PGF5003: Classical Electrodynamics I
Problem Set 1

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(Due to April 13, 2021)

1 Question (1 point)

Given the following vector field $\mathbf{F} = f\vec{\nabla}g$*, with* f *and* g *scalar functions, prove the first Green identity:*

$$\int_V dV \left(f\nabla^2 g + \vec{\nabla} f \cdot \vec{\nabla} g \right) = \oint_{S(V)} d\vec{S} \cdot \left(f \vec{\nabla} g \right). \tag{1}$$

What do we need to suppose about f *and* g *to deduce this identity?*

2 Question (1 point)

Show that

$$\nabla^2 \left(\frac{1}{r} \right) = -4\pi\delta(\mathbf{r}). \tag{2}$$

3 Question (1 point)

Using the second Green identity:

$$\int_V dV \left(f\nabla^2 g - g\nabla^2 f \right) = \oint_{S(V)} d\mathbf{S} \cdot \left(f\vec{\nabla} g - g\vec{\nabla} f \right), \tag{3}$$

taking $f = \phi(\mathbf{x}')$ *(for the electrostatic potential* $\mathbf{E} = \vec{\nabla}\phi$*) and* $g = 1/R = 1/|\mathbf{x} - \mathbf{x}'|$*, show that*

$$\phi(\mathbf{x}) = \int_V d^3x' \frac{\rho(\mathbf{x}')}{R} + \frac{1}{4\pi} \oint_{S(V)} d\mathbf{S}' \cdot \left[\frac{1}{R} \vec{\nabla}' \phi(\mathbf{x}') - \phi(\mathbf{x}') \vec{\nabla}' \frac{1}{R} \right], \tag{4}$$

where $\vec{\nabla}'$ corresponds to differential operation related to $\mathbf{x}'$.

4 Question (1 point)

Consider the electric field

$$\mathbf{E} = \frac{Ae^{r/r_0}}{r}\hat{r}. \tag{5}$$

a) *Determine the density of charge.*

b) *Determine the total charge into a radius R.*

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5 Question (1 point)

Consider two infinite plates (with zero thickness) with the distributions of charge σ and $-\sigma$, respectively. The plates are orthogonal to each other.

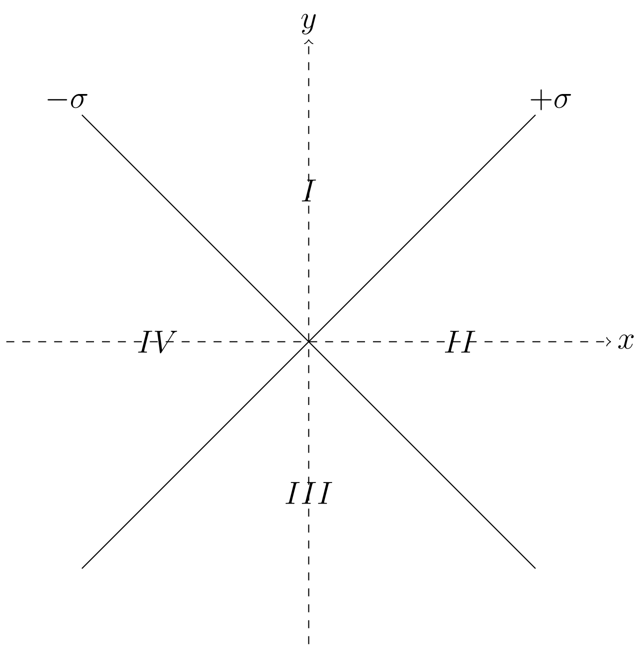

Figure 1: Figure for the question 5.

a) *Find the electric field in all the regions (I, II, III, IV and total space).*

b) *Draw a figure, representing the electrical field.*

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6 Question (1 point)

A spherical shell of radius R is made with isolating material and has a surface density of charge σ (that, in principle, we do not know). The electric potential outside the sphere is $V_{out}(r) = V_0\left(\frac{R}{r}\right)^2\cos\theta$, where V_0 is a constant. The electric field $\mathbf{E}_{ins}(\mathbf{r}) = \frac{-V_0}{R}\hat{z}$. Compute:

a) *the electric field outside the sphere* $\mathbf{E}_{out}(\mathbf{r})$ *and the electric potential inside the sphere* V_{ins}*;*

b) *the superficial density of charge* σ*;*

c) *the force per unity of area* $\mathbf{f}$ *over the surface of the sphere;*

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7 Question (2 points)

Consider a potential problem in the half-space defined by $z \geq 0$*, with Dirichlet boundary conditions on the plane* $z = 0$ *(and at infinity).*

a) *Write down the appropriate Green's function* $G(\mathbf{r}, \mathbf{r}')$.

b) *If the potential on the plane* $z = 0$ *is specified to be* $\Phi = V$ *inside a circle of radius* a *centered at the origin, and* $\Phi = 0$ *outside that circle, find an integral expression for the potential at a point* P *specified in terms of cylindrical coordinates* ρ, ϕ, z.

