Lei de Formação para as Fatorações

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 - b^3 = (a-b).(a^2 + ab + b^2)$$

$$a^4 - b^4 = (a^2 + b^2).(a^2 - b^2)$$

$$a^4 - b^4 = (a-b).(a+b).(a^2 + b^2)$$

$$a^4 - b^4 = (a-b).(a^3 + ab^2 + a^2 b + b^3)$$

$$a^4 - b^4 = (a-b).(a^3 + a^2 b + ab^2 + b^3)$$

$$a^5 - b^5 = (a-b).(a^4 + a^3 b + a^2 b^2 + ab^3 + b^4)$$

$$a^6 - b^6 = (a-b).(a^5 + a^4 b + a^3 b^2 + a^2 b^3 + a b^4 + b^5)$$

$$a^7 - b^7 = (a-b).(a^6 + a^5 b + a^4 b^2 + a^3 b^3 + a^2 b^4 + a b^5 + b^6)$$

Então tem-se:

$$\boxed{a^m - b^m = (a-b).(a^{m-1} + a^{m-2} b + a^{m-3} b^2 + \dots + ab^{m-2} + b^{m-1})}$$

Exemplos

a) $$\lim_{x \to 1} \frac{\sqrt{2x} - \sqrt{x+1}}{x-1} = \frac{0}{0}$$

$$= \lim_{x \to 1} \frac{\sqrt{2x} - \sqrt{x+1} \cdot (\sqrt{2x} + \sqrt{x+1})}{(x-1) \cdot (\sqrt{2x} + \sqrt{x+1})}$$

$$= \lim_{x \to 1} \frac{2x + \cancel{\sqrt{2x} \cdot \sqrt{x+1}} - \cancel{\sqrt{2x} \cdot \sqrt{x+1}} - x - 1}{(x-1) \cdot (\sqrt{2x} + \sqrt{x+1})}$$

$$= \lim_{x \to 1} \frac{\cancel{x-1}}{\cancel{(x-1)} \cdot (\sqrt{2x} + \sqrt{x+1})}$$

$$= \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$

b) $$\lim_{x \to 2} \frac{\sqrt{2x^2 - 3x + 2} - 2}{\sqrt{3x^2 - 5x - 1} - 1} = \frac{0}{0}$$

$$= \lim_{x \to 2} \frac{\sqrt{2x^2 - 3x + 2} - 2 \cdot (\sqrt{2x^2 - 3x + 2} + 2) \cdot \sqrt{3x^2 - 5x - 1} + 1}{\sqrt{3x^2 - 5x - 1} - 1 \cdot \sqrt{2x^2 - 3x + 2} + 2) \cdot \sqrt{3x^2 - 5x - 1} + 1}$$

$$\lim_{x \to 2} \frac{(2x^2 - 3x + 2 - 4) \cdot (\sqrt{3x^2 - 5x - 1} + 1)}{(3x^2 - 5x - 1 - 1) \cdot (\sqrt{2x^2 - 3x + 2} + 2)}$$

$2x^2 - 3x + 2 - 4 = 0$

$2x^2 - 3x - 2 = 0$

$x_1 = 2$

$x_2 = -1/2$

$3x^2 - 5x - 2 = 0$

$x_1 = 2$

$x_2 = \frac{-1}{3}$

$a(x - x_1) \cdot (x - x_2)$

$$= \lim_{x \to 2} \frac{2(x-2)(x+1/2)\cdot(\sqrt{3x^2-3x+2}+1)}{3(x-2)\cdot(x+1/2)\cdot(\sqrt{2x^2-3x+2}+2)}$$

$$= \frac{10}{28} = \frac{5}{14}$$

c) $$\lim_{x \to -2} \frac{\sqrt[3]{2-3x}-2}{1+\sqrt[3]{2x+3}} = \frac{0}{0}$$

$$a^3 - b^3 = (a-b).(a^2+ab+b^2)$$

$$a^3 + b^3 = (a+b).(a^2-ab+b^2)$$

$$\left(\sqrt[3]{2-3x}\right)^3 - (2)^3 = \left(\sqrt[3]{2-3x}-2\right).\left(\sqrt[3]{(2-3x)^2}+2\sqrt[3]{2-3x}+4\right)$$

$$(1)^3 + \left(\sqrt[3]{2x+3}\right)^3 = \left(1+\sqrt[3]{2x+3}\right).\left(1-\sqrt[3]{2x+3}+\sqrt[3]{(2x+3)^2}\right)$$

$$\lim_{x \to -2} \frac{\left(\sqrt[3]{2-3x}-2\right).\left(\sqrt[3]{(2-3x)^2}+2\sqrt[3]{2-3x}+4\right).\left(1-\sqrt[3]{2x+3}+\sqrt[3]{(2x+3)^2}\right)}{\left(1+\sqrt[3]{2x+3}\right).\left(1-\sqrt[3]{2x+3}+\sqrt[3]{(2x+3)^2}\right).\left(\sqrt[3]{(2-3x)^2}+2\sqrt[3]{2-3x}+4\right)}$$

$$\lim_{x \to -2} \frac{(\overbrace{2-3x-8}^{-3x-6}).\left(1-\sqrt[3]{2x+3}+\sqrt[3]{(2x+3)^2}\right)}{(1+2x+3).\left(\sqrt[3]{(2-3x)^2}+2\sqrt[3]{2-3x}+4\right)}$$

$$\lim_{x \to -2} \frac{-3\cancel{(x+2)}.\left(1-\sqrt[3]{2x+3}+\sqrt[3]{(2x+3)^2}\right)}{2\cancel{(x+2)}.\left(\underbrace{\sqrt[3]{(2-3x)^2}}+2\sqrt[3]{2-3x}+4\right)} = \frac{-\cancel{9}^{\,3}}{2\cdot \cancel{12}_{\,4}} =$$

$$\sqrt[3]{64} = \sqrt[3]{2^6}$$

$$-\frac{3}{8}$$

d) $\lim_{x \to 0} \frac{(x+a)^n - a^n}{x}$

$\boxed{a^m - b^m = (a-b).(a^{m-1} + a^{m-2}.b + a^{m-3}.b^2 + \dots + ab^{m-2} + b^{m-1})}$

