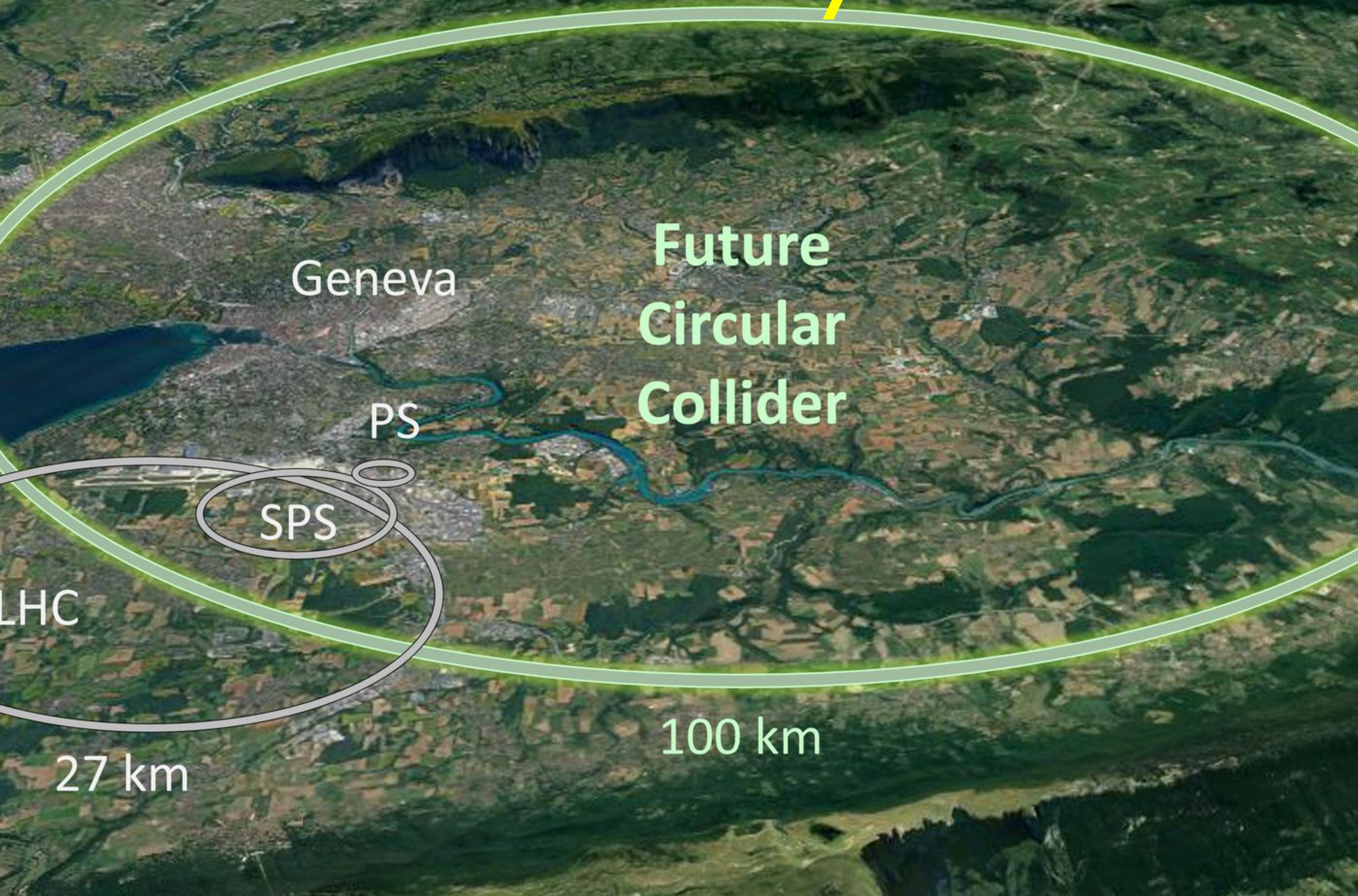
Collider Physics



II.A Some observables

- rapidity and pseudo-rapidity
- invariant mass
- transverse mass
- M_{T2}

Rapidity and pseudo-rapidity

- Given a 4 vector: $p^\mu = E(1, \beta \sin\theta \ \cos\phi, \beta \sin\theta \ \sin\phi, \beta \cos\theta)$

we define the rapidity as $y \equiv \frac{1}{2} \ln \left[\frac{E + p_z}{E - p_z} \right]$

we define the pseudo-rapidity as $\eta \equiv \frac{1}{2} \ln \left[\frac{1 + \cos\theta}{1 - \cos\theta} \right]$

Notice that $\eta \to y \text{ for } \beta \to 1$

It is useful to trade the polar angle by the pseudo-rapidity

Rapidity and pseudo-rapidity

•Given a 4 $\beta\cos\theta)$

we define

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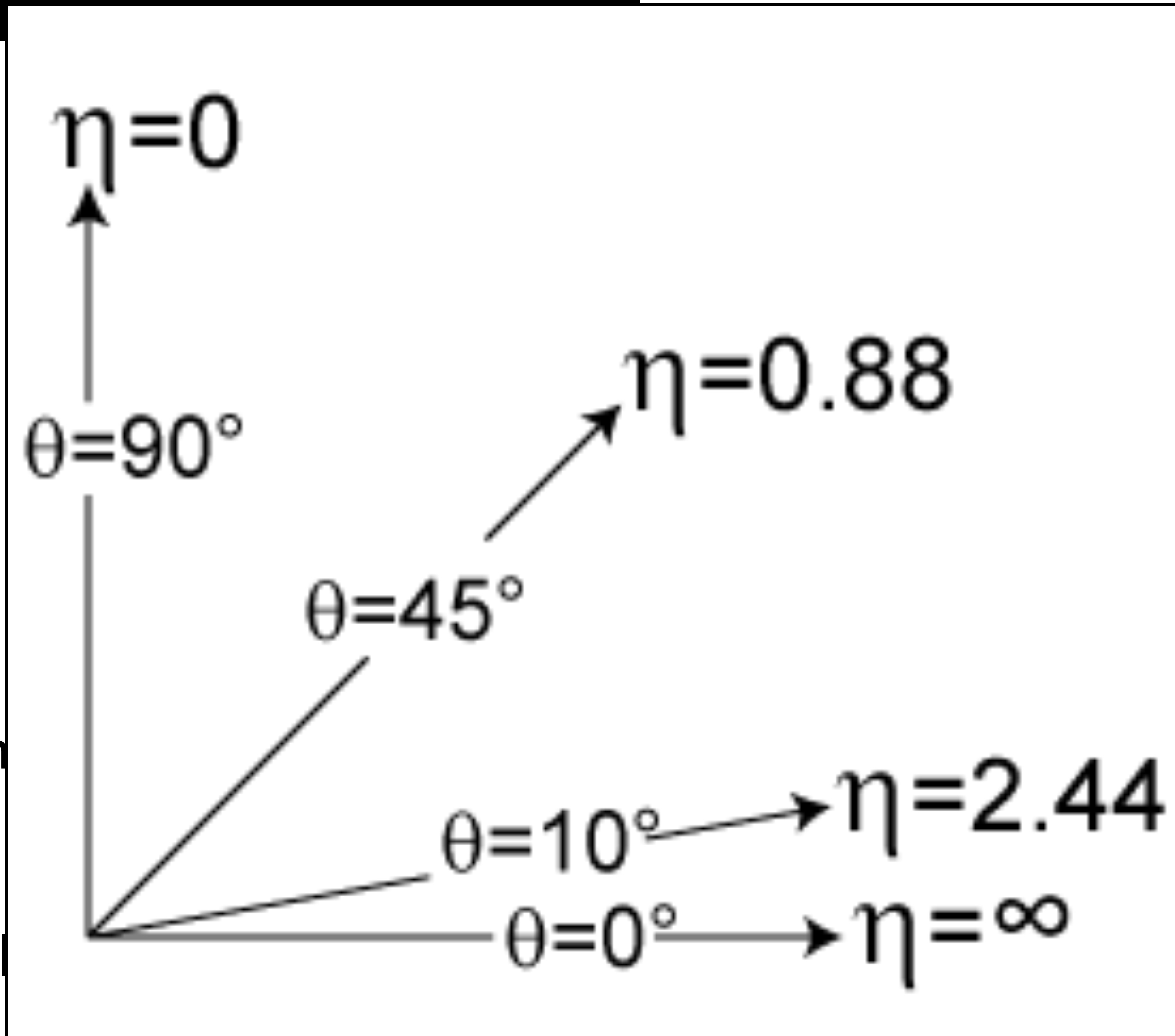
It is useful



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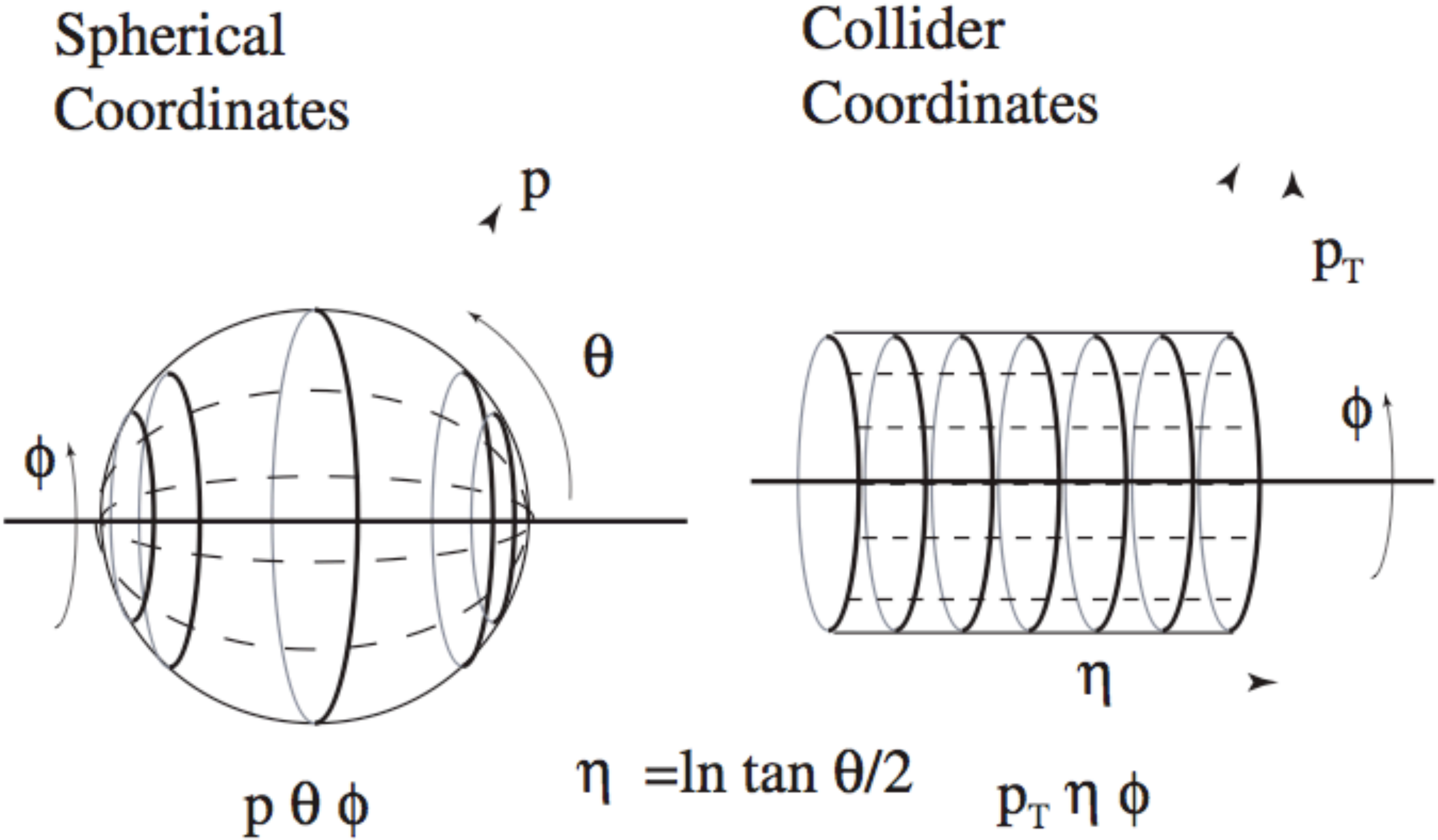
Rapidity and pseudo-rapidity



