• Vetor de Bloch (Felix Bloch)

• Sistemas 2 níveis

Possível expressar qualquer estado de 2-níveis:

$$\boxed{\hat{\rho} = \frac{\hat{\mathbb{1}} + \vec{r}\cdot\vec{\sigma}}{2}}$$

Vetor de Bloch: $\vec{r}$

$\rightarrow \|\vec{r}\| \leq 1$ $\begin{cases} \|\vec{r}\| = 1 & \text{(puro)} \\ \|\vec{r}\| < 1 & \text{(mistura)} \end{cases}$

$\{r_x, r_y, r_z\} \in \mathbb{R}$

$$\vec{r} = (r_x, r_y, r_z)$$

$$\vec{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$$

$$\vec{r}\cdot\vec{\sigma} = r_x\hat{\sigma}_x + r_y\hat{\sigma}_y + r_z\hat{\sigma}_z$$

$$\hat{\rho} = \frac{1}{2}\left(\hat{\mathbb{1}} + r_x\hat{\sigma}_x + r_y\hat{\sigma}_y + r_z\hat{\sigma}_z\right);$$

$$\begin{cases} r_x = Tr(\hat{\rho}\hat{\sigma}_x) \\ r_y = Tr(\hat{\rho}\hat{\sigma}_y) \\ r_z = Tr(\hat{\rho}\hat{\sigma}_z) \end{cases}$$

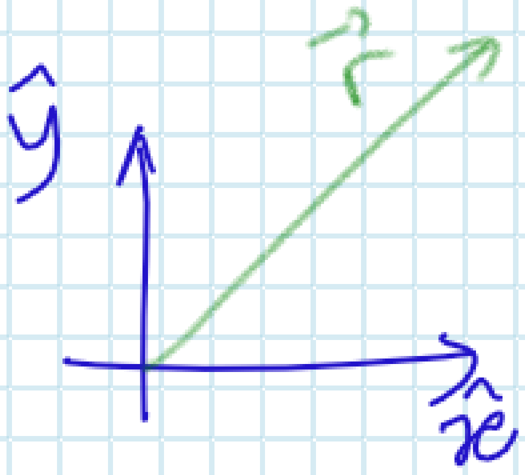
• **Sugestão:**

$$\rho = \begin{pmatrix} 2/3 & 1/6 - \frac{1}{3}i \\ 1/6 + 1/3\, i & 1/3 \end{pmatrix}$$

a) Encontre $\vec{r}$

b) Estado puro ou mistura? (usando $\vec{r}$: Vetor de Bloch)

