ESCOLA POLITÉCNICA DA UNIVERSIDADE DE SÃO PAULO
DEPARTAMENTO DE ENGENHARIA MECÂNICA

PME3403 - Laboratório de Vibração e Controle

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Balanceamento em um Plano sem Medição de Fase

Balanceamento sem Medição de Fase

O extrato da seção *47. Balancing of Solid Rotors* do capítulo *VI: Rotating Machinary* do livro *Mechanical Vibrations* de *Den Hartog (1934)* temos o procedimento para balanceamento em um plano de correção sem medição de fase:

> A more reliable method is based on observations of the amplitude only. It consists of conducting three test runs with the rotor in three different conditions: (1) without any additions to the rotor, (2) with a unit unbalance weight placed in an arbitrary hole of the rotor, and (3) with the same unbalance weight placed in the diametrically opposite hole. In Fig. 1 let OA represent to a certain scale the original unbalance in the rotor and also, to another scale, the vibrational amplitude observed as a result of this unbalance at a certain speed. Similarly let OB represent vectorially the total unbalance of the rotor after the unit addition has been placed in the first hole. It is seen that the vector OB may be considered as the sum of the vectors OA and AB, where AB now represents the extra unbalance introduced. If now this unbalance is removed and replaced in the diametrically opposite hole, necessarily the new additional unbalance is represented by the vector AC equal and opposite to AB, and consequently the vector OC, being the sum of the original unbalance AC, represents the complete unbalance in the third run.

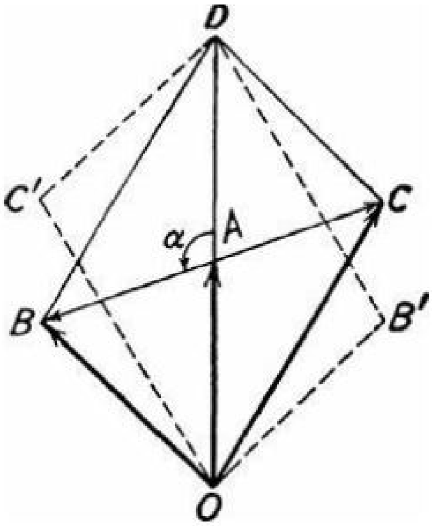

Figura 1: Vector diagram for determining the unbalance in a plane by three or four observations of amplitude.

As a result of the amplitude observations in these three runs, we know the relative lengths of the vectors OB, OA, and OC, but we do not as yet know their absolute lengths or their angular relationships. However, we do know that OA must be the median of the triangle OBC and the problem therefore consists in constructing a triangle OBC, of which are known the ratios of two sides and a median. Its construction by Euclid's geometry is carried out by doubling the length OA to OD and then observing that in the triangle ODC the side DC is equal to OB, so that in triangle OCD all three sides are known. Thus the triangle can be constructed, and as soon as this has been done. we know the relative lengths of AB and OA. Since AB represents a known unbalance weight artificially introduced, we can deduce from it the magnitude of the original unknown unbalance OA. Also the angular location α of the original unbalance OA with respect to the known angular location AB is known. There is one ambiguity in this construction. In finding the original triangle OCD, we might have obtained the triangle $OC'D$ instead. Consequently we would have obtained the direction $C'B'$ instead of the direction CB for our artificially introduced unbalances. This ambiguity can be removed by a fourth run which also will act as a check on the accuracy of the previous observations. It is noted that in the construction of Fig. 1 no other assumptions have been made than that the system is linear, i.e., that all vibration amplitudes are proportional to the unbalance masses. This relation is not entirely true for actual rotors but it is a good approximation to the truth. If after going through the motions shown in Fig. 1 and if after inserting the correction weight so found there still is vibration present in the machinery, that vibration will be very much less than the original one and the process of Fig. 1 may be repeated once more.

Hartog, J. P. Den. Mechanical Vibrations . Read Books Ltd.. Edição do Kindle.