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Notice that $\eta \to y \text{ for } \beta \to 1$

It is useful to trade the polar angle by the pseudo-rapidity

- We can write $p^\mu = (E_T \cosh\eta, \vec{p}_T, E_T \sinh\eta)$

  with $E_T = \sqrt{m^2 + \vec{p}_T^{\,2}}$

- Largely used due to $$\frac{d^3\vec{p}}{E} = p_T \; dp_T \; d\varphi \; d\eta$$

  with the φ (azimuthal angle), p_T (transverse momentum) and $d\eta$ being invariant under longitudinal boosts

- It is usual to represent deposit of energy in the (η, φ) plane

  and the separation $\mathbf{\Delta R} = \sqrt{\mathbf{\Delta}\varphi^{\mathbf{2}} + \mathbf{\Delta}\eta^{\mathbf{2}}}$

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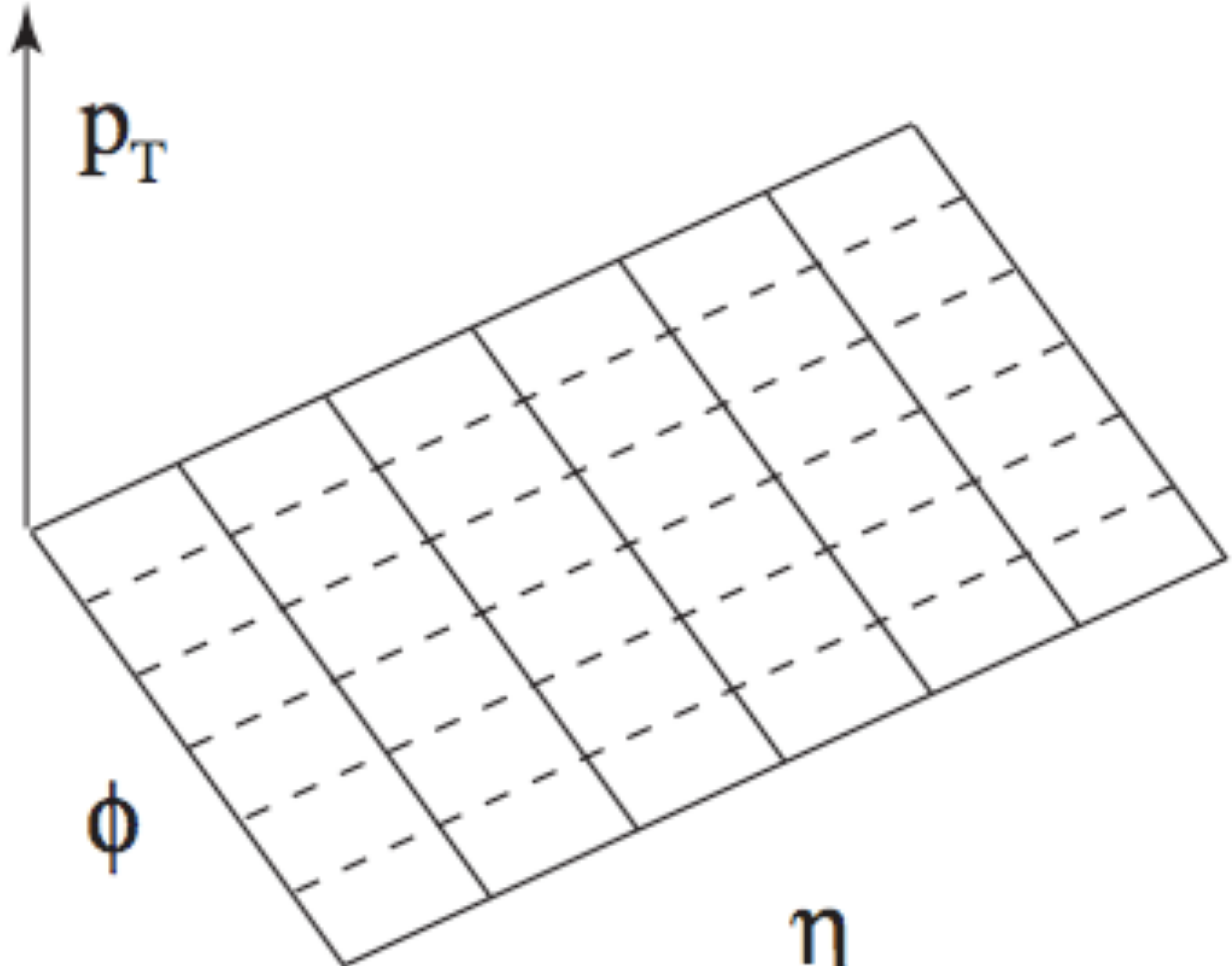


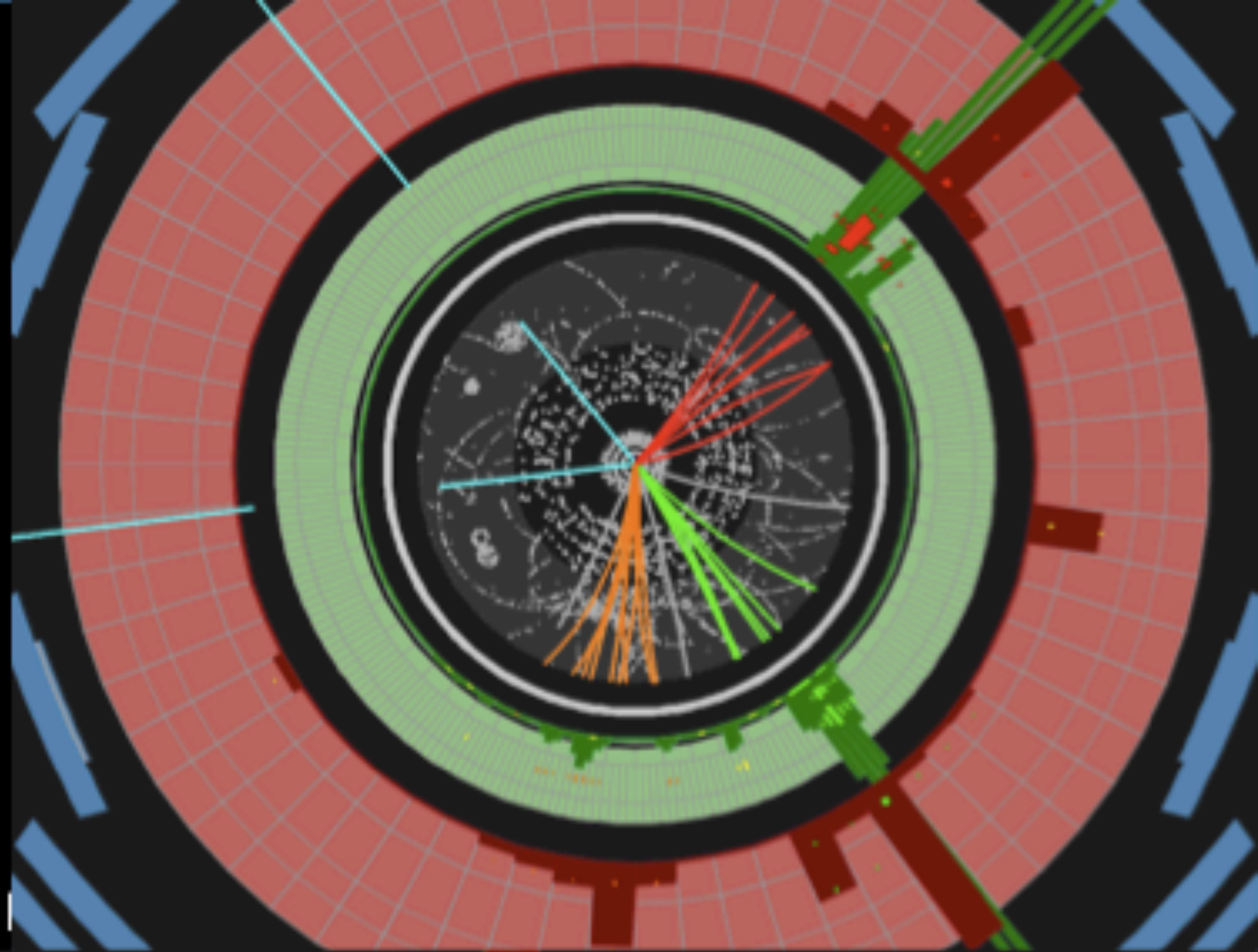
$d\eta$

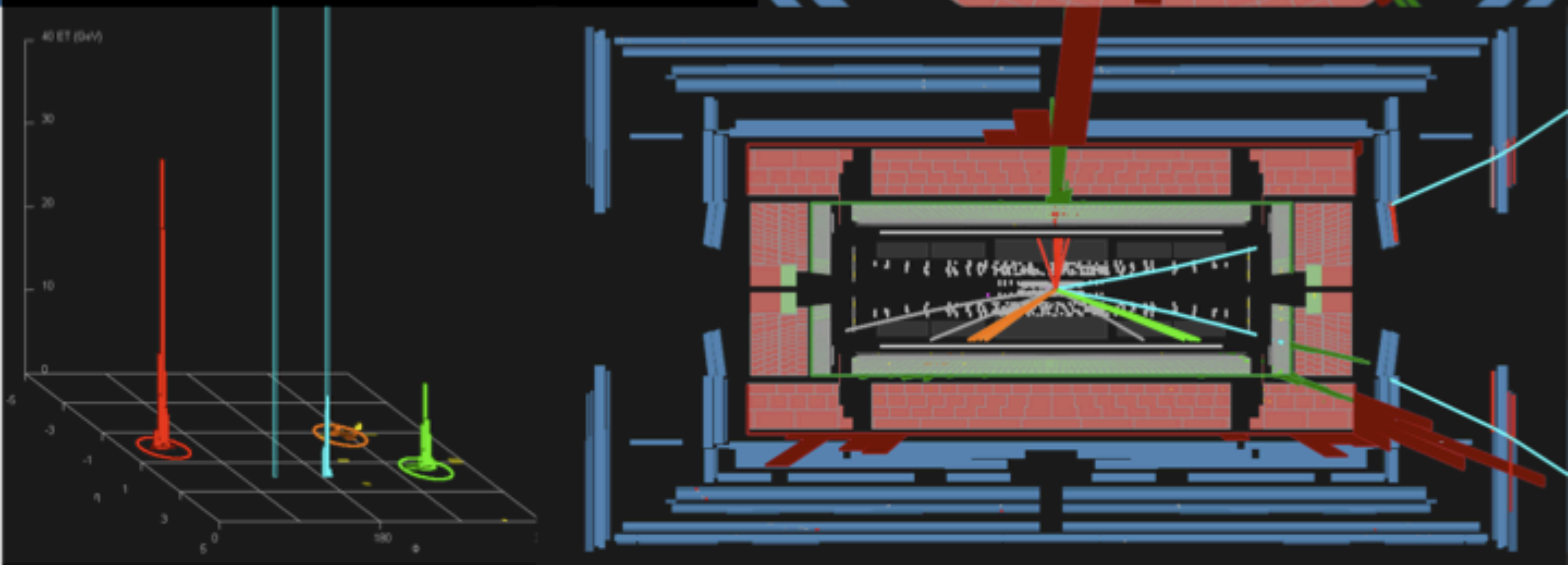
Lego Plot

ATLAS EXPERIMENT

$$Z \to \mu^- \mu^+ + 3 \text{ jets}$$

Run Number 158466, Event Number 4174272
Date: 2010-07-02 17:49:13 CEST





- In a hadron collision we call the subprocess COM energy $\sqrt{\hat{s}}$

- The four-momentum of the COM is

$$p^{\mu}_{CM} = \frac{\sqrt{s}}{2}\,(x_1 + x_2, 0, 0, x_1 - x_2) \Longrightarrow \hat{s} = p^{\mu}_{CM} p^{CM}_{\mu} = x_1 x_2 s$$