c) *Show that, along the axis of the circle (*$\rho = 0$*), the potential is given by*

$$\Phi = V\left(1 - \frac{z}{\sqrt{a^2 + z^2}}\right). \tag{6}$$

d) *Show that at large distances (*$\rho^2 + z^2 \gg a^2$*) the potential can be expanded in a power series in* $(\rho^2 + z^2)^{-1}$*, and that the leading terms are*

$$\Phi = \frac{Va^2}{2}\frac{z}{(\rho^2 + z^2)^{3/2}}\left[1 - \frac{3a^2}{4(\rho^2 + z^2)} + \frac{5(3\rho^2 a^2 + a^4)}{8(\rho^2 + z^2)^2} + \ldots\right]. \tag{7}$$

Verify that the result of (c) is consistent with this results.

8 Question (2 points)

An infinite metallic plate has a spherical overhang of radius a. This plate is grounded. A charge $+q$ is placed over the hemisphere of the overhang, with a distance d of the center of the sphere. Show that the induced charge on the overhang is

$$q' = -q\left[1 - \frac{(d^2 - a^2)}{d\sqrt{d^2 + a^2}}\right]. \tag{8}$$

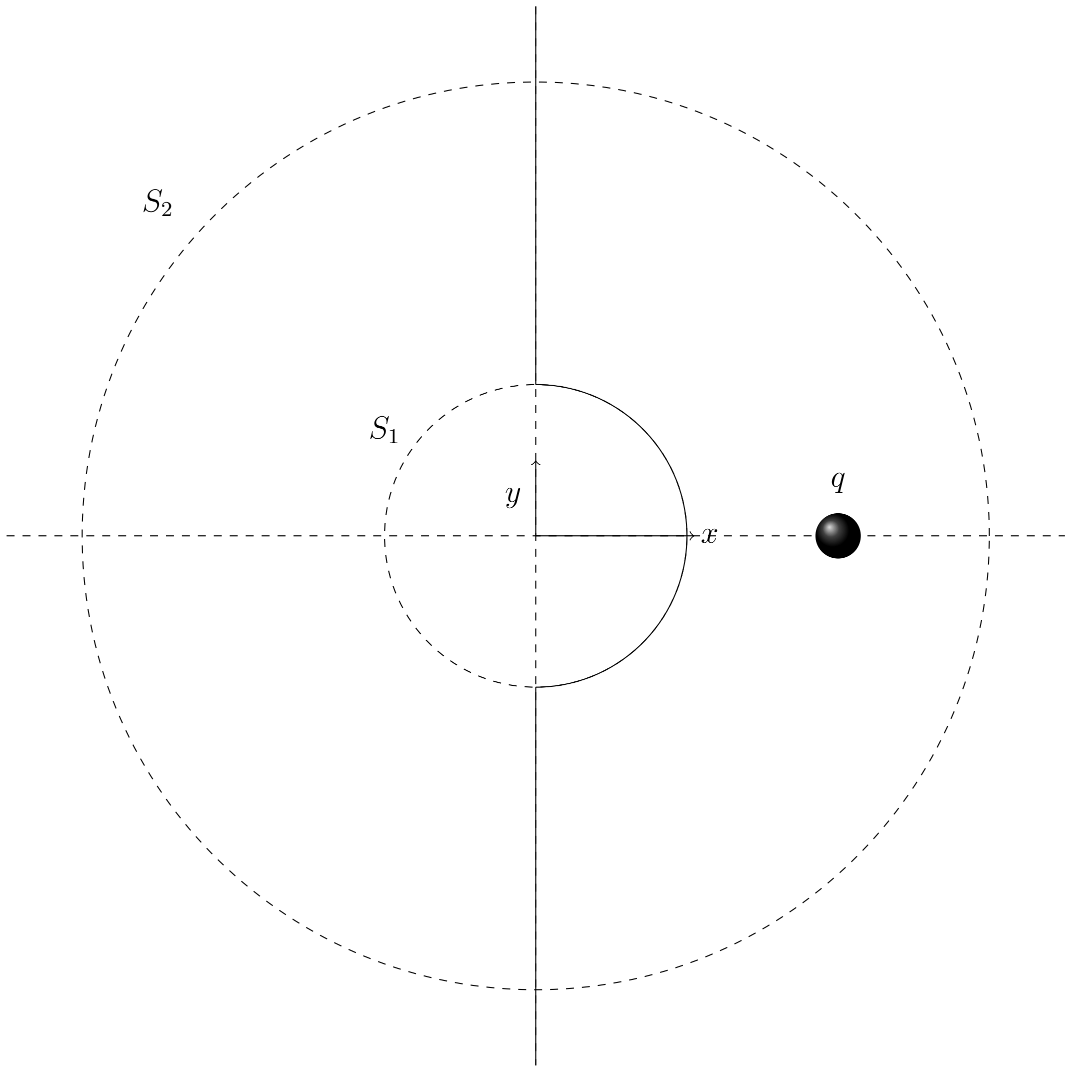

Figure 2: Figure for the question 8.