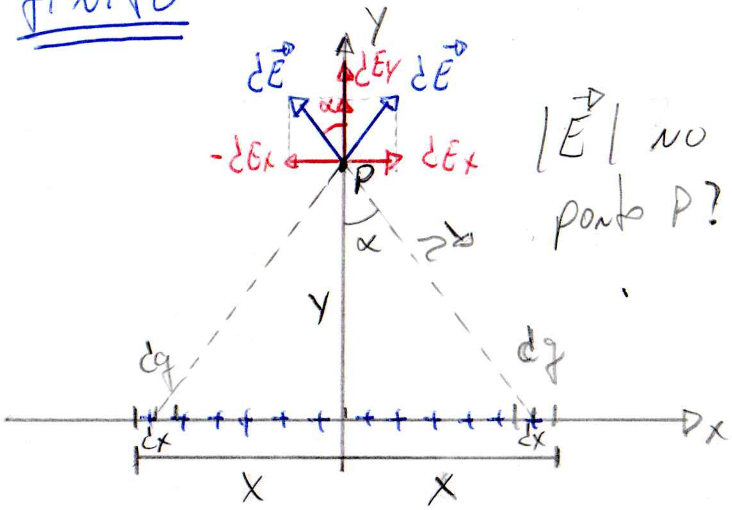
Campo em um fio carregado *finito*



$|\vec{E}|$ no ponto P?

$$dq = \lambda\, dx \qquad |\vec{r}| = \sqrt{x^2+y^2}$$

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r^2}$$

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda\, dx}{x^2+y^2}$$

dE_x se anulam por simetria.

$$dE_y = dE \cos\alpha$$

$$dE_y = \frac{1}{4\pi\epsilon_0} \frac{\lambda\, dx}{x^2+y^2} \cdot \cos\alpha ,$$

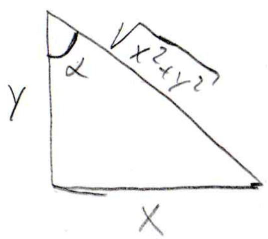
onde $\cos\alpha = \dfrac{y}{r} = \dfrac{y}{\sqrt{x^2+y^2}}$

$$dE_y = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda . dx . y}{(x^2+y^2)^{3/2}}$$

$$|\vec{E}| = E = \int_{-x}^{x} dE_y = 2\int_0^x dE_y$$

$$E = 2\int_0^x \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda\, y\, dx}{(x^2+y^2)^{3/2}}$$

Substituição trigonométrica



$$x = y . tg\,\alpha$$

$$dx = y \sec^2\alpha\, d\alpha$$

$$x^2+y^2 = y^2 tg^2\alpha + y^2$$

$$x^2+y^2 = y^2(1+tg^2\alpha) = y^2 \sec^2\alpha$$

$$E = \frac{2\lambda y}{4\pi\epsilon_0} \cdot \int \frac{y \sec^2\alpha\, d\alpha}{(y^2 \sec^2\alpha)^{3/2}}$$

$$E = \frac{\lambda y}{2\pi\epsilon_0} \int \frac{y \sec^2\alpha\, d\alpha}{y^3 \sec^3\alpha}$$

$$E = \frac{\lambda}{2\pi\epsilon_0} \cdot \frac{1}{y} \cdot \int \frac{d\alpha}{\sec\alpha}$$

$$E = \frac{\lambda}{2\pi\epsilon_0} \cdot \frac{1}{y} \int \cos\alpha\, d\alpha$$

$$E = \frac{\lambda}{2\pi\epsilon_0} \frac{1}{y} \operatorname{sen}\alpha \Big|_0^{\alpha}$$

$$E = \frac{\lambda}{2\pi\epsilon_0} \cdot \frac{1}{y} \cdot \operatorname{sen}\alpha$$

Mas $\operatorname{sen}\alpha = \dfrac{x}{\sqrt{x^2+y^2}}$

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0} \cdot \frac{1}{y} \cdot \frac{x}{\sqrt{x^2+y^2}}\, \hat{y}$$

Para distâncias do fio muito maiores que x:

$$y \gg x \Rightarrow \sqrt{x^2+y^2} \rightarrow y$$

$$\vec{E} \simeq \frac{\lambda \cdot x}{2\pi\varepsilon_0 y^2}\, \hat{y}, \quad \text{mas} \quad \lambda = \frac{q}{2x}$$

$$\vec{E} \simeq \frac{q}{2x} \cdot \frac{x}{2\pi\varepsilon_0 y^2}\, \hat{y}$$

$$\boxed{\vec{E} \simeq \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{y^2}\, \hat{y}}$$

→ carga pontual.

Para um fio infinito:

$$\lim_{x \to \infty} \frac{x}{\sqrt{x^2+y^2}} = 1$$

$$\vec{E} \simeq \frac{\lambda}{2\pi\varepsilon_0 y}\, \hat{y}$$