Lab3 - Experimento de Stern-Gerlach

June 14, 2020

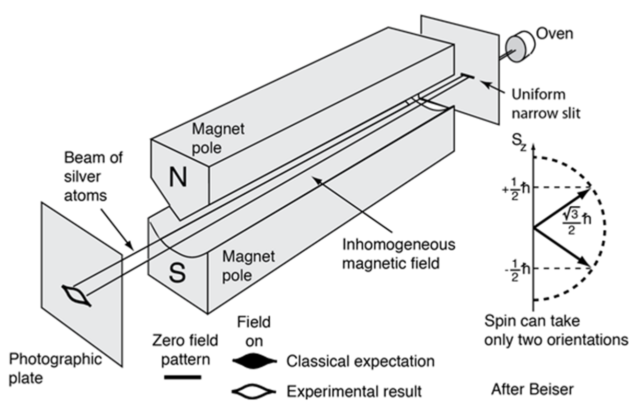
1 Lab3 - Experimento de Stern-Gerlach

Neste notebook aprenderemos um pouco mais sobre o famoso experimento de Otto Stern e Walther Gerlach, em 1922, que, além da sua importância histórica, ilustra bem como estados quânticos são diferentes de estados clássicos. Mais do que simplesmente apresentar os resultados, **iremos simular**, de fato, **o experimento aqui**, pelo menos os elementos mais fundamentais, para mostrar como os resultados são diferentes entre os sistemas clássico e quântico.

Para concentrar a discussão na física, ao invés da programação, vou usar o **QuTiP** (*Quantum Toobox in Python*), um framework para cálculos quânticos em Python. Isso permiterá usar ferramentas profissionais que definem e encapsulam em comandos simples toda a **álgebra que aprendemos nas aulas**, de uma forma mais direta e prática. Também é uma oportunidade de lhes apresentar mais uma ferramenta computacional bastante útil. Depois da aula você poderá explorar o notebook e a documentação online do QuTip para aprender mais, se tiver interesse.

Créditos: na preparação dessa aula usei como base um dos tutoriais do QuTip, apesentado na EuroSciPy 2019 por Simon Cross, disponível neste link.

1.0.1 Aparato experimental



Fonte: http://hyperphysics.phy-astr.gsu.edu/hbase/spin.html

Veja também:

- https://plato.stanford.edu/entries/physics-experiment/app5.html
- https://en.wikipedia.org/wiki/Stern-Gerlach_experiment#History

Carregando as bibliotecas necessárias do Python...

[1]:
```
%matplotlib inline
from collections import namedtuple
import matplotlib.pyplot as plt
import numpy as np
import qutip
from qutip import Bloch, ket
```

[2]:
```
# About QuTip  -- visit hhtp://qutip.org
qutip.about()
```

```
QuTiP: Quantum Toolbox in Python
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Copyright (c) QuTiP team 2011 and later.
Original developers: R. J. Johansson & P. D. Nation.
Current admin team: Alexander Pitchford, Paul D. Nation, Nathan Shammah,
Shahnawaz Ahmed, Neill Lambert, and Eric Giguère.
Project Manager: Franco Nori.
Currently developed through wide collaboration. See https://github.com/qutip for
details.

QuTiP Version:      4.4.1
Numpy Version:      1.18.1
Scipy Version:      1.4.1
Cython Version:     0.29.17
Matplotlib Version: 3.1.3
Python Version:     3.7.4
Number of CPUs:     2
BLAS Info:          INTEL MKL
OPENMP Installed:   False
INTEL MKL Ext:      True
Platform Info:      Windows (AMD64)
Installation path:  C:\Users\SergioMuniz\Anaconda3\lib\site-packages\qutip
==============================================================================
Please cite QuTiP in your publication.
==============================================================================
For your convenience a bibtex reference can be easily generated using
`qutip.cite()`
```
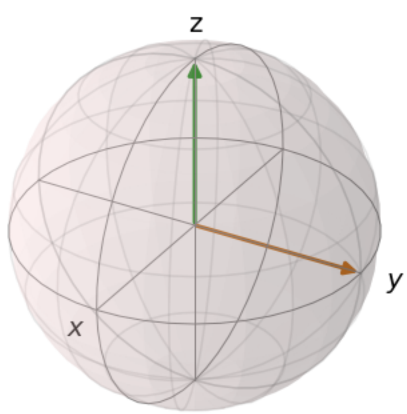