- The CM rapidity is $\quad y_{cm} = \frac{1}{2}\ln\left(\frac{x_1}{x_2}\right)$

- The rapidities of a particle in CM and LAB frames are related by

$$y = y^{\star} + y_{cm}$$

lab

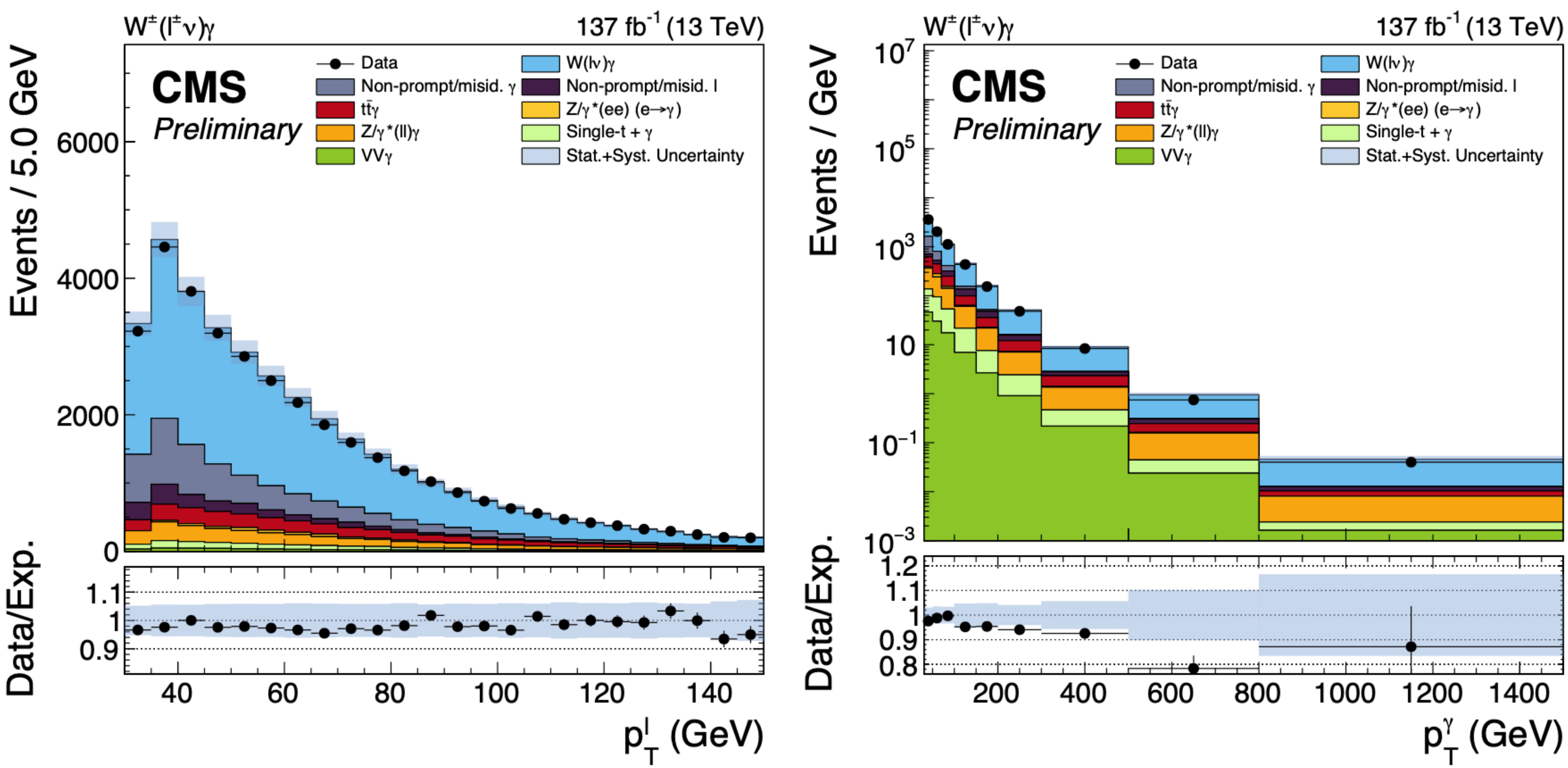
in the COM

- A useful change of variables is

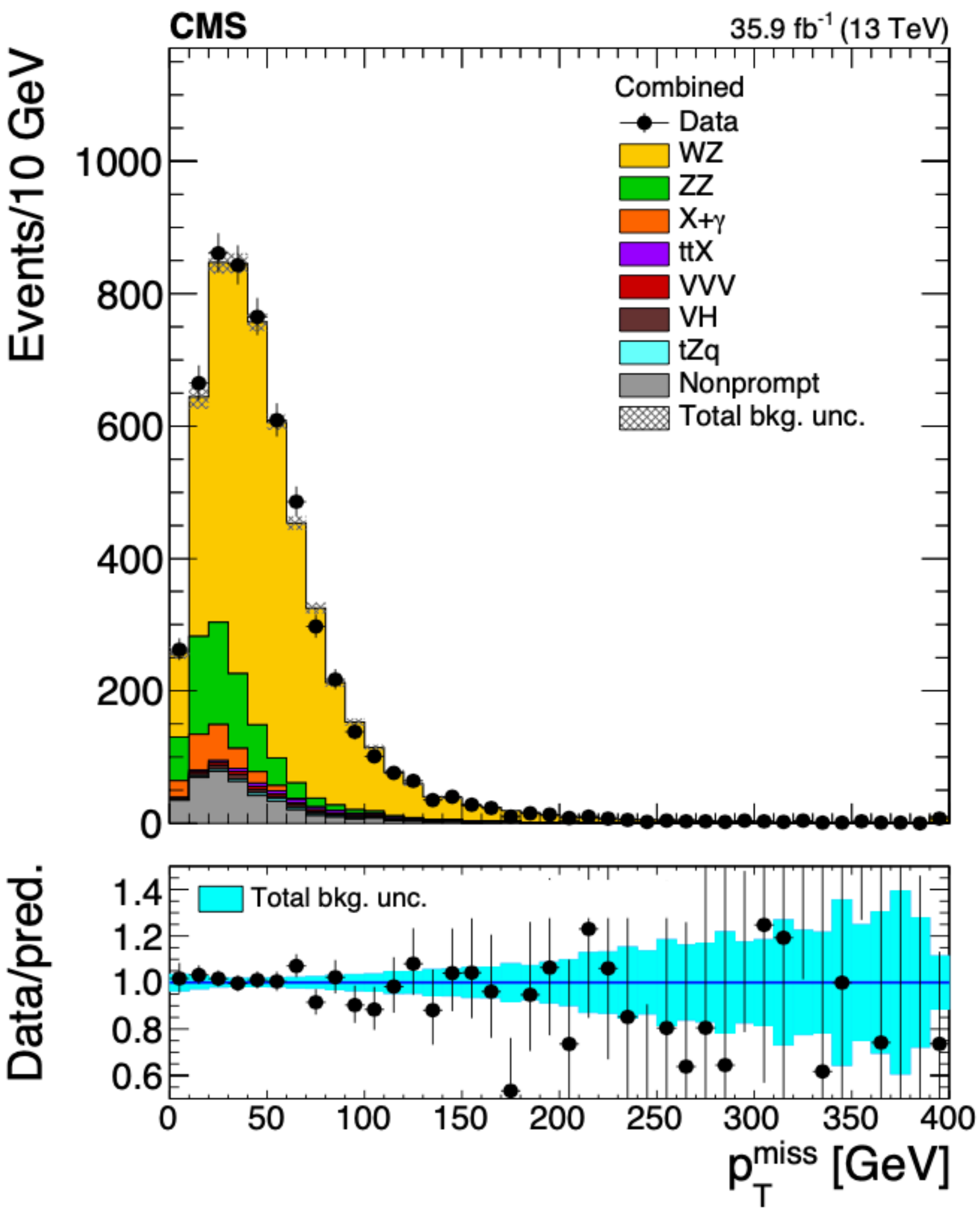
$$\mathbf{x_{1,2}} = \sqrt{\tau}\ \mathbf{e}^{\pm \mathbf{y_{cm}}} \quad \Longrightarrow \quad \int_{\tau_{\mathbf{0}}}^{\mathbf{1}} \mathbf{dx_1} \int_{\tau_{\mathbf{0}}/\mathbf{x_1}}^{\mathbf{1}} \mathbf{dx_2} = \int_{\tau_{\mathbf{0}}}^{\mathbf{1}} \mathbf{d}\tau \int_{\frac{\mathbf{1}}{\mathbf{2}}\ln\tau}^{-\frac{\mathbf{1}}{\mathbf{2}}\ln\tau} \mathbf{dy_{cm}}$$

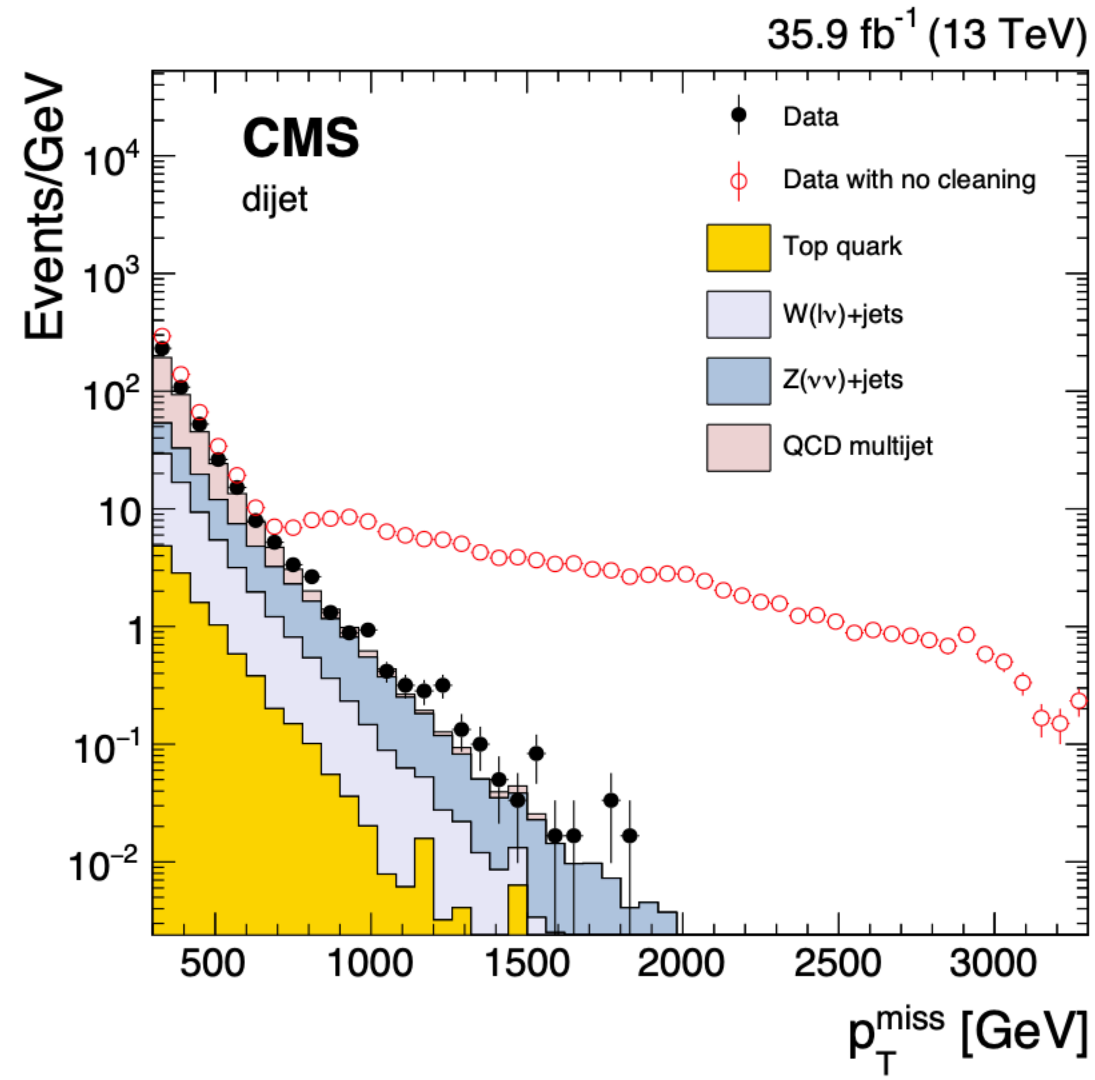
Transverse Momentum

- The momentum perpendicular to the bin is a Lorentz invariant
- It is used to perform many analysis



