 0, Q_2^Z \geq 30 \end{cases}$$

X, Y specialize in good 1; Z produces both

The next slide presents the Relative Supply curve, taking into account the cases just described.



QUESTION 2

- d) Let's guess that the relative demand curve crosses the relative supply curve in the region $P_1/P_2 \in \left(\frac{1}{3}, 1\right)$. In this case:

$$\frac{Q_1^X + Q_1^Y + Q_1^Z}{Q_2^X + Q_2^Y + Q_2^Z} = \frac{30}{45} = \frac{2}{3}$$

From the relative demand curve:

$$\frac{P_1}{P_2} = \frac{1}{2} \frac{Q_2^X + Q_2^Y + Q_2^Z}{Q_1^X + Q_1^Y + Q_1^Z} = \frac{1}{2} \frac{3}{2} = 3/4$$

Which is indeed between $1/3$ and 1 . Guess is therefore verified.

QUESTION 2

$$\frac{P_1}{P_2} = 3/4$$

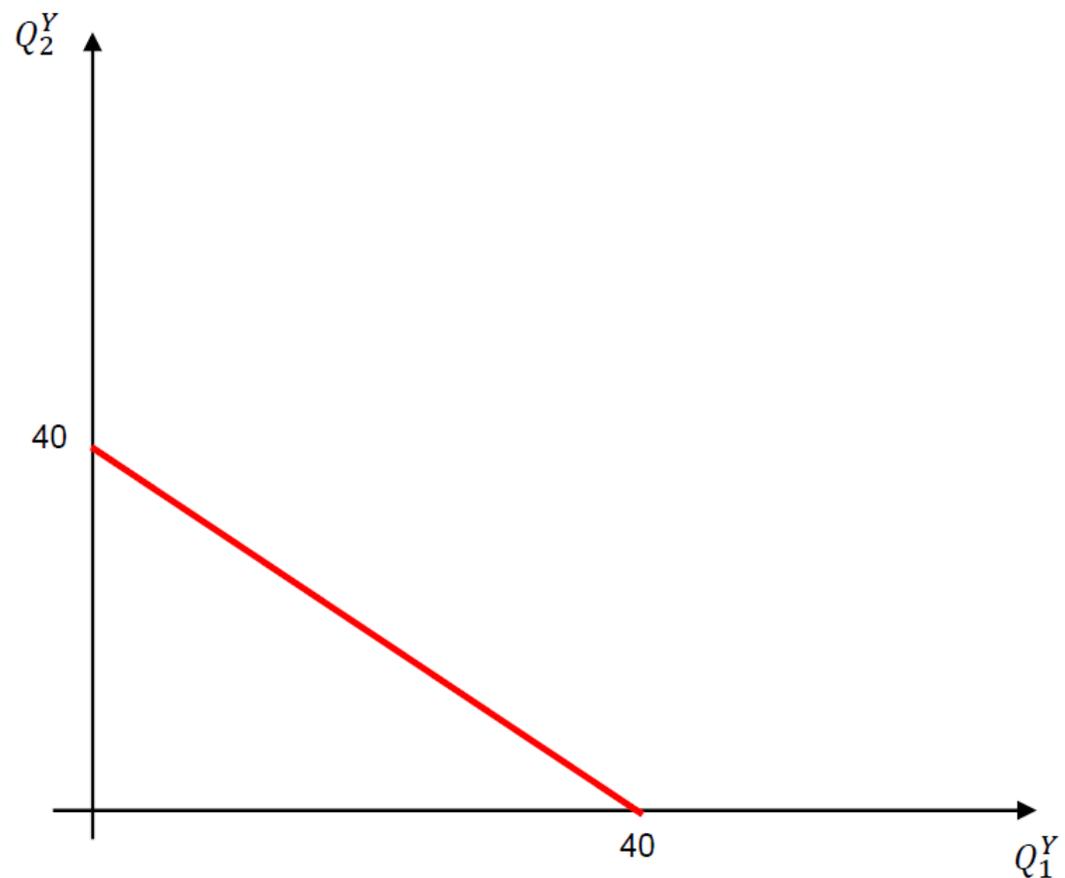
In this region:

- Country X: produces good 1; exports good 1; imports good 2
- Country Y: produces good 2; exports good 2; imports good 1
- Country Z: produces good 2; exports good 2; imports good 1

QUESTION 2

e) The new PPF for country Z is:

$$2Q_1^Y + 2Q_2^Y = 80$$



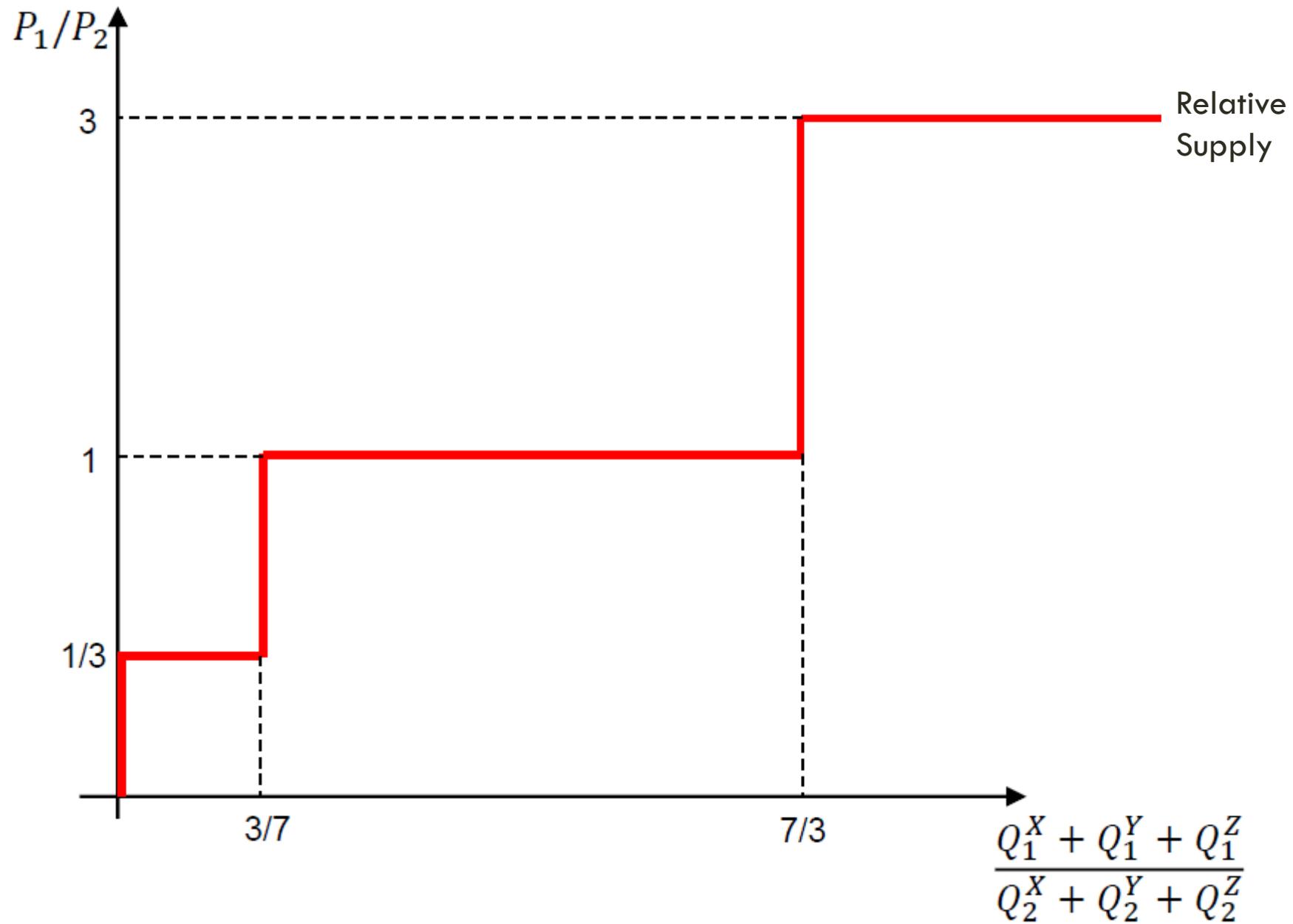
QUESTION 2

We have to adjust the following parts of the relative supply curve:

$$\bullet P_1/P_2 \in (1/3, 1) : \begin{cases} Q_1^X = 30, Q_2^X = 0 \\ Q_1^Y = 0, Q_2^Y = 40 \\ Q_1^Z = 0, Q_2^Z = 30 \end{cases} \implies \frac{Q_1^X + Q_1^Y + Q_1^Z}{Q_2^X + Q_2^Y + Q_2^Z} = \frac{30}{70} = \frac{3}{7}$$

$$\bullet P_1/P_2 \in (1, 3) : \begin{cases} Q_1^X = 30, Q_2^X = 0 \\ Q_1^Y = 40, Q_2^Y = 0 \\ Q_1^Z = 0, Q_2^Z = 30 \end{cases} \implies \frac{Q_1^X + Q_1^Y + Q_1^Z}{Q_2^X + Q_2^Y + Q_2^Z} = \frac{70}{30} = \frac{7}{3}$$

New relative supply curve in the next slide



QUESTION 2

Once more, guess $P_1/P_2 \in \left(\frac{1}{3}, 1\right)$. Using the relative demand curve:

$$\frac{P_1}{P_2} = \frac{1 Q_2^X + Q_2^Y + Q_2^Z}{2 Q_1^X + Q_1^Y + Q_1^Z} = \frac{17}{23} = 7/6 \notin (1/3, 1)$$

Guess is not verified.

Now guess that $P_1/P_2 \in (1, 3)$. Using the relative demand curve:

$$\frac{P_1}{P_2} = \frac{1 Q_2^X + Q_2^Y + Q_2^Z}{2 Q_1^X + Q_1^Y + Q_1^Z} = \frac{13}{27} = 3/14 \notin (1, 3)$$

This guess is also not verified.

QUESTION 2

Therefore, the relative demand curve crosses the relative supply curve at the flat segment with:

$$\frac{P_1}{P_2} = 1$$

In this region:

- Country X: produces good 1; exports good 1; imports good 2
- Country Y: produces both goods
- Country Z: produces good 2; exports good 2; imports good 1

We still need to determine country Y's exporting and importing goods.

QUESTION 2

Using the relative demand curve:

$$1 = \frac{P_1}{P_2} = \frac{1 Q_2^X + Q_2^Y + Q_2^Z}{2 Q_1^X + Q_1^Y + Q_1^Z} = \frac{1 \cdot 0 + Q_2^Y + 30}{2 \cdot 30 + Q_1^Y + 0}$$

Or:

$$\begin{aligned} Q_2^Y + 30 &= 60 + 2Q_1^Y \\ Q_2^Y &= 30 + 2Q_1^Y \quad (*) \end{aligned}$$

Using country Y's PPF:

$$Q_1^Y + Q_2^Y = 40 \quad (**)$$

QUESTION 2

Combining (*) and (**):

$$Q_1^Y + 30 + 2Q_1^Y = 40$$

$$Q_1^Y = 10/3$$

$$Q_2^Y = 40 - 10/3 = 110/3$$

In other words:

$$\frac{Q_1^Y}{Q_2^Y} = \frac{1}{11}$$

QUESTION 2

In the world economy, total production of a good has to equal to its total consumption. Then:

$$\frac{P_1}{P_2} = \frac{1 Q_2^X + Q_2^Y + Q_2^Z}{2 Q_1^X + Q_1^Y + Q_1^Z} = \frac{1 C_2^X + C_2^Y + C_2^Z}{2 C_1^X + C_1^Y + C_1^Z}$$

Since preferences are homogenous across countries

$$\frac{P_1}{P_2} = \frac{1 C_2^X + C_2^Y + C_2^Z}{2 C_1^X + C_1^Y + C_1^Z} = \frac{1 C_2^Y}{2 C_1^Y}$$

QUESTION 2

It then follows that:

$$\frac{C_1^Y}{C_2^Y} = 1/2$$

In other words:

$$\frac{C_1^Y}{C_2^Y} > \frac{Q_1^Y}{Q_2^Y}$$

Which implies that country Y imports good 1 and exports good 2.