2 O experimento de Stern-Gerlach (1922)

3 Esfera de Bloch

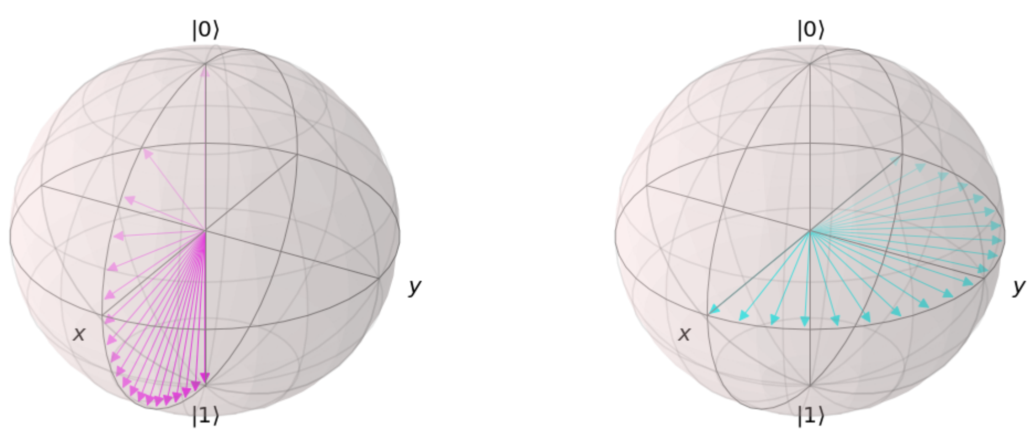
3.0.1 representação do vetor de estado $|\psi\rangle$ no espaço SU(2)

[3]:
```
z = np.array([0, 0, 1])
mu = np.array([0, 1, 0])

bloch = Bloch()
bloch.zlabel=("z", "")
bloch.add_vectors([z, mu])
bloch.show()
```