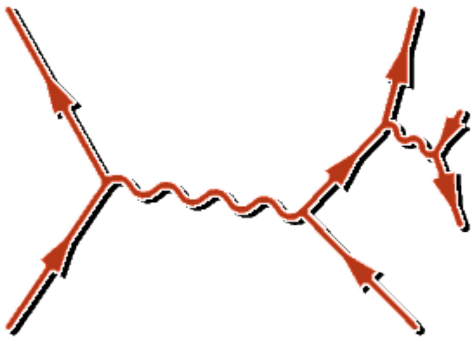
- Missing transverse momentum $\vec{p}_T^{\,miss} = - \sum_{j visible} \vec{p}_T^{\,j}$





Invariant mass

- Consider an unstable particle ($\mathbf{X = Z,\ W^{\pm},\ t}$) decaying $\mathbf{X \to ab}\ldots$



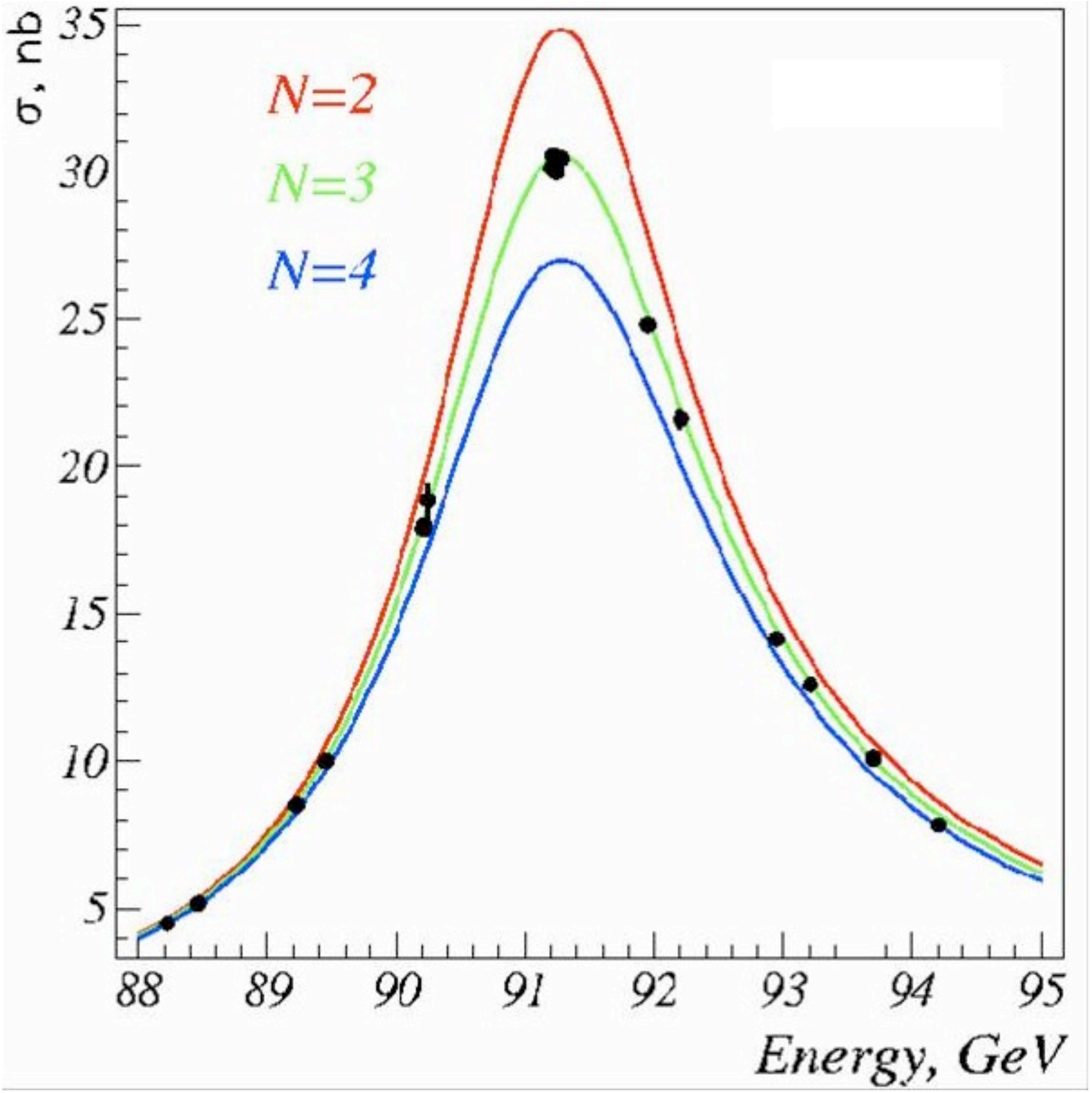
$$\frac{\mathbf{d\sigma}}{\mathbf{dM_{ab\ldots}}} \propto \frac{\mathbf{1}}{(\mathbf{M^2_{ab\ldots} - M^2_X})^{\mathbf{2}} + \mathbf{\Gamma^2_X M^2_X}}$$

and exhibits a peak for $\mathbf{M^2_{ab\ldots}} = (\mathbf{p_a} + \mathbf{p_b} + \ldots)^{\mathbf{2}} = (\ \sum_{\mathbf{i}}^{\mathbf{n}} \mathbf{p_i}\ )^{\mathbf{2}} \approx \mathbf{M^2_X}$

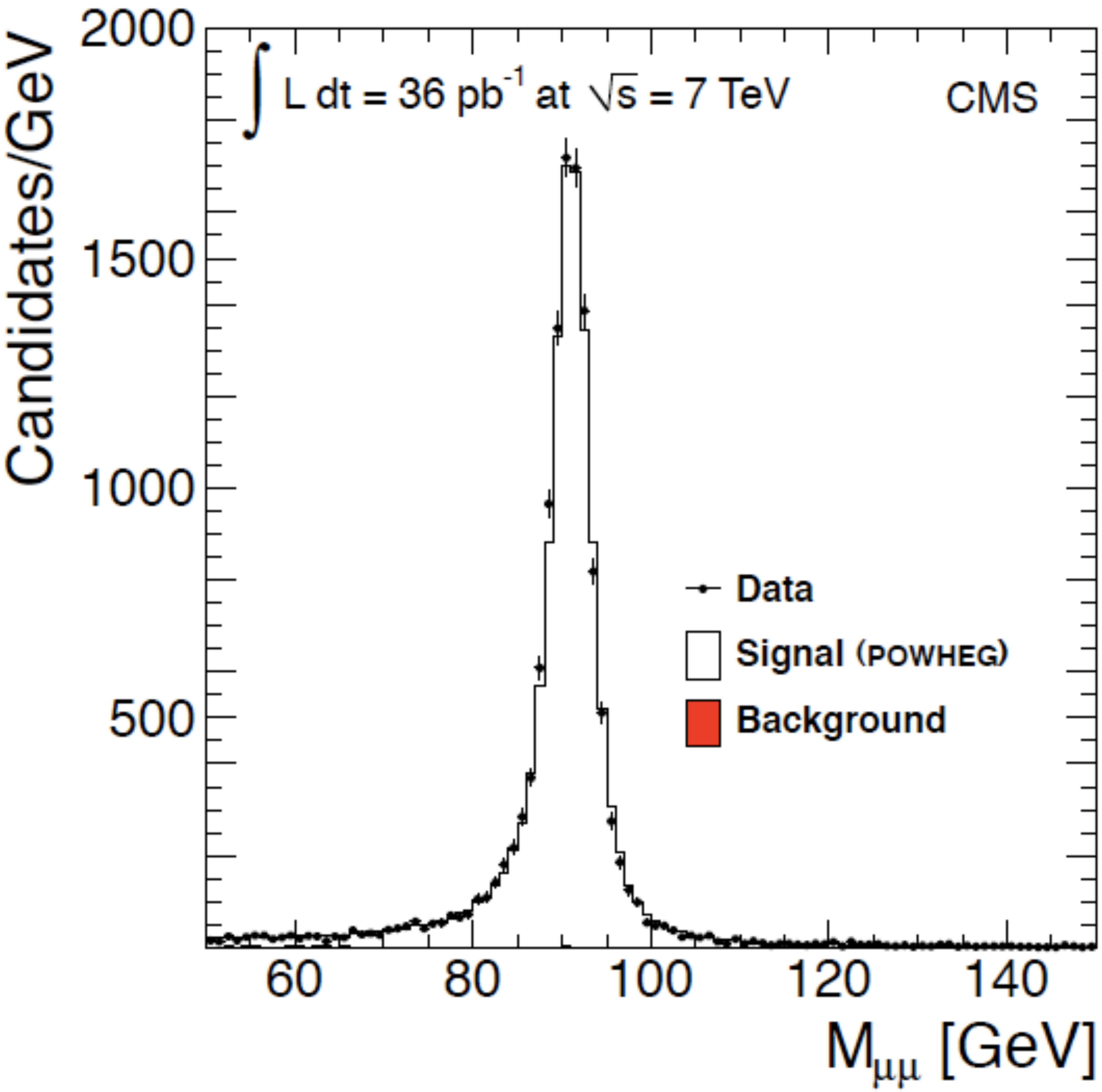
- For the same reason the production $\mathbf{ab} \to \mathbf{X}+$ anything exhibits a peak for $\mathbf{M^2_{ab}} \simeq \mathbf{M^2_X}$

- If the decays products are observable $\Longrightarrow$ we can reconstruct $\mathbf{M_{ab\ldots}}$, e.g. $\mathbf{Z} \to \mathbf{e^+e^-},\ \mathbf{b\bar{b}},\ \ldots$

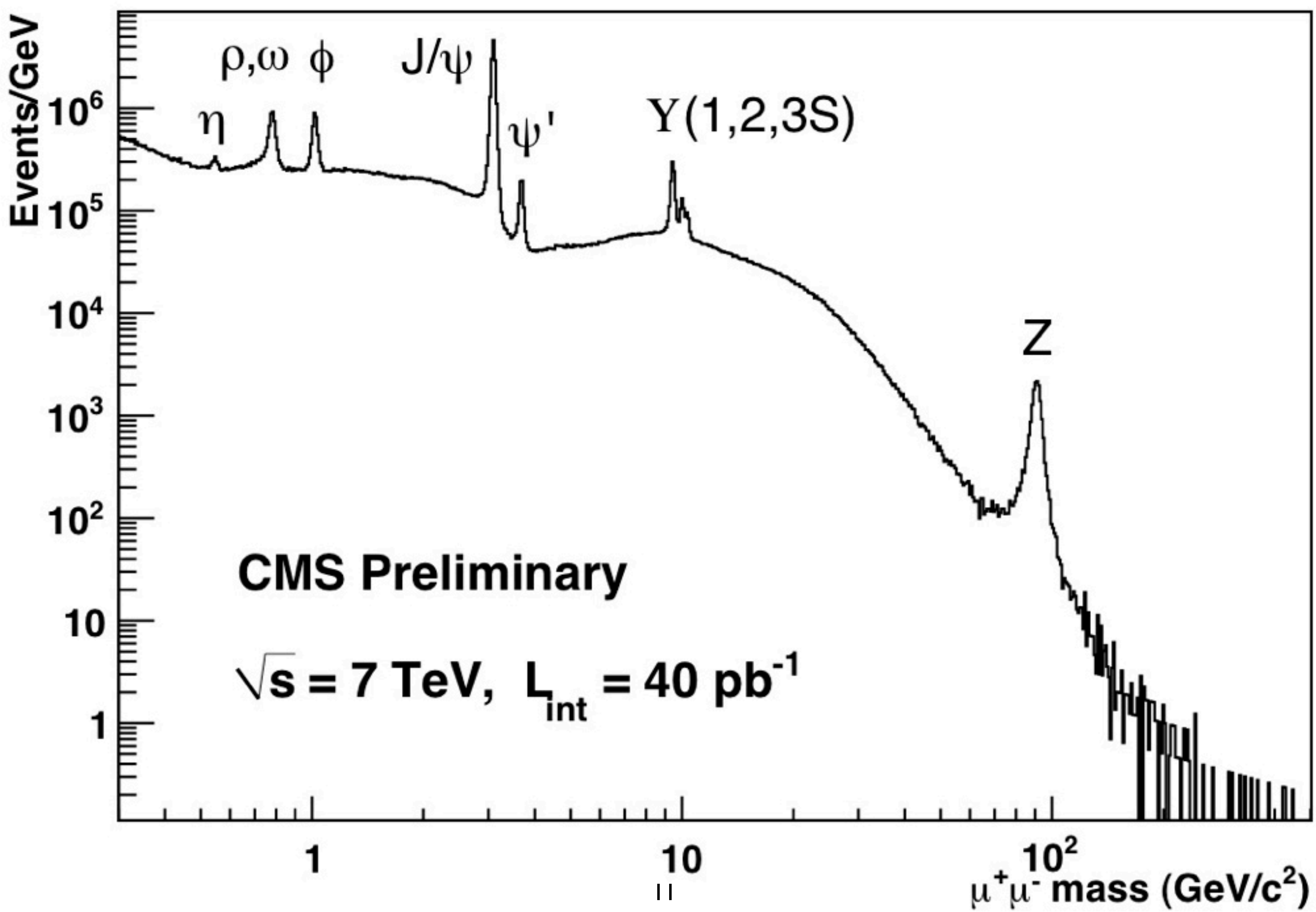
- $\mathbf{e^+e^- \to Z}$: in this case $\mathbf{M_{e^+e^-} = \sqrt{s}}$