QUESTION 3

Unit-value isoquants:

$$P_A z_A F_A(K_A, L_A) = 1$$
$$P_T z_T F_T(K_T, L_T) = 1$$

Or:

$$F_A(K_A, L_A) = \frac{1}{P_A z_A}$$
$$F_T(K_T, L_T) = \frac{1}{P_T z_T}$$

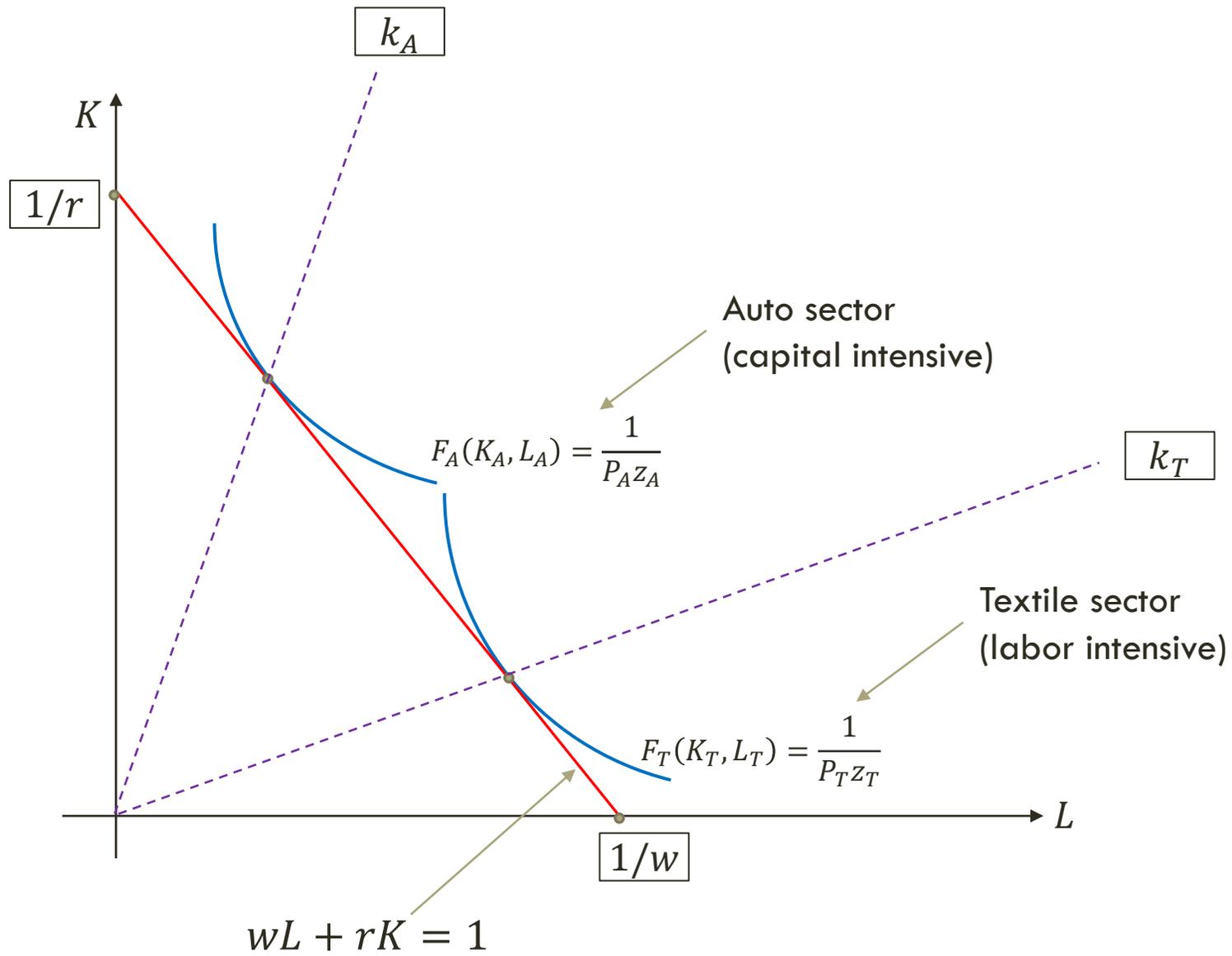
QUESTION 3

Assume that the economy produces both goods.

We must then find a single isocost line that is tangent to both unit-value isoquants:

$$wL + rK = 1$$

Intercepts ($1/r$ and $1/w$) determine factor prices (see next slide).