3.0.2 exemplo de evolução temporal de vetores de Bloch

Bloch Sphere



Changing relative magnitude

Changing relative angle

4 Simulando Stern-Gerlach no Python!

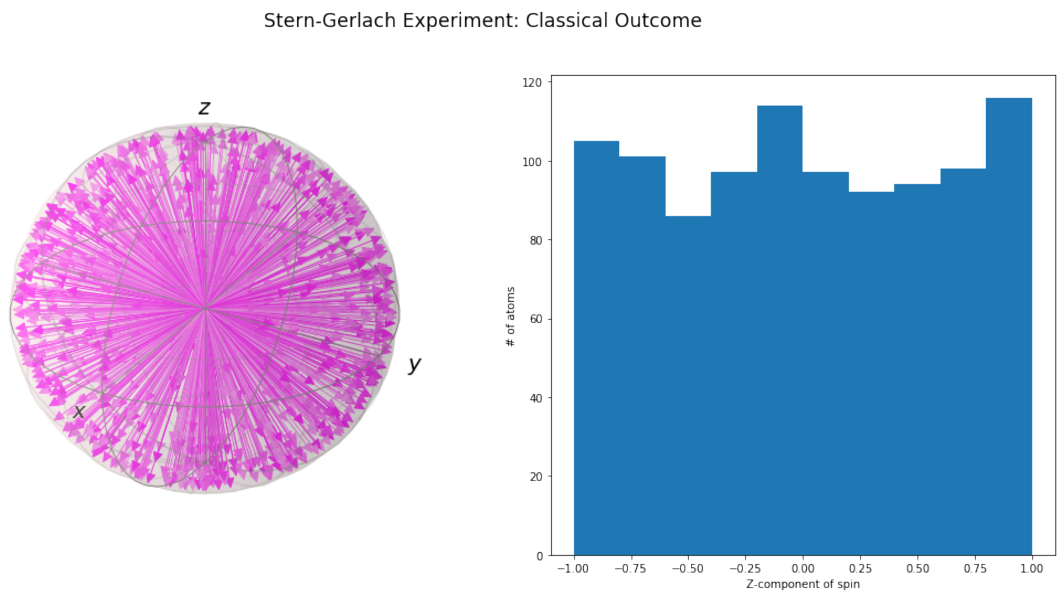
[4]:
```
# Definições gerais comuns

Direction = namedtuple("Direction", ["theta", "phi"])


def random_direction():
    """ Generate a random direction. """
    # See http://mathworld.wolfram.com/SpherePointPicking.html
    r = 0
    while r == 0:
        x, y, z = np.random.normal(0, 1, 3)
        r = np.sqrt(x**2 + y**2 + z**2)
    phi = np.arctan2(y, x)
    theta = np.arccos(z / r)
    return Direction(theta=theta, phi=phi)


def plot_classical_results(atoms, spins):
    fig = plt.figure(figsize=(18.0, 8.0))
    fig.suptitle("Stern-Gerlach Experiment: Classical Outcome",␣
↪fontsize="xx-large")

    ax1 = plt.subplot(1, 2, 1, projection='3d')
    ax2 = plt.subplot(1, 2, 2)

    b = Bloch(fig=fig, axes=ax1)
    b.vector_width = 1
    b.vector_color = ["#ff{:x}0ff".format(i, i) for i in range(10)]
    b.zlabel = ["$z$", ""]
    b.add_vectors(atoms)
    b.render(fig=fig, axes=ax1)

    ax2.hist(spins)
    ax2.set_xlabel("Z-component of spin")
    ax2.set_ylabel("# of atoms")


def plot_quantum_results(atoms, spins):
    fig = plt.figure(figsize=(18.0, 8.0))
    fig.suptitle("Stern-Gerlach Experiment: Quantum Outcome",␣
↪fontsize="xx-large")

    ax1 = plt.subplot(1, 2, 1, projection='3d')
    ax2 = plt.subplot(1, 2, 2)
```

```
    b = Bloch(fig=fig, axes=ax1)
    b.vector_width = 1
    b.vector_color = ["#{:x}0{:x}0ff".format(i, i) for i in range(10)]
    b.add_states(atoms)
    b.render(fig=fig, axes=ax1)

    ax2.hist(spins)
    ax2.set_xlabel("Z-component of spin")
    ax2.set_ylabel("# of atoms")
```

4.1 Simulando o caso Clássico

[5]:
```
# Simulation of expected results in the classical case

classical_up = np.array([0, 0, 1])

def classical_state(d):
    """ Prepare a spin state given a direction. """
    x = np.sin(d.theta) * np.cos(d.phi)
    y = np.sin(d.theta) * np.sin(d.phi)
    z = np.cos(d.theta)
    return np.array([x, y, z])


def classical_spin(c):
    """ Measure the z-component of the spin. """
    return classical_up.dot(c)


def classical_stern_gerlach(n):
    """ Simulate the Stern-Gerlach experiment """
    directions = [random_direction() for _ in range(n)]
    atoms = [classical_state(d) for d in directions]
    spins = [classical_spin(c) for c in atoms]
    return atoms, spins
```

```
[6]: atoms, spins = classical_stern_gerlach(1000)
     plot_classical_results(atoms, spins)
```



4.2 Os resultados experimentais (reais) observados

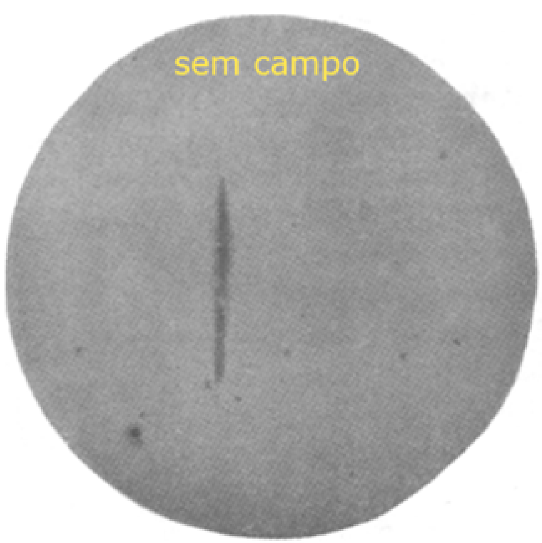


Fig. 2.