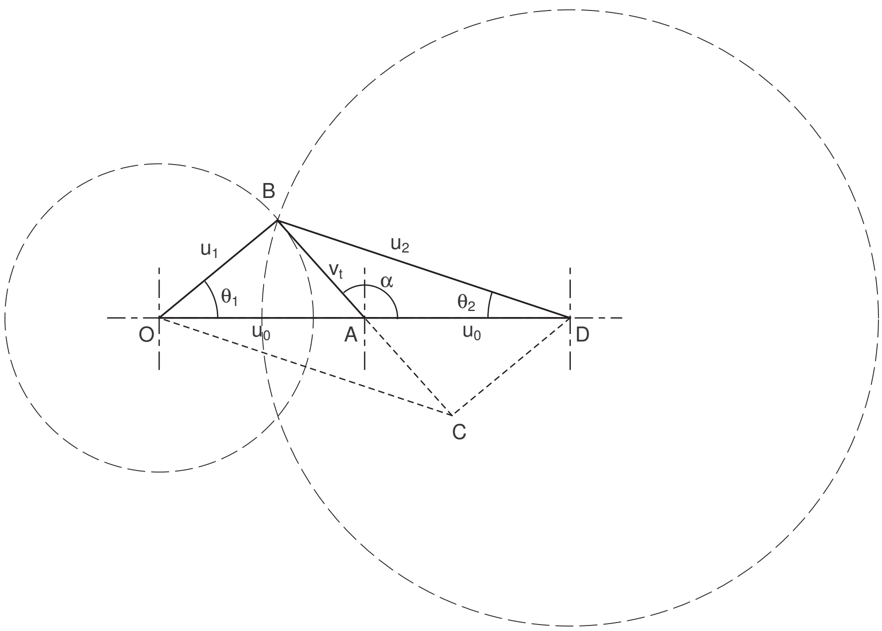

Figura 2: Construção geométrica do balanceamento sem medição de fase

Desse procedimento verificamos que o triângulo escaleno OBD mostrado na figura 2 tem lados iguais a OD, proporcional ao dobro da magnitude do desbalanceamento original (u_0), OB proporcional à magnitude do desbalanceamento produzido com a massa de teste na primeira posição (u_1) e BD proporcional à magnitude do desbalanceamento produzido com a massa colocada a 180^o da posição original (u_2).

Assim aplicando a *Lei dos Cossenos* ao triângulo OBD temos:

$$u_2^2 = (2 \cdot u_0)^2 + u_1^2 - 2 \cdot 2 \cdot u_0 \cdot u_1 \cdot \cos\theta_1$$

ou seja:

$$\cos\theta_1 = \frac{(2 \cdot u_0)^2 + u_1^2 - u_2^2}{4 \cdot u_0 \cdot u_1}$$

ou analogamente, para o ângulo θ_2 temos:

$$\cos\theta_2 = \frac{(2 \cdot u_0)^2 + u_2^2 - u_1^2}{4 \cdot u_0 \cdot u_2}$$

E aplicando-se a *Lei dos Cossenos* ao triângulo OAB temos:

$$v_t^2 = u_0^2 + u_1^2 - 2 \cdot u_0 \cdot u_1 \cdot \cos\theta_1$$

Assim:

$$v_t^2 = u_0^2 + u_1^2 - \frac{(2 \cdot u_0)^2 + u_1^2 - u_2^2}{2} = \frac{u_1^2}{2} + \frac{u_2^2}{2} - u_0^2$$

E finalmente, aplicando a *Lei dos Cossenos* ao triângulo ABD temos:

$$u_2^2 = u_0^2 + v_t^2 - 2\, u_0\, v_t\, \cos\alpha$$

e assim:

$$\frac{u_2^2 - u_1^2}{2} = -2\, u_0 \sqrt{\frac{u_1^2}{2} + \frac{u_2^2}{2} - u_0^2}\, \cos\alpha$$