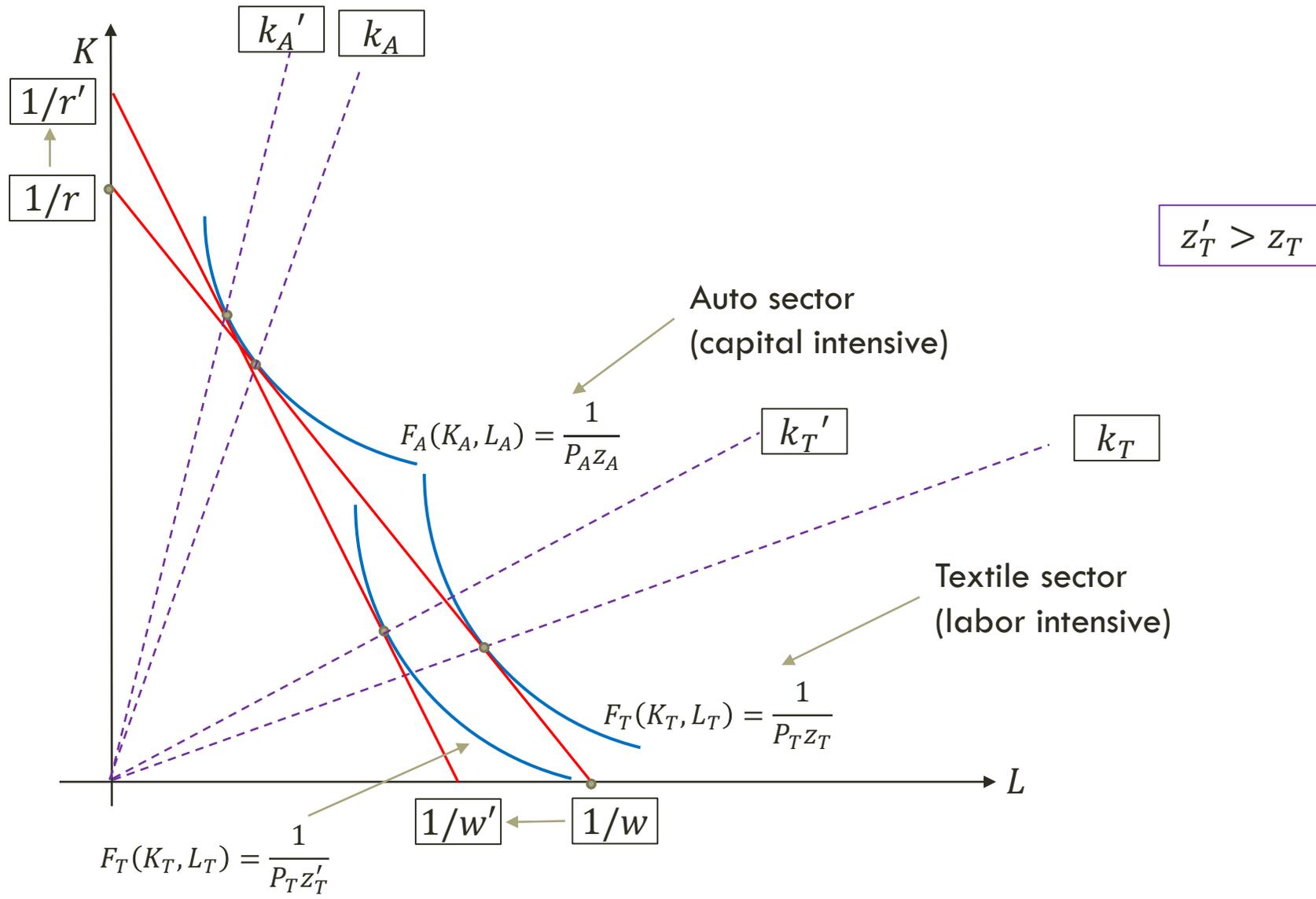
QUESTION 3

Increase in Z_T given Z_A

- Similar to an increase in the price of the labor-intensive good, discussed in class (see Stolper-Samuelson Theorem)
- Unit value isoquant for T sector shifts in (next slide)
- Wage rises; rental rate of capital falls

Mechanism:

- Sector T firms expand; they demand capital and labor, but relatively more labor, since tech is labor intensive
- Sector A shrinks; releases capital and labor, but relatively more capital, since tech is capital intensive
- At initial prices, there is excess demand for labor and excess supply of capital
 - Wage increases, rental rate falls
- We end up with more capital and labor in sector T , less in sector A ; output of sector T goes up and output of sector A goes down.



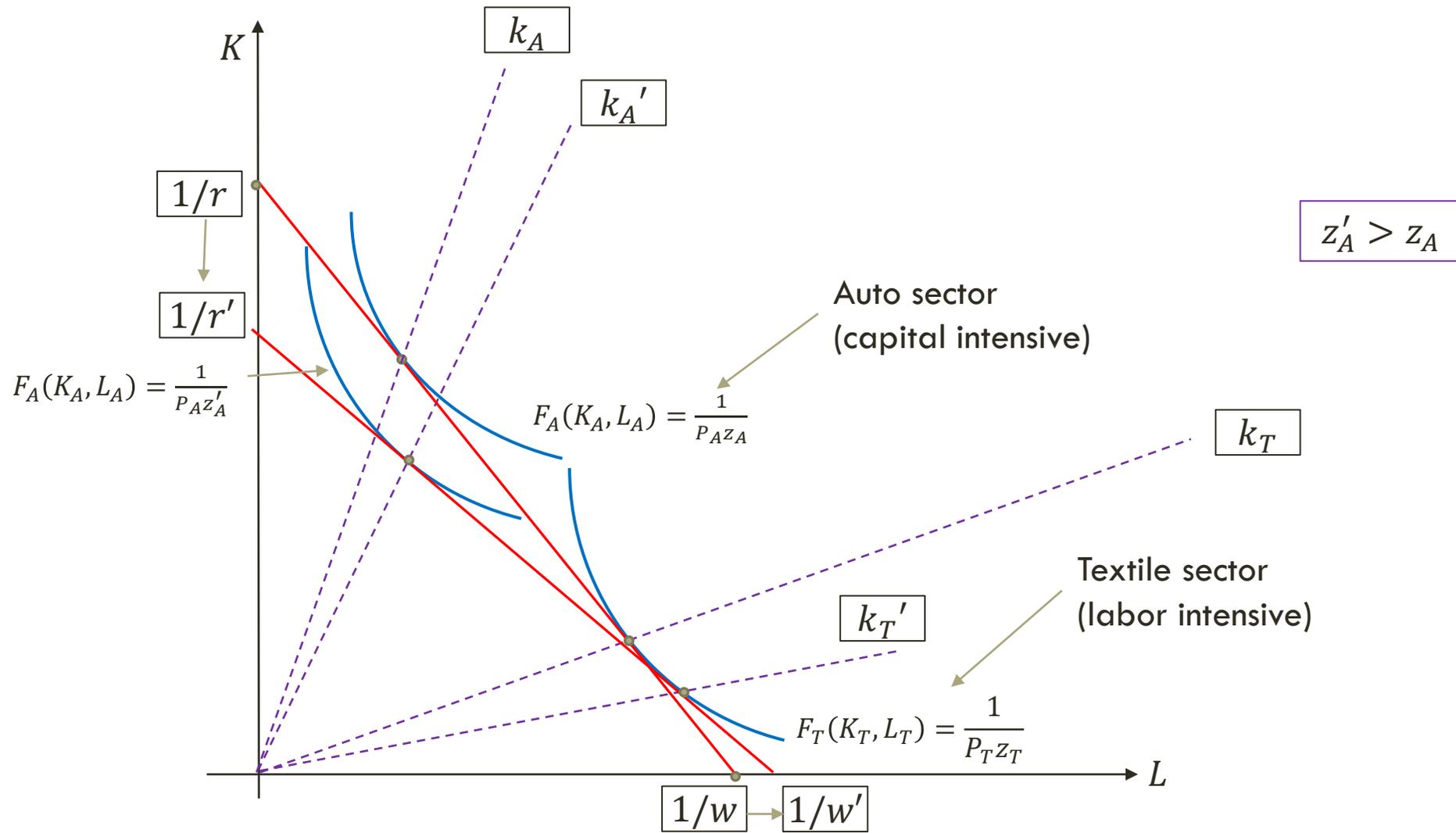
QUESTION 3

Increase in Z_A given Z_T

- Similar to an increase in the price of the capital-intensive good, discussed in class (see Stolper-Samuelson Theorem)
- Unit value isoquant for A sector shifts in (next slide)
- Wage falls; rental rate of capital rises

Mechanism:

- Sector A firms expand; they demand capital and labor, but relatively more capital, since tech is capital intensive
- Sector T shrinks; releases capital and labor, but relatively more labor, since tech is labor intensive
- At initial prices, there is excess demand for capital and excess supply of labor
 - Wage falls, rental rate increases
- We end up with more capital and labor in sector A , less in sector T ; output of sector A goes up and output of sector T goes down.