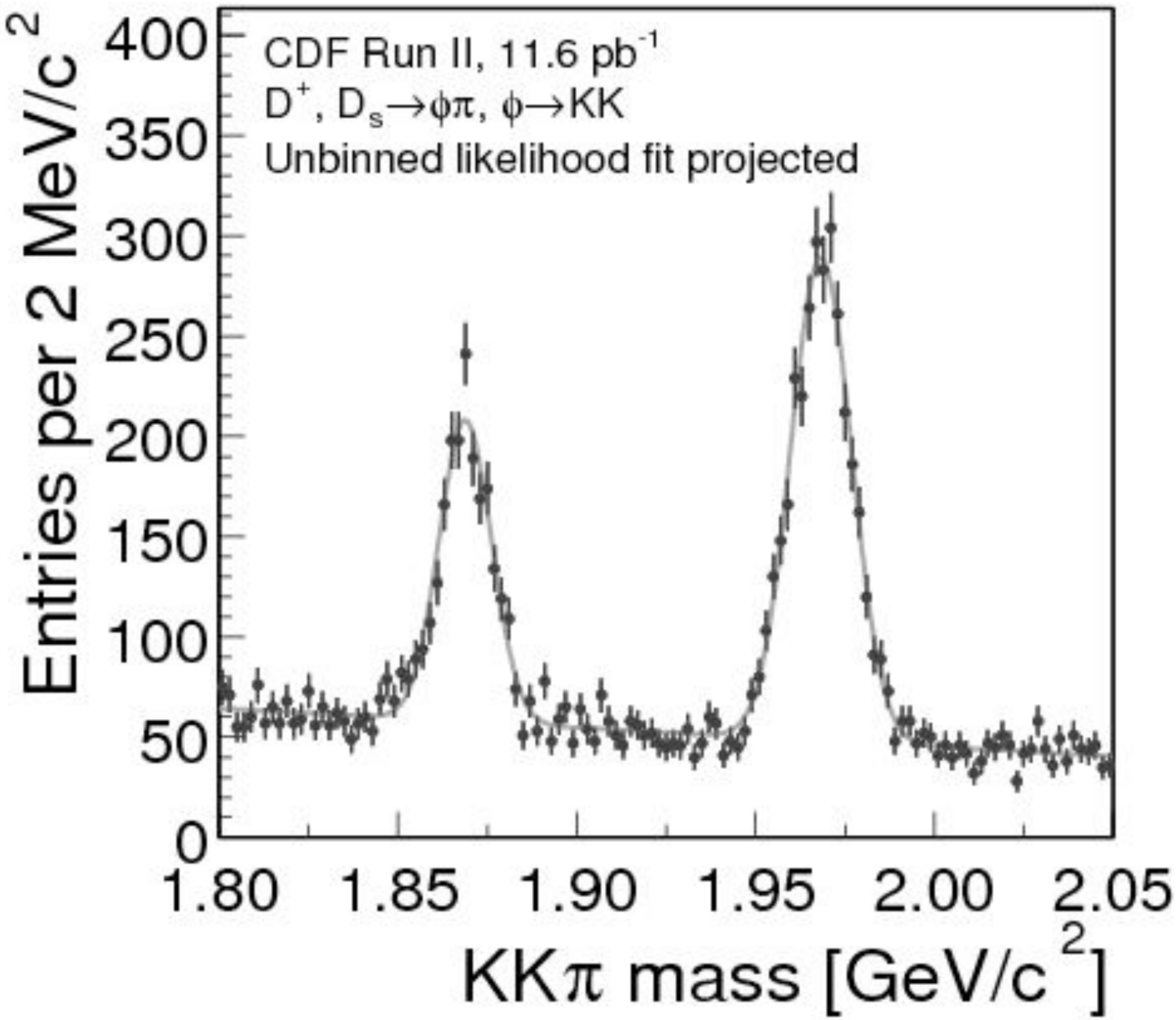
- At the CMS $pp \to \mathbf{Z} + \mathbf{X} \to \mu^+\mu^-$



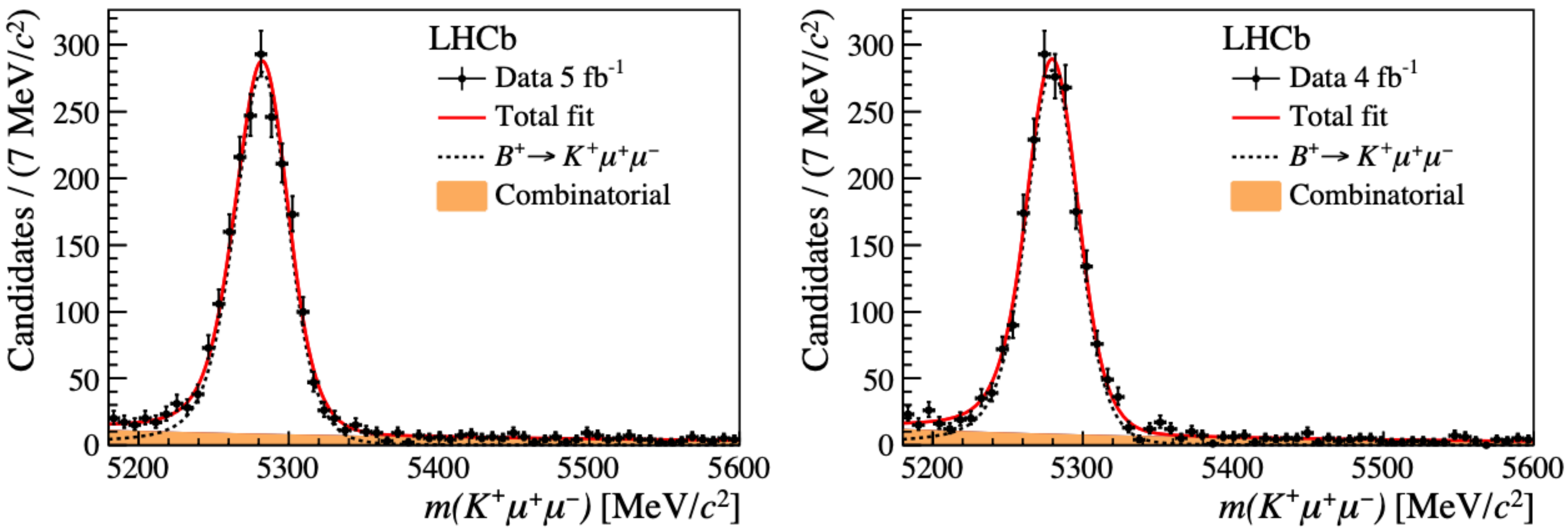
much more can be done with dileptons!


Events/GeV
10^6
10^5
10^4
10^3
10^2
10
1
η
ρ,ω
ϕ
J/ψ
ψ'
Υ(1,2,3S)
Z
CMS Preliminary
$\sqrt{s}$ = 7 TeV, L_{int} = 40 pb^{-1}
1
10
10^2
$\mu^+\mu^-$ mass (GeV/c^2)

“colliders can study hadron physics”

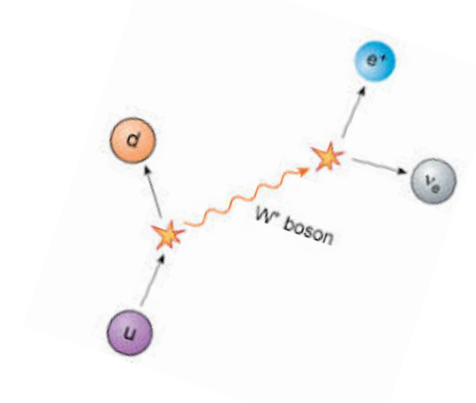


- LHCb search for LFU violation



Transverse mass

- The presence of invisible particle prevents the mass measurement in a single event



- Consider the process $pp \to WX \to e\nu X$

$$M_W^2 = (E_e + E_\nu)^2 - (\vec{p}_e + \vec{p}_\nu)^2 = 2(E_e E_\nu - \vec{p}_T^{\,e} \cdot \vec{p}_T^{\,\nu} - p_L^e p_L^\nu)$$

but $\vec{p}_\nu$ is not measurable!

- We can infer that $\vec{p}_T^{\,\nu} \simeq \vec{\not{p}}_T = - \sum_{j \in visible} \vec{p}_T^{\,j}$

- Analogously $E_T^\nu = \not{E}_T$

- We define the transverse mass [UA1 and UA2]

$$m_T^2 \equiv (E_T^e + E_T^\nu)^2 - (\vec{p}_T^{\,e} + \vec{p}_T^{\,\nu})^2 = 2E_T^e E_T^\nu (1 - \cos\varphi_{e\nu})$$

this definition uses all we can measure in the transverse plane

- Important property $0 \leq m_T \leq m_W$

In fact, remembering that $M_W^2 = 2(E_e E_\nu - \vec{p}_T^{\,e} \cdot \vec{p}_T^{\,\nu} - p_L^{\,e} p_L^{\,\nu})$

however (prove it) $E_e E_\nu - p_L^{\,e} p_L^{\,\nu} \geq E_T^e E_T^\nu$

so $M_W^2 \geq 2(E_T^e E_T^\nu - \vec{p}_T^{\,e} \cdot \vec{p}_T^{\,\nu}) = m_T^2$