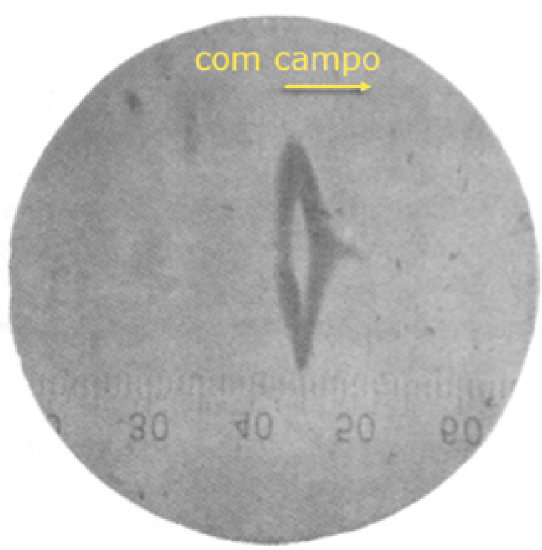


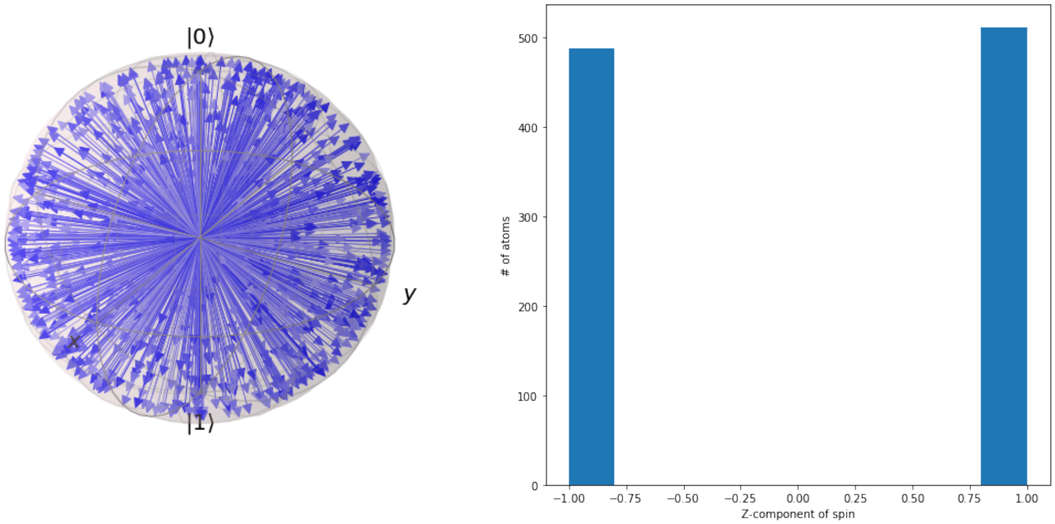
Fig. 3.

4.3 Simulando o caso Quântico

[7]:
```python
# Simulation of expected results in the quantum case
# usando a biblioteca qutip
up = ket('0')
down = ket('1')


def quantum_state(d):
    """ Prepare a spin state given a direction. """
    return np.cos(d.theta / 2) * up + np.exp(1j * d.phi) * np.sin(d.theta / 2) *
 ↪down


def quantum_spin(q):
    """ Measurement the z-component of the spin. """
    a_up = (up.dag() * q).tr()
    prob_up = np.abs(a_up) ** 2
    return 1 if np.random.uniform(0, 1) <= prob_up else -1


def quantum_stern_gerlach(n):
    """ Simulate the Stern-Gerlach experiment """
    directions = [random_direction() for _ in range(n)]
    atoms = [quantum_state(d) for d in directions]
    spins = [quantum_spin(q) for q in atoms]
    return atoms, spins
```

[8]:
```python
atoms, spins = quantum_stern_gerlach(1000)
plot_quantum_results(atoms, spins)
```

Stern-Gerlach Experiment: Quantum Outcome



5 Leitura complementar

1. QuTiP documentation [https://qutip.org]
2. History of the Stern-Gerlach experiment [https://plato.stanford.edu/entries/physics-experiment/app5.html]
3. Quantum Computing for the Determined by Michael Nielsen [http://michaelnielsen.org/blog/quantum-computing-for-the-determined]