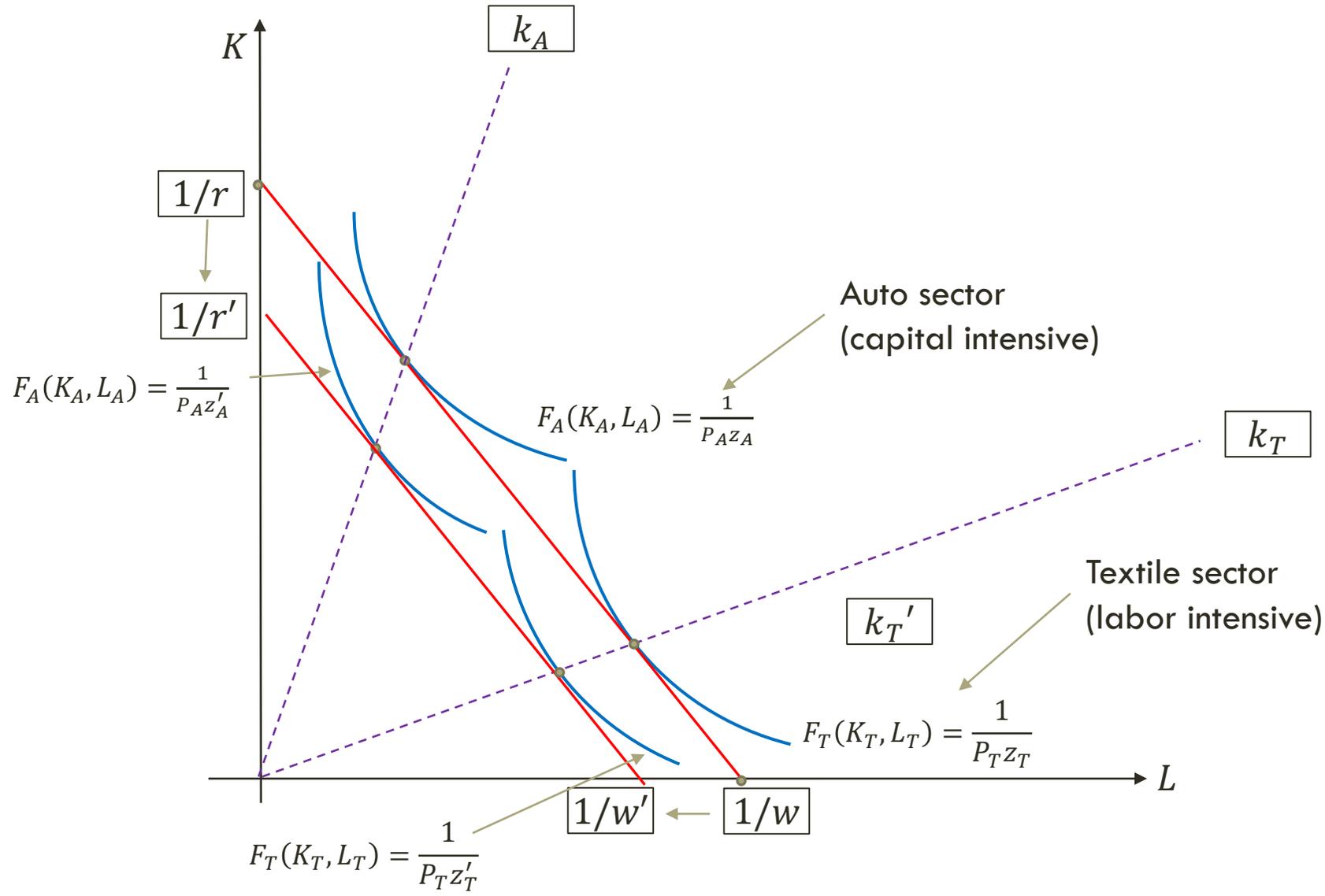
QUESTION 3

Z_T and Z_A increase in the same proportion

- Both unit value isoquants shift in proportionally
- Wage and rental rate increase

Mechanism:

- No sector has differential advantage now, so they are not able to change their relative sizes
- Allocation of capital and labor remains the same
- Since both capital and labor are more productive, firms are willing to pay more for them – both factor prices increase
- Outputs of both sectors go up, since their techs have improved