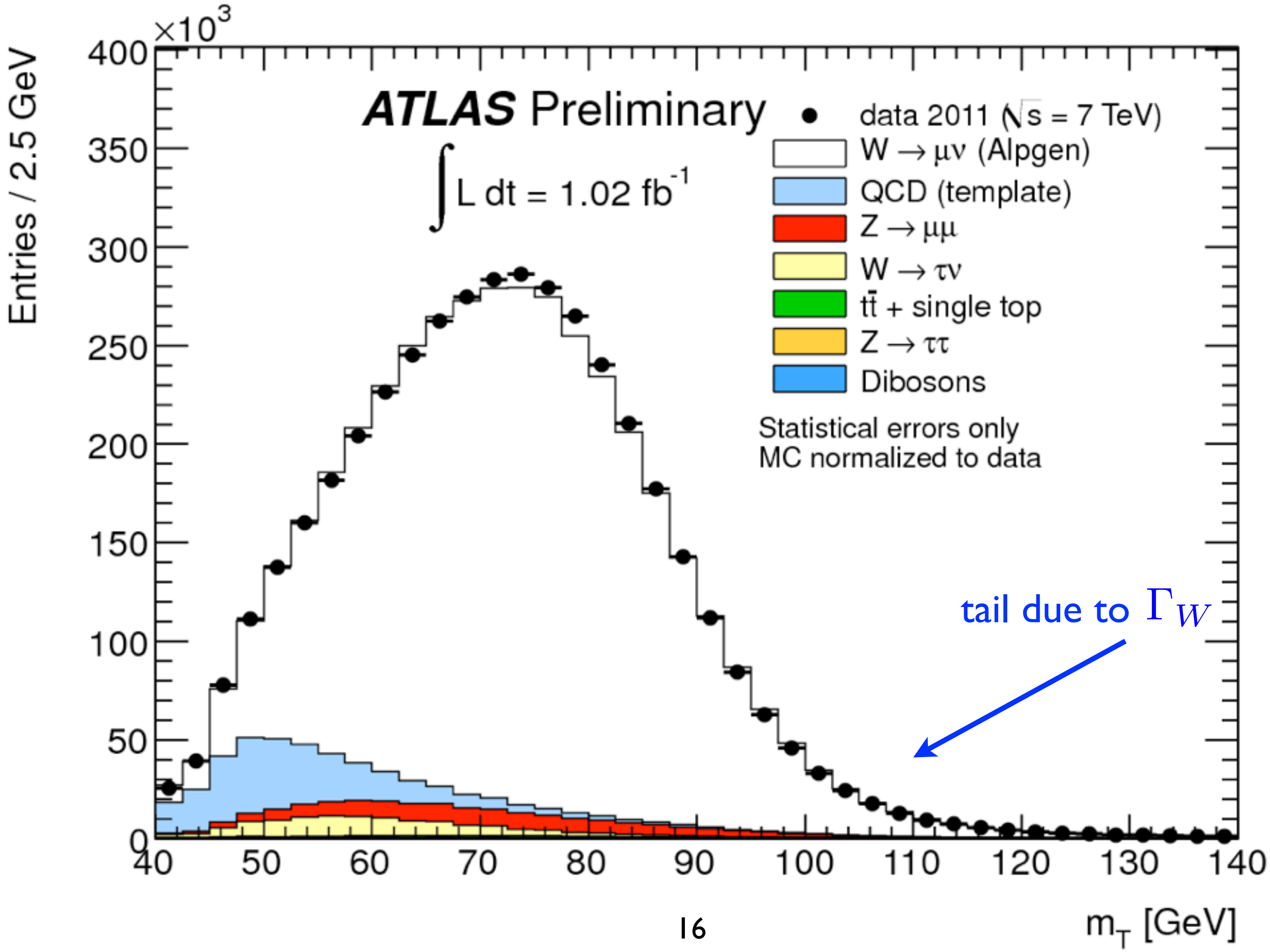
- It is straightforward the generalization for massive particles

$$M_W^2 = m_\ell^2 + m_\nu^2 + 2(E_T^\ell E_T^\nu \cosh(\Delta\eta) - \vec{p}_T^{\,\ell} \cdot \vec{p}_T^{\,\nu})$$

$$M_T^2 = m_\ell^2 + m_\nu^2 + 2(E_T^\ell E_T^\nu - \vec{p}_T^{\,\ell} \cdot \vec{p}_T^{\,\nu})$$

- For $q\bar{q}' \to W^* \to e\nu$ there is a Jacobian peak.

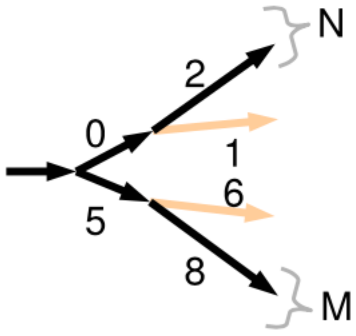
$$\frac{\mathbf{d}\hat{\sigma}}{\mathbf{dm^2_{e\nu,T}}} \propto \frac{\mathbf{\Gamma_W M_W}}{(\mathbf{m^2_{e\nu}} - \mathbf{M^2_W})^2 + \mathbf{\Gamma^2_W M^2_W}} \frac{\mathbf{1}}{\sqrt{\mathbf{m^2_{e\nu}} - \mathbf{m^2_{e\nu,T}}}}.$$



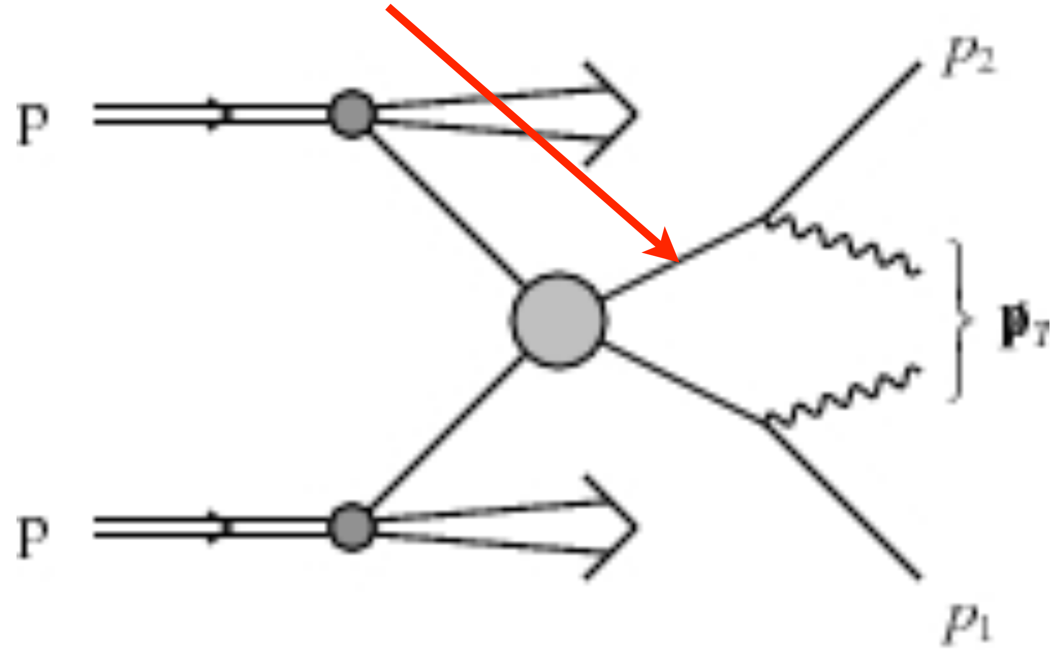
M_{T2} [Lester & Summer hep-ph/9906349]

- A more complex situation is when there are 2 invisible particles

- For instance in searches for BSM physics



$$pp \rightarrow X + \tilde{l}_R^+ \tilde{l}_R^- \rightarrow X + l^+ l^- \tilde{\chi}_1^0 \tilde{\chi}_1^0$$



- For each particle

$$M_{T,i}^2 = m_\ell^2 + m_{\tilde{\chi}_i}^2 + 2(E_T^\ell E_T^{\tilde{\chi}_i} - \vec{p}_T^{\,\ell} \cdot \vec{p}_T^{\,\tilde{\chi}_i}) \leq M_{\tilde{\ell}}^2$$

- But we don't observe the $\tilde{\chi}$!!! $\quad \not{p}_T = \vec{p}_T^{\,\tilde{\chi}_1} + \vec{p}_T^{\,\tilde{\chi}_2}$

- We know that $M_{\tilde{\ell}}^2 \geq \max\left[M_T^2(\vec{p}_T^{\,\ell_1}, \vec{p}_T^{\,\tilde{\chi}_1}), M_T^2(\vec{p}_T^{\,\ell_2}, \vec{p}_T^{\,\tilde{\chi}_2}) \right]$

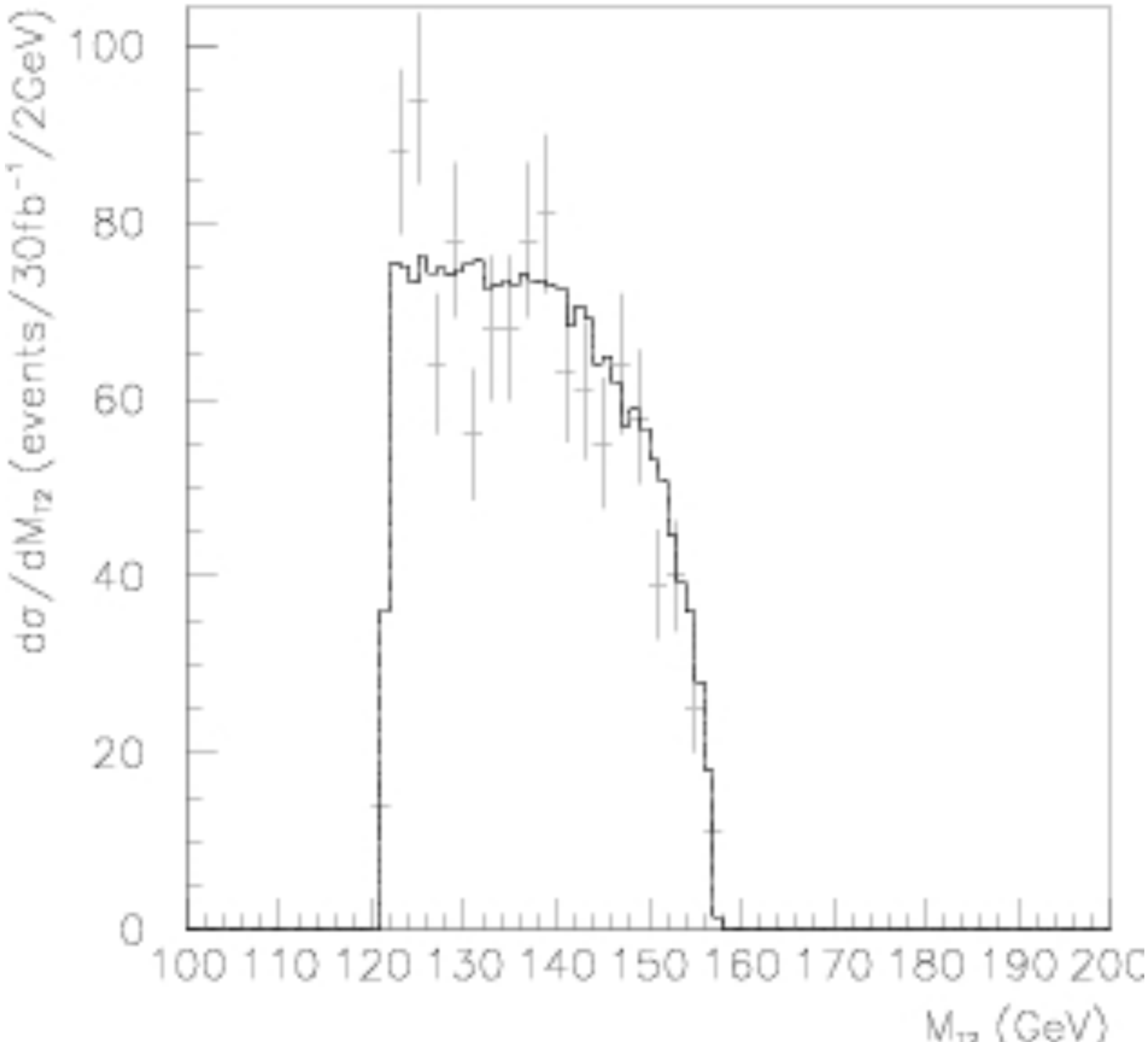
unknown

- The best we can do is

$$M_{\tilde{\ell}}^2 \geq M_{T2}^2 \equiv \min_{\vec{p}_T^{\,\tilde{\chi}_1} + \vec{p}_T^{\,\tilde{\chi}_2} = \not{p}_T} \left\{ \max\left[M_T^2(\vec{p}_T^{\,\ell_1}, \vec{p}_T^{\,\tilde{\chi}_1}), M_T^2(\vec{p}_T^{\,\ell_2}, \vec{p}_T^{\,\tilde{\chi}_2}) \right] \right\}$$