$\lim_{x \to 0} \frac{(\cancel{x} \cancel{+a} \cancel{-a}) \left[(x+a)^{m-1} + a(x+a)^{m-2} + a^2(x+a)^{m-3} + \dots + a^{m-2}(x+a) + a^{m-1}\right]}{\cancel{x}}$

$= a^{m-1} + a.a^{m-2} + a^2.a^{m-3} + a^{m-2}.a + a^{m-1}$

$= a^{m-1} + a^{m-1} + a^{m-1} + a^{m-1} + a^{m-1}$

$= 5 a^{m-1}$ ou

$= n a^{m-1}$

e) $\lim_{x \to a} \frac{x^m - a^m}{x^n - a^n}$

$\lim_{x \to a} \frac{\cancel{(x-a)}.(x^{m-1} + ax^{m-2} + a^2x^{m-3} + \dots + a^{m-2}.x + a^{m-1})}{\cancel{(x-a)}.(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \dots + a^{n-2}.x + a^{n-1})}$

$= \frac{a^{m-1} + a.a^{m-2} + a^2.a^{m-3} + \dots + a^{m-2}.a + a^{m-1}}{a^{n-1} + a.a^{n-2} + a^2a^{n-3} + \dots + a^{n-2}.a + a^{n-1}}$

$= \frac{a^{m-1} + a^{m-1} + a^{m-1} + \dots + a^{m-1} + a^{m-1}}{a^{n-1} + a^{n-1} + a^{n-1} + \dots + a^{n-1} + a^{n-1}}$

$= \frac{\alpha a^{m-1}}{\beta a^{n-1}} = \frac{\alpha}{\beta} a^{m \cancel{-1} - n \cancel{+1}}$

$= \frac{\alpha}{\beta} a^{m-n}$

Demonstração do Teorema do Confronto

Hipótese $\quad g(x) \leqslant f(x) \leqslant h(x)$

$$\lim_{x \to a} g(x) = L \qquad \lim_{x \to a} h(x) = L$$

Tese: $\lim_{x \to a} g(x) = L$

Pela hipótese existem $\delta_1 > 0$ e $\delta_2 > 0$

$|g(x) - L| < \varepsilon$ sempre que $0 < |x-a| < \delta_1$

$L - \varepsilon < g(x) < L + \varepsilon$ " " $0 < |x-a| < \delta_1$

$|h(x) - L| < \varepsilon$ sempre que $0 < |x-a| < \delta_1$

$L - \varepsilon < h(x) < L + \varepsilon$ " " $0 < |x-a| < \delta_2$

Considerando o valor mínimo de δ, tem-se:

$$\delta_{mín} = \{\delta_1 , \delta_2\}$$

Então pela hipótese tem-se:

$L - \varepsilon < g(x) \leqslant f(x) \leqslant h(x) < L + \varepsilon$ sempre que

$$0 < |x-a| < \delta$$

De forma particular $L - \varepsilon < f(x) < L + \varepsilon$ sempre que

$$0 < |x-a| < \delta$$

Portanto $\boxed{\lim_{x \to a} f(x) = L}$ //

Exemplos: Teorema do Confronto

a) Mostrar que $\lim\limits_{x \to 0} \underbrace{\sqrt{x^3+x^2}}_{t} \operatorname{sen} \frac{\pi}{x} = 0$

$$-1 \leq \operatorname{sen} t \leq 1 \quad \times \sqrt{x^3+x^2}$$

$$-\sqrt{x^3+x^2} \leq \sqrt{x^3+x^2} \operatorname{sen} \frac{\pi}{x} \leq \sqrt{x^3+x^2}$$

$$\underbrace{\lim_{x \to 0} -\sqrt{x^3+x^2}}_{=0} \leq \lim_{x \to 0} \sqrt{x^3+x^2} \operatorname{sen} \frac{\pi}{x} \leq \underbrace{\lim_{x \to 0} \sqrt{x^3+x^2}}_{=0}$$

$$0 \leq \lim_{x \to 0} \sqrt{x^3+x^2} \operatorname{sen} \frac{\pi}{x} \leq 0$$

Então $\lim\limits_{x \to 0} \sqrt{x^3+x^2} \operatorname{sen} \frac{\pi}{x} = 0$

b) Seja uma função definida em $\mathbb{R}$ tal que para todo $x \neq 1$, tem-se:

$$-x^2+3x \leq f(x) \leq \frac{x^2-1}{x-1}$$

$$\underbrace{\lim_{x \to 1} (-x^2+3x)}_{2} \leq \lim_{x \to 1} f(x) \leq \underbrace{\lim_{x \to 1} \frac{x^2-1}{x-1}}_{2} \rightarrow \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$$

Então $\lim\limits_{x \to 1} f(x) = 2$