$$z'_A > z_A$$

$$z'_T > z_T$$

$$\frac{z'_A}{z'_T} = \frac{z_A}{z_T}$$

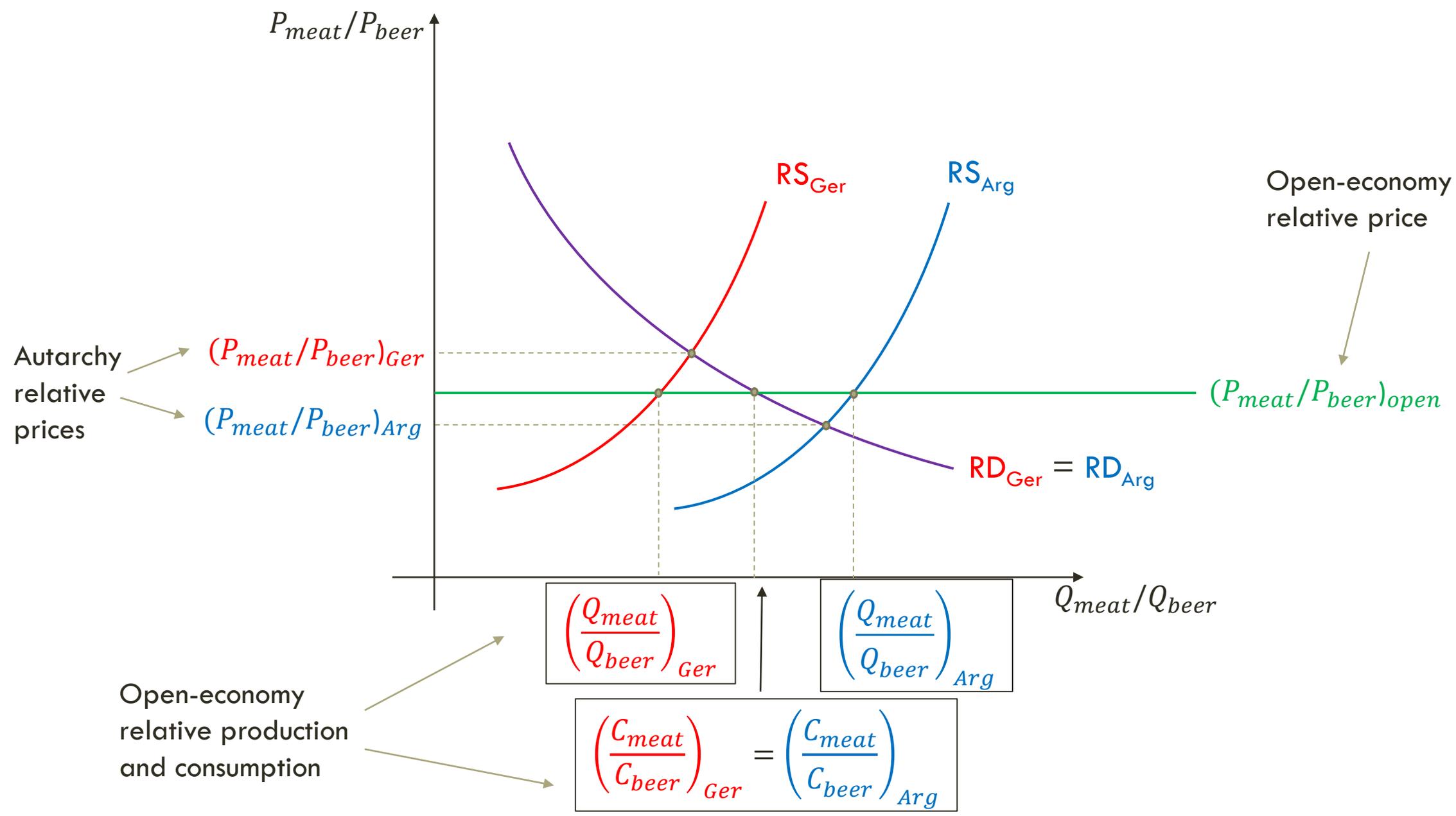
QUESTION 4

Case I:

- In the diagram, let's use price of meat relative to beer, and quantity of meat relative to beer
- Argentina is land abundant; therefore, for a given relative price, it will produce relatively more meat (land intensive) than Germany. This means that Argentina's relative supply curve is to the right of Germany's.
- In autarchy, relative price of meat is higher in Germany.

Open economy:

- Relative price will be somewhere in between the two autarchy prices
- In Germany, relative consumption of meat is higher than relative production = Germany imports meat, exports beer
- In Argentina, relative consumption of meat is lower than relative production = Argentina exports meat, imports beer



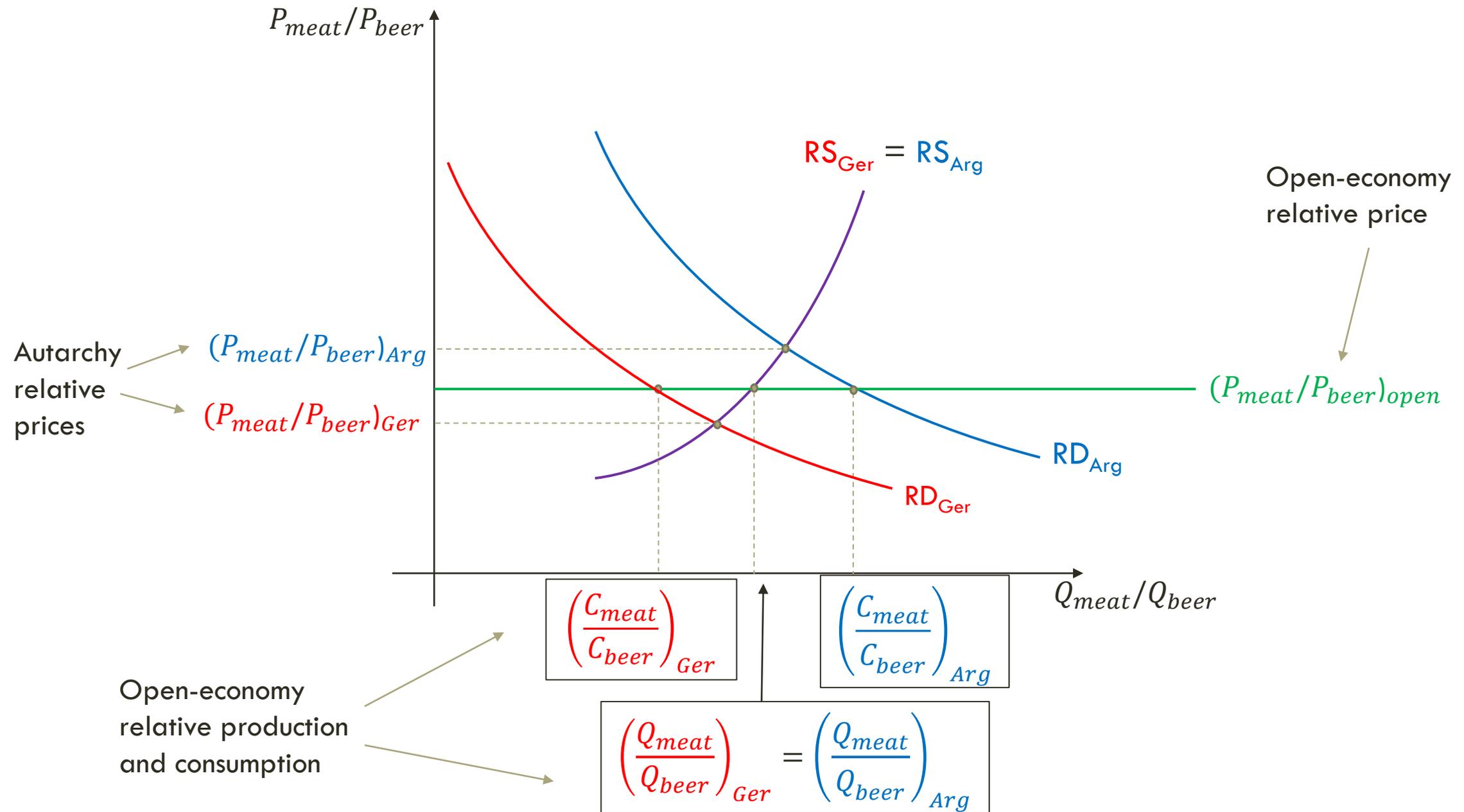
QUESTION 4

Case II:

- Countries have the same endowments of land and labor; therefore, their relative supply curves coincide
- Since Argentinians have stronger preference for meat, their relative demand curve will be to the right of Germany's
- In autarchy, relative price of meat is higher in Argentina

Open economy:

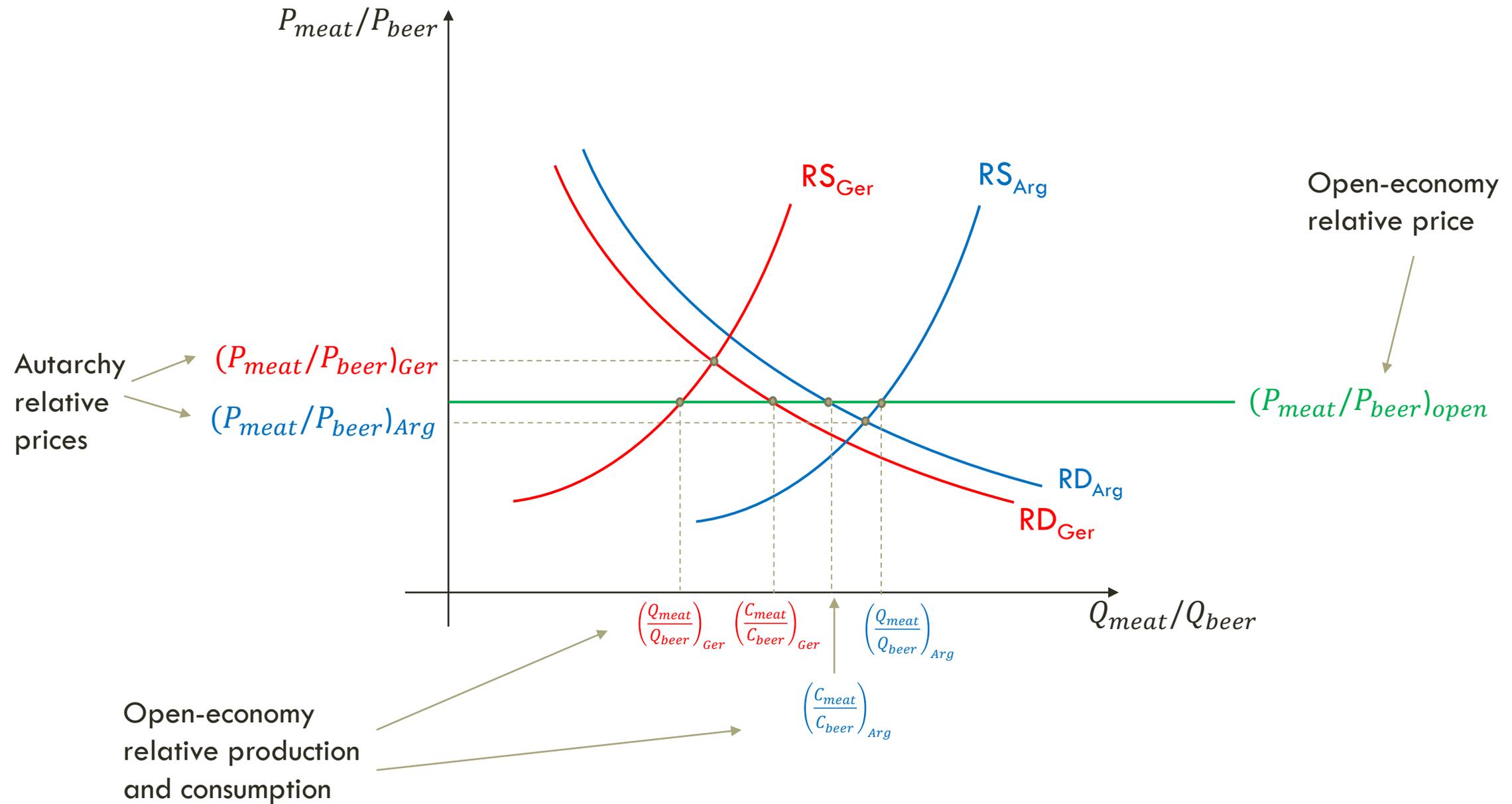
- Relative price will be somewhere in between the two autarchy prices
- In Germany, relative consumption of meat is lower than relative production = Germany imports beer, exports meat
- In Argentina, relative consumption of meat is higher than relative production = Argentina exports beer, imports meat



QUESTION 4

Case III:

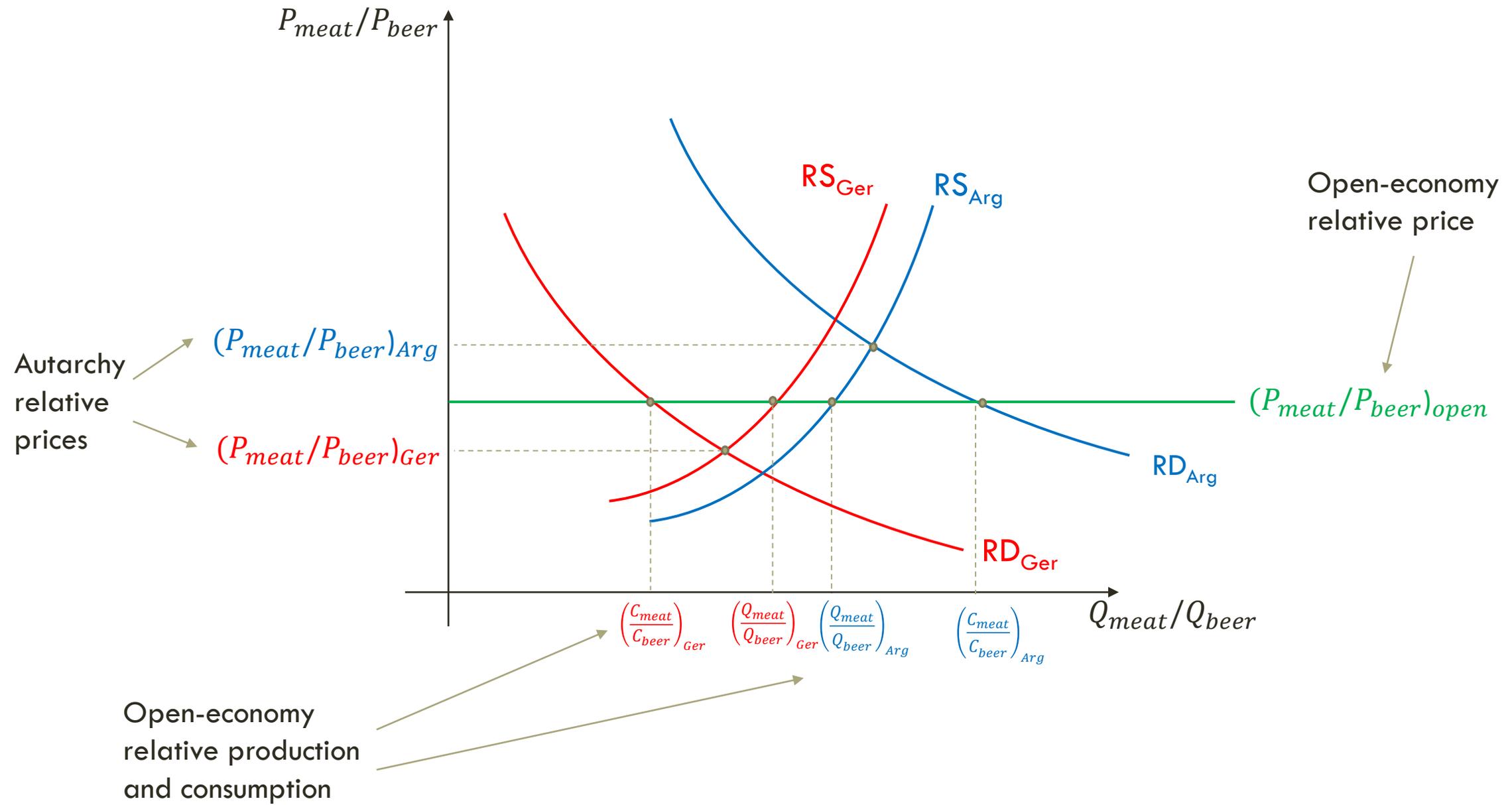
- Here we have a mix of the previous two effects
- Differences in factor endowments induce Argentina to be a meat exporter, and Germany to be a beer exporter
- But differences in preferences work in the opposite way
- Therefore, if differences in factor endowments are large, compared to differences in preferences:
 - Argentina exports meat, imports beer
 - Germany exports beer, imports meat
- This is illustrated in the next slide:
 - Relative consumption of meat is higher than relative production in Germany
 - Relative consumption of meat is lower than relative production in Argentina