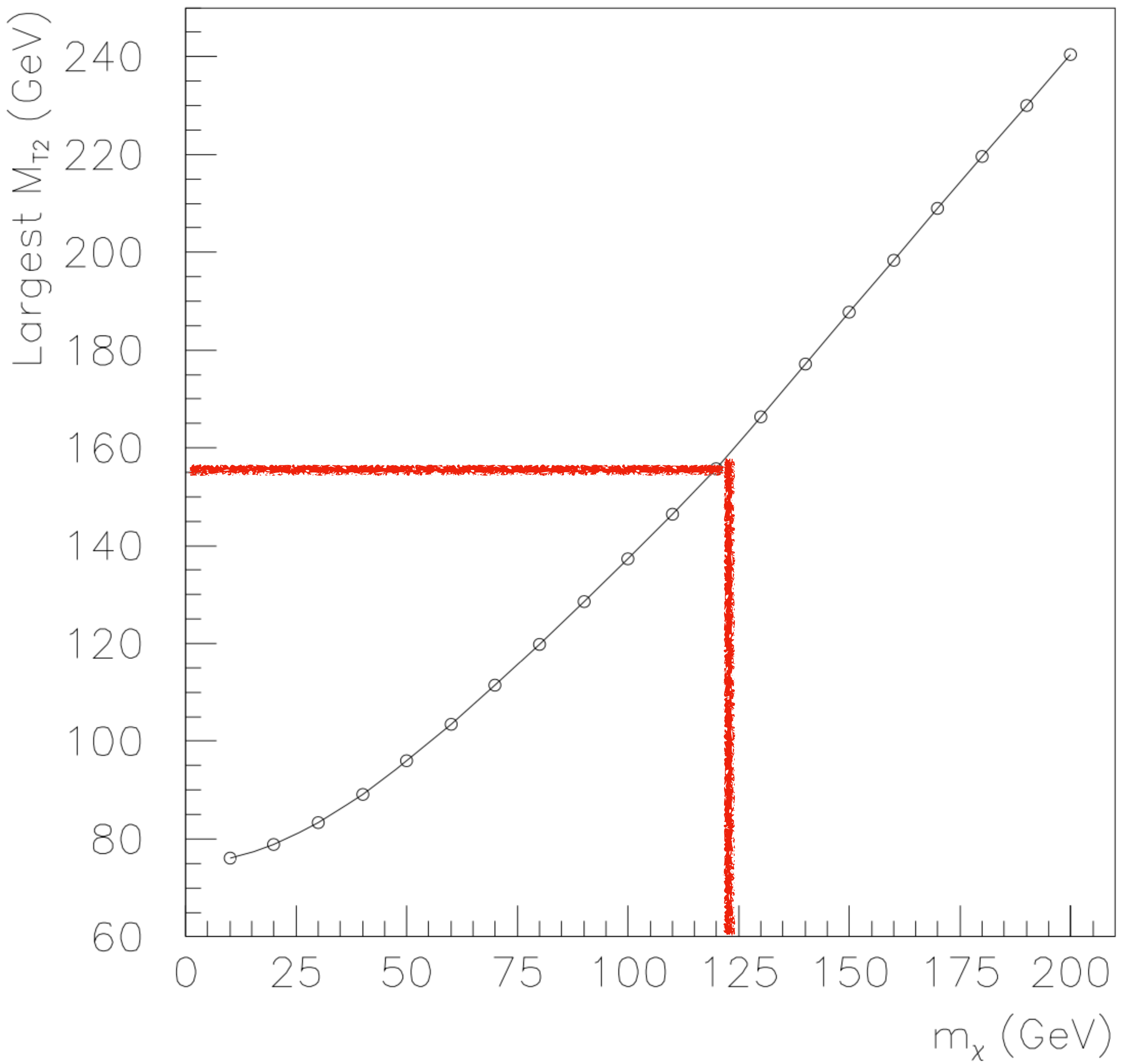
then we look for endpoints in M_{T2}

- For $m_{\tilde{\ell}} = 157$ GeV and $m_{\tilde{\chi}} = 122$ GeV



assuming the we know $m_{\tilde{\chi}} = 122$ GeV

- For $m_{\tilde{\ell}} = 157$ GeV and $m_{\tilde{\chi}} = 122$ GeV



- To learn more about M_{T2}

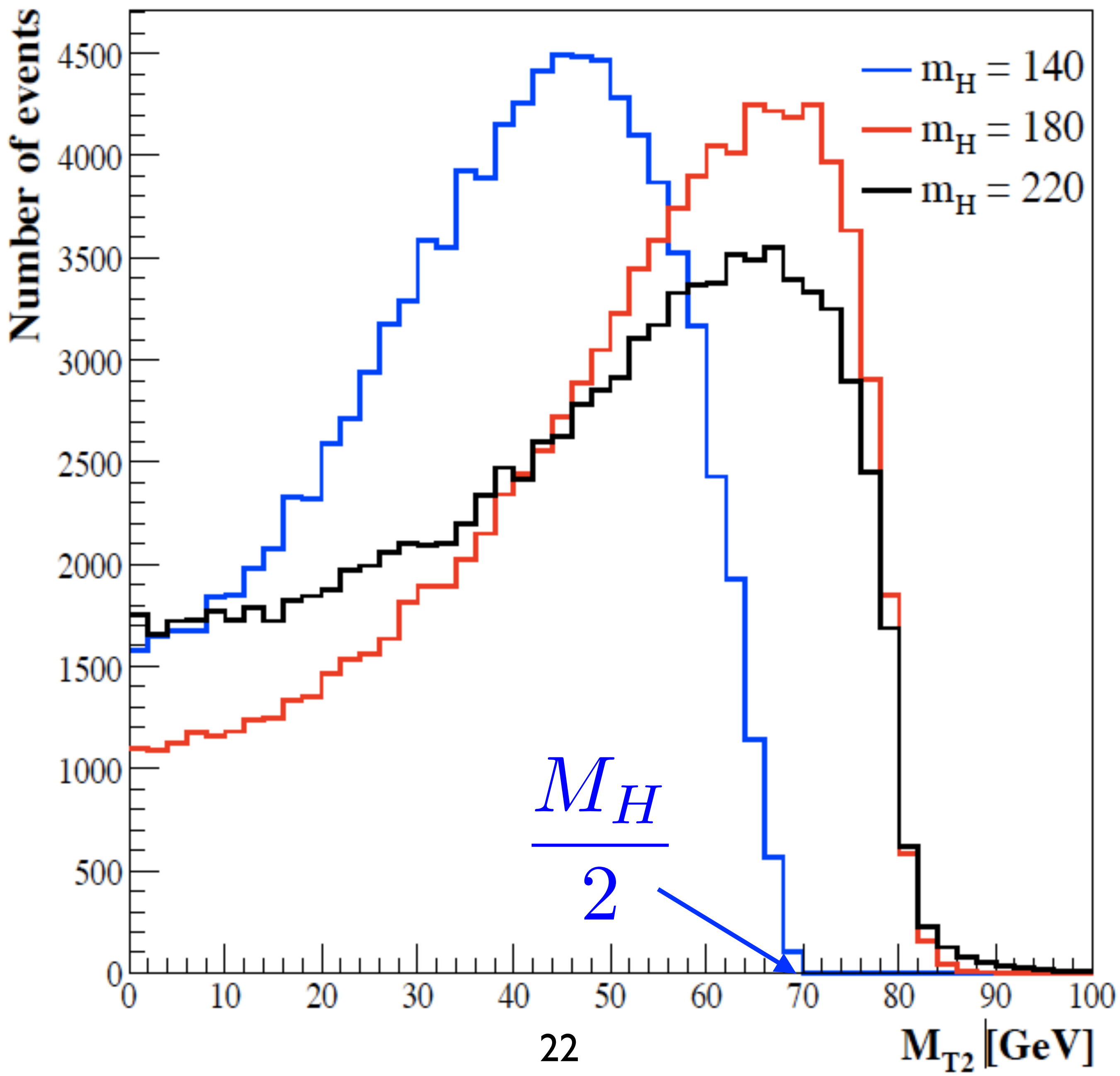
1. Lester and Summer: hep-ph/9906349
2. Barr, Gripaios, Lester: arXiv: 0711.4008
3. Cho, Choi, Kim, Park: arXiv:0711.4526

M_{T2} assisted on-shell (MAOS) reconstruction

[Choi,Choi,Lee,Park]
arXiv: 0908.0079

- How can we reconstruct $\mathbf{pp} \to \mathbf{X} \to \mathbf{W^+W^-} \to \ell^+\ell^-\not{\mathbf{p}}_\mathbf{T}$?

- M_{T2} gives information on M_W



- We can partially reconstruct the neutrinos:

for $\mathbf{W}^+(\mathbf{p_1}+\mathbf{p_2})\mathbf{W}^-(\mathbf{k_1}+\mathbf{k_2}) \to \ell^+(\mathbf{p_1})\nu(\mathbf{p_2})\ell^-(\mathbf{k_1})\nu(\mathbf{k_2})$

$$M_{T2} \equiv \min_{\mathbf{p_2}_T+\mathbf{k_2}_T=\not{p}_T} \left[\max\left\{M_T(\mathbf{p_1}_T,\mathbf{p_2}_T), M_T(\mathbf{k_1}_T,\mathbf{k_2}_T)\right\}\right]$$

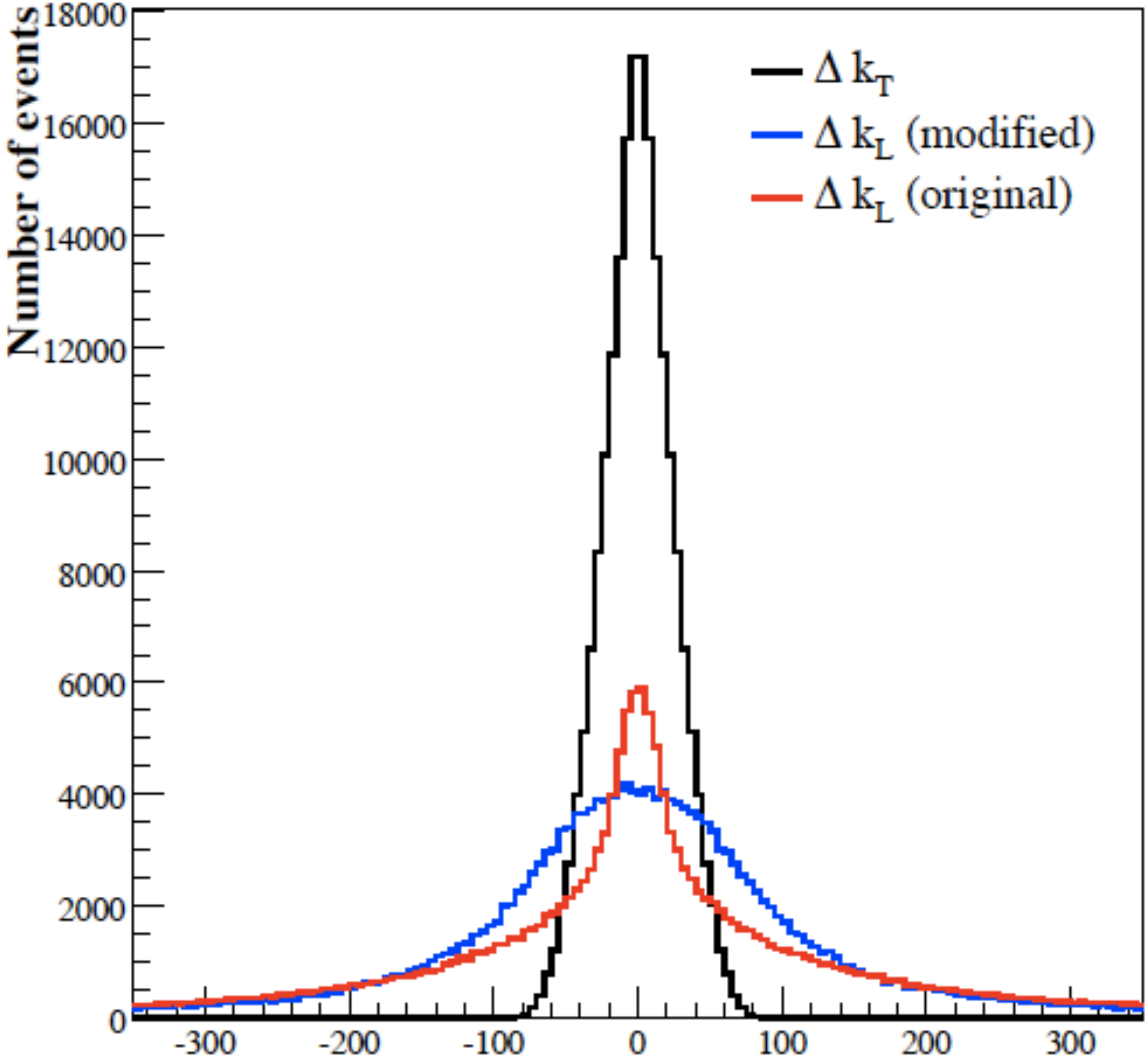
without ISR: $\mathbf{p_2}_{\mathbf{T}}^{\mathrm{maos}} = -\mathbf{k_1}_{\mathbf{T}}, \quad \mathbf{k_2}_{\mathbf{T}}^{\mathrm{maos}} = -\mathbf{p_1}_{\mathbf{T}}$