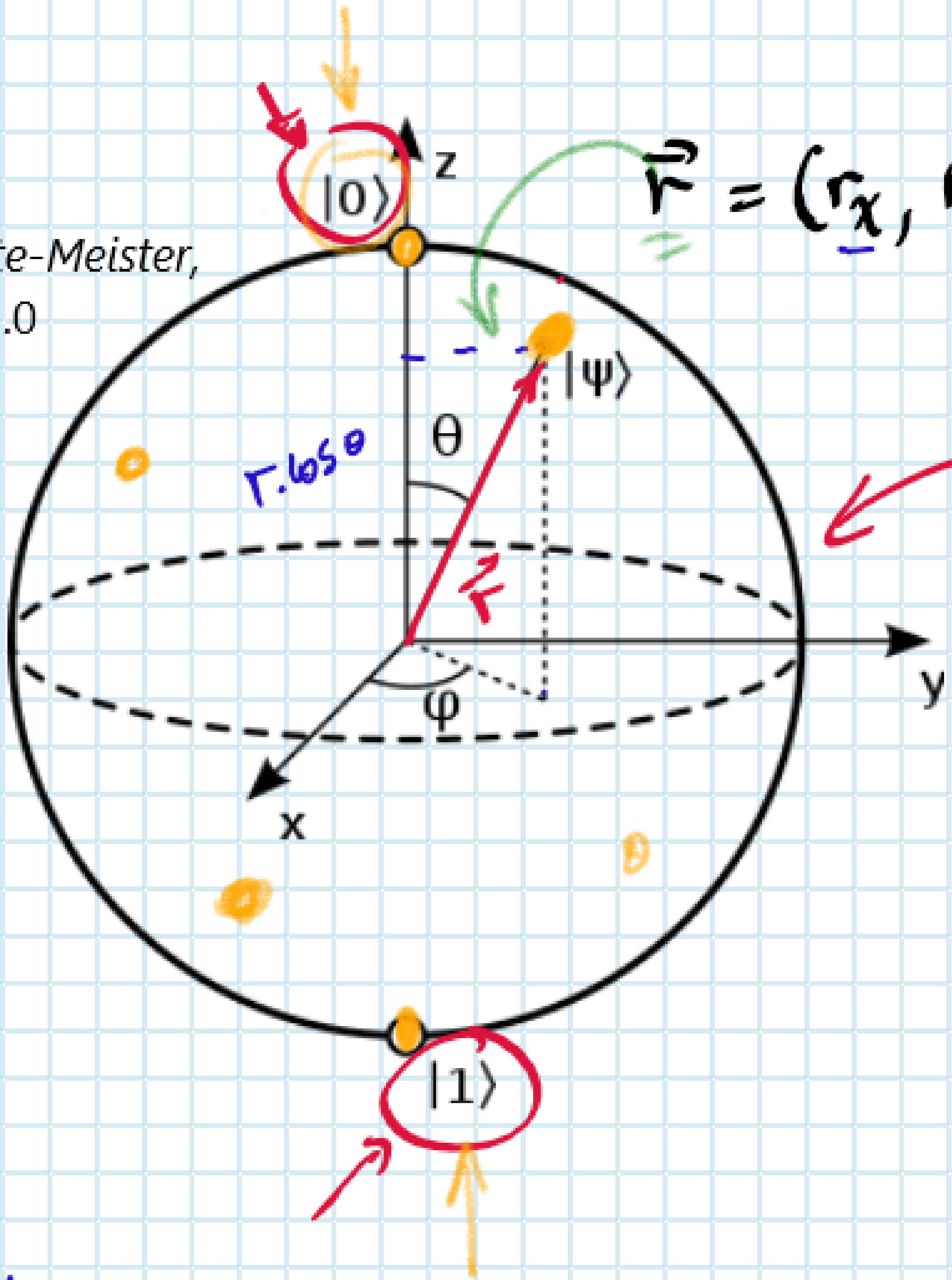
$\mathbb{R}^2$:



$\vec{r} = (r_x, r_y)$; $r_x = \vec{r}\cdot\hat{x}$, $r_y = \vec{r}\cdot\hat{y}$

$\mathbb{R}^3$:

Fig. by Smite-Meister, CC BY-SA 3.0



$\vec{r} = (r_x, r_y, r_z)$

esfera de Bloch

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |\uparrow\rangle$

$|\downarrow\rangle = |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|x\rangle = |0\rangle + |1\rangle$

Qualquer estado $|\psi\rangle$ de 1-qbit pode ser representado por um vetor de Bloch $\vec{r}$

• Vetores com a ponta na superfície da esfera representam estados puros $\rightarrow \|\vec{r}\| = 1$

• Vetores no interior da esfera: $\|\vec{r}\| < 1$ são estados mistos.

$\langle 0|1\rangle = 0$

Em geral $\left\{ \begin{array}{l} |\psi\rangle = \cos\left(\frac{\theta}{2}\right)|0\rangle + e^{i\varphi}\,\mathrm{sen}\left(\frac{\theta}{2}\right)|1\rangle \\ |\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle \end{array}\right.$

$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

Exemplos

$|0\rangle \rightarrow \theta = 0 \;;\; \varphi$ qualquer

$|1\rangle \rightarrow \theta = \pi \;;\; \varphi$ "

$|x\rangle = |+x\rangle = |+\rangle = \theta = \pi/2 \,,\; \varphi = 0 \rightarrow \vec{r} = \begin{pmatrix}1\\0\\0\end{pmatrix}$