QUESTION 4

Case III (cont.):

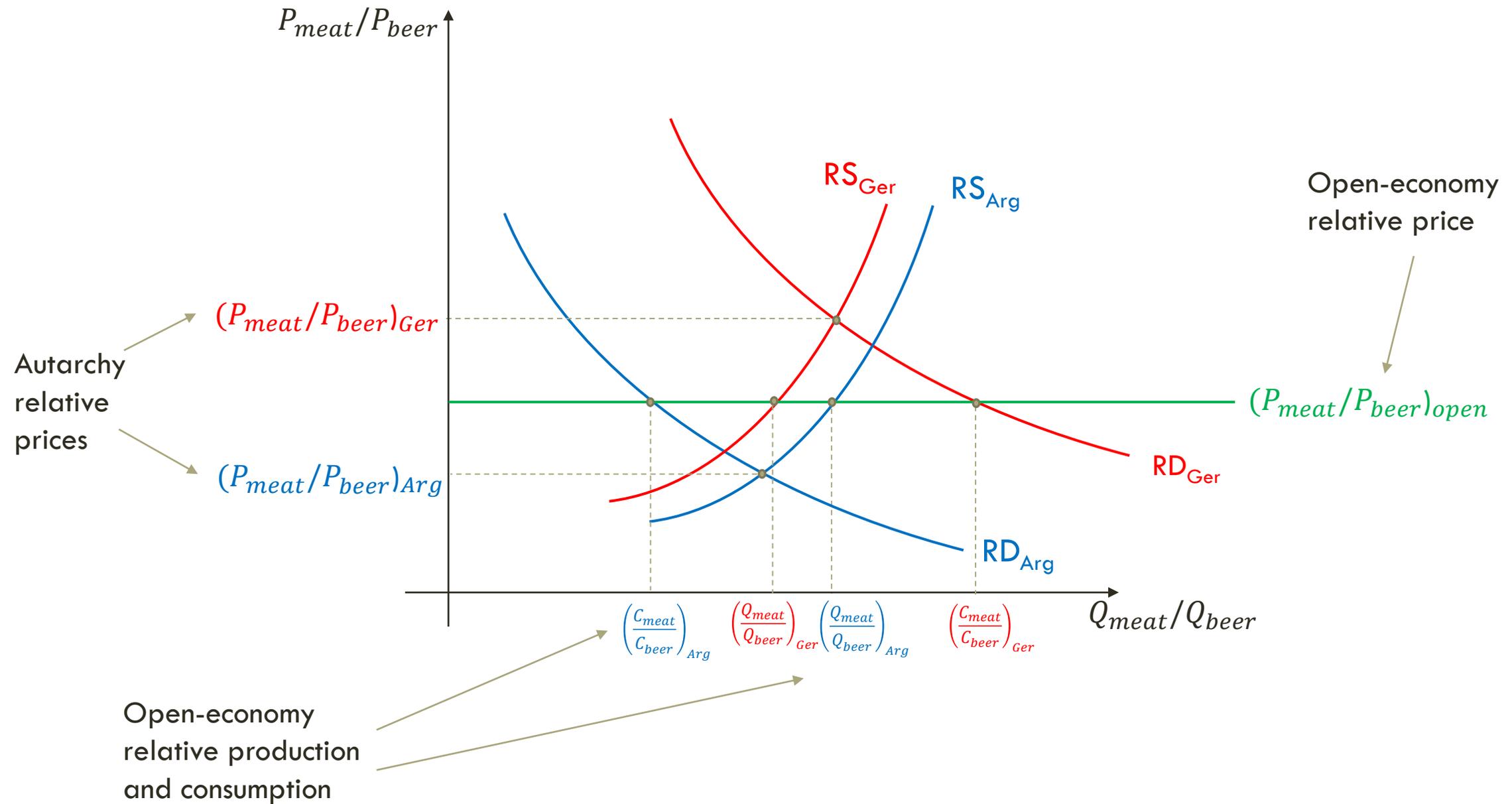
- But if differences in preferences are large, compared to differences in factor endowments:
 - Argentina exports beer, imports meat
 - Germany exports meat, imports beer
- This is illustrated in the next slide:
 - Relative consumption of meat is lower than relative production in Germany
 - Relative consumption of meat is higher than relative production in Argentina



QUESTION 4

Case IV:

- Now differences in factor endowments and in preferences reinforce each other
 - Argentina exports meat, imports beer
 - Germany exports beer, imports meat
- This is illustrated in the next slide:
 - Relative consumption of meat is higher than relative production in Germany
 - Relative consumption of meat is lower than relative production in Argentina



QUESTION 5

See presentation “Heckscher-Ohlin Model – Part IV”
(slides 21-30)