- Now we impose the lepton-neutrino pairs reconstruct a W

$$p_{2L}^{\mathrm{maos}}(\pm) = \frac{1}{|\mathbf{p_1}_T|^2}\left[p_{1L}\,A \pm \sqrt{|\mathbf{p_1}_T|^2+p_{1L}^2}\,\sqrt{A^2-|\mathbf{p_1}_T|^2|\mathbf{p_2}_T^{\mathrm{maos}}|^2}\right],$$

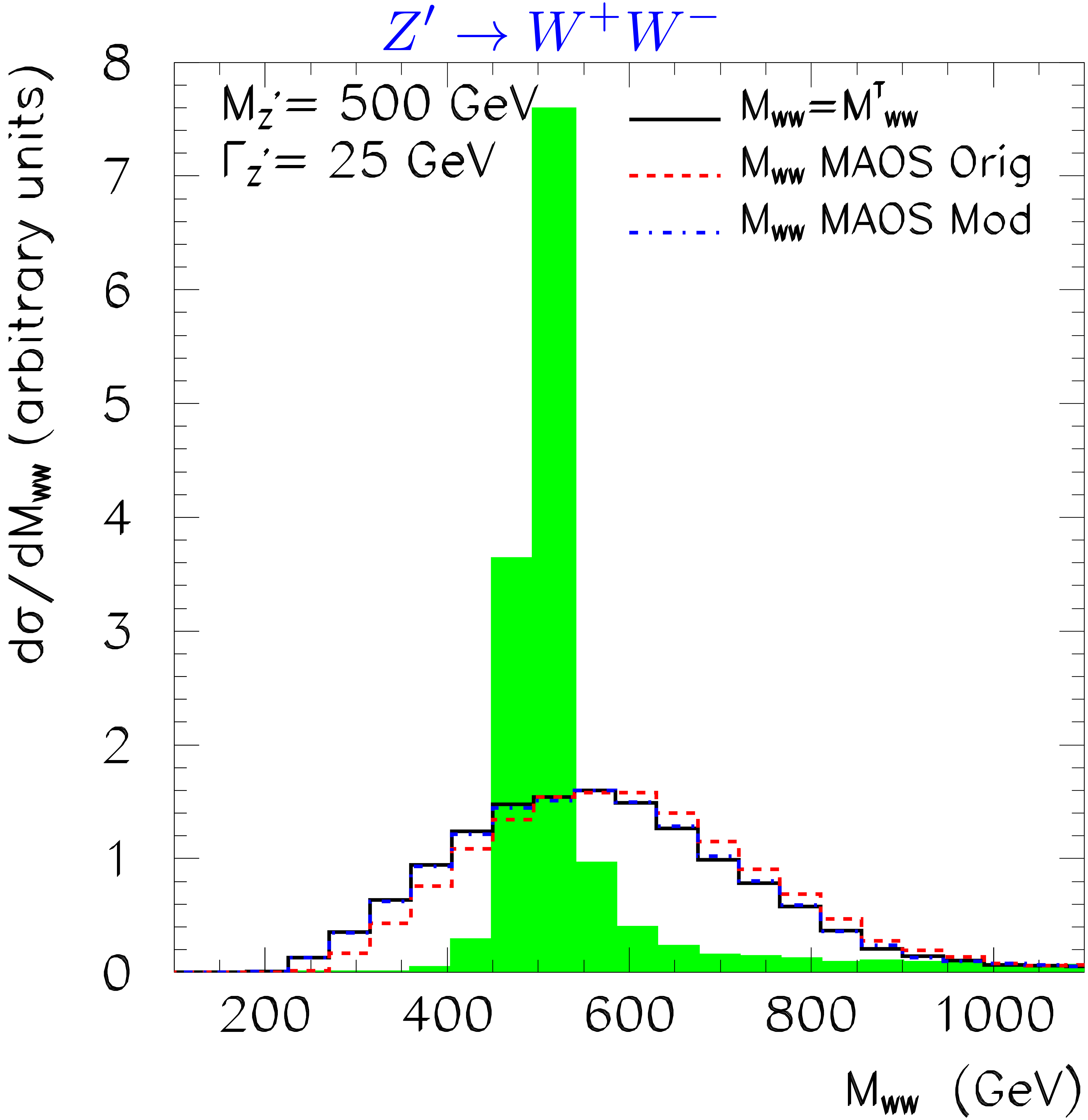
$$k_{2L}^{\mathrm{maos}}(\pm) = \frac{1}{|\mathbf{k_1}_T|^2}\left[k_{1L}\,B \pm \sqrt{|\mathbf{k_1}_T|^2+k_{1L}^2}\,\sqrt{B^2-|\mathbf{k_1}_T|^2|\mathbf{k_2}_T^{\mathrm{maos}}|^2}\right],$$

where $A \equiv M_W^2/2 + \mathbf{p_1}_T\cdot\mathbf{p_2}_T^{\mathrm{maos}}$ and $B \equiv M_W^2/2 + \mathbf{k_1}_T\cdot\mathbf{k_2}_T^{\mathrm{maos}}$.



Number of events
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
-300
-200
-100
0
100
200
300
Δ k_T
Δ k_L (modified)
Δ k_L (original)
Δ k_{T,L} [GeV]

M_H=180 GeV



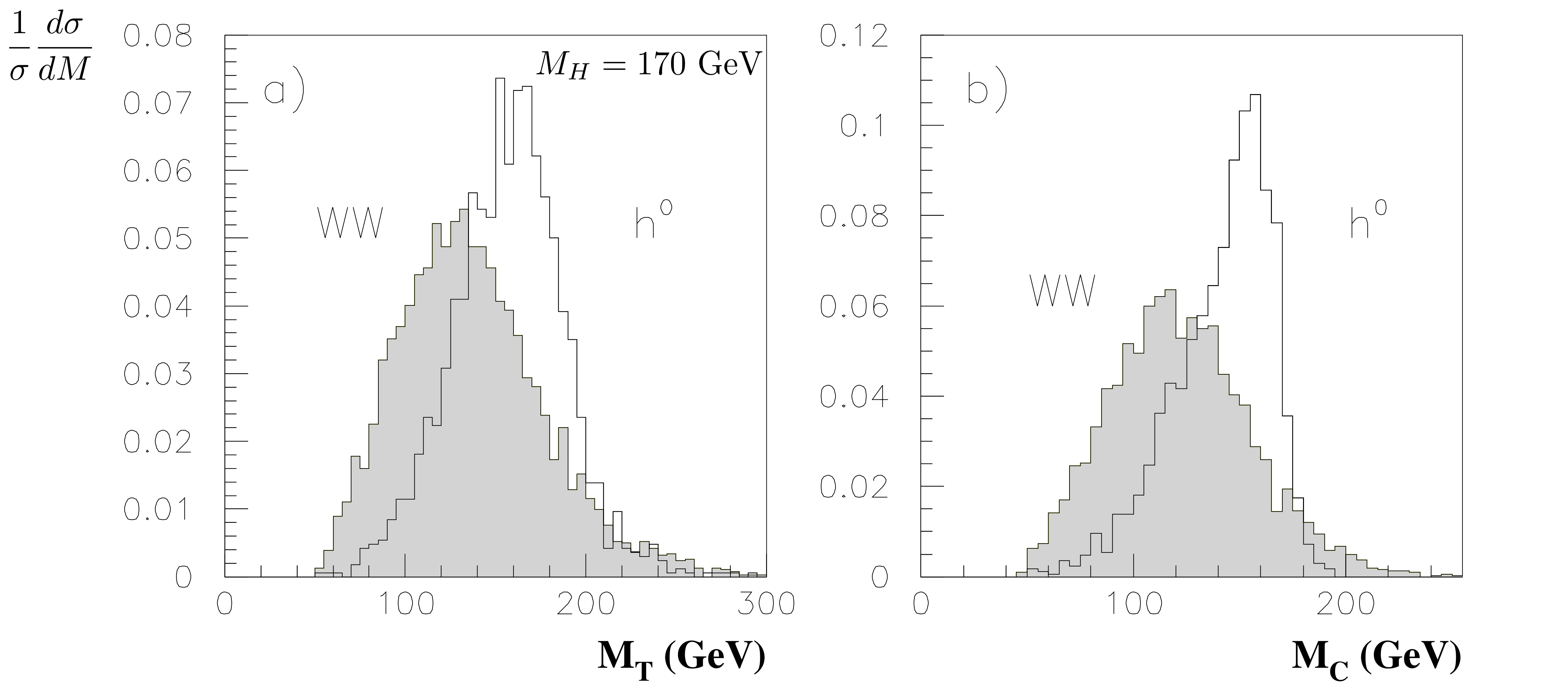
$Z' \to W^+W^-$
$M_{Z'}$= 500 GeV
$\Gamma_{Z'}$= 25 GeV
$M_{WW}=M^T_{WW}$
M_{WW} MAOS Orig
M_{WW} MAOS Mod
$d\sigma/dM_{WW}$ (arbitrary units)
M_{WW} (GeV)
0
1
2
3
4
5
6
7
8
200
400
600
800
1000

- A more complex case is $\mathbf{H} \to \mathbf{W_1 W_2} \to \ell_1\nu_1\ \ell_2\nu_2$:

$$\mathbf{M^2_{C,WW}} = \left(\sqrt{\mathbf{p^2_{T,\ell\ell}} + \mathbf{M^2_{\ell\ell}}} + \tilde{\not{\mathbf{p}}}_{\mathbf{T}}\right)^{\mathbf{2}} - (\tilde{\mathbf{p}}_{\mathbf{T},\ell\ell} + \tilde{\not{\mathbf{p}}}_{\mathbf{T}})^{\mathbf{2}}$$

an alternative is

$$\mathbf{M_{T,WW}} \approx \mathbf{2}\sqrt{\mathbf{p^2_{T,\ell\ell}} + \mathbf{M^2_{\ell\ell}}}$$



SM is narrow for this mass

- Let's consider a more complex case $\mathbf{H} \rightarrow \mathbf{W_1 W_2} \rightarrow \mathbf{q_1 \bar{q}_2}\ \mathbf{e}\nu$: A natural choice is

$$\mathbf{M'^2_{T,WW}} = \left(\sqrt{\mathbf{p^2_{T,jj}} + \mathbf{m^2_{jj}}} + \sqrt{\mathbf{p^2_{T,e\nu}} + \mathbf{m^2_{e\nu T}}}\right)^{\mathbf{2}} - (\tilde{\mathbf{p}}_{\mathbf{T,jje}} + \tilde{\not{\mathbf{p}}}_{\mathbf{T}})^{\mathbf{2}}$$

since the $\mathbf{W}$ can be offshell

W's can be off shell

- We can also cluster the electron and jets together

$$\mathbf{M^2_{C,WW}} = \left(\sqrt{\mathbf{p^2_{T,jje}} + \mathbf{m^2_{jje}}} + \not{\mathbf{p}}_{\mathbf{T}}\right)^{\mathbf{2}} - (\tilde{\mathbf{p}}_{\mathbf{T,jje}} + \tilde{\not{\mathbf{p}}}_{\mathbf{T}})^{\mathbf{2}}$$

- Here we can also "reconstruct" the neutrino, assuming it comes from a W

$$p_L^\nu = \frac{1}{2{p_T^l}^2}\left\{[M_W^2 + 2(\vec{p_T^l} \cdot \vec{\not{p}_T})]p_L^l \pm \sqrt{[M_W^2 + 2(\vec{p_T^l} \cdot \vec{\not{p}_T}]^2|\vec{p^l}|^2 - 4(p_T^l E^l \not{E}_T)^2}\right\}$$