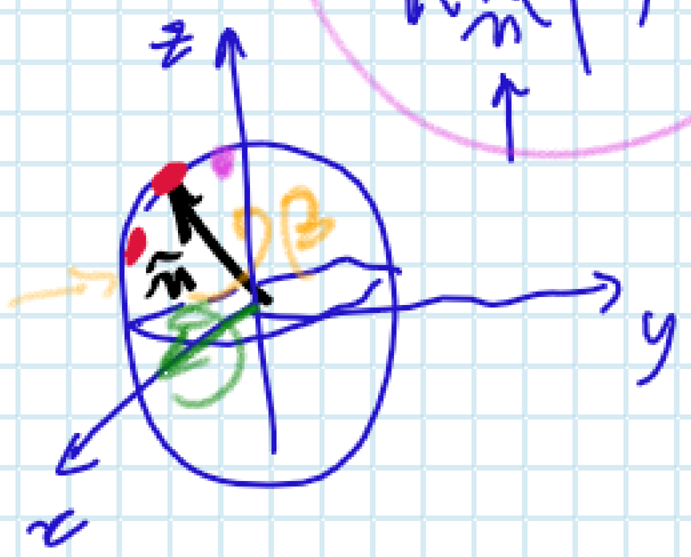
$|-x\rangle = |-\rangle \Rightarrow \theta = \pi/2 \;;\; \varphi = \pi \longrightarrow \vec{r} = \begin{pmatrix}-1\\0\\0\end{pmatrix}$

$|y\rangle = |+i\rangle = |+y\rangle \Rightarrow \theta = \frac{\pi}{2} \rightarrow \varphi = \pi/2 \rightarrow \vec{r} = \begin{pmatrix}0\\1\\0\end{pmatrix}$

$\Rightarrow$ Qualquer 1-qbit "quantum gate", corresponde uma transf. unitária,
$\hookrightarrow$ equivalente a uma rotação do qbit

$\boxed{\text{op. 1-qbit} \Rightarrow \text{Rotações}}$

$\hat{R}_{\hat{n}}(\beta) = \,?$



Relembrando

$e^{i\beta} = \cos(\beta) + i\,\mathrm{sen}(\beta)$

$e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^n}{n!} + \cdots$

$e^{A} = 𝟙 + A + \frac{A \cdot A}{2} + \frac{A^3}{3!} + \cdots \frac{A^k}{k!}$

$A^{\dagger}A = 𝟙$
$A = A^{\dagger}$ $\rightarrow \boxed{A^2 = 𝟙}$; $i = \sqrt{-1}$; $\left\{ e^{i\beta A} = 𝟙 + (i\beta A) + \frac{(i\beta A)^2}{2} + \cdots \right.$

$\boxed{e^{i\beta A} = \cos(\beta)\,𝟙 + i\,\mathrm{sen}(\beta)\,A}$

$\boxed{R_{\hat{n}}(\beta) = e^{-i\left(\frac{\beta}{2}\right)\vec{n}\cdot\vec{\sigma}}}$

$\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$

$\vec{n} = (n_x, n_y, n_z)$

$\sigma_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$; $\sigma_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}$; $\sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$

$R_n(\beta) \propto R_z(\alpha)\, R_y(\beta)\, R_z(\gamma)$

Exemplos:

$R_x(\beta) = \begin{pmatrix}\cos(\beta/2) & 0\\0 & \cos\beta/2\end{pmatrix} + \begin{pmatrix}0 & -i\,\mathrm{sen}\,\beta/2\\-i\,\mathrm{sen}\,\beta/2 & 0\end{pmatrix}$

$R_x(\beta) = \begin{pmatrix}\cos\beta/2 & -i\,\mathrm{sen}(\beta/2)\\-i\,\mathrm{sen}\,\beta/2 & \cos\beta/2\end{pmatrix}$

$R_y(\beta) = \begin{pmatrix}\cos\beta/2 & -\mathrm{sen}\,\beta/2\\\mathrm{sen}\,\beta/2 & \cos\beta/2\end{pmatrix}$

$R_z(\beta) = \begin{pmatrix}e^{-i\beta/2} & 0\\0 & e^{i\beta/2}\end{pmatrix}$