Assim o ângulo α entre a posição de adição inicial da massa de teste e a posição angular do desbalanceamento inicial é dado por:

$$\cos\alpha = \frac{u_1^2 - u_2^2}{4\, u_0\, v_t}$$

onde:

$$v_t = \sqrt{\frac{u_1^2}{2} + \frac{u_2^2}{2} - u_0^2}$$

Pela ambiguidade da figura geométrica, essa expressão também poderia fornecer o ângulo complementar, i.e. $\frac{\pi}{2} - \alpha$. Ou ainda, bastaria trocar u_1 por u_2 e vice-versa, na expressão anterior para obter o ângulo complementar.

Tal ambiguidade pode ser eliminada pela adição da massa de teste numa posição a 90^o em relação à posição original, avançada em relação ao sentido de rotação.

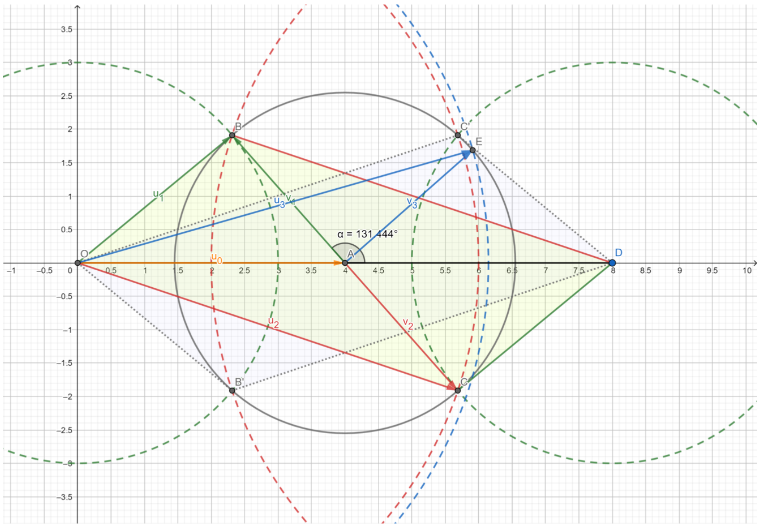

Figura 3: Geometria construída com o software *Geogebra*

Assim obtemos os vetores dos desbalanceamentos:

$$\vec{u}_0 = u_0 \angle\, (\alpha)$$

$$\vec{u}_1 = u_1 \angle\, (\alpha - \theta_1)$$

$$\vec{u}_2 = u_2 \angle\, (\alpha + \theta_2)$$

em relação à posição angular onde foi colocada originalmente a massa de teste.

Obviamente, mantida a proporcionalidade, temos a massa de correção dada por:

$$m_0 = \frac{u_0}{v_t} m_t$$

Na figura 3 é apresentado um exemplo da geometria do balanceamento sem medição de fase construído com o software *Geogebra*.

No exemplo temos:

$$\begin{aligned} u_0 &= 4 \text{ m/s}^2 \\ u_1 &= 3 \text{ m/s}^2 \\ u_2 &= 6 \text{ m/s}^2 \\ u_3 &= 6,147 \text{ m/s}^2 \end{aligned}$$

que fornece:

$$\cos\theta_1 = \frac{(2\cdot 4)^2 + 3^2 - 6^2}{4\cdot 4\cdot 3} = 0,77083 \qquad \Rightarrow \qquad \theta_1 = 39,571^o$$

$$\cos\theta_2 = \frac{(2\cdot 4)^2 + 6^2 - 3^2}{4\cdot 4\cdot 6} = 0,94792 \qquad \Rightarrow \qquad \theta_2 = 18,573^o$$

$$v_t = \sqrt{\frac{3^2}{2} + \frac{6^2}{2} - 4^2} = \sqrt{6,5} = 2,550$$

$$\cos\alpha = \frac{3^2 - 6^2}{4\cdot 4\cdot 2,550} = -0,66181 \qquad \Rightarrow \qquad \alpha = 131,444^o$$